Karlsruher Institut für Technologie

SU(2) Yang-Mills thermodynamics: Violation of conformal temperature - scale factor relation and dynamical breaking of CMB statistical isotropy at low redshifts

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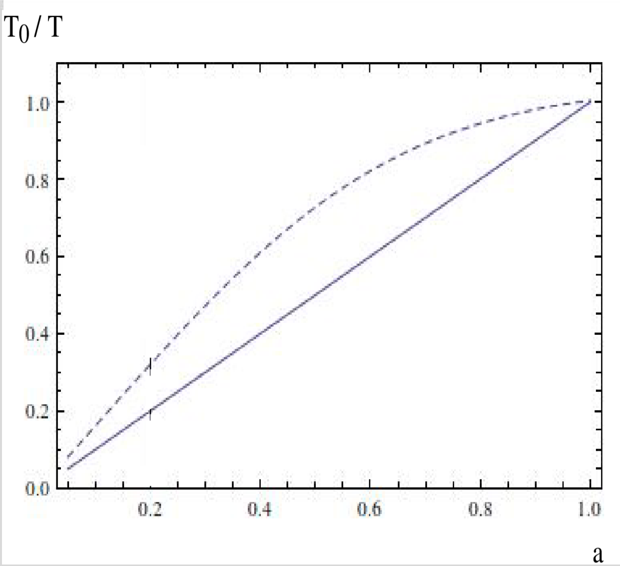
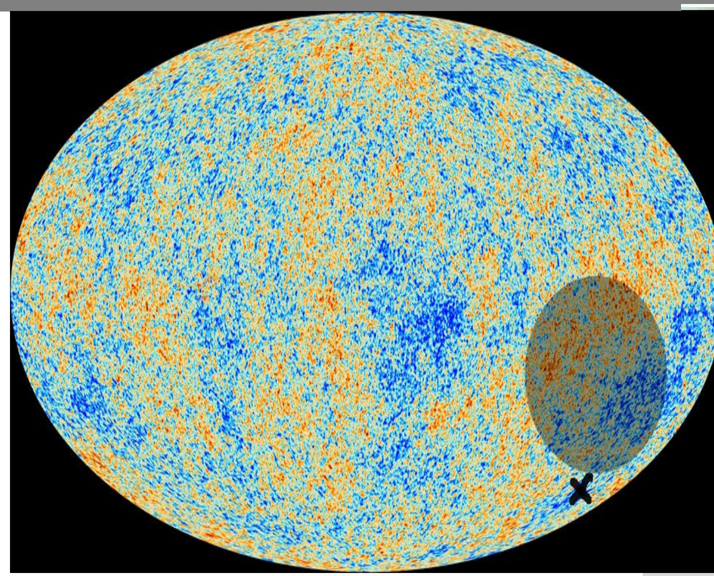




Outline

- **essentials, thermal ground state:**
  coarse-graining over nonpropagating
  (anti-)calorons of winding number unity, effective action
- **adjoint Higgs mechanism:**
  massive vector modes and kinematic constraints (1),
  coupling, deconf.-preconf. phase boundary, (anti-)caloron action
- **pressure and energy density on one-loop level**
- **small radiative corrections:**
  kinematic constraints (2), polarisation tensor of massless mode,
  longitudinal and transverse thermal dispersion,
  „photon-photon“ scattering
- **SU(2) postulate for photon propagation:**
  Yang-Mills scale or critical temperature
  (radio-frequency CMB observations), black-body spectral
  and integral anomaly
- **Scaling violation and CMB large-angle anomalies (WMAP, Planck):**
  CMB temperature vs FLRW scale factor, early reionisation,
  SU(2) transverse dispersion relation and „axis of evil“,
  intergalactic magnetic fields (longitudinal dispersion),
  cosmic neutrinos (neutrino temperature and mass)

motivation

- Andrei Linde (1980):
  „Infrared Problem in the Thermodynamics of the Yang-Mills Gas“

  - soft magnetic sector screened weakly in perturbation theory (infrared instability)

  - no „convergence“ of series since kinetic and interaction energies comparable in this sector

- issue of finite-volume constraints in lattice gauge theory

  - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

  - nonperturbative, lattice β function

nonperturbative Yang-Mills thermodynamics: SU(2)

[Herbst et Hofmann (2004), Hofmann (2005-2007), Giacosa et Hofmann (2006), Schwarz et al. (2007), Ludescher et Hofmann (2008), Falquez et al. (2010- 2011), Hofmann (2012)]

thermal ground state at high temperature:

- **Euclidean action:**

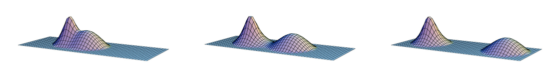
$$S = \frac{\mathrm{tr}}{2}\int_0^\beta d\tau \int d^3x\, F_{\mu\nu}F_{\mu\nu}\,, \qquad (\,\beta \equiv 1/T\,)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ [Schafer et Shuryak (1996)]

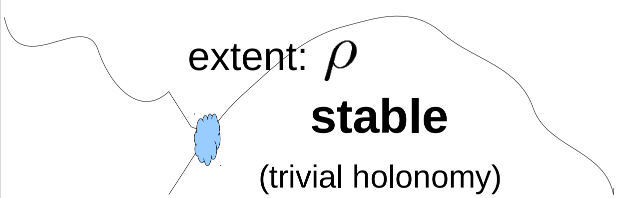
- **(anti)selfdual gauge fields:** $F_{\mu\nu}[A] = \pm\tilde{F}_{\mu\nu}[A] \;\Rightarrow\; \theta_{\mu\nu}[A] \equiv 0\,.$

field configs. stabilized by winding: $S_3 \rightarrow SU(2) = S_3$

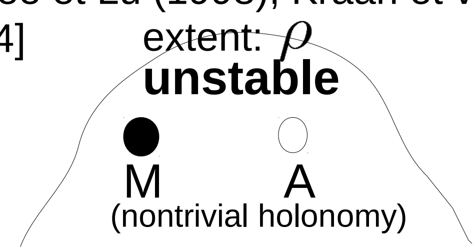
- **in particular:** (anti)calorons of winding number unity



[Harrington et Shepard (1977)]



[Nahm (1981-84), Lee et Lu (1998), Kraan et v. Baal (1998), Diakonov et al. 2004]



spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field ϕ

$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0}) \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} F_{\mu\nu}(\tau,\vec{x}) \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, dimensionless definition of **family of phases**, where

$$\left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} \equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu\, A_\mu(z)\right]$$ and

$$\left\{(\tau,\vec{x}),(\tau,\vec{0})\right\} \equiv \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\}^\dagger$$

- magnetic-magnetic correlations contribute only [Herbst et RH, 2004]

- uniquely determined, annihilating operator: $D = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$

- $\{\hat{\phi}^a\}$ sharply dominated by cut-off for ρ integration, later!

spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field

- no explicit β dependence in ϕ field dynamics (caloron action!)

- absorb β dependence of operator D into potential V

(BPS and EL yield: $\frac{\partial V(|\phi|^2)}{\partial |\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2} \Rightarrow$

$$\boxed{V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}}$$

(Yang-Mills scale)

and

$$\boxed{|\phi| = \sqrt{\frac{\Lambda^3 \beta}{2\pi}}}$$

- BPS equation:

$$\partial_\tau \phi = \pm 2i\,\Lambda^3\, t_3\, \phi^{-1} = \pm i\, V^{1/2}(\phi)$$

$\Rightarrow$ no **additive** ambiguity for V !

effective action (deconfining phase)

- $$\mathcal{L}_{\rm eff}[a_\mu] = {\rm tr}\left(\frac{1}{2}G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

((i) perturbative renormalizability (G^2 highest power in action)
(ii) ϕ 's inertness – no higher dim., mixed operators to mediate
4-momentum transfer between ϕ and a_μ
(iii) gauge invariance)

- effective YM equation $D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]$ has ground-state solution:

$$a_\mu^{\rm gs} = \mp\delta_{\mu 4}\frac{2\pi}{e\beta}t_3 \qquad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0)$$

$$\Longrightarrow \quad P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T\,.$$

interacting small and transient holonomy (anti)calorons, (collapsing monopole-antimonopole pairs)

(vanishing entropy density of ground state!)

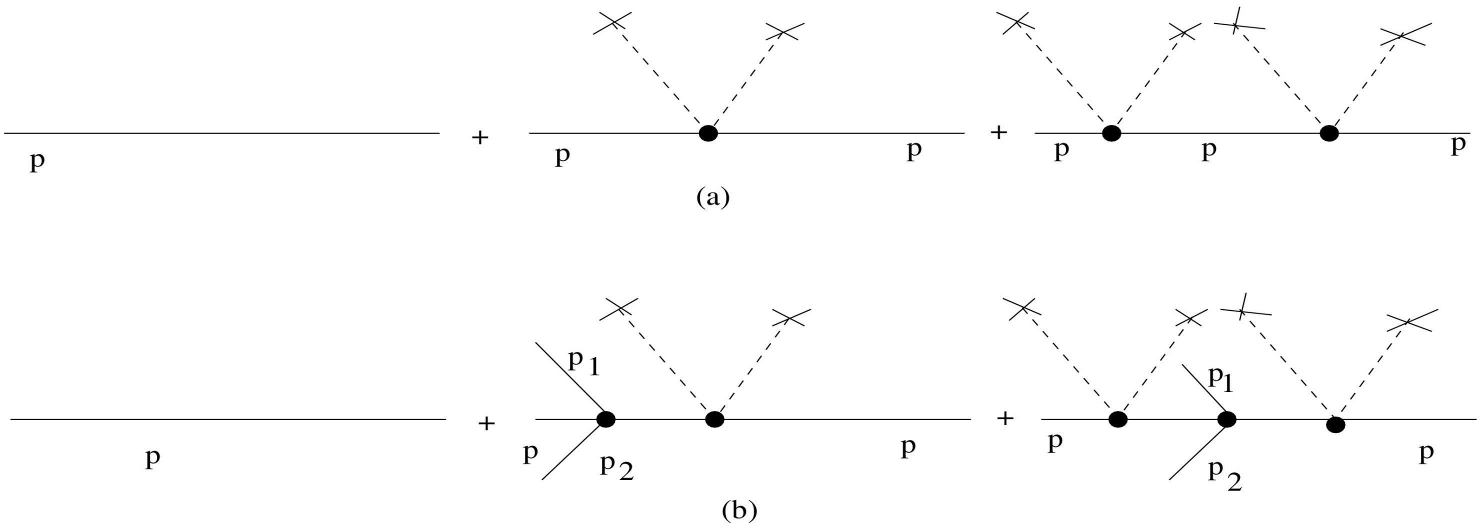
adjoint Higgs (deconfining phase)

(SU(2) → U(1))

- from effective action:

$$\boxed{m_a^2 = -2e^2 \mathrm{tr}\,[\phi, t_a][\phi, t_a]} \xrightarrow{\text{unitary gauge}} \boxed{m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}\,,\ m_3 = 0}$$

- no momentum transfer to ϕ, **but this infinitely often** (Dyson series for mass generation):



