SU(2) Yang-Mills thermodynamics:
Calorons, the deconfining thermal ground state, and its permittivity/permeability

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$$S_\beta = \mathrm{tr}\,\frac{1}{2}\int_0^\beta d\tau \int d^3x\, F_{\mu\nu}F_{\mu\nu}$$

motivation

- Andrei Linde (1980):
  „Infrared Problem in the Thermodynamics of the Yang-Mills Gas“

  - soft magnetic sector screened weakly in perturbation theory (infrared instability), *actually:* foliation of momentum regimes by virtue of powers of a small coupling is a useless concept, see talk by T. Grandou tomorrow

  - no „convergence“ of series since kinetic and interaction energies comparable in this sector

- issue of finite-volume constraints in lattice gauge theory

  - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

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nonperturbative Yang-Mills thermodynamics: SU(2)

[Herbst & RH (2004), RH (2005-2007), Giacosa & RH (2006), Schwarz, Giacosa & RH (2007), Ludescher & RH (2008), Falquez, Baumbach & RH (2010- 2011), RH (2012), Krasowski & RH (2013), Grandou & RH (2015), RH (2016)]

thermal ground state at high temperature:

- **Euclidean action:**

$$S = \frac{\mathrm{tr}}{2}\int_0^\beta d\tau \int d^3x\, F_{\mu\nu}F_{\mu\nu}\,,$$

($\beta \equiv 1/T$)

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ [Schafer et Shuryak (1996)]

- **(anti)selfdual gauge fields: [(anti)calorons]**

$$F_{\mu\nu}[A] = \pm\tilde{F}_{\mu\nu}[A] \;\Rightarrow\; \theta_{\mu\nu}[A] \equiv 0\,.$$

(A_μ periodic)

field configs. stabilized by gauge-field winding: $\partial\mathbf{R}^4 = S_3 \to SU(2) = S_3$

- **in particular:** (anti)calorons of winding number **unity**

Calorons of top. charge unity (selfdual field configs. on $S_1 \times \mathbf{R}_3$): (singular gauge)

['t Hooft, Rebbi & Jackiw (1977)]

Harrington-Shepard (1977): (trivial holonomy)

$$\boxed{A_\mu = \bar{\eta}^a_{\mu\nu} t_a \partial_\nu \log \Pi(\tau, r)}$$

with $$\Pi = \begin{cases} \left(1 + \frac{1}{3}\frac{s}{\beta}\right) + \frac{\rho^2}{x^2} & (|x| \ll \beta) \\ 1 + \frac{s}{r} & (r \gg \beta) \end{cases}$$

and $$\boxed{s \equiv \frac{\pi\rho^2}{\beta}\,, \quad \beta \equiv \frac{1}{T}\,.}$$

[Gross, Pisarski & Yaffe (1981)]

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$\Rightarrow$ $F_{\mu\nu}$ that of singular-gauge instanton with $\rho'^2 = \frac{\rho^2}{1 + \frac{1}{3}\frac{s}{\beta}}$ $(|x| \ll \beta)$

(action: $S_c = \frac{8\pi^2}{g^2} \int_{S_3^\delta} d\Sigma_\mu \, K_\mu = \frac{8\pi^2}{g^2}$ localised about instanton center in $S_1 \times \mathbf{R}_3$)

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$$E^a_i = B^a_i = s\,\frac{\delta^a_i - 3\,\hat{x}^a \hat{x}^i}{r^3} \qquad (r \gg s)\,.$$

(static selfdual dipole-field with dipole moment: $p^a_i = s\,\delta^a_i$)

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Nahm (1983), Lee-Lu-Kraan-van-Baal (1998):
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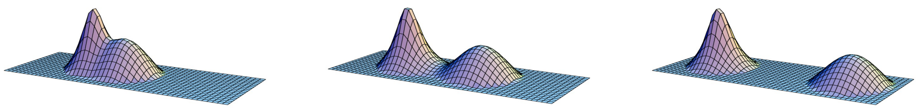
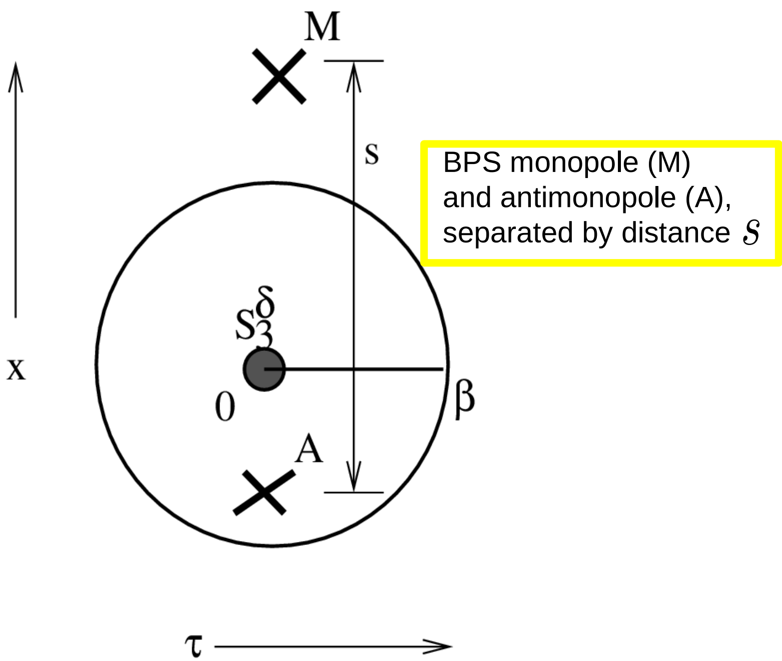
$$\boxed{m_M = 4\pi u\,, m_A = 4\pi\left(\frac{2\pi}{\beta} - u\right)}$$



(action density on spatial slice)



(M-A separation, caloron center)

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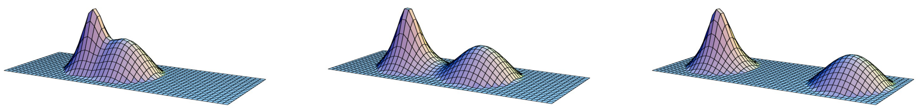

