



Thermodynamics of SU(2) quantum Yang-Mills theory and CMB anomalies

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Outline



motivation: nonperturbative, analytical approach to YMTD

essentials, thermal ground state:

coarse-graining over nonpropagating

(anti-)calorons of winding number unity, effective action

adjoint Higgs mechanism:

massive vector modes and kinematic constraints (1),

coupling, deconf.-preconf. phase boundary, (anti-)caloron action,

radiative corrections:

kinematic constraints (2), polarisation tensor of massless mode, longitudinal and transverse thermal dispersion

SU(2) postulate for photon propagation:

Yang-Mills scale or critical temperature (radio-frequency CMB observations)

CMB large-angle anomalies (WMAP, Planck):

possible explanation via SU(2) dispersion, onset of dynamical breaking of statistical isotropy at redshift unity, SU(2) vector modes and cosmic neutrinos

motivation



- Andrei Linde (1980): "Infrared Problem in the Thermodynamics of the Yang-Mills Gas"
 - soft magnetic sector screened weakly in perturbation theory (infrared instability)
 - no "convergence" of series since kinetic and interaction energies comparable in this sector
- issue of finite-volume constraints in lattice gauge theory
 - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

- nonperturbative, lattice $\,\beta\,$ function

nonperturbative Yang-Mills thermodynamics: SU(2)

[Herbst et Hofmann (2004), Hofmann (2005-2007), Giacosa et Hofmann (2006), Schwarz et al. (2007), Ludescher et Hofmann (2008), Falquez et al. (2010-2011), Hofmann (2012)]

thermal ground state at high temperature:

- Euclidean action:

$$S = \frac{\mathrm{tr}}{2} \int_{0}^{\beta} d\tau \int d^{3}x \, F_{\mu\nu} F_{\mu\nu} \,, \qquad (\beta \equiv 1/T)$$

where $F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig[A_{\mu}, A_{\nu}]$ [Schafer et Shuryak (1996)]

- (anti)selfdual gauge fields: $F_{\mu\nu}[A] = \pm \tilde{F}_{\mu\nu}[A] \Rightarrow \theta_{\mu\nu}[A] \stackrel{\bullet}{=} 0$.

field configs. stabilized by winding: $S_3
ightarrow SU(2) = S_3$

- in particular: (anti)calorons of winding number unity



spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field ϕ



$$\{\hat{\phi}^{a}\} \equiv \sum_{\pm} \operatorname{tr} \int d^{3}x \int d\rho \, t^{a} \, F_{\mu\nu}(\tau,\vec{0}) \, \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} \, F_{\mu\nu}(\tau,\vec{x}) \, \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, dimensionless definition of family of phases, where

$$\left\{ (\tau, \vec{0}), (\tau, \vec{x}) \right\} \equiv \mathcal{P} \exp \left[i \int_{(\tau, \vec{0})}^{(\tau, \vec{x})} dz_{\mu} A_{\mu}(z) \right] \quad \text{and}$$
$$\left\{ (\tau, \vec{x}), (\tau, \vec{0}) \right\} \equiv \left\{ (\tau, \vec{0}), (\tau, \vec{x}) \right\}^{\dagger}$$

- magnetic-magnetic correlations contribute only
- uniquely determined, annihilating operator:

$$D = \partial_{\tau}^2 + \left(\frac{2\pi}{\beta}\right)^2$$

- $\{\hat{\phi}^a\}$ sharply dominated by cut-off for ho integration, later!

spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field



- no explicit eta dependence in ϕ field dynamics (caloron action!)