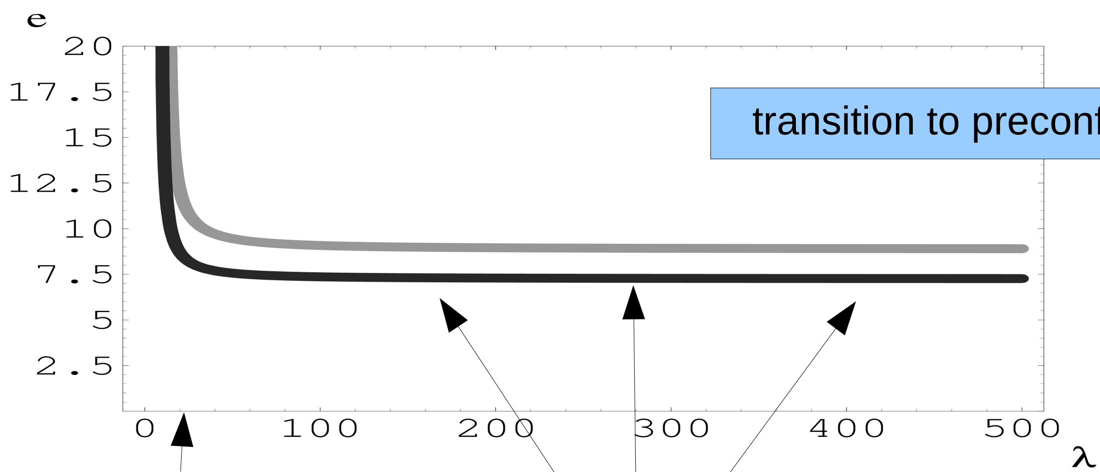
- no off-shell propagation of massive modes (otherwise: momentum transfer to ϕ !)

effective gauge coupling

- evolution of effective gauge coupling:

e

thermodynamical consistency



transition to preconfining phase

[Dolan et Jackiw (1974)]

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

coarse-graining dominated by $\rho \sim |\phi|^{-1}$

- restore $\hbar$

$$e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}\,.$$

$$S_{C/A} = \hbar\,.$$

[Brodsky et al. (2011);
Kaviani et Hofmann 2012,
Hofmann (2012,2013)]

electric-magnetically dual interpretation:

- if SU(2) something to do with photons (later!) then **electric-magnetically dual** interpretation required, **argument goes as:** in units

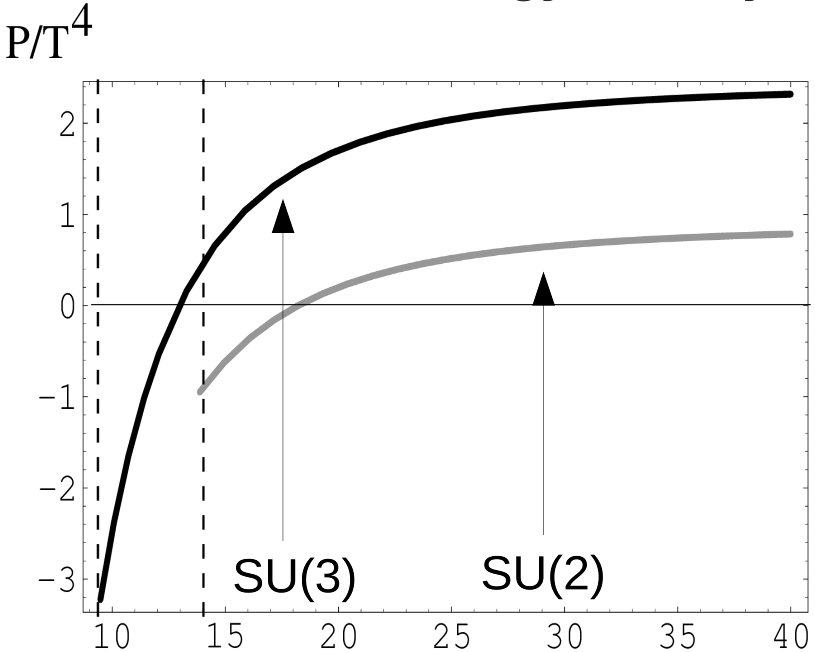
$c = \epsilon_0 = \mu_0 = k_B = 1$ $\Longrightarrow$ fine-structure constant $\boxed{\alpha = \frac{Q^2}{4\pi\hbar}\,,}$

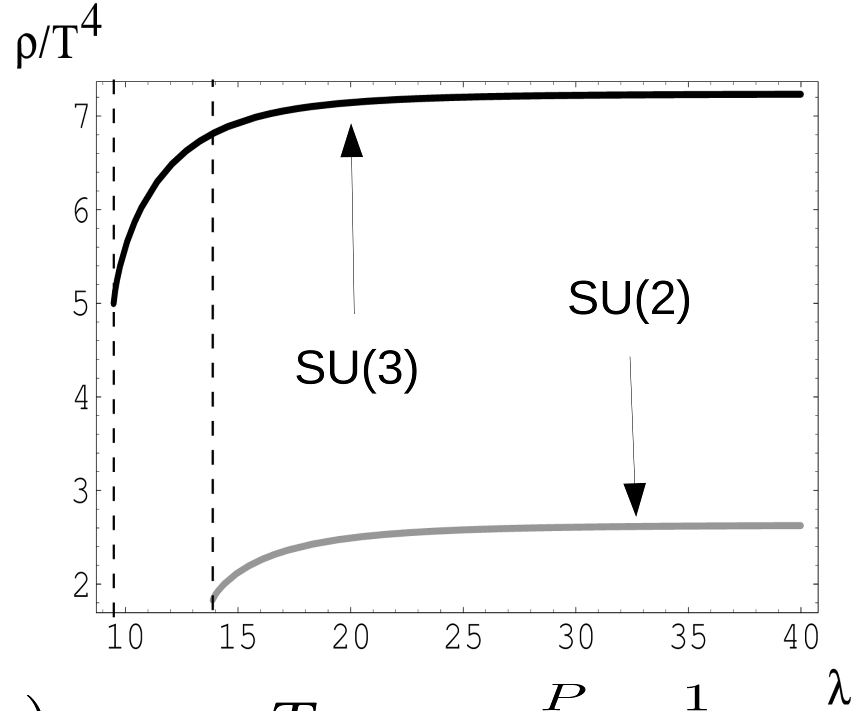
for α to be unitless: $\Longrightarrow$ $\boxed{Q \propto \frac{1}{e}\,.}$

But: magnetic coupling in SU(2) $\boxed{g = \frac{4\pi}{e}\,.}$

$\Longrightarrow$ SU(2) to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole, etc.)

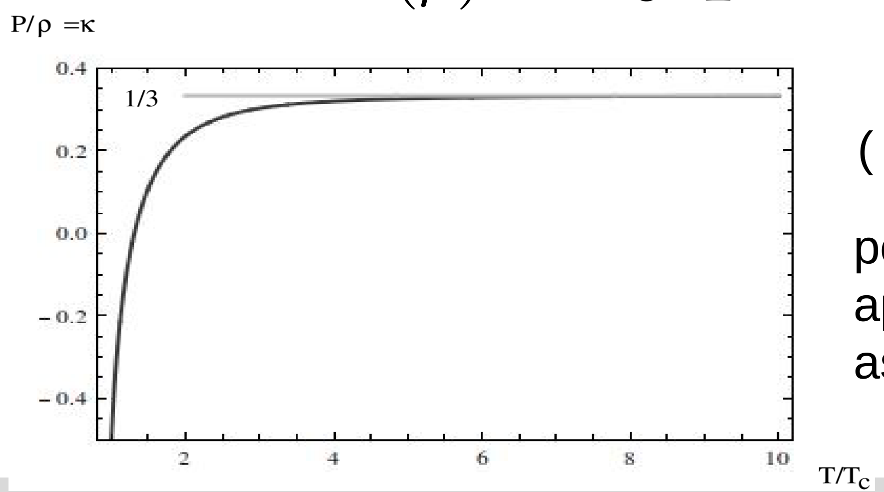
Pressure and energy density:





$\Longrightarrow$ equation of state: $P = P(\rho)$, for high T we have $\frac{P}{\rho} = \frac{1}{3}$

indeed:

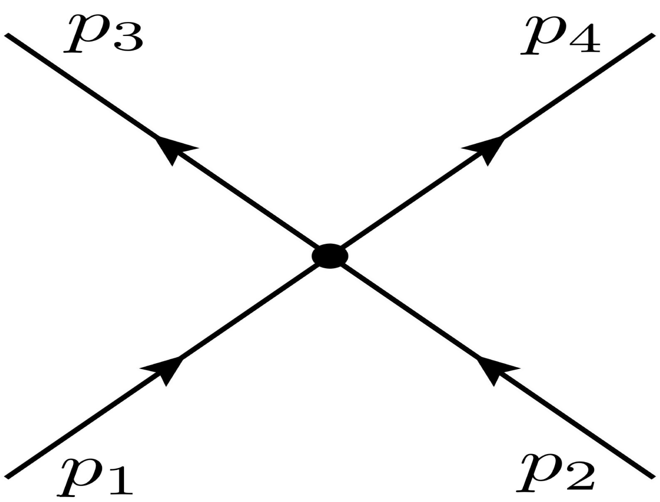


(SU(2))

powerlike fast approach to SB asymptote

radiative corrections (deconfining phase): Feynman rules

- constrained momentum transfer in effective 4-vertex (unitary-Coulomb gauge):



s-channel: $\boxed{|(p_1+p_2)^2| \leq |\phi|^2}$

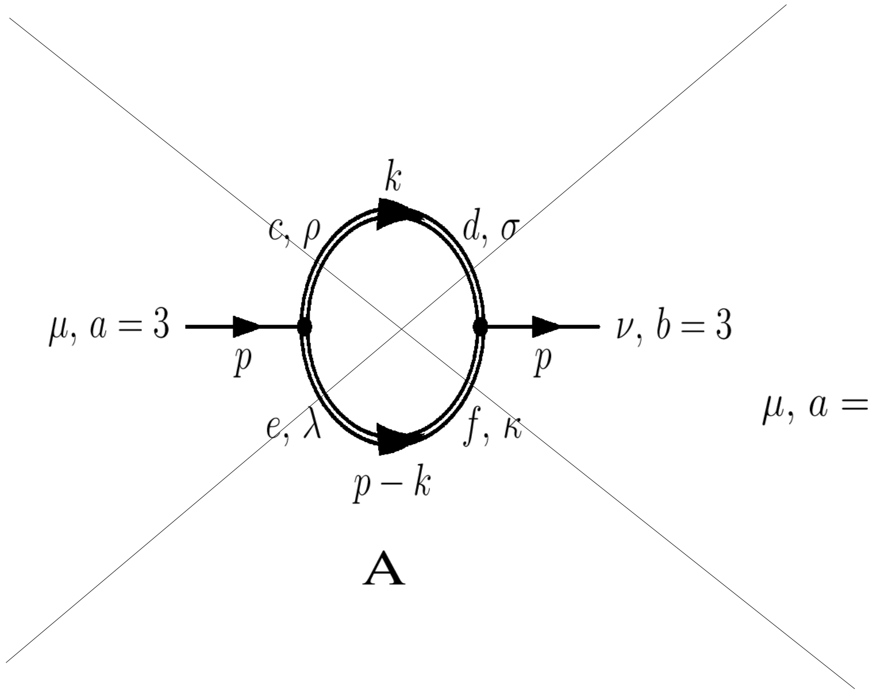
t-channel: $\boxed{|(p_1-p_3)^2| \leq |\phi|^2}$