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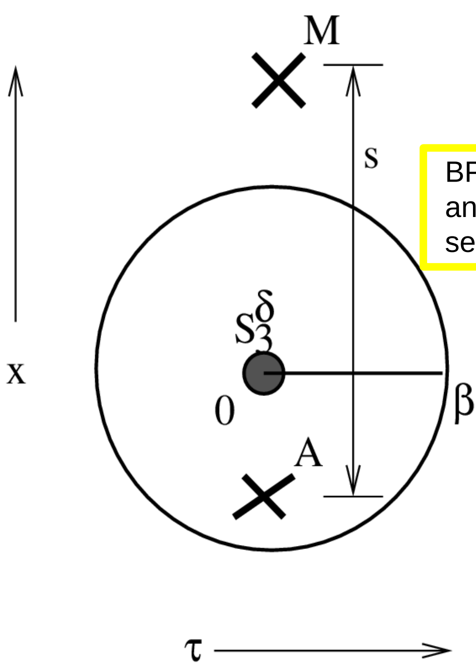

BPS monopole (M)
and antimonopole (A),
separated by distance s

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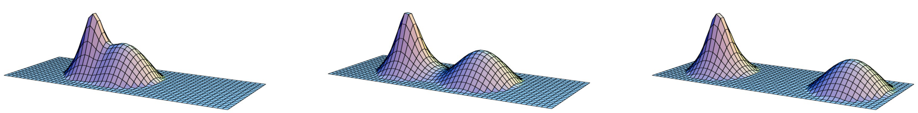

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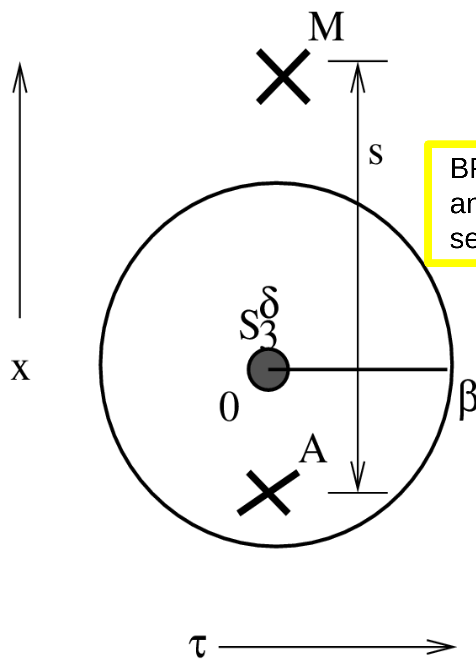

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(small holonomy u, likely)

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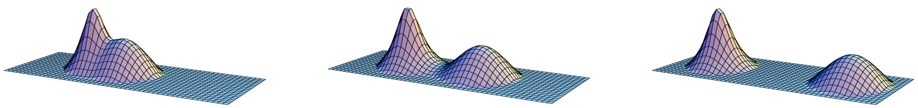

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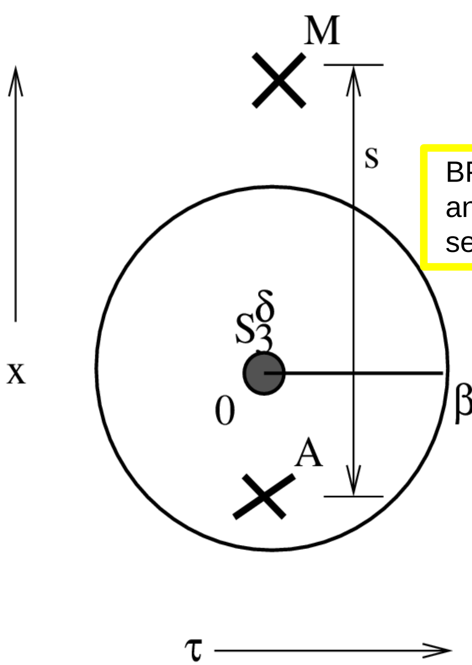

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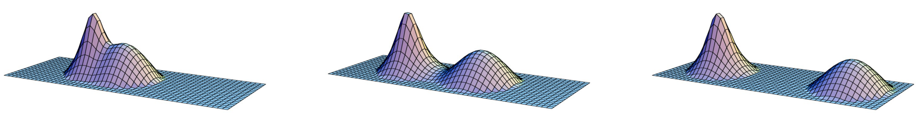

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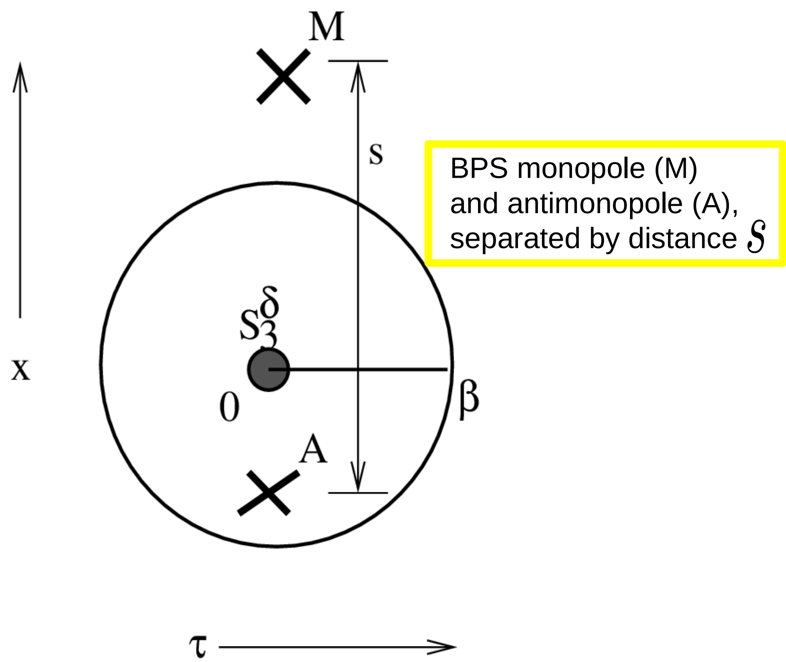

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- locus of action within S_3^δ $(\delta \to 0)$

- trivial-holonomy limit:
  M massless, A still massive, stable

spatial coarse-graining over pair of trivial-hol. (anti-)calorons: inert, adjoint scalar field ϕ

[Herbst & RH (2004)]

$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0}) \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} F_{\mu\nu}(\tau,\vec{x}) \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, dimensionless definition of **family of phases**, where

$$\left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} \equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu\, A_\mu(z)\right]$$ and

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-$\{\hat{\phi}^a\}$ sharply dominated by cut-off for ρ integration

spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field ϕ

- no explicit β dependence in ϕ field dynamics (caloron action!)