- absorb β dependence of D into potential V(BPS and EL yield: $\frac{\partial V(|\phi|^2)}{\partial |\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2} \Longrightarrow$ $V(|\phi|^2) = \frac{\Lambda^2}{|\phi|^2}$ (Yang-Mills scale) $|\phi| = \sqrt{\frac{\Lambda^3\beta}{2\pi}}$ and

- BPS equation:

$$\partial_\tau \phi = \pm 2i \Lambda^3 t_3 \phi^{-1} = \pm i V^{1/2}(\phi)$$

no **additive** ambiguity for V !

effective action (deconfining phase)



$$\mathcal{L}_{\rm eff}[a_{\mu}] = \operatorname{tr} \left(\frac{1}{2} G_{\mu\nu} G_{\mu\nu} + (D_{\mu}\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

- ((i) perturbative renormalizability (ii) ϕ 's inertness – no higher dim. operators to mediate 4-momentum transfer between ϕ and a_{μ} (iii) gauge invariance)
- effective YM equation $D_{\mu}G_{\mu\nu} = ie[\phi, D_{\nu}\phi]$ has ground-state solution:

$$a^{\rm gs}_{\mu} = \mp \delta_{\mu 4} \frac{2\pi}{e\beta} t_3 \qquad (D_{\nu}\phi \equiv G_{\mu\nu} \equiv 0)$$

$$\Rightarrow P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T.$$

interacting small-holonomy (anti)calorons (collapsing monopoleantimonopoel pairs)



- no off-shell propagation of massive modes (otherwise: momentum transfer to ϕ !)



electric-magnetically dual interpretation:



- if SU(2) something to do with photons (later!) then electric-magnetically dual interpretation required: in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\alpha = \frac{Q^2}{4\pi\hbar} \,,$$

for α to be unitless:

$$Q \propto rac{1}{e} \, .$$

But: magnetic coupling in SU(2)

$$g = \frac{4\pi}{e} \,.$$

SU(2) is to be interpreted in an electric-magnetically dual way.
 (e.g., magnetic monopole <--> electric monopole, etc.)

radiative corrections (deconfining phase)



- momentum transfer in effective 4-vertex (unitary-Coulomb gauge):



coherent average over all three channels _____
 thermodynamical quanties: 2-loop/1-loop (<10⁻³), 3-loop/1-loop (<10⁻⁷), loop expansion into 1-PI diagrams probably terminates at finite order

radiative corrections (deconfining phase)

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- polarization tensor of massless mode:



(excluded by kinematic constraints (1) and (2))

screening functions G, F as solutions of respective gap equations

radiative corrections (deconfining phase)

- transverse photons, screening function G: [Schwarz et al. (2007), Ludescher et Hofmann (2008)]



(spectral) radiative corrections (deconfining phase)



- spectral distribution of energy density, massless mode – transverse propagation at $T=2T_{\rm O}$



(integrated) radiative corrections (deconfining phase)





difference between energy density of SU(2) and U(1), massless mode – transverse propagation

(**positive** slope \leftarrow bias for **negative** temperature fluctuations, later!)

(spectral) radiative corrections (deconfining phase)



- low-momentum-support dispersion law, massless mode - longitudinal propagation



(charge-density waves: real-world magnetic modes, intergalactic magnetic fields [Falquez et al (2011)])

SU(2) postulate for photon propagation

- What is T_c ?



$$T(\nu) = T_0 + T_R \left(\frac{\nu}{\nu_0}\right)^{\beta}$$

[Fixsen et al. (2009), Haslam et al. (1981), Reich et Reich (1986), Roger et al. (1999), Maeda et al. (1999)]

where:

$$T_0 = 2.725 \,\mathrm{K}; \ \nu_0 = 1 \,\mathrm{GHz};$$

 $\beta = -2.62 \pm 0.04; T_R = (1.19 \pm 0.14) \,\mathrm{K}.$

(radio-frequency surveys of CMB yield line temperatures as:

source	$ u[{ m GHz}]$	T[K]
Roger	0.022	21200 ± 5125
Maeda	0.045	4355 ± 520
Haslam	0.408	16.24 ± 3.4
Reich	1.42	3.213 ± 0.53
Arcade2	3.20	2.792 ± 0.010
Arcade2	3.41	2.771 ± 0.009 .



evanescent low-frequency modes



Yang-Mills scale of SU(2)_{CMB}:





some CMB large-angle anomalies: WMAP and Planck

- dipolar power asymmetry (extends from $l = 2, \dots, 600$ in blocks of $\Delta l = 100$) [Hansen et al. (2009), Ade et al. (2013), etc.]
- low variance on ecliptic North, associated with I=2,3 [Monteserin et al. (2008), Cruz et al., (2011), Ade et al. (2013), etc.]
- anomalous alignment of I=2,3 (3°-9°)

