Mass effects in the Higgs transverse momentum distribution

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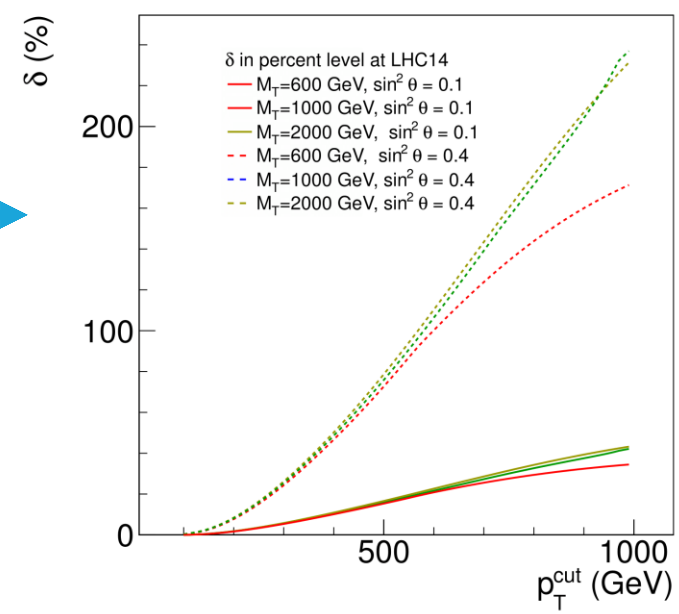
Karlsruher Institut für Technologie

Higgs Couplings, Heidelberg, 06-10 Nov, 2017

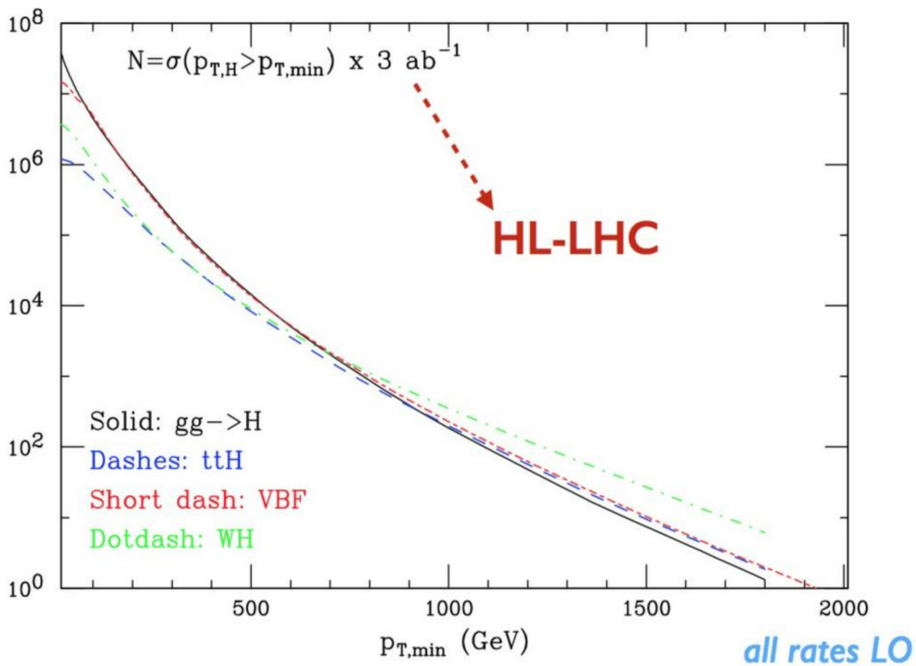
Introduction

- *Higgs couplings* form a crucial part of LHC physics program
- Higgs $p_{T,H}$ distributions can give more constraints than inclusive rate measurements (example: light-quark Yukawa couplings) [Bishara, Monni et al '16; Soreq et al '16]
- Alternative to $t\bar{t}H$ for studying the top Yukawa coupling: $H+j$ distributions at large $p_{T,H}$
- Furthermore: Higgs coupling to top-partners can be constrained by studying Higgs distribution at large $p_{T,H}$
- Experiments have already begun searching for boosted $H \to b\bar{b}$ decay
- Theoretical caveat: usual HEFT approach breaks down starting at very large $p_{T,H} \sim 400$ GeV
- At large $p_{T,H}$: top mass corrections cannot be neglected

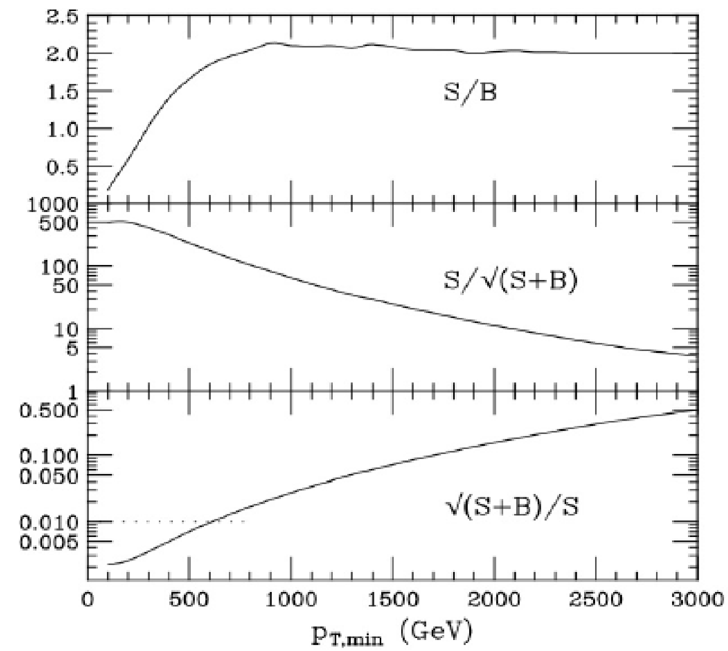


[Banfi, Martin, Sanz, arXiv:1308.4771]