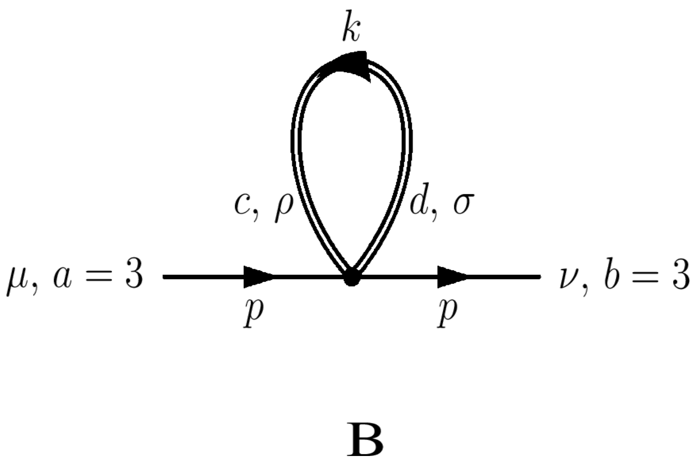
u-channel: $\boxed{|(p_1-p_4)^2| \leq |\phi|^2}$

- coherent average over all three channels $\longrightarrow$
  thermodynamical quanties: 2-loop/1-loop ($<10^{-3}$), 3-loop/1-loop ($<10^{-7}$),
  conjecture:
  loop expansion into 1-PI diagrams probably terminates at finite order [RH 2006]

radiative corrections (deconfining phase): photon polarisation

- polarisation tensor of massless mode (Coulomb gauge):





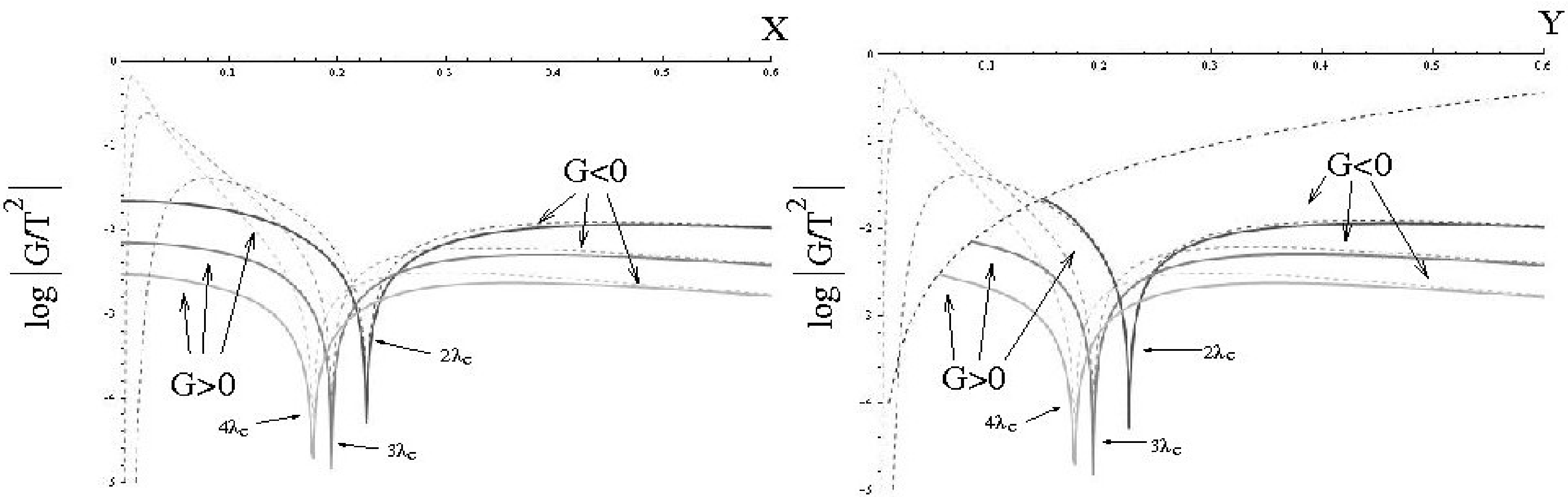
(excluded by kinematic constraints:
on-shellness of vector mode,
restricted off-shellness of massless mode)

$\longrightarrow$ screening functions G, F as solutions of respective gap equations

radiative corrections (deconfining phase): transverse photon dispersion relation

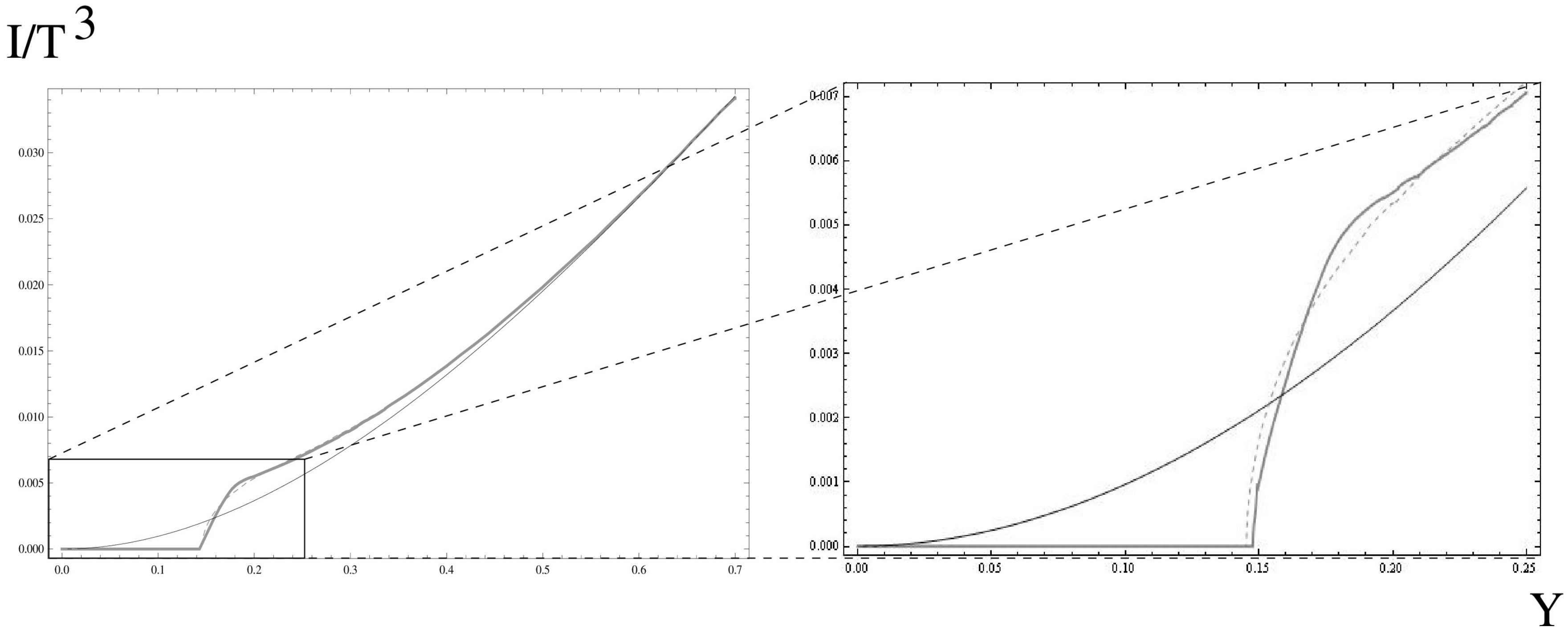
- transverse photons, screening function G :

[Schwarz et al. (2007), Ludescher et Hofmann (2008)]



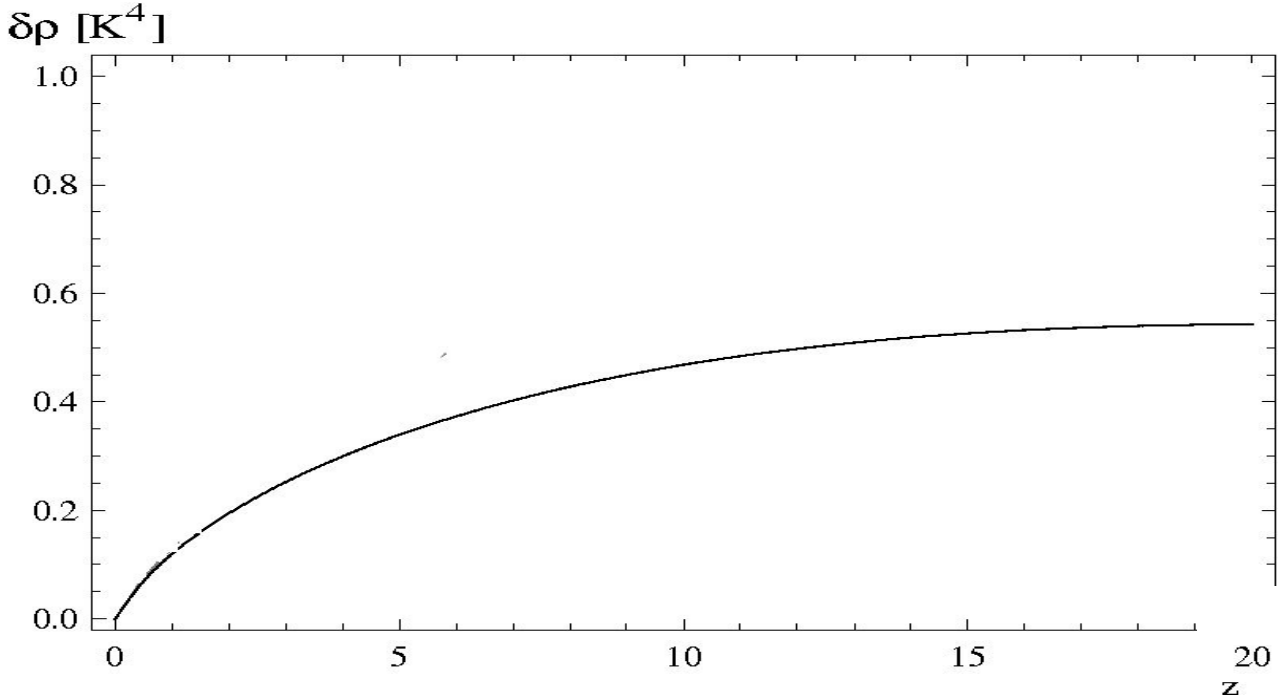
radiative corrections (deconfining phase): modified Rayleigh-Jeans part of black body spectrum

- spectral distribution of energy density, massless mode – transverse propagation at $T = 2T_0$



radiative corrections (deconfining phase): Integral black body anomaly

- difference between energy density of SU(2) and U(1), massless mode – transverse propagation (presuming that $T_c = T_0 = 2.725\,\mathrm{K}$, later)