[Tegmark et al. (2003), de Oliveira-Costa et al. (2004), Ade et al. (2013), etc. (estimator of axis: maximum of angular momentum dispersion), Copi et al. (2004), Schwarz et al. (2004), Bielewicz et al. (2005,2009), Copi et al. (2010), etc. (multipole vector decomposition)]

- cold spot (-73µK@4°; -20µK@10°; l,b=207.8°,-56.3°)

[Viela et al. (2004), Cruz et al. 2005, Rudnick et al. (2007), Ade et al. (2013), etc.]

- hemispherical asymmetry (for I=2-40 max. larger power on hemisphere I,b=237°,-20°) [Eriksen et al. (2004), Hansen et al. (2004), Park (2004), Ade et al. (2013), etc.]
- mirror parity violation (plane of max. antisymmetry: I,b=262°,-14°) [Finelli et al.(2012); Ben-David et al. (2012), etc.]
- suppression of $\langle TT \rangle(\theta) \equiv C(\theta)$ for $\theta \geq 60^{\circ}$ on ecliptic North [Spergel et al. (2003), Copi et al. (2004,2007,2009,2010), Ade et al. (2013), etc.]

cold spot





TT suppression on ecliptic North



successful phenomenological attempt at explanation: multiplicative, dipolar modulation model



[Gordon et al. (2005), Eriksen et al. (2007), Hoftuft et al. (2009), Ade et al. (2013)]



maximum likelihood at: $A\sim 0.07;~l_p\sim 220^\circ; b_p\sim -21^\circ$

- robust against change of foreground treatment and experiment (WMAP,Planck)
- comparison with CMB cold spot: $~~l_{cs}\sim 207.8^\circ; b_{cs}\sim -56.3^\circ$

$$\Rightarrow \angle \vec{p}, \vec{e}_{cs} \sim 36^{\circ}$$



 $(2\pi a)^{3}$

(Silk cutoff)

- integrated blackbody anomaly due to SU(2)_{CMB} :

•
$$\delta \rho(T) \equiv \rho_{\rm SU(2)_{CMB}} - \rho_{\rm U(1)}$$

• $T = \bar{T}(t) + \delta T(t, \vec{x})$

 \clubsuit SU(2)_{CMB} bias factor $F(\bar{T},\delta T)$ for $~\delta T~$ in phys. voxel volume $\Delta V~$

$$F(\bar{T}, \delta T) = \frac{P_{\rm SU(2)}}{P_{\rm U(1)}}$$

where

$$P = \frac{\exp(-\rho\Delta V/\bar{T})}{\int_{T_0}^{\infty} dT \, \exp(-\rho\Delta V/\bar{T})}$$

(in comoving Fourier-space simulation: use convolution $\tilde{F} * \delta \tilde{T}$ for conventionally evolved $\delta \tilde{T}$ at $\{z_n\}$, then projection)

Since slope of $\delta
ho$ positive \implies negative δT favoured!



- semiquantitative model: effective $SU(2)_{CMB}$ evolution

$$\sqrt{-g} \mathcal{L}_{\rm CMB} = \left(\frac{\bar{T}_0}{\bar{T}}\right)^3 \left(k \,\partial_\mu \delta T \partial^\mu \delta T - \delta \rho(T)\right)$$

- assuming 3D spherical symmetry, causal boundary conditions

$$0 = \partial_{\tau} \partial_{\tau} \delta T - \left(\frac{\mathrm{d}a}{a\,\mathrm{d}\tau}\right)^{2} \left[\partial_{\sigma} \partial_{\sigma} \delta T + \frac{2}{\sigma} \partial_{\sigma} \delta T\right] - \frac{3}{\bar{T}} \partial_{\tau} \bar{T} \partial_{\tau} \delta T + \frac{T_{0}^{2}}{kH_{0}^{2}} \left[\frac{1}{2} \frac{\mathrm{d}^{2}\hat{\rho}}{\mathrm{d}T^{2}}\Big|_{T=\bar{T}} \delta T + \frac{1}{2} \frac{\mathrm{d}\hat{\rho}}{\mathrm{d}T}\Big|_{T=\bar{T}}\right]$$
source term





dynamical breaking of statistical isotropy:



- low variance, power asymmetry:

(simplified, instantaneous light propagation for projection)



- suppression of TT for $\,\theta\geq 60^\circ\,$:

rapid build-up of profile for $\ z \leq 1$

- dynamical contribution in measured (kinematically dominated) CMB dipole

$$\begin{aligned} |\vec{D}_{dyn}| &= \frac{1}{2} \left(\delta T_L - \delta T_R \right) \\ - \text{ offset} &= \frac{1}{2} \left(\delta T_L + \delta T_R \right) & \longrightarrow \text{ cold spot} \end{aligned}$$

$$ightarrow ec{d_{cs}} || ec{e}_{ ext{mirror antisymm}}$$

 $ec{d_{cs}} || ec{e}_{ ext{hemisph asymmetry}}$

Planck results:
$$\angle \vec{e}_{\text{mirror antisym}}, \vec{e}_{cs} \sim 42^{\circ} - 56^{\circ};$$

 $\angle \vec{e}_{\text{hemisph asym}}, \vec{e}_{cs} \sim 42^{\circ}.$



SU(2) vector modes and neutrinos:



from Planck:

$$N_{
m eff} = 3.30 \pm 0.27$$

But have 2 x 3 ~ N $_{\rm eff}$ x 2 rel. d.o.f. from SU(2) $_{\rm CMB}$ vector modes.

Too many rel. d.o.f ? Do vector modes play role of cosmological neutrinos? Neutrinos (luke-)warm dark matter?

massive cosmic neutrino equation of state:





Summary



- SU(2) thermodynamics nonperturbatively:
- caloron, thermal ground state, adjoint Higgs mechanism, caloron action

- blackbody anomaly:
 - thermal photon dispersion, critical temperature for dec.-prec. PT from low-frequency spectral anomaly (Arcade2, terrestial radio-frequency CMB observations)

- CMB large-angle anomalies:

Yang-Mills favours **negative temperature fluctuations**, semiquantitative model, cosmic neutrinos and relativistic vector modes

Thank you.





successful phenomenological attempt at explanation: multiplicative, dipolar modulation model



[Gordon et al. (2005), Eriksen et al. (2007), Hoftuft et al. (2009), Ade et al. (2013)]



maximum likelihood at: $A\sim 0.07;~l_p\sim 220^\circ; b_p\sim -21^\circ$

- robust against change of foreground treatment and experiment (WMAP,Planck)
- comparison with CMB cold spot: $~~l_{cs}\sim 207.8^\circ; b_{cs}\sim -56.3^\circ$

$$\Rightarrow \angle \vec{p}, \vec{e}_{cs} \sim 36^{\circ}$$

two more facts on CMB sky:



$$\angle \vec{e}_{\text{mirror antisym}}, \vec{e}_{cs} \sim 42^{\circ} - 56^{\circ};$$

 $\angle \vec{e}_{\text{hemisph asym}}, \vec{e}_{cs} \sim 42^{\circ}.$

CMB at low frequencies: ARCADE 2 and terrestial radio

observations [Fixsen et al. (2009), Haslam et al. (1981), Reich et Reich (1986), Roger et al. (1999), Maeda et al. (1999)]



- strong increase of CMB line temperture below $\,\nu=3\,$ GHz

$$T(\nu) = T_0 + T_R \left(\frac{\nu}{\nu_0}\right)^{\beta}$$

where:

$$T_0 = 2.725 \,\mathrm{K}; \ \nu_0 = 1 \,\mathrm{GHz};$$

 $\beta = -2.62 \pm 0.04.$

- notice also: radiosurveys of CMB yield line temperatures as:

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Deconfining SU(2) Yang-Mills thermodynamics