Experimental motivation: boosted Higgs



$H \to \gamma\gamma$ signal-background ratio



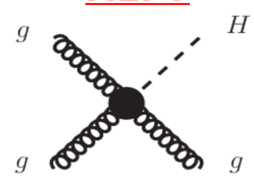
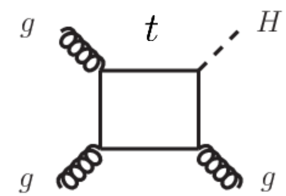
Plots taken from: Mangano talk at Higgs Couplings 2016

- HL-LHC at large $p_{T,H} > 400$ GeV : many events can be expected ~ 10^4
- Boosted Higgs advantage: large signal to background ratio → **Exploit large boosted Higgs differential measurements to constrain Higgs couplings**

→ **This project**: focus on H production recoiling against a jet

Theoretical interest: boosted $H + j$

- $H + j$ at LHC proceeds largely through quark loops, historically computed in HEFT limit $m_t \to \infty$

$p_{T,H}$ range:	below top threshold	close to threshold	above top threshold
$\mathcal{A}^{LO}_{gg\to gH} \sim$	**HEFT** 	increasing $p_{T,H}$	
Expansion parameters:	$\frac{m_h^2}{4m_t^2}, \frac{p_\perp^2}{4m_t^2} \ll 1$	$1 - \frac{4m_t^2}{\hat{s}} \ll 1$	$\frac{m_h^2}{4m_t^2}, \frac{4m_t^2}{p_\perp^2} \ll 1$

Above top threshold:

$$m_h^2 \ll (2m_t)^2 \ll (p_\perp)^2$$

HEFT expansion fails

- At $\boldsymbol{p_{T,H}}$ larger than twice the top mass, the $gggH$ coupling is not point-like anymore

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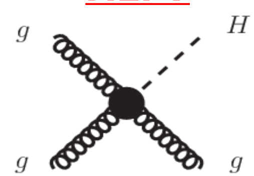
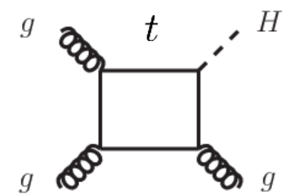
$\mathcal{A}^{LO}_{gg\to gH} \sim$	**HEFT** 	increasing $p_{T,H}$	
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Above top threshold:

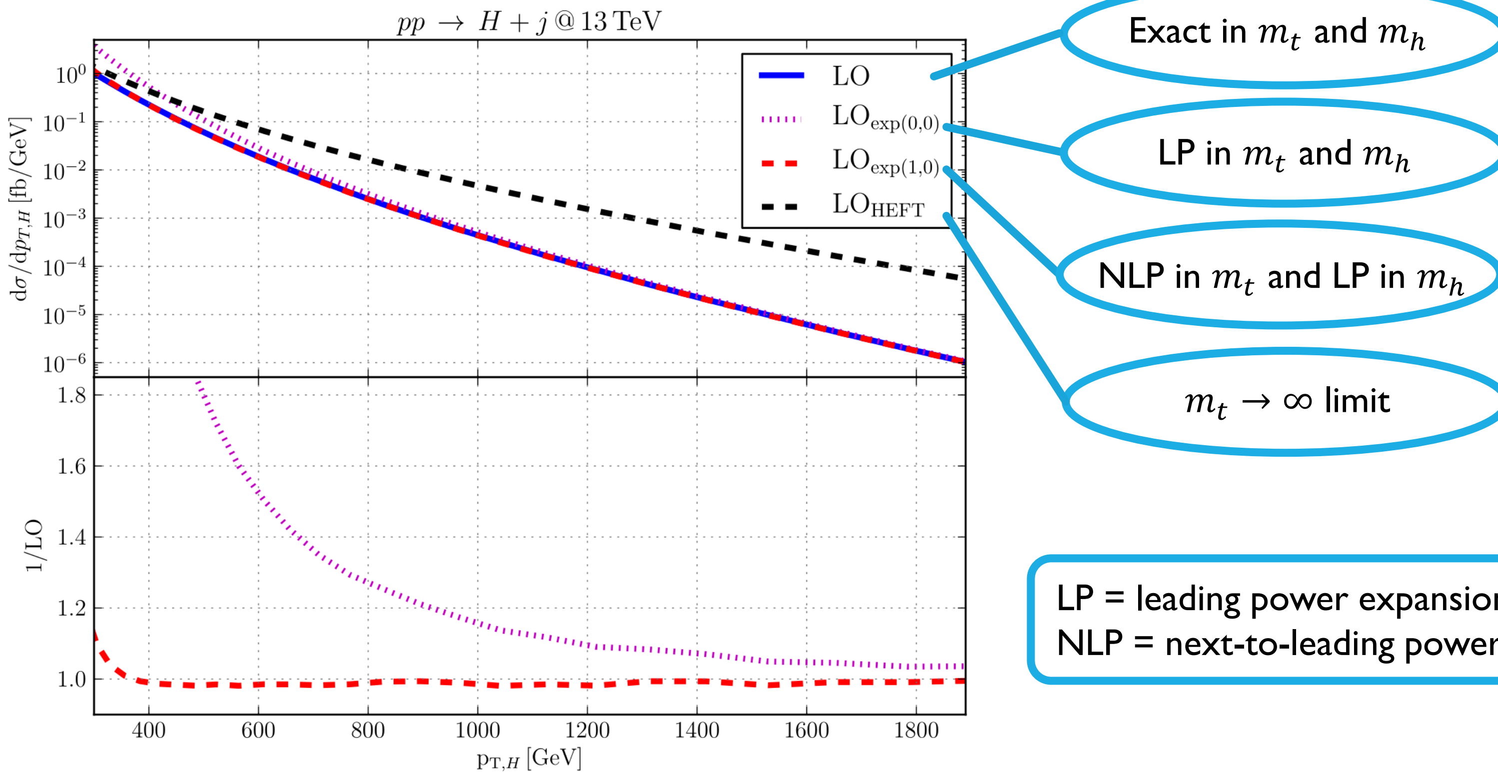
$$m_h^2 \ll (2m_t)^2 \ll (p_\perp)^2 \longrightarrow \boxed{\text{HEFT expansion fails}}$$