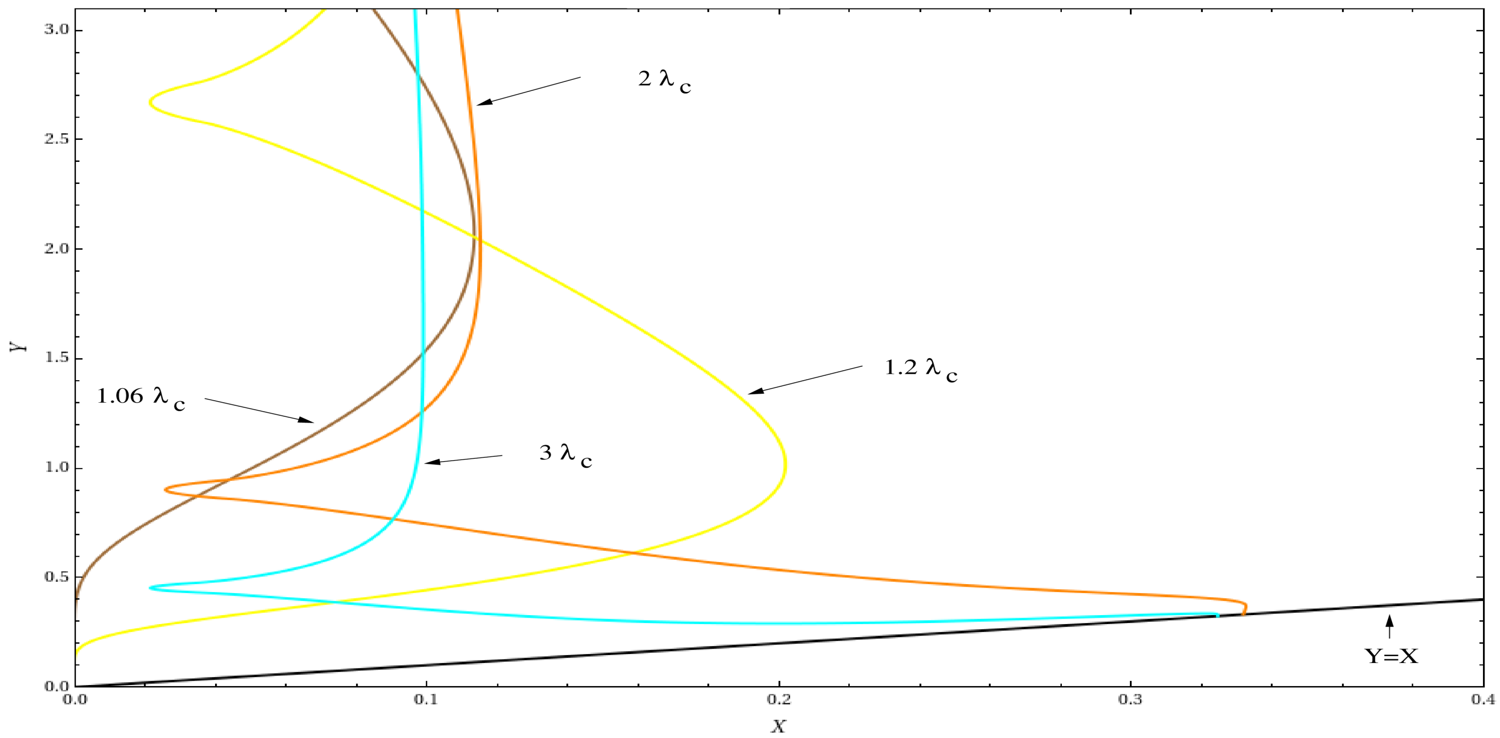


$T/T_c - 1$

(**positive** slope $\longleftrightarrow$ bias for **negative** temperature fluctuations, later!)

radiative corrections (deconfining phase): branches of longitudinal propagation of magnetic fields

- low-momentum-support dispersion relation, massless mode - longitudinal propagation

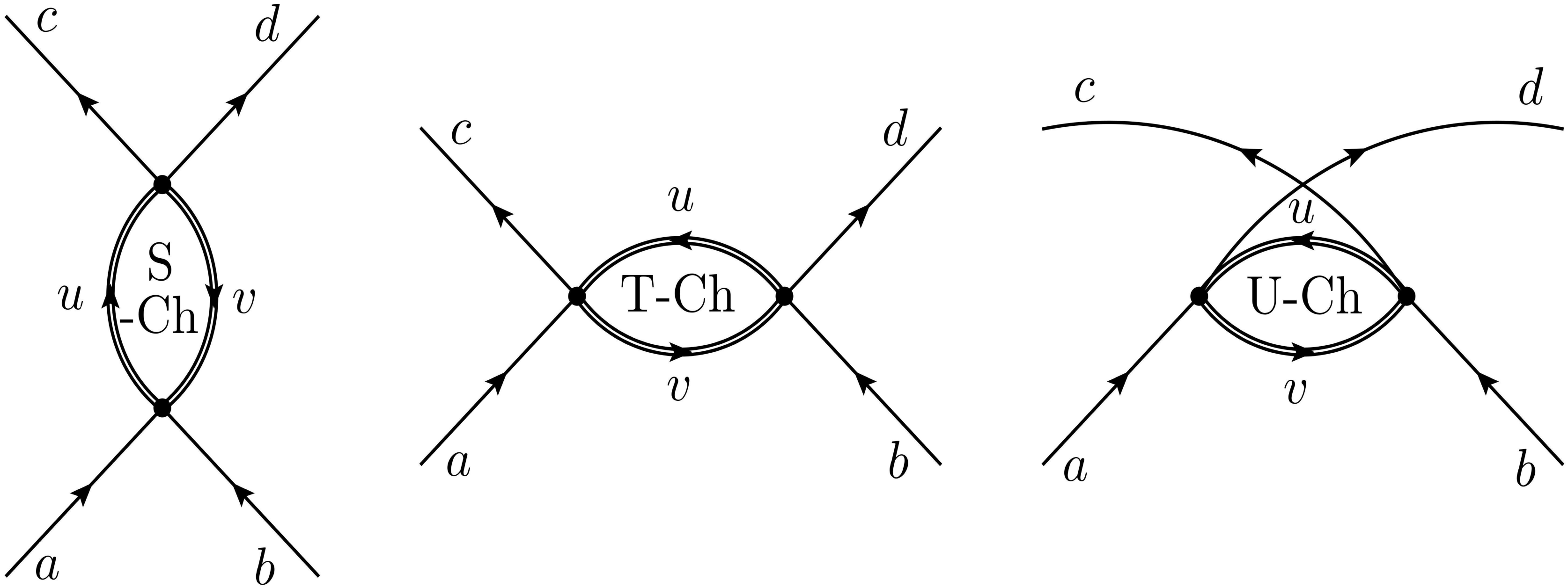


(charge-density waves: real-world magnetic modes, cosmic magnetic fields, $B \sim 10^{-8}\,\mathrm{G}$, [Falquez et al (2011)])

radiative corrections (deconfining phase): Photon-photon scattering

- „photon-photon“ scattering [Krasowski et Hofmann (2013)]

due to kinematic constraints only topology with two 4-vertices contributes



radiative corrections (deconfining phase): Photon-photon scattering

- analysis of forbidden sign-combinations of u_0, v_0 leads to exclusion tables for each of overall S, T, or U channels

  for example:

Table 1
Forbidden combinations of energy flow (marked with a X) in all scattering channel combinations of vertex 1 and vertex 2 in the overall S-channel.

Vertex 2 \ Vertex 1	s-ch.		t-ch.		u-ch.	
s- ch.		X	X	X	X	X
	X		X	X	X	X
t-ch.	X	X	X		X	
	X	X	X	X	X	X
u-ch.	X	X	X		X	
	X	X	X	X	X	X

Tool for eventual proof of termination of loop expansion at finite one-particle irreducible loop order.

SU(2) postulate for photon propagation

- What is T_c ?

- strong increase of CMB line temperture below $\nu = 3$ GHz

$$T(\nu) = T_0 + T_R \left(\frac{\nu}{\nu_0}\right)^\beta$$

[Fixsen et al. (2009), Haslam et al. (1981), Reich et Reich (1986), Roger et al. (1999), Maeda et al. (1999)]

where: $T_0 = 2.725\,\mathrm{K}\,;\ \nu_0 = 1\,\mathrm{GHz}\,;$
$\beta = -2.62 \pm 0.04\,; T_R = (1.19 \pm 0.14)\,\mathrm{K}\,.$

(radio-frequency surveys of CMB yield line temperatures as:

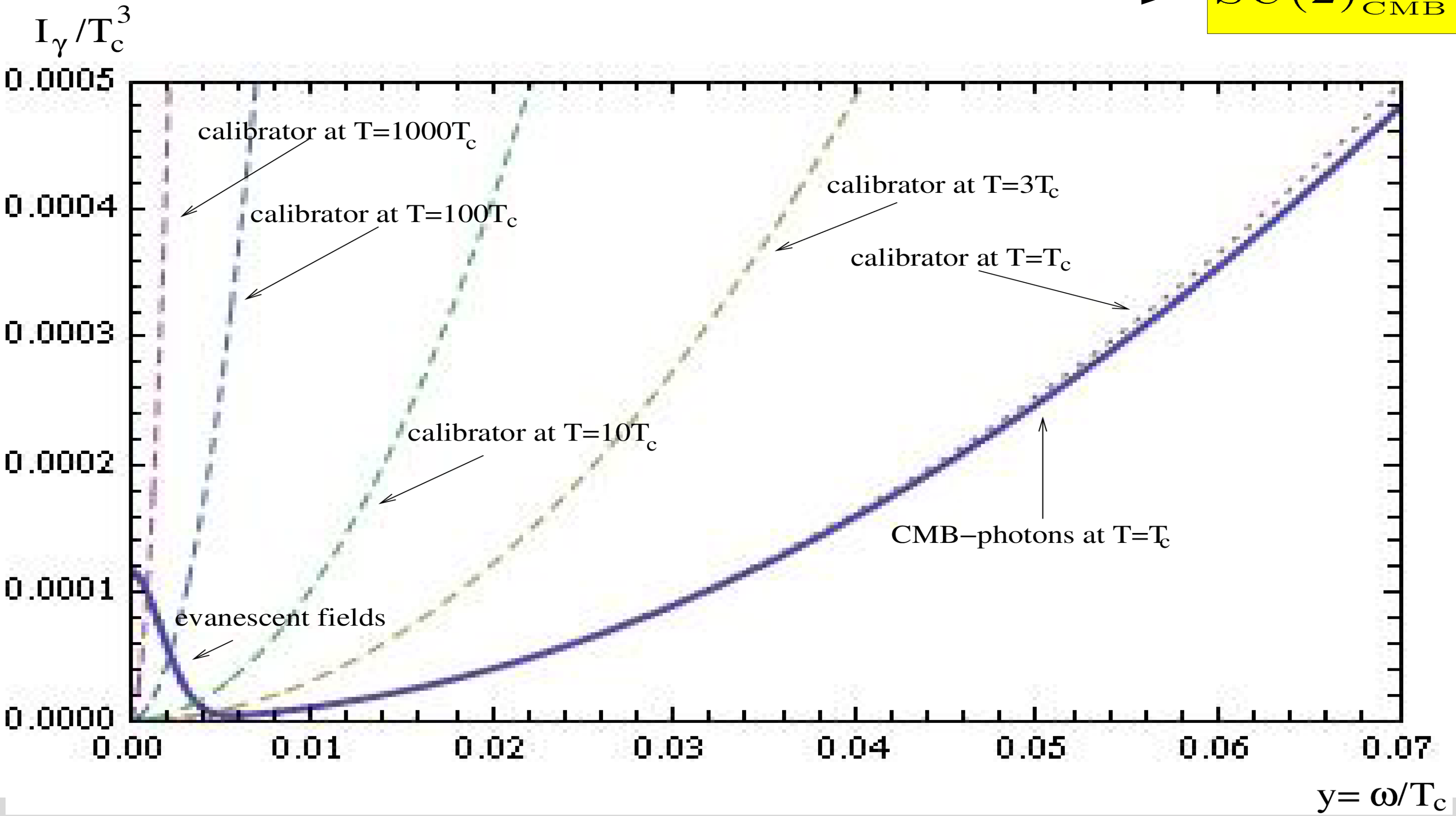
source	ν[GHz]	T[K]
Roger	0.022	21200 ± 5125
Maeda	0.045	4355 ± 520
Haslam	0.408	16.24 ± 3.4
Reich	1.42	3.213 ± 0.53
Arcade2	3.20	2.792 ± 0.010
Arcade2	3.41	$2.771 \pm 0.009\,.$