- absorb β dependence of operator D into potential V

(BPS and EL yield: $\frac{\partial V(|\phi|^2)}{\partial |\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2} \Rightarrow$

$$\boxed{V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}}$$

(Yang-Mills scale constant of integr.)

and

$$\boxed{|\phi| = \sqrt{\frac{\Lambda^3 \beta}{2\pi}}}$$

(Euclidean time dependence of HS (anti)caloron centers coarse-grains into a time dependence of ϕ which can be made trivial by singular but admissible gauge trafo.)

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- BPS equation:

$$\partial_\tau \phi = \pm 2i\,\Lambda^3\,t_3\,\phi^{-1} = \pm i\,V^{1/2}(\phi)$$

$\Rightarrow$ no **additive** ambiguity in V !

effective action (deconfining phase), thermal ground state

- $$\mathcal{L}_{\rm eff}[a_\mu] = {\rm tr}\left(\frac{1}{2}\,G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

  (i) perturbative renormalizability (G^2 highest power in effect. action,
  propagating part of a_μ adiabatic excitation of thermal ground state)
  (iil) ϕ 's inertness – no higher dim., mixed operators to mediate
  4-momentum transfer between ϕ and a_μ
  (iii) gauge invariance

  [see also RH (2016)]

- effective YM equation $D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]$ has ground-state solution:

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$$a_\mu^{\rm gs} = \mp\delta_{\mu4}\frac{2\pi}{e\beta}t_3 \qquad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0)$$

(centers of HS (anti)calorons packed densely, static peripheries overlap to form $a_\mu^{\rm gs}$)

$$\Longrightarrow \quad P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T\,.$$

interacting small and transient-holonomy (anti)calorons, (collapsing monopole-antimonopole pairs)

(vanishing entropy density of ground state!)

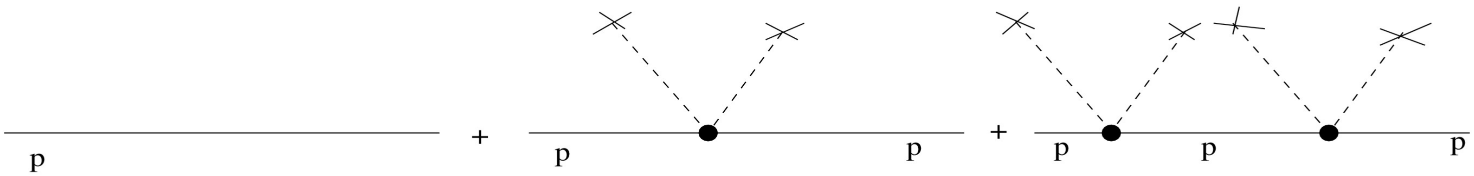
adjoint Higgs mechanism (deconfining phase)

(SU(2) → U(1))

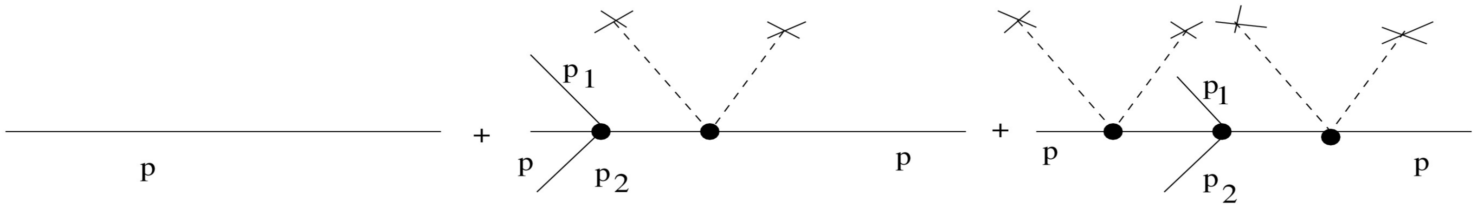
- from effective action:

$$\boxed{m_a^2 = -2e^2 \mathrm{tr}\,[\phi, t_a][\phi, t_a]} \xrightarrow{\text{unitary gauge}} \boxed{m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}\,, \; m_3 = 0}$$

- no momentum transfer to ϕ, **but this infinitely often** (Dyson series for mass generation):



(a)

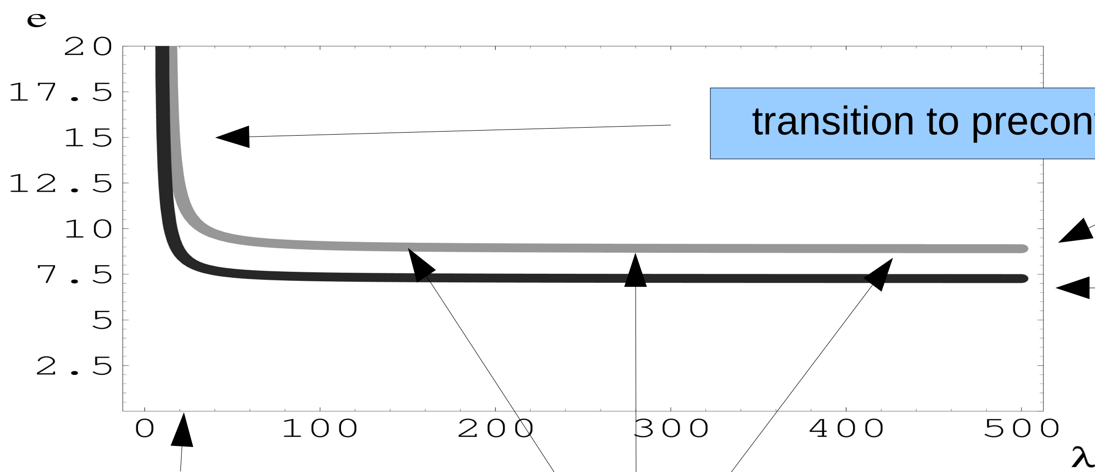


(b)