- At $\boldsymbol{p_{T,H}}$ larger than twice the top mass, the ggH coupling is not point-like anymore
- In fact: top amplitude *contains enhanced Sudakov-like logarithms* above top threshold

$$\mathcal{A}^{(L)}_{gg\to Hg}(m_h^2, m_t^2, s, t) = \frac{y_t m_t}{p_\perp} \sum_{j,l\geq 0} \left(\frac{m_h^2}{4m_t^2}\right)^j \left(\frac{4m_t^2}{p_\perp^2}\right)^l \sum_{n=0}^{2L} A_{jln}(s,t) \boxed{\times \log^n\left(\frac{4m_t^2}{p_\perp^2}\right)}$$

$\longrightarrow$ Top mass effects cannot be neglected at large $p_{T,H}$

Boosted Higgs $p_{T,H}$-distribution at LO



LP = leading power expansion
NLP = next-to-leading power

LO: Expanding to LP in m_h and NLP in m_t gives very good description down to $p_{T,H}$ of at least 400 GeV

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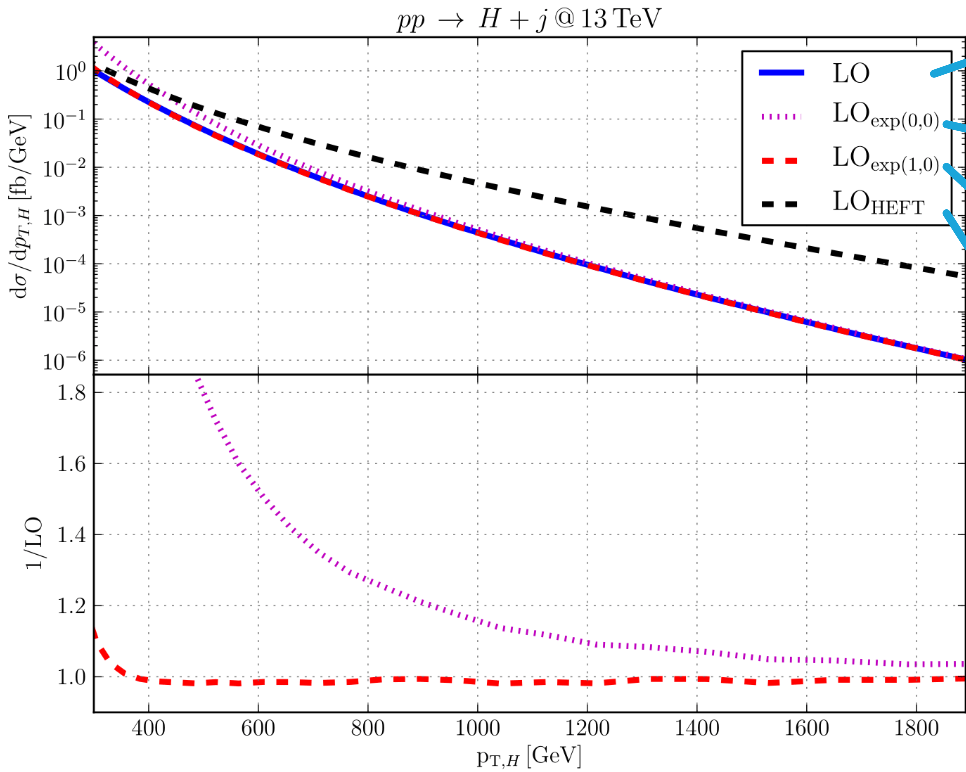