)

evanescent low-frequency modes

- bump from evanescent modes ($\omega < m_\gamma$),
  m_γ photon Meissner mass (condensation of electric monopoles)
- T_c very close to present CMB temperature T_0 (onset of dec.-prec. PT)
  [Hofmann (2009)]

$\longrightarrow$ $\mathrm{SU(2)_{CMB}}$



Yang-Mills scale of $SU(2)_{CMB}$:

$$T_c = \frac{13.87}{2\pi}\Lambda_{\mathrm{CMB}} = 2.725\,\mathrm{Kelvin} \sim 2 \times 10^{-4}\,\mathrm{eV}$$

some CMB large-angle anomalies: WMAP and Planck

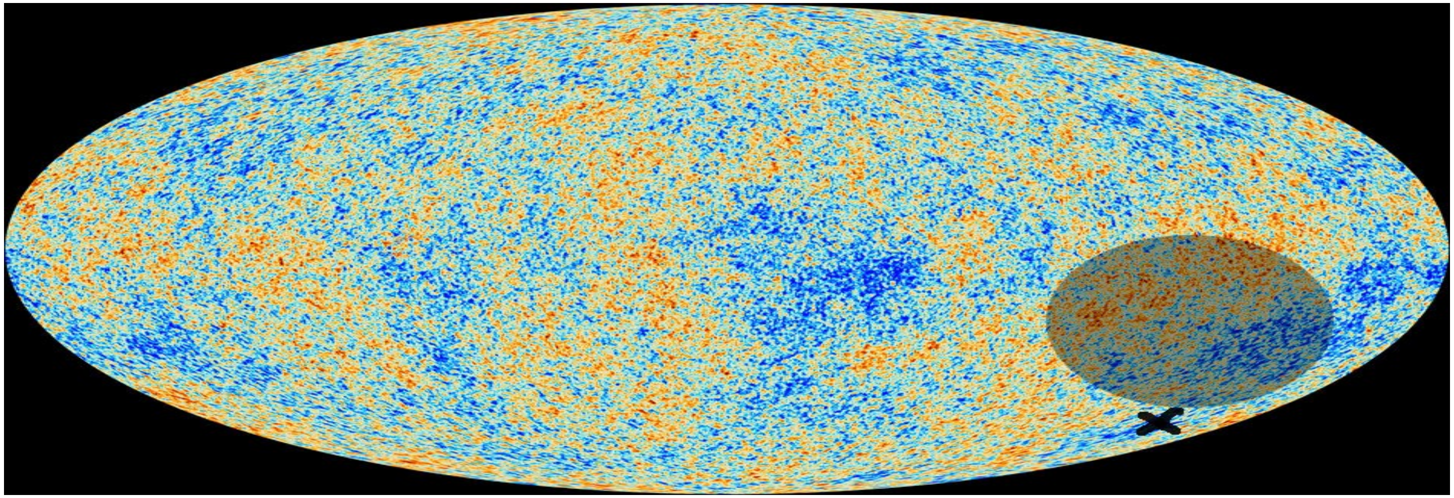
- dipolar power asymmetry (extends from $l = 2, \cdots, 600$ in blocks of $\Delta l = 100$)
  [Hansen et al. (2009), Ade et al. (2013), etc.]

- low variance on ecliptic North, associated with l=2,3
  [Monteserin et al. (2008), Cruz et al., (2011), Ade et al. (2013), etc.]

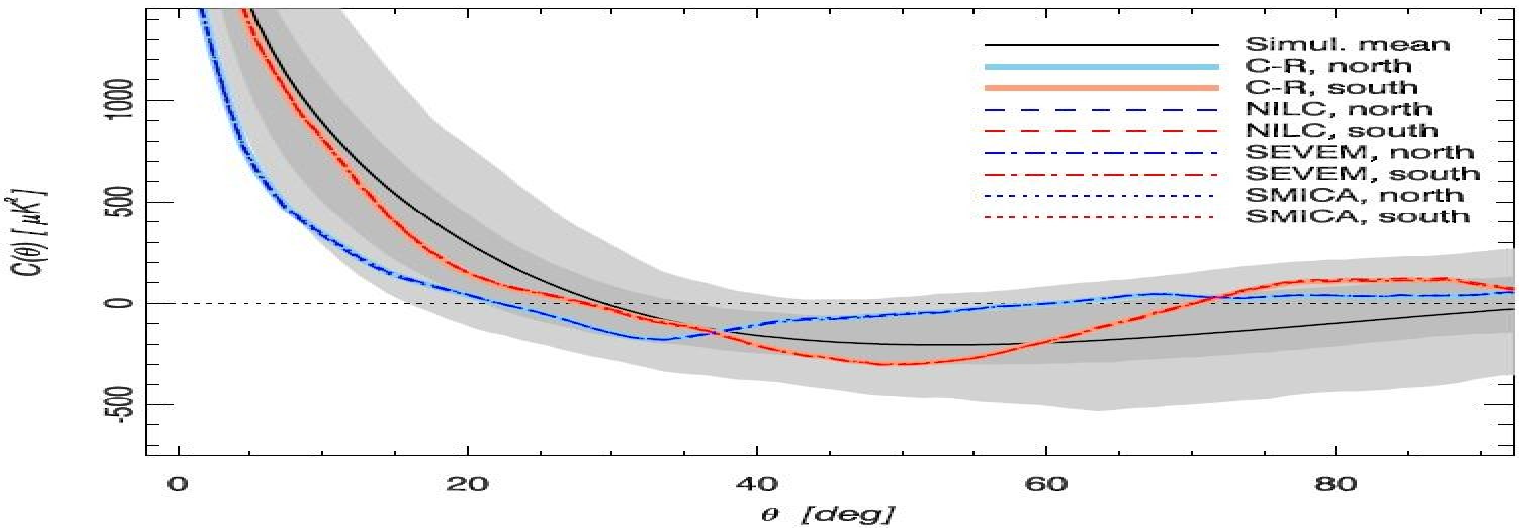
- anomalous alignment of l=2,3 (3°-9°)
  [Tegmark et al. (2003), de Oliveira-Costa et al. (2004), Ade et al. (2013), etc.
  (estimator of axis: maximum of angular momentum dispersion),
  Copi et al. (2004), Schwarz et al. (2004), Bielewicz et al. (2005,2009), Copi et al. (2010), etc.
  (multipole vector decomposition)]

- cold spot (-73μK@4°; -20μK@10°; l,b=207.8°,-56.3°)
  [Viela et al. (2004), Cruz et al. 2005, Rudnick et al. (2007), Ade et al. (2013), etc.]
- hemispherical asymmetry
  (for l=2-40 max. larger power on hemisphere l,b=237°,-20°)
  [Eriksen et al. (2004), Hansen et al. (2004), Park (2004), Ade et al. (2013), etc.]
- mirror parity violation (plane of max. antisymmetry: l,b=262°,-14°)
  [Finelli et al.(2012); Ben-David et al. (2012), etc.]
- suppression of $\langle TT\rangle(\theta) \equiv C(\theta)$ for $\theta \geq 60^\circ$ on ecliptic North
  [Spergel et al. (2003), Copi et al. (2004,2007,2009,2010), Ade et al. (2013), etc.]

cold spot



TT suppression on ecliptic North



dynamical breaking of statistical isotropy:
Temperature fluctuations in Cosmic Microwave Background

- CMB temperature fluctuations expanded into spherical harmonics

$$\delta T(\phi,\theta)=\sum_{l,m} a_{lm} Y_l^m(\phi,\theta)$$

- a_{lm} assumed to be independent Gaussian random variables

Is this really so for all l ?

successful phenomenological attempt at explanation: multiplicative, dipolar modulation model

[Gordon et al. (2005), Eriksen et al. (2007), Hoftuft et al. (2009), Ade et al. (2013)]

$$\boxed{\vec{d}(\vec{n}) = (1 + A\vec{p}\cdot\vec{n})\vec{s}_{\rm iso} + \vec{n}}$$

dipole amplitude   dipole direction   instrumental noise   isotropic CMB sky

maximum likelihood at: $A \sim 0.07;\;\; l_p \sim 220^\circ; b_p \sim -21^\circ$

- robust against change of foreground treatment and experiment (WMAP,Planck)
- comparison with CMB cold spot: $l_{cs} \sim 207.8^\circ; b_{cs} \sim -56.3^\circ$

$$\boxed{\Rightarrow \angle\vec{p},\vec{e}_{cs} \sim 36^\circ}$$

dynamical breaking of statistical isotropy:

- integrated blackbody anomaly due to SU(2)$_{\rm CMB}$:

- $\boxed{\delta\rho(T) \equiv \rho_{\rm SU(2)_{CMB}} - \rho_{\rm U(1)}}$
- $T = \bar{T}(t) + \delta T(t,\vec{x})$
- SU(2)$_{\rm CMB}$ bias factor $F(\bar{T},\delta T)$ for δT in phys. voxel volume $\Delta V \sim \frac{(2\pi a)^3}{k_s^3}$ (Silk cutoff)

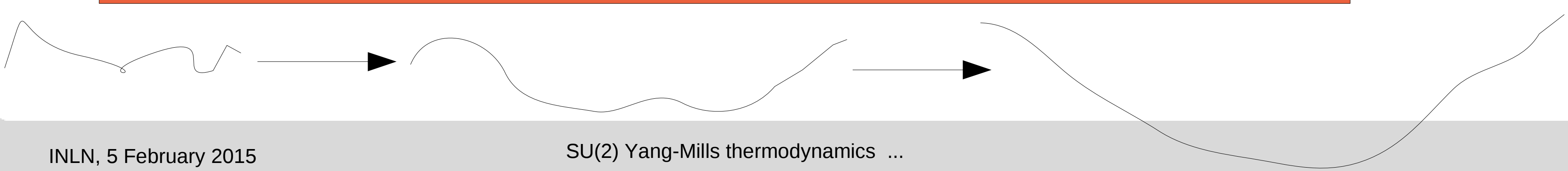
$$\boxed{F(\bar{T},\delta T) = \frac{P_{\rm SU(2)}}{P_{\rm U(1)}}}$$

where

$$\boxed{P = \frac{\exp(-\rho\Delta V/\bar{T})}{\int_{T_0}^{\infty} dT\, \exp(-\rho\Delta V/\bar{T})}}$$

(in comoving Fourier-space simulation:
use convolution $\boxed{\tilde{F} * \tilde{\delta T}}$ for conventionally evolved $\tilde{\delta T}$ at $\{z_n\}$,
then projection)

Since slope of $\delta\rho$ positive $\Longrightarrow$ negative δT favoured!



dynamical breaking of statistical isotropy:

- semiquantitative model: effective SU(2)$_{\mathrm{CMB}}$ evolution

$$\sqrt{-g}\,\mathcal{L}_{\mathrm{CMB}} = \left(\frac{\bar{T}_0}{\bar{T}}\right)^3 \left(k\,\partial_\mu \delta T \partial^\mu \delta T - \delta\rho(T)\right)$$