- no off-shell propagation of massive modes (otherwise: momentum transfer to ϕ !)

effective gauge coupling

e ← thermodynamical consistency



transition to preconfining phase

SU(2)

SU(3)

small-mass expansion, pressure
[Dolan et Jackiw (1974)]

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

effective gauge coupling

e

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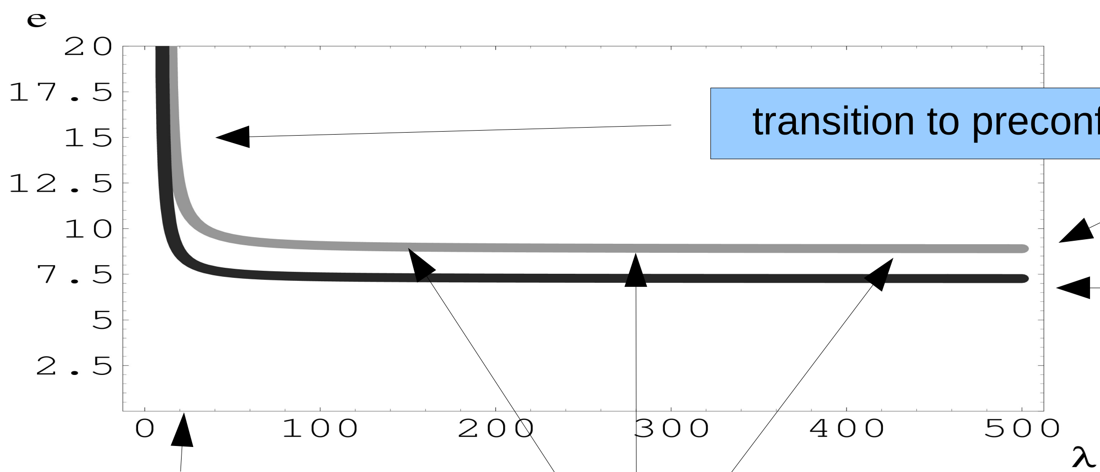

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coarse-graining dominated by $\rho \sim |\phi|^{-1}$

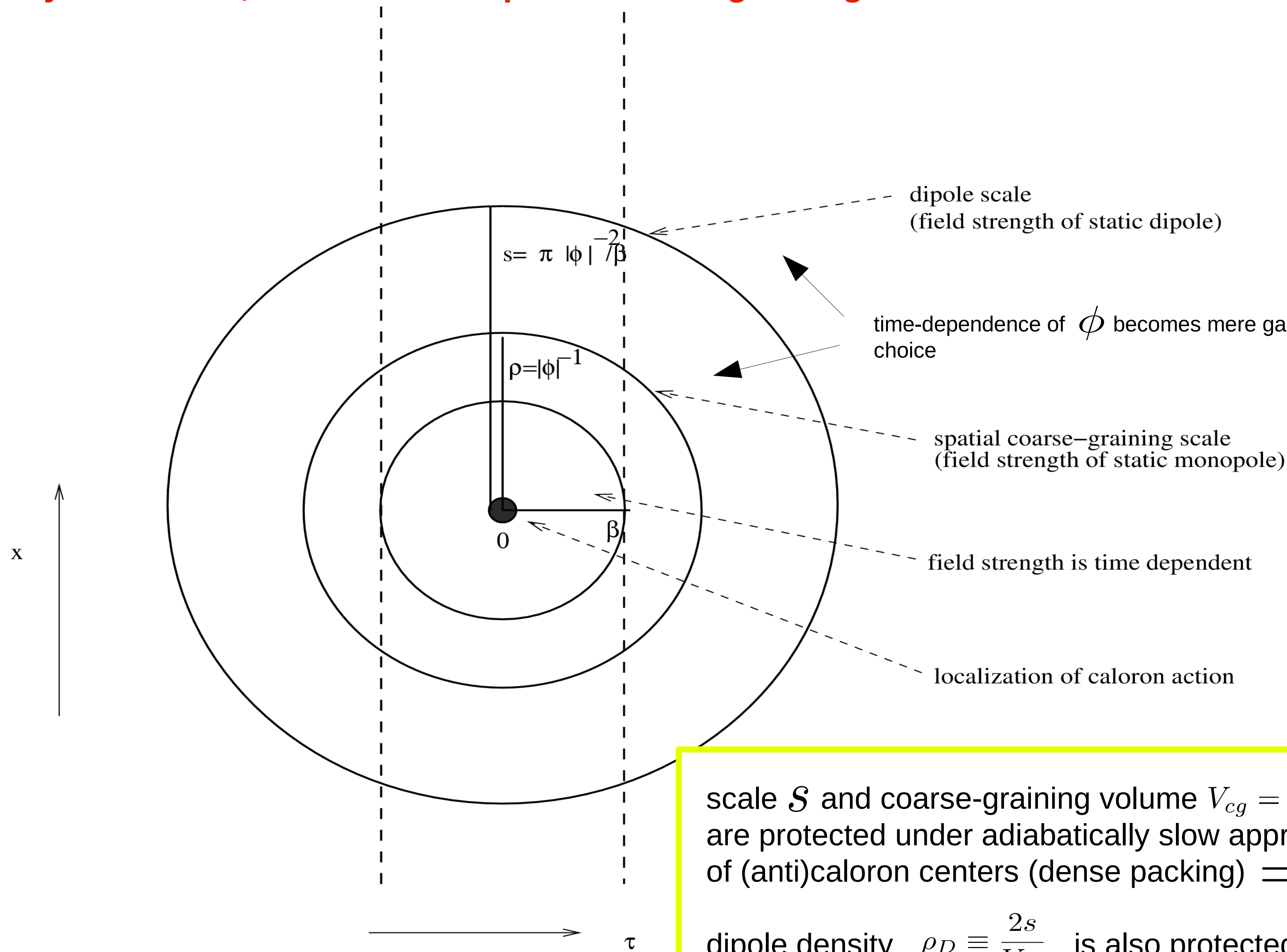
- restore $\hbar$

$$e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}\,.$$

$$S_{C/A} = \hbar\,.$$

[Brodsky et al. (2011); Kaviani & RH (2012), RH (2012,2013)]

anatomy of caloron, inferred after spatial coarse-graining:



scale S and coarse-graining volume $V_{cg} = \frac{4}{3}\pi|\phi|^{-3}$
are protected under adiabatically slow approacn
of (anti)caloron centers (dense packing) $\Longrightarrow$

dipole density $\rho_D \equiv \frac{2s}{V_{cg}}$ is also protected

summary: induced, effective thermal QFT; convergence of loop expansions

defining Yang-Mills action: classical, Euclidean gauge-field theory on $S_1 \times \mathbf{R}_3$