Exact in m_t and m_h

~~LP in m_t and m_h~~

NLP in m_t and LP in m_h

~~$m_t \to \infty$ limit~~

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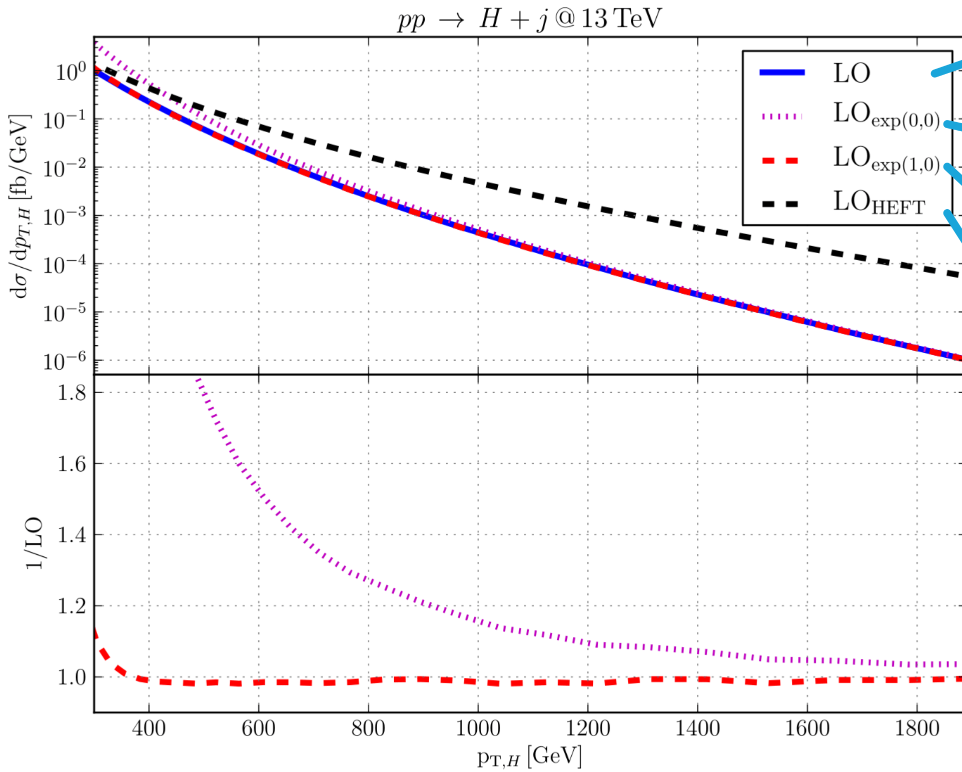

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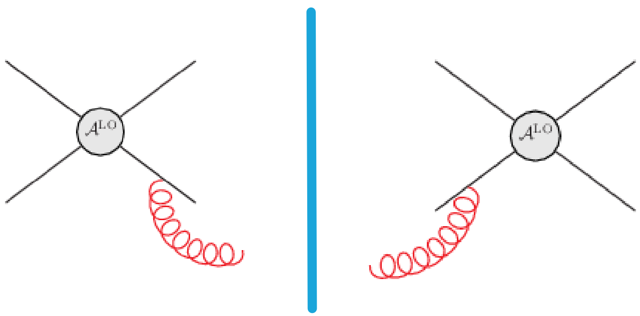
LO: Expanding to LP in m_h and NLP in m_t gives very good description down to $p_{T,H}$ of at least 400 GeV

What about NLO corrections above top threshold with finite top mass? Potentially large!

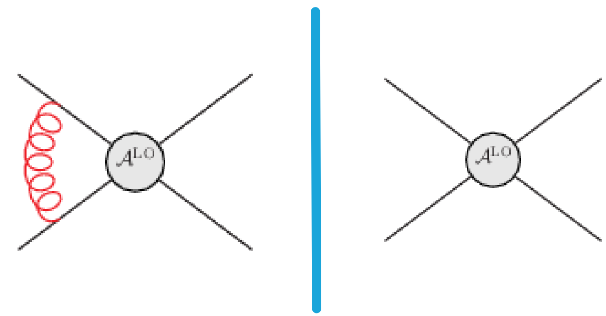
- Two-loop amplitude adds terms with <u>higher powers of enhancing logarithms</u> $\log^{(3,4)}(4m_t^2/p_\perp^2)$
- NLO corrections below top threshold very large ~ 100% → possibly large also above

Calculation at NLO

- Real (2 to 3) and virtual (2 to 2) contributions need to be combined, very well understood at NLO



Real corrections



Virtual corrections

- Peculiarity in this case: *LO is already 1-loop*

- Real corrections receive contributions from kinematical regions where one parton become soft or collinear to another parton, so a numerically stable approach required

- Real corrections computed in Openloops with exact top mass dependence

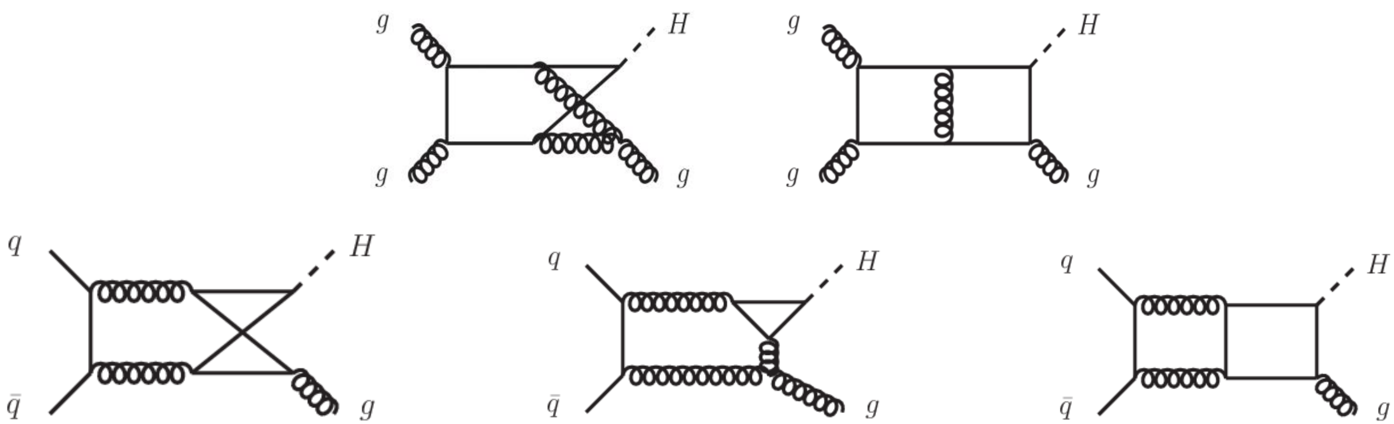
[Cascioli, Lindert, Pozzorini et al '12-17; Denner et al '03-'17]