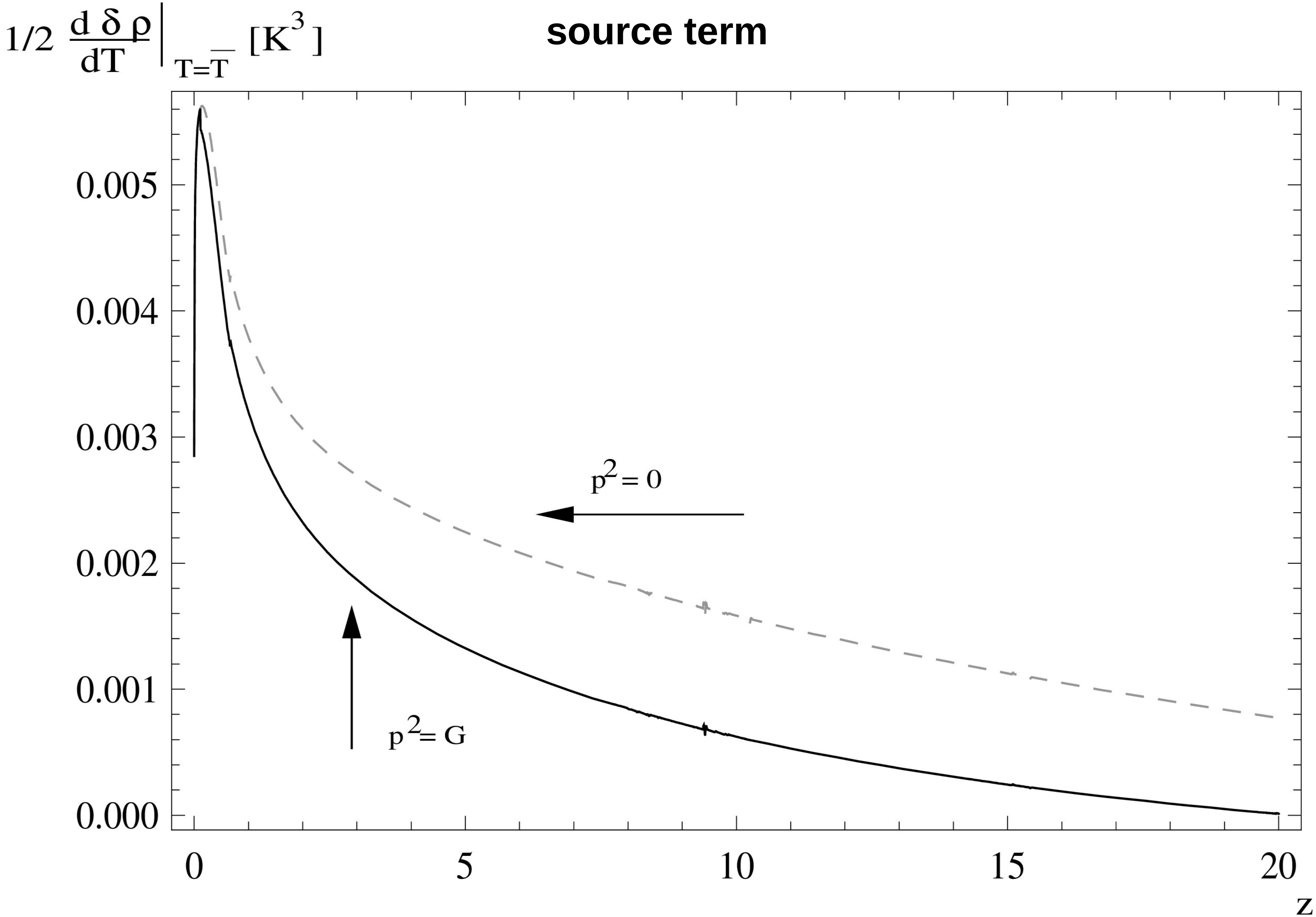
- assuming 3D spherical symmetry, causal boundary conditions

$$0 = \partial_\tau \partial_\tau \delta T - \left(\frac{\mathrm{d}a}{a\,\mathrm{d}\tau}\right)^2 \left[\partial_\sigma \partial_\sigma \delta T + \frac{2}{\sigma}\partial_\sigma \delta T\right] - \frac{3}{\bar{T}}\partial_\tau \bar{T}\,\partial_\tau \delta T + \frac{T_0^2}{k H_0^2}\left[\frac{1}{2}\left.\frac{\mathrm{d}^2\hat{\rho}}{\mathrm{d}T^2}\right|_{T=\bar{T}} \delta T + \frac{1}{2}\left.\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}T}\right|_{T=\bar{T}}\right]$$

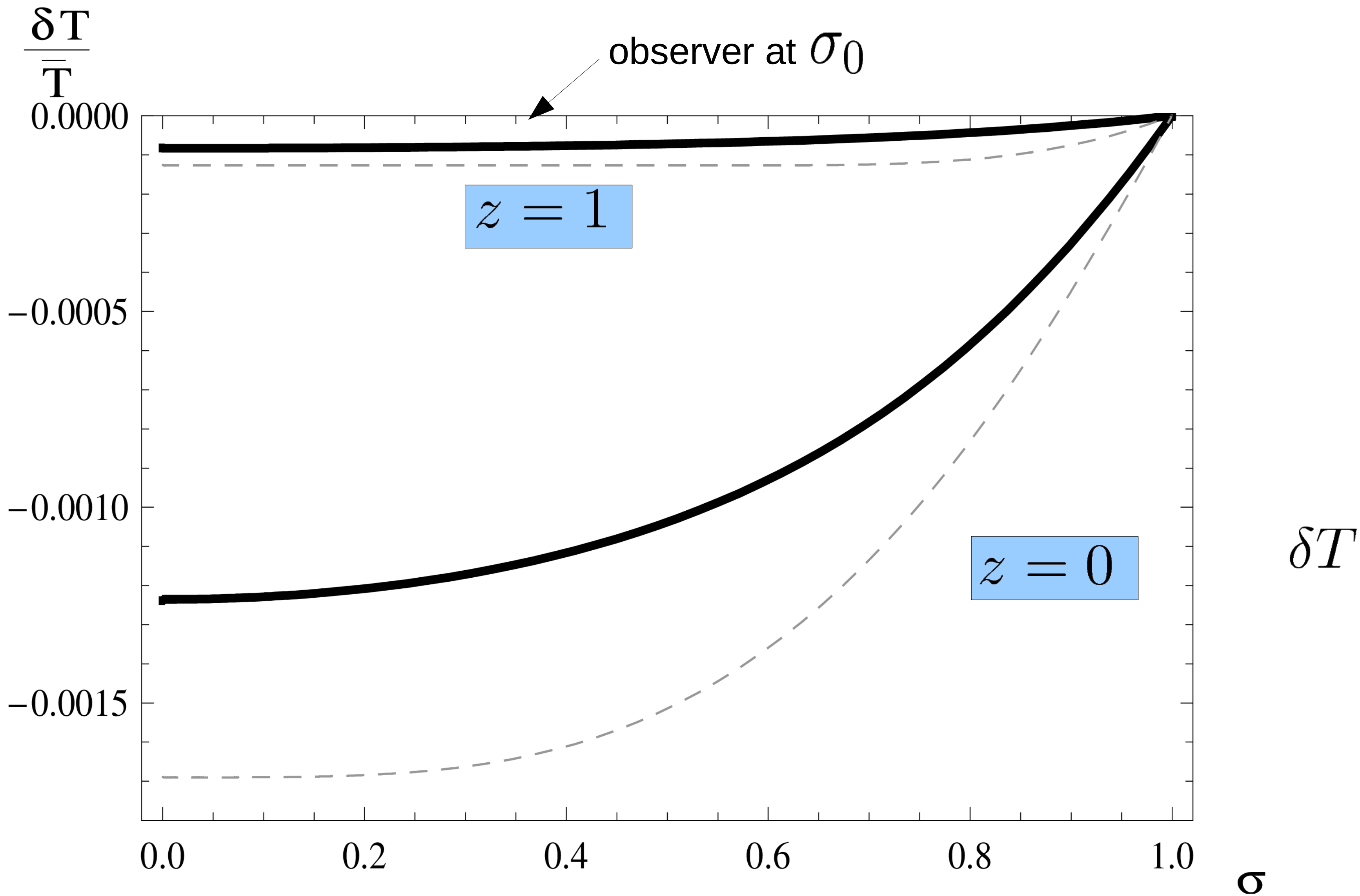
[Szopa, RH, 2007]

source term

dynamical breaking of statistical isotropy:



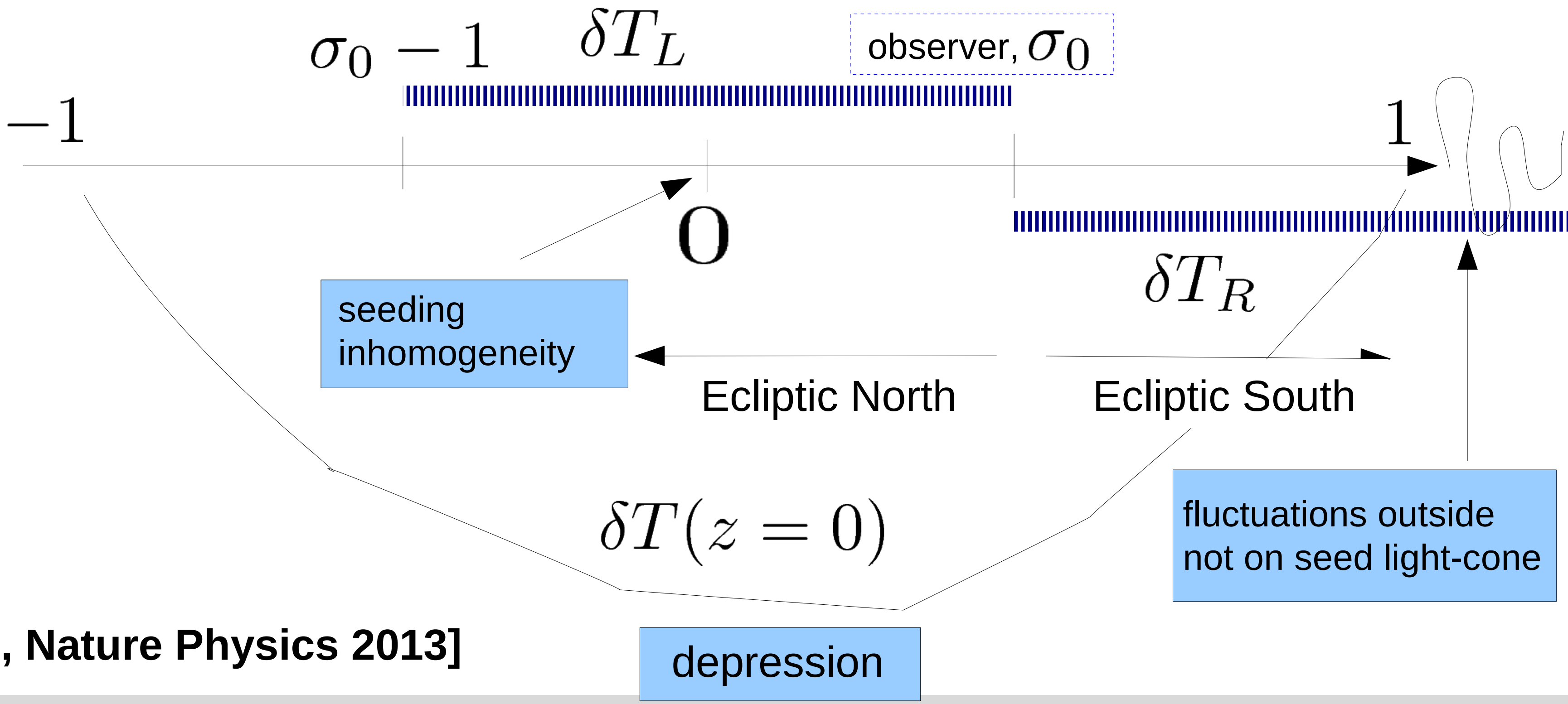
dynamical breaking of statistical isotropy:



dynamical breaking of statistical isotropy:

- **low variance, power asymmetry:**
  (simplified, instantaneous light propagation for projection)

$$\delta T_L \equiv \int_{\sigma_0}^{1} d\xi\, \delta T(z=0,\xi)\,, \quad \delta T_R \equiv \int_{\sigma_0-1}^{\sigma_0} d\xi\, \delta T(z=0,\xi)$$



[RH, Nature Physics 2013]

dynamical breaking of statistical isotropy:

- **suppression of TT for** $\theta \geq 60^\circ$ **:**

  rapid build-up of profile for $z \leq 1$

- **dynamical contribution in measured (kinematically dominated) CMB dipole** $\longrightarrow$

$$\boxed{|\vec{D}_{dyn}| = \frac{1}{2}(\delta T_L - \delta T_R)}$$

- $\boxed{\text{offset} = \frac{1}{2}(\delta T_L + \delta T_R)}$ $\longrightarrow$ **cold spot**

$\longrightarrow$ $\boxed{\vec{d}_{cs}||\vec{e}_{\text{mirror antisymm}}}$ $\boxed{\vec{d}_{cs}||\vec{e}_{\text{hemisph asymmetry}}}$

Planck results**:**

$$\angle \vec{e}_{\text{mirror antisym}}, \vec{e}_{cs} \sim 42^\circ - 56^\circ\,;$$

$$\angle \vec{e}_{\text{hemisph asym}}, \vec{e}_{cs} \sim 42^\circ\,.$$

Violation of conformal scale factor – temperature relation in low-z CMB:

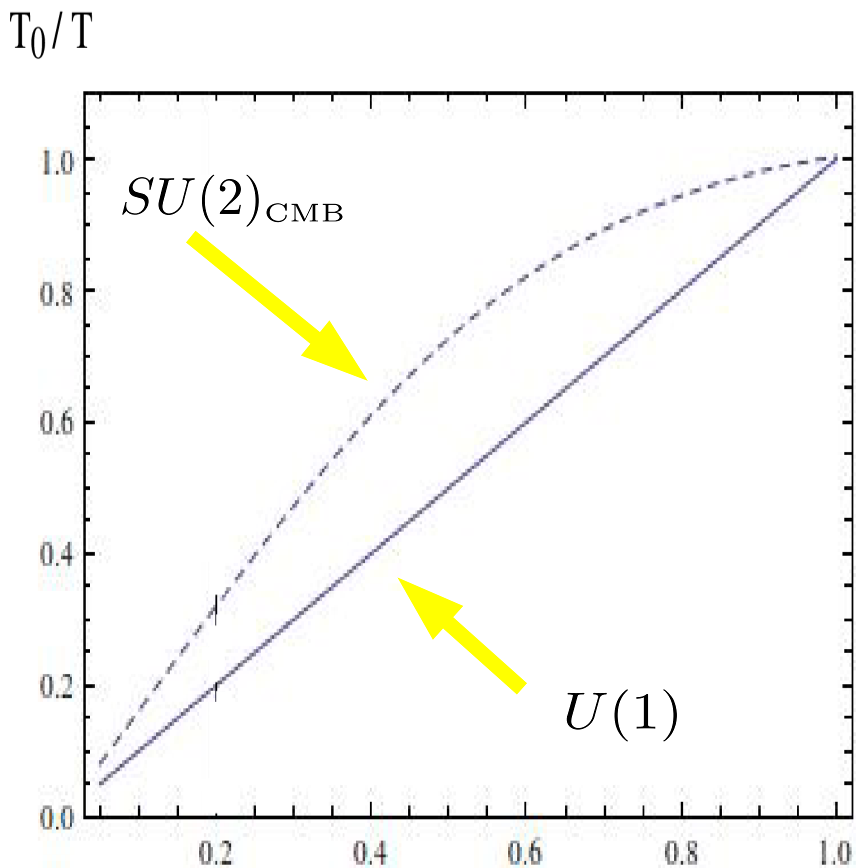
[RH, 2014]

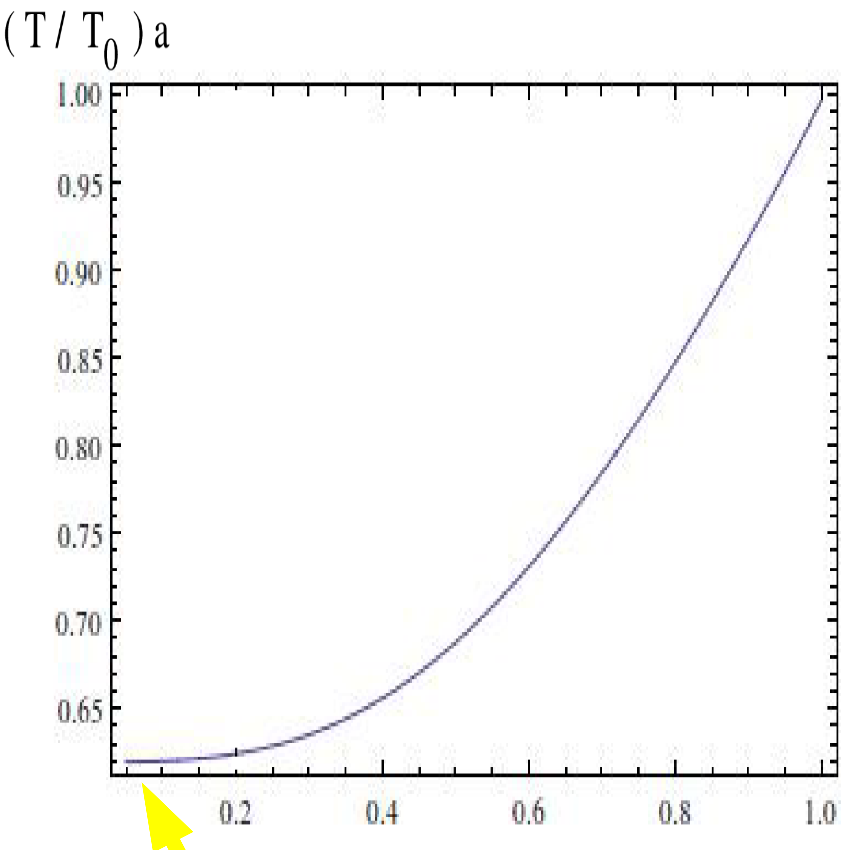
- energy conservation in a Friedmann-Lemaitre-Robertson-Walker Universe: (SU(2) fluid does not dominate the expansion from CMB decoupling onwards)

$$\boxed{\frac{d\rho}{da} = -\frac{3}{a}(P+\rho)}$$

- equation of state $P = P(\rho)$ from deconfining SU(2) $\Longrightarrow$ $\rho(a)$
- with $\rho(T) \Longrightarrow T(a) \overset{(a_0=1)}{\Longrightarrow}$

Violation of conformal scale factor – temperature relation in low-z CMB:





photon gas heats up slower with increasing a compression of Universe than in conventional theory

restoration of conformal scaling for $a < \frac{1}{10}$

but: $\frac{\rho}{\rho_\gamma} = 0.591\,, \quad (a < \frac{1}{10})$

Violation of conformal scale factor – temperature relation in low-z CMB: early reionisation

- due to structure formation, ignition of first stars
- ultraviolet and harder spectral components in starlight ionise diffuse hydrogen also in between galaxies
- makes imprint on angular power spectrum of TT, TE correlations in CMB
- also shows up as a spectral suppression effect in quasar light (Gunn-Peterson trough)
- following discrepancy:

$$\boxed{z_{\mathrm{re,CMB}} = 11.3 \pm 1.1} \longleftrightarrow \boxed{z_{\mathrm{re,\ quasar}} = 6.3 \pm 0.3}$$

assumes conformal scaling

[Ade et al., 1303.5076v3 (2013)]

observes **spectral** redshift,
not affected by $SU(2)_{\mathrm{CMB}}$
[Becker et al., Astron. J. (2001)]

Violation of conformal scale factor – temperature relation in low-z CMB:

- $z_{\mathrm{re,\ quasar}} = 6.3 \pm 0.3$ is physical and can be taken at face value

- if converted by conventional scaling $\frac{T}{T_0} = a^{-1}$ into a CMB temperature

  then $T_{\mathrm{re}} \sim 7.3\, T_0$

- converting this into a redshift according to $SU(2)_{\mathrm{CMB}}$ scaling,

  one obtains $\boxed{z_{\mathrm{re}} = 10.77}$

(consistent with $z_{\mathrm{re,CMB}} = 11.3 \pm 1.1$)

$\Rightarrow$ **argument in favour of** $SU(2)_{\mathrm{CMB}}$:

making a **false** scaling assumption to convert a **physical** redshift into a **false** CMB temperature and converting this false CMB temperature **physically** back into a redshift links the discrepant values of z_{re}

Violation of conformal scale factor – temperature relation in low-z CMB: What about $N_{\rm eff}$ (cosmic neutrinos)?

first scenario:

neutrinos are massless and form their own cosmic fluid

- due to 8 instead of 2 relativistic degrees of freedom in $SU(2)_{\rm CMB}$ at e^+e^- annihilation we have

$$\boxed{\frac{T_\nu}{T} = \left(\frac{16}{23}\right)^{1/3}} \quad \text{instead of} \quad \frac{T_\nu}{T} = \left(\frac{4}{11}\right)^{1/3}$$

- however, today we would have

$$N_{\rm eff} = \frac{\frac{7}{8}\,N_\nu(0.62)^4\,\left(\frac{16}{23}\right)^{4/3}}{\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}}$$

and using $N_\nu = 3$ (missing width in Z decay) would predict

$$\boxed{N_{\rm eff} = 1.053}$$

(value lower than 3.36 from Planck, depends strongly on redshift at which extracted!)