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small-holonomy (anti)calorons of action $\hbar$ **constitute** effective **thermal ground state, mediate interactions** (vertices) between effectively propagating modes (BE distributed QF – massive; low-frequency waves, high-frequency BE distr. QF - massless) [Kaviani & RH (2012), Krasowski & RH (2013), Grandou & RH (2015), RH (2016)]

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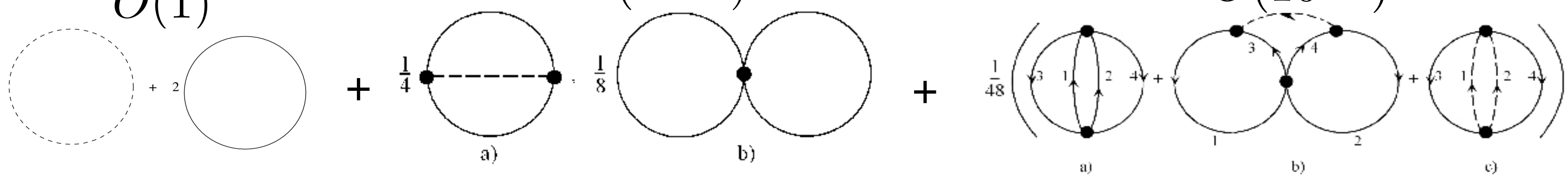
kinematic constraints in (totally fixed) unitary-Coulomb gauge imply that radiative corrections are extremely **well controlled**
[Schwarz, Giacosa, & RH (2006), Ludescher & RH (2008)]

expansion of thermodyn. quantities into **1PI loops** probably **terminates** at finite order, say, pressure
[RH (2006)]

$O(1)$ $\quad O(10^{-2})$ $\quad O(10^{-7})$



real-world implications

electric-magnetically dual interpretation of U(1) charge:

if SU(2) something to do with photons [RH (2005), Grandou & RH (2015), etc]
then **electric-magnetically dual** interpretation required:
in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\boxed{\alpha = \frac{Q^2}{4\pi\hbar}\,,}$$

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for α to be unitless:

$$\Big(e = \frac{\sqrt{8\pi}}{\sqrt{\hbar}}\,. \Big) \qquad \boxed{Q \propto \frac{1}{e}\,.}$$

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But: magnetic coupling in SU(2)

$$\boxed{g = \frac{4\pi}{e}\,.}$$

$\Rightarrow$ SU(2) to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole, etc.)

electric/magnetic dipole density (permittivity/permeability of vacuum):

[temperature a fictitious quantity]

$$\boxed{|\mathbf{D}_e| = \frac{2s}{V_{\rm cg}} \propto T^{1/2}}$$

external electric field strength (plane wave):

$$\boxed{\rho_{\rm gs} = 4\pi T\Lambda^3 = \rho_{\rm EM} = \epsilon_0 \mathbf{E}_e^2 \Rightarrow |\mathbf{E}_e| \propto T^{1/2}}$$

$$\Longrightarrow \boxed{\epsilon_0 \equiv \frac{|\mathbf{D}_e|}{|\mathbf{E}_e|} \neq f(T)}$$

[Grandou & RH (2015)]

[also applies to Rayleigh-Jeans regime in thermal situation; subject to frequency dependent charge screening, however [RH (2016)]]

similarily for magnetic permeability μ_0 .

electric/magnetic dipole density (permittivity/permeability of vacuum):

moreover: imposing the condition that λ larger than dipole scale s $\Rightarrow$

$$\boxed{\mathbf{E}_e^4 \nu \ll 8\Lambda^9} \qquad \text{(UR)}$$

[Grandou & RH (2015)]

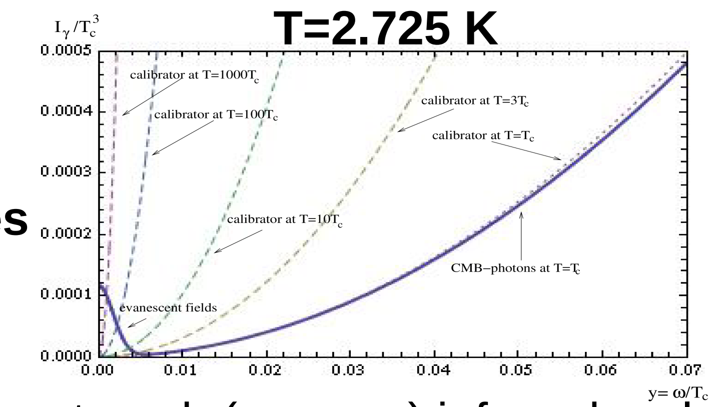
with $\Lambda_{\mathrm{CMB}} \sim 10^{-4}\,\mathrm{eV}$ (thermal photon gases, later) (UR) is violated for almost all experimentally/observationally surveyed (super radio frequency) phenomena linked to e-m wave propagation. At $T_0 = 2.73$ K we'd need $\nu < 1.7$ GHz for wave propagation, see later.