- One new ingredient are two-loop virtual corrections

Virtual corrections

$$d\sigma^{\text{virt}} \sim \text{Re}\left[\frac{\alpha_s}{2\pi}\left(A^{(2L)}A^{(1L)*}\right)\right]$$

- Typical two-loop Feynman diagrams are:



- Exact mass dependence in two-loop Feynman Integrals currently out of reach [planar diagrams: Bonciani et al '16]

Scale hierarchy: $m_h^2 \ll (2m_t)^2 \ll (p_\perp)^2$ →

- Expand amplitudes in small parameters by using ***differential equation method (DE)***
- Expansion with DE approach already used successfully for small bottom mass loop [Mueller & Ozturk '15; Lindert, Melnikov, Tancredi, CW '17]
- Bonus of DE approach: extending to higher powers in m_h and m_t is very algorithmic
- In this work: expand amplitudes to LP in m_h (effectively $m_h = 0$) and NLP in m_t

Computing virtual two-loop amplitudes

[Melnikov, Tancredi, CW '17]

- Virtual amplitude made up of *complicated two-loop tensor Feynman integrals*
- Powerful tool for scalar integrals: *IBP reduction* to minimal set of *Master Integrals* (**MI**)

$\longrightarrow$ project amplitude onto form factors

$$\mathcal{A}_{H\to ggg}\,(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3}\;\epsilon_1^{\mu}\,\epsilon_2^{\nu}\,\epsilon_3^{\rho}\;(F_1\,g^{\mu\nu}\,p_2^{\rho} + F_2\,g^{\mu\rho}\,p_1^{\nu} + F_3\,g^{\nu\rho}\,p_3^{\mu} + F_4\,p_3^{\mu}p_1^{\nu}p_2^{\rho}\,)$$

- Form factors F_i expressed in terms of *scalar integrals*

Three families flashing by

$$\mathcal{I}_{\text{top}}(a_1, a_2, ..., a_8, a_9) = \int \frac{\mathfrak{D}^d k \mathfrak{D}^d l}{[1]^{a_1}[2]^{a_2}[3]^{a_3}[4]^{a_4}[5]^{a_5}[6]^{a_6}[7]^{a_7}[8]^{a_8}[9]^{a_9}}$$

Prop.	Topology PL1	Topology PL2	Topology NPL
[1]	k^2	$k^2 - m_t^2$	$k^2 - m_t^2$
[2]	$(k-p_1)^2$	$(k-p_1)^2 - m_t^2$	$(k+p_1)^2 - m_t^2$
[3]	$(k-p_1-p_2)^2$	$(k-p_1-p_2)^2 - m_t^2$	$(k-p_2-p_3)^2 - m_t^2$
[4]	$(k-p_1-p_2-p_3)^2$	$(k-p_1-p_2-p_3)^2 - m_t^2$	$l^2 - m_t^2$
[5]	$l^2 - m_t^2$	$l^2 - m_t^2$	$(l+p_1)^2 - m_t^2$
[6]	$(l-p_1)^2 - m_t^2$	$(l-p_1)^2 - m_t^2$	$(l-p_3)^2 - m_t^2$
[7]	$(l-p_1-p_2)^2 - m_t^2$	$(l-p_1-p_2)^2 - m_t^2$	$(k-l)^2$
[8]	$(l-p_1-p_2-p_3)^2 - m_t^2$	$(l-p_1-p_2-p_3)^2 - m_t^2$	$(k-l-p_2)^2$
[9]	$(k-l)^2 - m_t^2$	$(k-l)^2$	$(k-l-p_2-p_3)^2$

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→ project amplitude onto form factors

$$\mathcal{A}_{H\to ggg}(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3}\, \epsilon_1^{\mu}\, \epsilon_2^{\nu}\, \epsilon_3^{\rho}\, (F_1\, g^{\mu\nu}\, p_2^{\rho} + F_2\, g^{\mu\rho}\, p_1^{\nu} + F_3\, g^{\nu\rho}\, p_3^{\mu} + F_4\, p_3^{\mu} p_1^{\nu} p_2^{\rho})$$

- Form factors F_i expressed in terms of *scalar integrals*
- *Integration by parts* (IBP) identities $\displaystyle\int \Big(\prod_i d^d k_i\Big) \frac{\partial}{\partial k_j^{\mu}} \Big(v^{\mu} I\Big) = \text{Boundary term} \overset{DR}{=} 0$
- Reduce to set of *MI* is *very* difficult, naïve reduction with public codes failed
- Performed in steps: top topology to subtopology reduction with Form+Reduze, then FIRE

$$\mathcal{I}_{a_1\cdots a_n}(s) = \sum_{(b_1\cdots b_n)\in \text{Master Integrals}} \text{Rational}_{a_1\cdots a_n}^{b_1\cdots b_n}(s,d)\,\text{MI}_{b_1\cdots b_n}(s)$$

MI with DE method for $m_h^2 \ll (2m_t)^2 \ll s, t$

[Kudashkin, Melnikov, CW '17]

- System of partial differential equations (**DE**) in $\boldsymbol{m_h^2, m_t^2, s, t}$ with IBP relations

$$\frac{\partial}{\partial \tilde{s}_k} \vec{\mathcal{I}}^{MI}(\tilde{s}, \epsilon) \stackrel{\text{IBP}}{=} \overline{\overline{M}}_k(\tilde{s}, \epsilon).\vec{\mathcal{I}}^{MI}(\tilde{s}, \epsilon)$$