[Herbst et Hofmann (2004), Hofmann (2005,2006), Giacosa et Hofmann (2006), Schwarz et al. (2007), Ludescher et Hofmann (2008), Falquez et al. (2010, 2011), Hofmann (2012)]

- Euclidean action:

$$S = \frac{\mathrm{tr}}{2} \int_0^\beta d\tau \int d^3 x \, F_{\mu\nu} F_{\mu\nu} \,,$$

where
$$F_{\mu
u}\equiv\partial_{\mu}A_{
u}-\partial_{
u}A_{\mu}-ig[A_{\mu},A_{
u}]$$

- (anti)selfdual gauge fields: $F_{\mu\nu}[A] = \pm \tilde{F}_{\mu\nu}[A] \Rightarrow \theta_{\mu\nu}[A] \equiv 0$.

Nontrivial configs. stabilized by winding $\,S_3
ightarrow SU(2) = S_3\,$.

- in particular: (anti)calorons of winding number unity, localized action density











• if SU(2) will have to do something with photons then electric-magnetically dual interpretation required: in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\alpha = \frac{Q^2}{4\pi\hbar} \,,$$

for α to be unitless:

$$Q \propto rac{1}{e}$$
 .

But: magnetic coupling in SU(2)

$$g = \frac{4\pi}{e} \,.$$

In real world: SU(2) is to be interpreted in an electric-magnetically dual way.

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- radiative corrections (feeble interaction of vectors with photon):
 - thermodynamical quanties: 2-loop/1-loop (<10⁻³), 3-loop/1-loop (<10⁻⁷)
 - polarization tensor of massless mode:





• transverse photons, screening function G: [Schwarz et al. (2007), Ludescher et Hofmann (2008)]





Iongitudinal "photons" (purely magnetic), dispersion law : [Falquez et al. (2011)]





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Spectral black-body anomaly



spectral distribution of energy density, $T=2T_0$



Spectral black-body anomaly





Effective theory of BB anomaly-induced δT in large-angle CMB

- integral BB anomaly:

 $\bullet \ \delta \rho(T) \equiv \rho_{\rm SU(2)_{CMB}} - \rho_{\rm U(1)}$

•
$$T = \overline{T}(t) + \delta T(t, \vec{x})$$

• in simulations bias factor $F(T, \delta T)$ for δT in phys. voxel volume ΔV :

where $P = \frac{\exp(-\rho\Delta V/\bar{T})}{\int_{T_0}^{\infty} dT \, \exp(-\rho\Delta V/\bar{T})}$

.

δρ potential for scalar field δT ——
 Manton's programme ∂_μδT∂^μδT ——
 introduce kinetic term
 action density:

 $\frac{P_{\rm SU(2)}}{P_{\rm U(4)}}$

$$\sqrt{-g} \mathcal{L}_{\rm CMB} = \left(\frac{\bar{T}_0}{\bar{T}}\right)^3 \left(k \,\partial_\mu \delta T \partial^\mu \delta T - \delta \rho(T)\right)$$

where k empirically determined normalization

Effective theory, cntd.



- varying action, linearizing e.o.m., and coordinate change

$$\partial_{\tilde{\mu}}\partial^{\tilde{\mu}}\delta T - \frac{3}{\bar{T}}\,\partial_{\tau}\bar{T}\,\partial_{\tau}\delta T + \frac{T_0^2}{kH_0^2} \left[\frac{1}{2} \left.\frac{\mathrm{d}^2\hat{\rho}}{\mathrm{d}T^2}\right|_{T=\bar{T}} \left.\delta T + \frac{1}{2} \left.\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}T}\right|_{T=\bar{T}}\right] = 0\,,$$

where
$$\delta
ho=T_0^2\,\hat
ho$$
 and ct $ilde x_0\equiv au=H_0\,t\,,\,\,\, ilde x_i=rac{\mathrm{d}a}{\mathrm{d}t}\,x_i=rac{x_i}{H^{-1}}a\,.$

(time i.u. of today's age of universe; spatial coordinates i.u. size of actual universe)