Way out: Postulate that product $\mathrm{SU(2)_{CMB}} \times \mathrm{SU(2)}_e$ with $\Lambda_e \sim 0.5\,\mathrm{MeV}$, is responsible for photon and wave propagation beyond thermalization [RH (2015)], rotation into second factor for radiation far from thermal equlibrium allows for observed hard X-ray and γ-range spectra

$\longrightarrow$ description of thermal and nonthermal radiation with a **variable Weinberg angle** for mixing of the two U(1) subalgebras, dependent on degree of thermalisation

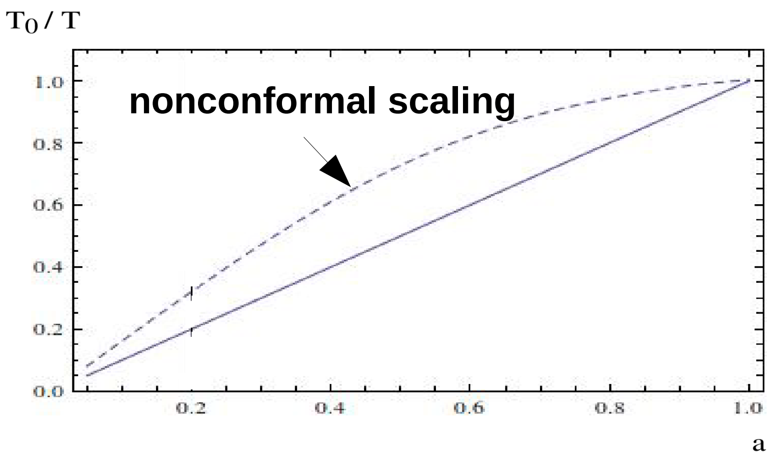
evidences for $SU(2)_{\rm CMB}$($\Lambda_{\rm CMB} \sim 10^{-4}\,{\rm eV}$): photon at tree level

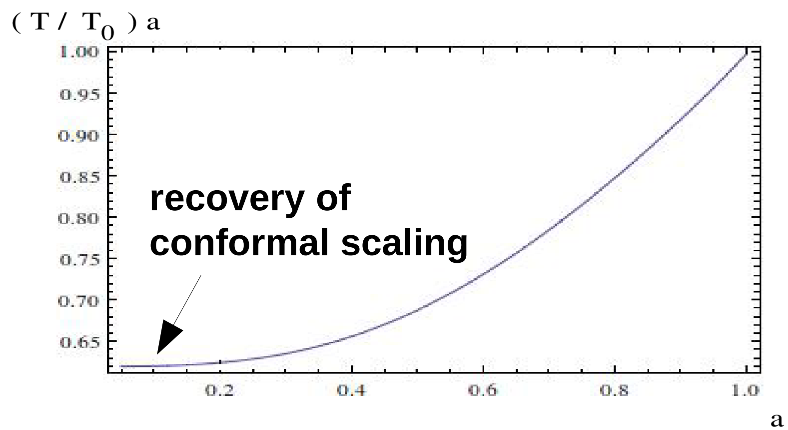
- cosmic radio background (UEGE), onset of Meissner effect $\longrightarrow$ **evanescent modes at low frequencies** [terrestial observ. (1981-1999), Arcade 2 (2009), RH (2009)]



- CMB angular spectrum vs. Gunn-Peterson trough (quasars) inferred early re-ionisation of intergalactic medium (z=11 vs. z=6 discrepancy), **non-conformal** $T-a$ **relation at late times**

[Becker et al. (2001), WMAP coll. (2004), Planck coll. (2013)]

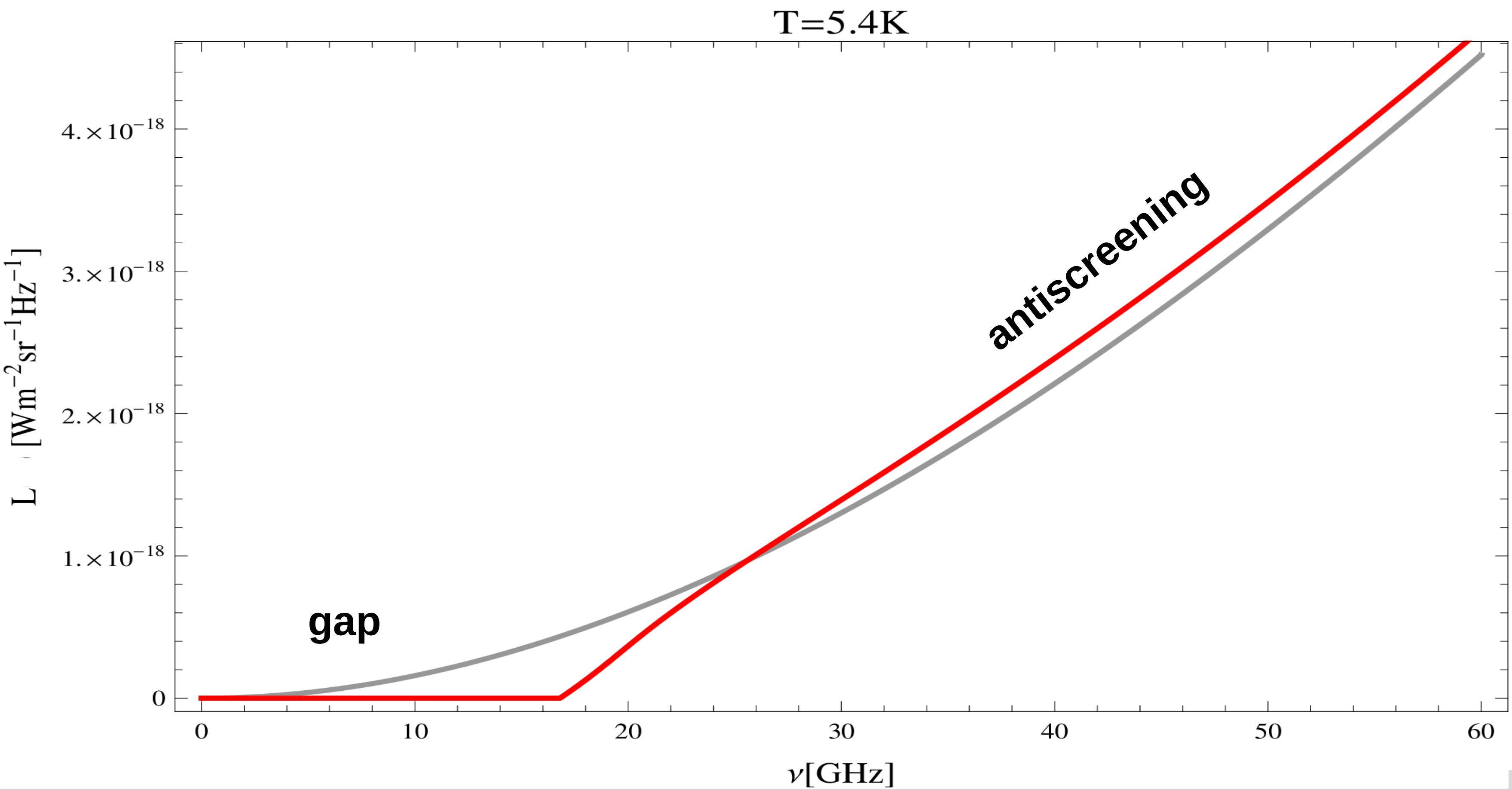




[RH (2014)]