- Interested in $m_h^2 \ll (2m_t)^2 \ll s, t$ expansion of Master integrals I^{MI}

→ expand homogeneous matrix M_k in small m_h, then small m_t

Solve DE in m_h and m_t

- Solve m_h *and* m_t DE with following ansatz

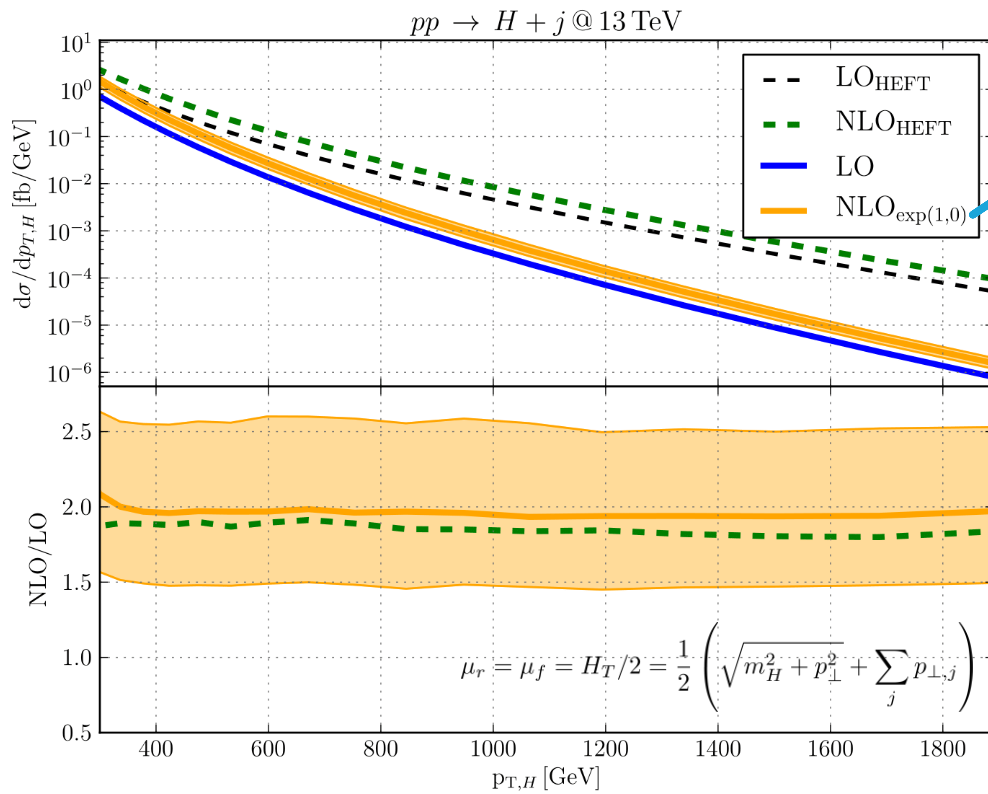
$$\mathcal{I}_i^{MI}(m_h^2, m_t^2, s, t, \epsilon) = \sum_{ijklmn} c_{ijklmn}(s, t, \epsilon) \left(\frac{m_h^2}{4m_t^2}\right)^{j-k\epsilon} \left(\frac{m_t^2}{s}\right)^{l-m\epsilon} \log^n\left(\frac{m_t^2}{s}\right)$$

- Plug into m_h *and* m_t DE and get constraints on coefficients $c_{i\ldots n}$
- Algorithmic: higher power expansion coefficients fixed from lower order power coefficients
- Remaining $c_{i\ldots n}(s, t, \epsilon)$ fixed by DE in (s, t) and expressed in: *Goncharov Polylogarithms*

Top mass corrections

[Kudashkin, Lindert, Melnikov, CW '17]

Preliminary



NLP in m_t and LP in m_h

PDF: NNPDF3.0

$$m_t = 173.2\,\mathrm{GeV}$$

$$m_H = 125\,\mathrm{GeV}$$

$$\mu = \{1/2, 2\} * \mu_0$$

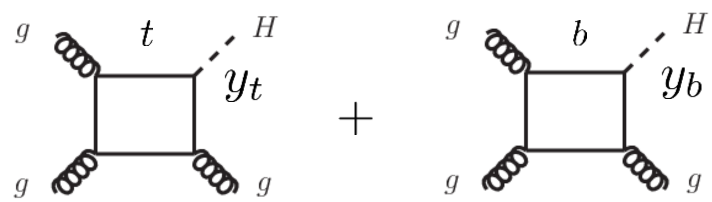
$\sigma(fb)$ / $p_{T,H}(GeV)$	LO(HEFT)	LO(full)	NLO(HEFT)	NLO(full)	K(HEFT)	K(full)
≥ 400	33.82	12.425	63.90	24.36	1.89	1.96
≥ 450	22.00	6.75	41.71	13.25	1.90	1.96
≥ 1000	0.628	0.042	1.149	0.080	1.83	1.93

Top-bottom interference

[Lindert, Melnikov, Tancredi, CW '17]

- What about bottom mass corrections to Higgs plus jet production?