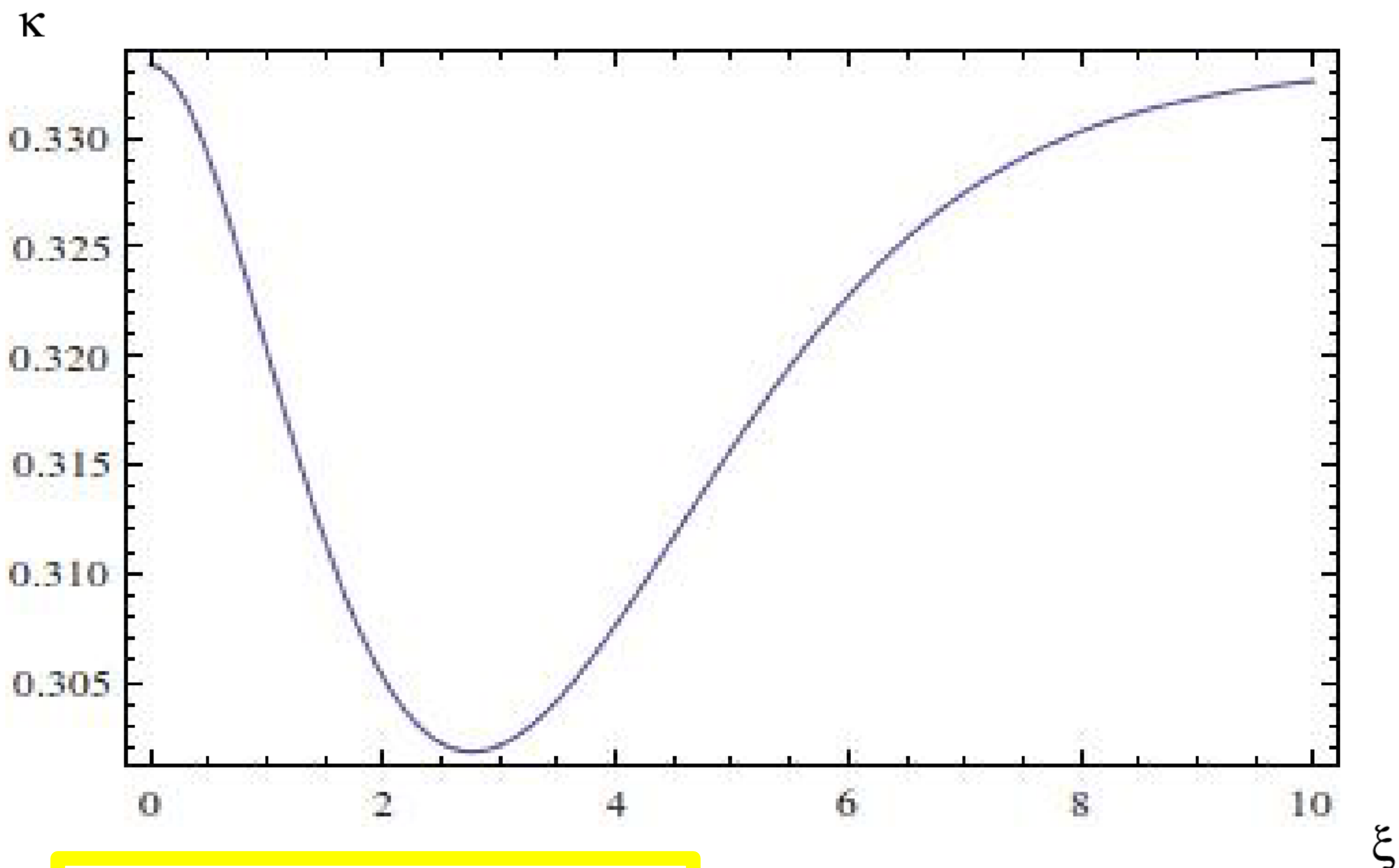
Violation of conformal scale factor – temperature relation in low-z CMB: What about $N_{\rm eff}$ (cosmic neutrinos)?

second scenario:
neutrinos are massive by interactions with the CMB, $T_\nu \equiv T$, and $m_\nu = \xi T\,, \quad (\xi = O(1))$, (neutrino fluid no longer separately conserved)
- we then have (thermodynamically inconsistent):

$$P_\nu = N_\nu T^4 \,\frac{1}{\pi^2}\int_0^\infty dx\, x^2 \log(1+\exp(-\sqrt{x^2+\xi^2})) \equiv N_\nu T^4\,\hat{P}_\nu(\xi)$$

$$\rho_\nu = N_\nu T^4\,\frac{1}{\pi^2}\int_0^\infty dx\,\frac{x^2\sqrt{x^2+\xi^2}}{1+\exp\sqrt{x^2+\xi^2}} \equiv N_\nu T^4\,\hat{\rho}_\nu(\xi)$$

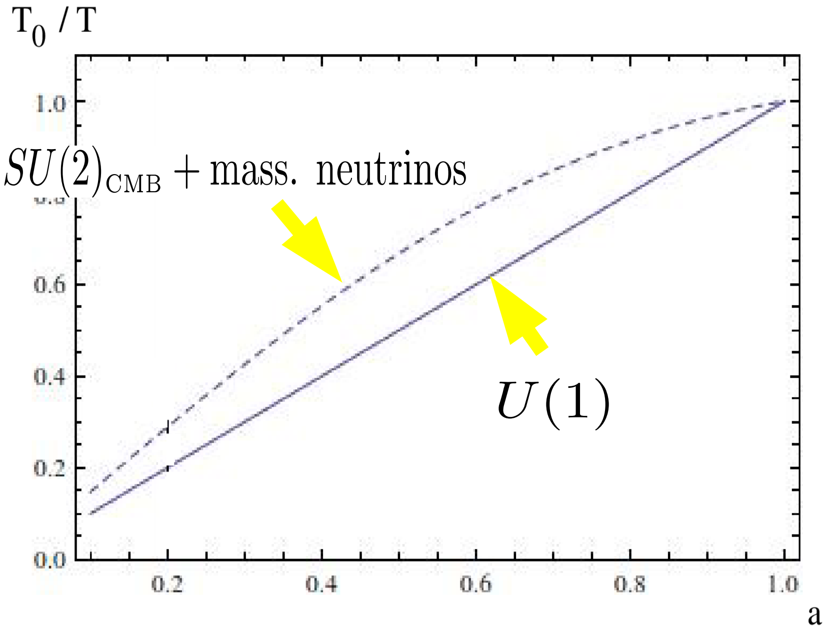
$$\boxed{\kappa = \frac{\frac{1}{3} + \frac{15}{4\pi^2} N_\nu \hat{P}_\nu(\xi)}{1 + \frac{15}{4\pi^2} N_\nu \hat{\rho}_\nu(\xi)}\,, \qquad (T \gg T_0)}$$

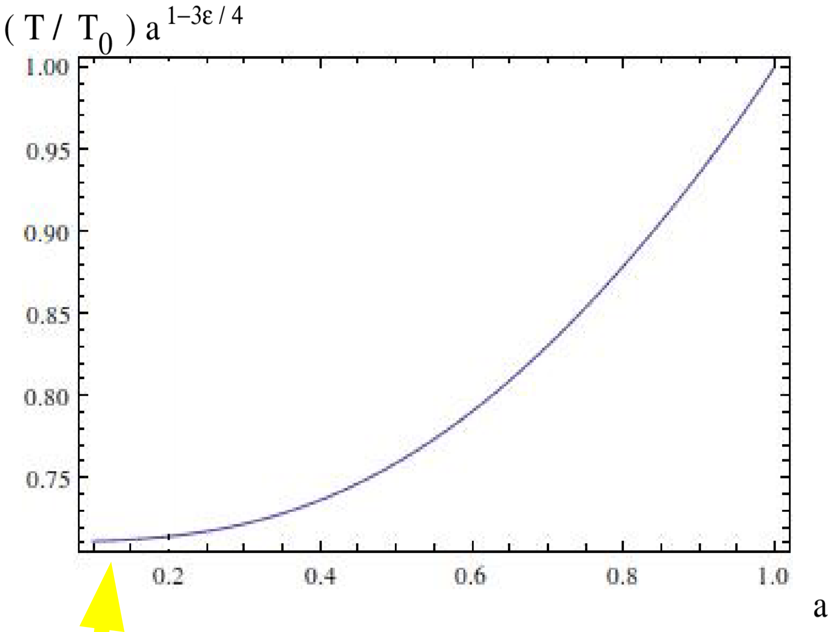


with $\boxed{N_{\rm eff}(\xi) = \frac{N_\nu \hat{\rho}_\nu(\xi)}{\frac{7}{8}\frac{\pi^2}{15}\left(\frac{4}{11}\right)^{4/3}}}$ (today) $\Longrightarrow$ $\boxed{\xi = 3.973}$

(to reproduce $N_{\rm eff} = 3.36$ with $N_\nu = 3$)

with $\boxed{\epsilon \equiv \frac{1}{3} - \kappa \;\Rightarrow T \propto a^{-1+\frac{3}{4}\epsilon}, \quad (T \gg T_0)}$





restoration of quasi-conformal scaling for $a < \frac{1}{10}$

$\Rightarrow \quad z_{\rm re} = 9.75$ (slightly at tension with $z_{\rm re,CMB} = 11.3 \pm 1.1$,
but interaction between neutrino fluid and CMB weaker at low z because lower part of Rayleigh-Jeans regime screened)

redshift of CMB decoupling:

$SU(2)_{\rm CMB}$ only:

$$z_{\rm dec}=\frac{1}{0.62}\,\frac{3000}{2.725}-1=1775$$

$SU(2)_{\rm CMB}+\text{mass. neutrinos}$:

$$z_{\rm dec}=10\left(\frac{3000}{6.8\times 2.725}\right)^{\frac{1}{1-\frac{3}{4}\epsilon}}-1=1793.5$$

But :

To keep ratio of radiation and matter energy densities the same as at $z_{\rm dec}=1089$ (same horizon size at decoupling, rough simplification!), latter would have to be rescaled by

$$\left(\frac{1089}{1775}\right)^3\sim 0.23 \quad\Longrightarrow\quad \Omega_m\sim 0.07$$

(close to baryonic part)

No corpuscular dark matter? Late-time cosmological dark-matter EoS $P_{DM}=0$ from coherently oscillating field whose local profiles explain rotation curves of galaxies ? [Giacosa, RH 2005; Krüger, Neubert, Wetterich 2008]

Summary

- SU(2) thermodynamics nonperturbatively:
  caloron, thermal ground state, adjoint Higgs mechanism, caloron action

- blackbody anomaly:
  thermal photon dispersion relations, critical temperature for dec.-prec. PT from low-frequency spectral anomaly (Arcade2, terrestial radio-frequency CMB observations)

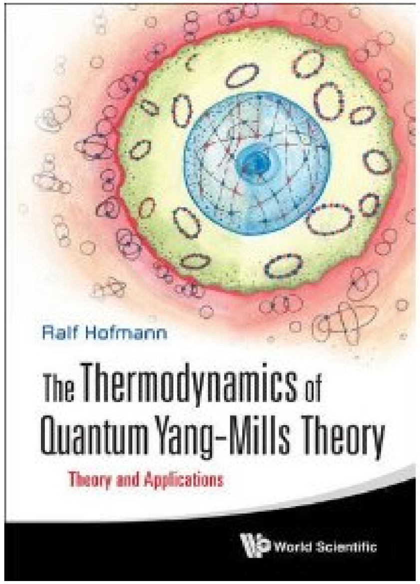
- CMB large-angle anomalies (WMAP, Planck):
  Yang-Mills favours **negative temperature fluctuations**, violation of conformal scaling, cosmic neutrinos and implications

Thank you.

more details:

Theory:

J. Ludescher, RH, Annalen der Physik (2009);
U. Herbst, RH, ISRN HEP (2012);
RH, D. Kaviani, Quant. Matt. (2012);
J. Moosmann, RH, ISRN MathPhys (2012);
N. Krasowski, RH, Ann. Phys. (2014);
C. Falquez, RH, T. Baumbach, Quant. Matt. (2012)



Cosmology:

F. Giacosa and RH, Eur. Phys. J. C (2005);
F. Giacosa, RH, M. Neubert, JHEP (2008);
M. Szopa, RH, JCAP (2008);
RH, Annalen d. Physik (2009);
RH, Nature Physics (2013);
RH, Annalen d. Physik (2015);