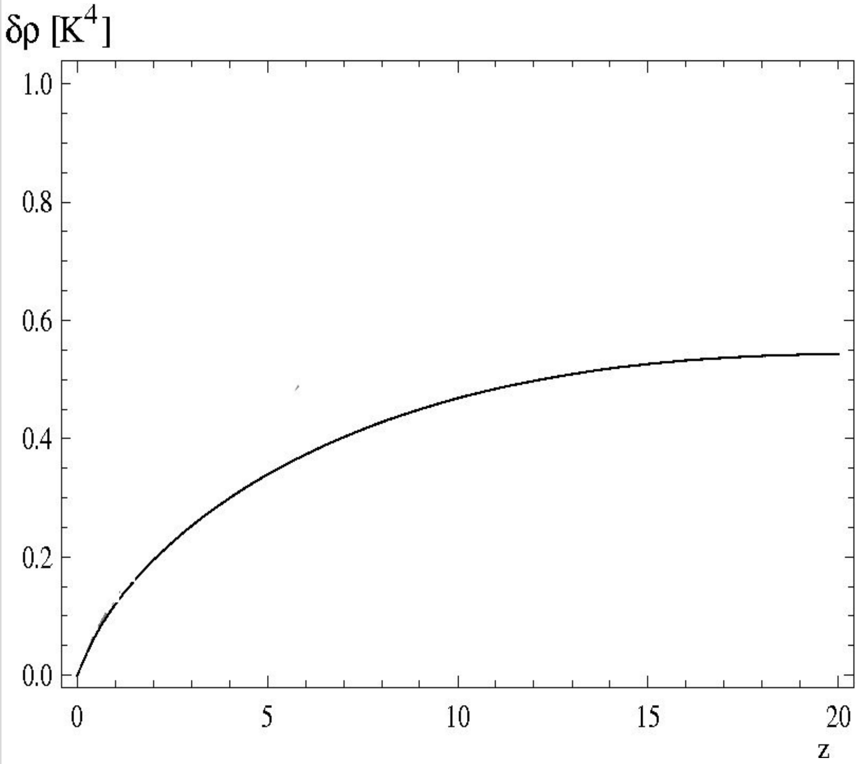
evidences for $SU(2)_{\mathrm{CMB}}$($\Lambda_{\mathrm{CMB}} \sim 10^{-4}\,\mathrm{eV}$): one-loop polarization

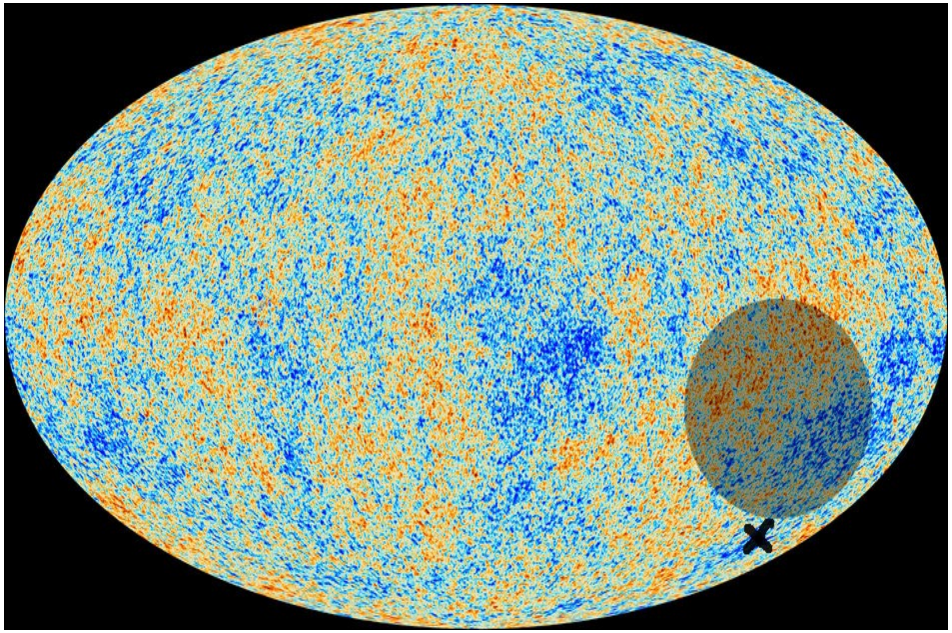
- **spectral blackbody anomaly:** max. gap in Rayleigh-Jeans reg. at $T \sim 5\,\mathrm{K}$, massless mode – transverse polarizations
  [Schwarz, Giacosa & RH (2006), Ludescher & RH (2008), Falquez, RH & Baumbach (2010,2011)]



evidences for $SU(2)_{\rm CMB}$($\Lambda_{\rm CMB} \sim 10^{-4}\,{\rm eV}$): one-loop polarization

- **integral blackbody anomaly:**
  difference $\delta\rho$ between energy density of SU(2) and U(1), massless mode – transverse polarizations