$\mathcal{A}^{LO}_{gg\to gH} \sim$



dominant bottom correction

- Differential cross section $\quad \mathrm{d}\sigma \sim |\mathcal{A}|^2 \to \mathrm{d}\sigma = \mathrm{d}\sigma_{tt} + \mathrm{d}\sigma_{tb} + \mathrm{d}\sigma_{bb}, \quad \mathrm{d}\sigma_{ij} \sim \mathcal{O}(y_i y_j)$

- LO study indicates large corrections *below top threshold* $\quad$ Scale hierarchy: $\quad 2m_b \ll p_\perp, m_h \ll 2m_t$

$$y_j \sim m_j \qquad m_b = 4.5\,\mathrm{GeV} \qquad m_t = 173\,\mathrm{GeV}$$

- Bottom amplitude similarly *contains enhanced Sudakov-like logarithms* above bottom threshold

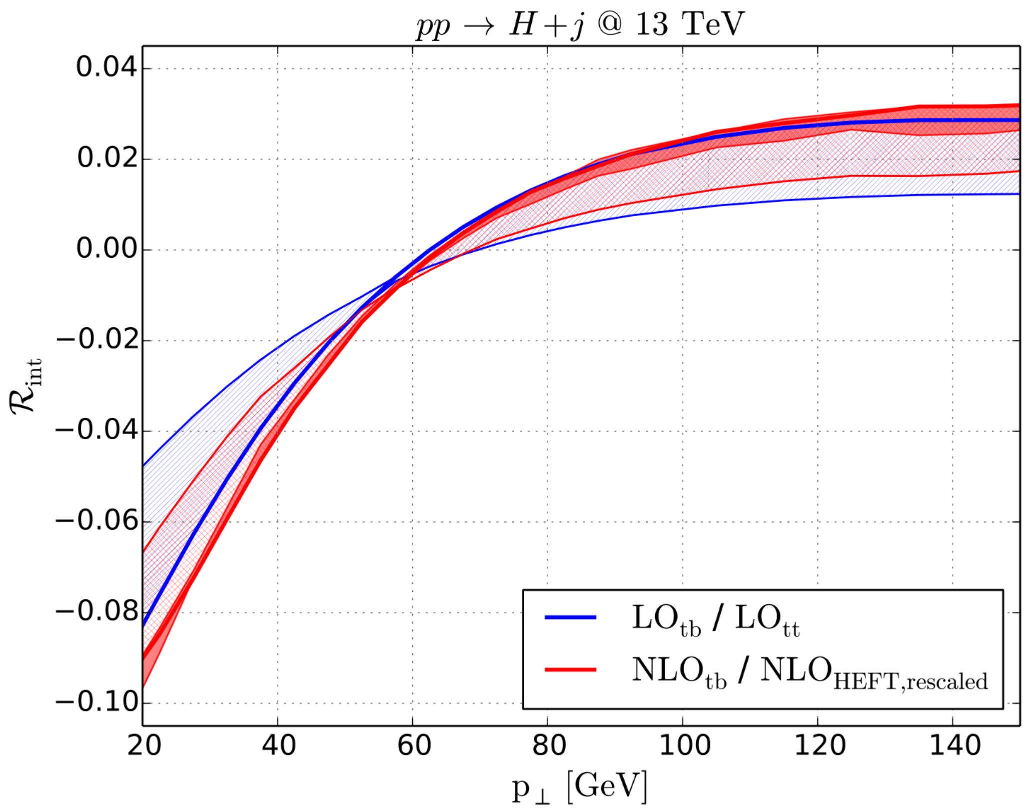
$$\mathcal{A}^{(L),\text{bottom-loop}}_{gg\to Hg}(m_b^2, m_h^2, s, t) = \frac{y_b m_b}{p_\perp} \sum_{l\geq 0} \left(\frac{4m_b^2}{p_\perp^2}\right)^l \sum_{n=0}^{2L} A_{ln}(m_h^2, s, t) \boxed{\times \log^n\left(\frac{4m_b^2}{p_\perp^2}\right)}$$

- Expansion of amplitude with DE approach applied in same way for *small bottom mass* loop (to LP in m_b^2)

→ Quantitatively, how large are the bottom corrections at NLO?

Bottom mass corrections

[Lindert, Melnikov, Tancredi, CW '17]



PDF: NNPDF3.0

$$m_t = 173.2\,\mathrm{GeV}$$

$$m_H = 125\,\mathrm{GeV}$$

$$\mu = \{1/2, 2\} * \mu_0$$

$$m_b^{\mathrm{OS}} = 4.75\,\mathrm{GeV}$$

$$m_b^{\overline{\mathrm{MS}}}(\mu = 100) = 3.07\,\mathrm{GeV}$$

- Top-bottom interference at moderate $p_{T,H}$=30 GeV: -6% at LO and -7% at NLO
- Large relative corrections to top-bottom interference ~ relative corrections to top-top ~ 40%
- Mass renormalization-scheme: reduction at small $p_{T,H}$ by a factor of two at NLO; less pronounced at larger $p_{T,H}$

Summary

Top mass corrections

- Boosted Higgs production: for $\boldsymbol{p_{T,H} > 2 * m_t \sim 350}$ GeV to get reliable LHC results for $H + j$ production we are forced to include finite top mass effects (HEFT breakdown)

- For first time computed fully differential NLO QCD corrections $H + j$ above top mass threshold, including finite top mass effects

- Two-loop integrals computed at leading power expansion in Higgs mass and next-to-leading power expansion in top mass expansion with differential equation method

- NLO K-factor of full result ~ 2, HEFT K-factor~1.9 flat on large $\boldsymbol{p_{T,H} > 400}$ GeV range

Bottom mass corrections

- Two-loop bottom integrals computed at leading power in bottom mass expansion with DE method

- NLO bottom contribution ~ [-10, -4] % of NLO top contribution at lower range of Higgs $\boldsymbol{p_{T,H}}$

- Large relative NLO corrections to top-bottom interference similar to pure top NLO corrections ~ 40% for Higgs $\boldsymbol{p_{T,H}}$ and rapidity distributions