- assuming 3D spherical symmetry

$$0 = \partial_{\tau}\partial_{\tau}\delta T - \left(\frac{\mathrm{d}a}{a\,\mathrm{d}\tau}\right)^{2} \left[\partial_{\sigma}\partial_{\sigma}\delta T + \frac{2}{\sigma}\partial_{\sigma}\delta T\right] - \frac{3}{\bar{T}}\partial_{\tau}\bar{T}\partial_{\tau}\delta T + \frac{T_{0}^{2}}{kH_{0}^{2}}\left[\frac{1}{2}\frac{\mathrm{d}^{2}\hat{\rho}}{\mathrm{d}T^{2}}\Big|_{T=\bar{T}}\delta T + \frac{1}{2}\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}T}\Big|_{T=\bar{T}}\right]$$
where

$$\sigma \equiv \sqrt{\tilde{x}_{1}^{2} + \tilde{x}_{2}^{2} + \tilde{x}_{3}^{2}}.$$
source term

source term:



δT depression:



- study evolution in terms of z subject to best-fit Λ CDM
- initial conditions: fluctuation of primordial norm., arbitr. width, speed of initial fluctuation zero for $z_i > 20$ or so
- boundary conditions: extremum at $\sigma=0$, zero at $\,\sigma=1\,$ (causal connection to $\,\sigma=0$)
- determine k phenomenologically (mismatch of Local Group motion with motion extracted kinemetically from measured CMB dipole or directly from a large-angle anomaly in dipole subtracted map, later) [Ludescher et Hofmann (2009), Erdogdu et al.(2006)]

δT depression, cntd.







cntd.



- CMB cold spot, low variance, power asymmetry: consider:



cntd.



now:



cntd.



- suppression of TT for $heta > 60^\circ$

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rapid build-up of profile at \,z \sim 1\,
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- alignment of quadrupole and octopule (axis of evil)
 - ~ along gradient to profile, $\nabla \delta T|_{z=0,\sigma_0}$:

$$\angle -\vec{e}_{aoe}, \vec{e}_{cs} \sim 49^{\circ}$$

dipolar power asymmetry:
 Planck: I-binned mean ~ 67 °
 concordance -model simulation: I-binned mean ~ 90 °

preferred direction over large range of angular resolution after dipole substraction: $\vec{\nabla}\delta T|_{z=0,\sigma_0}$ or \vec{e}_{cs}



- some oservational facts
- dipolar, multiplicative modulation model
- deconfining SU(2) YMTD
- dual interpretation
- photon propagation described by $\,{
 m SU(2)}_{_{
 m CMB}}\,$ rather than U(1)
- some evidence
- black-body anomaly
- effective theory for temperature fluctuations: rapid build-up of profile at $z\sim1$
- interpretation of results: accomodation of pot. dipole discrepancy, cold spot, variance and power asymmetries, and mirror antisymmetry preferred direction in the dipole subtracted CMB sky
- relax simplifying assumptions (spherical symmetry, instanteneous line-of-sight integrations) and do more realistic simulations
- BUT YOU CAN DO IT MUCH BETTER! (2-parameter model in simulations: $\sigma_0 ~~{\rm and}~ \xi$)

Thank you.



- excess of radiance at low frequencies:
- photon acquires Meissner mass $m_\gamma \sim 0.1\,{
 m GHz}$ by coupling to preconfining ground state [Hofmann 2009]
- for $\omega < m_{\gamma}$: photons become evanescent (standing waves) of Gaussian radiance distribution about zero mean _____ Interval the second stribution about zero mean _____ line temperature for $\omega \to 0$:

$$T \propto \omega^{-2}$$
 spectral index

(spectral index of line temperature ~ - 2.6 at $~\nu\sim 2\,GH_Z$, when lower $\nu~$ included in fit spectral index increases!)

massive cosmological neutrino equation of state:



Assume: $m_{\nu} = \xi T$ (neutrino single center-vortex loop of yet another but now confining-phase SU(2), neutrino mass induced by environment) [Moosmann,Hofmann 2008]