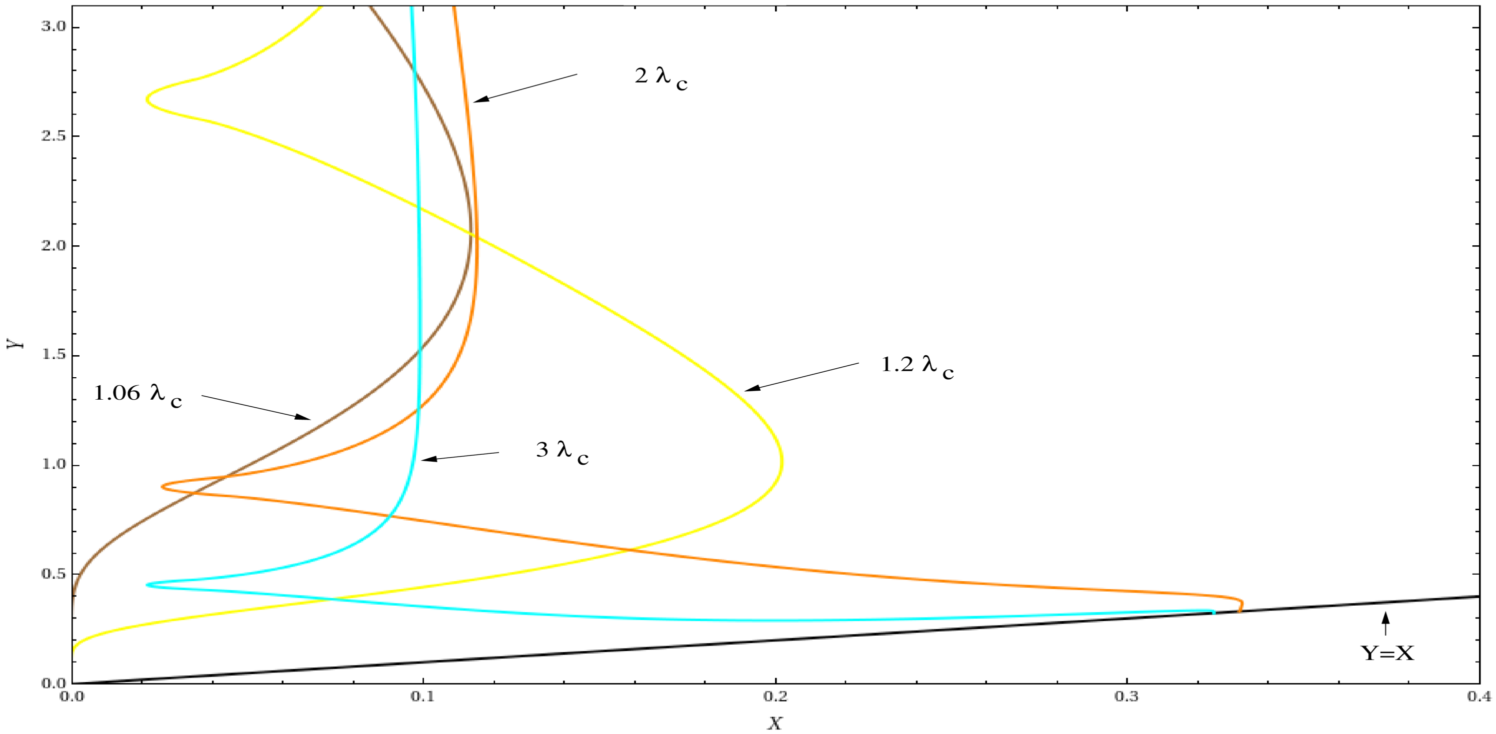


(**positive** slope of $\delta\rho$ bias for **negative** temperature fluctuations in late-time CMB)

[Szopa et al 2007, RH **Nature Physics** (2013)]

evidences for $SU(2)_{\rm CMB}$ ($\Lambda_{\rm CMB} \sim 10^{-4}\,{\rm eV}$): one-loop polarization

- low-momentum support of magnetic branches (dual interpretation)
  massless mode – longitudinal polarization



$\overset{?}{\Longrightarrow}$ intergalactic magnetic fields [Falquez et al. (2011)]

summary

- alternative to high-T perturbation theory:
  caloron induced dynamical gauge SB by thermal ground state

- effective thermal quantum field theory for deconfining phase of SU(2) YM

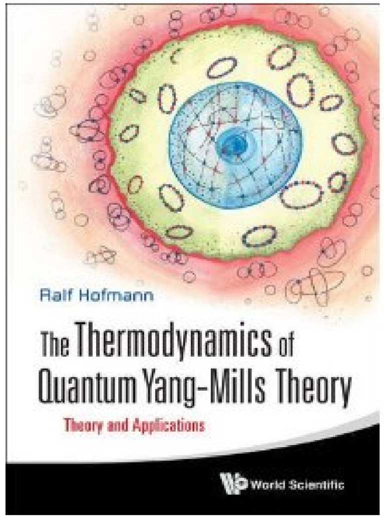
- effective coupling evolution: - caloron action $\hbar$,
  - caloron mediation of effective vertices,
  - e-m dual interpretation

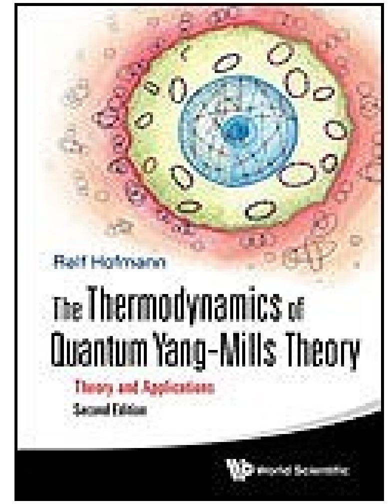
- effective radiative corrections: extremely well controlled

- SU(2) photons: tree-level and one-loop polarization anomalies
  → CMB anomalies
  - cosmic radio background
  - quasar vs CMB wrt reionization,
  - spectral & integral BB anomalies
    (CMB at large angles)

  → extragalactic magnetic fields

Theory: 1st ed. World Scientific, 2011 — 2nd ed. World Scientific, June 2016





Cosmological applications (CMB photons, dark energy, universe's eos):

F. Giacosa and RH, Eur. Phys. J. C (2005);
F. Giacosa, RH, M. Neubert, JHEP (2008);
M. Szopa, RH, JCAP (2008);
RH, Annalen d. Physik (2009);
RH, Nature Physics (2013);
RH, Annalen d. Physik (2015);
T. Grandou & RH, Adv. Math. Phys. (2015);
RH, Entropy (2016)

Thank you !