Backup slides

IBP reduction

[Melnikov, Tancredi, CW '16-'17]

- IBP reduction to Master Integrals

$$\mathcal{I}_{a_1\cdots a_n}(s) = \sum_{(b_1\cdots b_n)\in \text{Master Integrals}} \text{Rational}^{b_1\cdots b_n}_{a_1\cdots a_n}(s,d)\text{MI}_{b_1\cdots b_n}(s)$$

- *Reduction very non-trivial*: we were not able to reduce top non-planar integrals with $t=7$ denominators with FIRE5/Reduze
- Reduction fails because coefficients multiplying MI become too large to simplify ~ hundreds of Mb of text

- *Reduction for complicated t=7 non-planar integrals performed in two steps*:

1) FORM code reduction:

$$\mathcal{I}^{\text{NPL}}_{t=7} = \sum c_i \text{MI}^i_{t=7} + \sum d_i \mathcal{I}^i_{t=6}$$

2) Plug reduced integrals into amplitude, expand coefficients c_i, d_i in m_h, m_t

3) Reduce with FIRE/Reduze: $t=6$ denominator integrals $\mathcal{I}_{t=6}$

- Exact m_t dependence kept at intermediate stages. Algorithm for *solving IBP identities directly expanded in small parameter is still an open problem*

- *Expansion in m_t occurs at last step*: solving with Master integrals with *differential equation method*

How useful are expansions?

- NLO amplitudes require computing 2-loop Feynman integrals with massive quark loop
- If these integral are computed exactly in quark mass, results in *very complicated functions*

[planar diagrams: Bonciani et al '16]

$$
\begin{aligned}
&\log\left(x_3x_1^2 - x_1^2 + x_2x_1 - 4x_3x_1 + R_1(x_1)R_2(x_1)R_7(x)\right),\\
&\log\left(-x_2^2 + x_1x_2 - x_1x_3x_2 + 2x_3x_2 + 2x_1x_3 + R_1(x_2)R_2(x_2)R_7(x)\right),\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 4x_2x_3x_1 + R_1(x_3)R_5(x)R_6(x)x_1\right),\\
&\log\left(x_3R_1(x_2)R_2(x_2) + x_2R_1(x_3)R_2(x_3)\right),\\
&\log\left(x_1R_1(x_2)R_2(x_2) + x_2R_1(x_1)R_2(x_1)\right),\\
&\log\left(x_1R_1(x_3)R_2(x_3) - R_1(x_1)R_1(x_3)R_5(x)\right),\\
&\log\left(x_3R_1(x_1)R_2(x_1) - R_1(x_1)R_1(x_3)R_5(x)\right),\\
&\log\left(-x_2R_1(x_1)R_2(x_1) + x_3R_1(x_1)R_2(x_1) + x_1R_3(x_3)R_4(x_3)\right),\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_3R_1(x_2)R_2(x_2) + x_2R_3(x_3)R_4(x_3)\right),\\
&\log\left(-x_2R_1(x_3)R_2(x_3) + x_1R_1(x_3)R_2(x_3) + x_3R_3(x_1)R_4(x_1)\right),\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_1R_1(x_2)R_2(x_2) + x_2R_3(x_1)R_4(x_1)\right),\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 3x_2x_3x_1 + R_1(x_1)R_1(x_3)R_5(x)R_7(x)\right),\\
&\log\left(x_2R_1(x_1)R_1(x_3)R_5(x) - x_1x_3R_1(x_2)R_2(x_2)\right),\\
&\log\left(-x_2x_3 + x_1x_3 + R_1(x_2)R_2(x_2)x_3 - R_1(x_1)R_1(x_3)R_5(x)\right).
\end{aligned}
$$

$$
\begin{aligned}
&R_1(x_1) = \sqrt{-x_1},\; R_1(x_3) = \sqrt{-x_3},\; R_1(x_2) = \sqrt{-x_2},\\
&R_2(x_1) = \sqrt{4-x_1},\; R_2(x_3) = \sqrt{4-x_3},\; R_2(x_2) = \sqrt{4-x_2},\\
&R_3(x_1) = \sqrt{x_2-x_1},\; R_3(x_3) = \sqrt{x_2-x_3},\\
&R_4(x_1) = \sqrt{x_2-x_1-4},\; R_4(x_3) = \sqrt{x_2-x_3-4},\\
&R_5(x) = \sqrt{4x_2 + x_1x_3 - 4(x_1+x_3)},\\
&R_6(x) = \sqrt{2x_3(-2x_2+x_1+2x_3) - x_1x_3^2 - x_1},\\
&R_7(x) = \sqrt{2x_1x_3(x_2-x_1) + (x_2-x_1)^2 + (x_1-4)x_1x_3^2}.
\end{aligned}
$$

- Starting from weight three not possible to express in terms of usual GPL's anymore
- *Expanding* in small quark mass results in simple *2-dimensional harmonic polylogs*

[Vermaseren, Remiddi, Gehrmann]

Real corrections with Openloops

- Channels for real contribution to Higgs plus jet at NLO

$$gg \to Hgg,\; gg \to Hq\bar{q},\; qg \to Hqg,\; q\bar{q} \to Hgg,\; \cdots$$

- Receives contributions from kinematical regions where one parton become soft or collinear to another parton
- This requires a delicate approach of these regions in phase space integral
- Openloops algorithm is publicly available program which is capable of dealing with these singular regions in a numerically stable way
- Crucial ingredient is tensor integral reduction performed via expansions in small Gram determinants: Collier

[Cascioli et al '12, Denner et al '03-'17]

- Exact top mass dependence kept throughout for one-loop computations