High-loop order radiative corrections to the pressure in deconfining SU(2) Yang-Mills thermodynamics

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Motivation

- Perturbation theory problematic at finite T:
  - Infrared problem, in particular non-perturbative corrections from soft magnetic sector [Linde (1980)],
  - Impossibility to account for non-trivial topology
  - Questionable assumptions about free vacuum and emerging parameters
- Way out: Thermal ground state estimate from spatial coarse-graining over stable, topologically non-trivial Euclidean field configurations: Harrington-Shepard calorons [HS (1978)]
- Solves conceptual and technical problems:
  - Radiative corrections interpreted as small propagating disturbances of this ground state
  - Complete, physical gauge-fixing is possible and an ensuing physical quasi-particle mass avoids IR divergences (adjoint Higgs mechanism)
  - Natural UV cutoff abolishes the need for renormalisation

Outline

Thermal ground state estimate at high temperature

Euclidean effective action [Herbst, RH (2004)]

$$S_{\text{eff}}[a_\mu] = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3x \,\mathrm{tr}\, \left(\frac{1}{2} G_{\mu\nu} G_{\mu\nu} + (D_\mu \phi)^2 + \frac{\Lambda^6}{\phi^2} \right)$$

- $D_\mu \phi \equiv \partial_\mu \phi + ie[a_\mu, \phi]$
- $G_{\mu\nu}$: Field-strength of the coarse-grained, propagating trivial topology gauge fields a_μ
- ϕ: Inert, adjoint scalar field from spatial coarse-graining over topologically non-trivial field configurations
- Λ: Arbitrary integration constant of dimension mass (Yang-Mills scale)
- e: effective gauge coupling

Thermal ground state estimate at high temperature

- Time dependence of fundamental field configurations reduced by coarse-graining to phase of ϕ
- Natural gauge invariant resolution scale $|\phi|$
- Adjoint Higgs mechanism breaks SU(2)$\rightarrow$U(1) and assigns masses to the broken algebra directions
- Gauge fixing exhibits physical quasi-particle spectrum: Two massive (three polarisations) and one massless (two polarisations) gauge modes

Global gauge choice

$$\phi^a = \delta^{a3}|\phi|, \quad \partial_i a_i^3 = 0$$

In this gauge: $m_1^2 = m_2^2 = 4e^2|\phi|^2, \quad m_3 = 0$

Thermal ground state estimate at high temperature

Tree-level propagators

One can Wick rotate to Minkowski space and make use of the separation of propagators into vacuum and thermal parts:

Photon propagator

$$D^{\text{photon}}_{\mu\nu,ab}(p) = -\delta_{a3}\delta_{b3}\left(P^T_{\mu\nu}\left[\underbrace{\frac{i}{p^2+i0}}_{|p^2|\leq|\phi|^2} + \underbrace{2\pi\delta(p^2)n_B(|p_0|/T)}_{p^2\equiv 0}\right] - i\frac{u_\mu u_\nu}{\vec{p}^2}\right),$$

$$P^{00}_T = P^{0i}_T = P^{i0} = 0, \quad P^{ij}_T = \delta^{ij} - \frac{p^i p^j}{\mathbf{p}^2}, \quad i,j = 1,2,3$$

$$u = (1,0,0,0)$$

Massive gauge boson propagator

$$D^{\text{massive}}_{\mu\nu,ab}(p) = -\delta_{ah}\delta_{bh}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right)\underbrace{2\pi\delta(p^2-m^2)n_B(|p_0|/T)}_{\text{thermal only}}, \; h = 1,2$$

[RH (2016)]

Thermal ground state estimate at high temperature

Pressure and energy density (1-loop)

- Pressure and energy density (non-interacting) [RH (2016); Dolan, Jackiw (1974)]:

$$P(\lambda) = -\Lambda^4 \left(\frac{2\lambda^4}{(2\pi)^6} \left[2\bar{P}(0) + 6\bar{P}(2a) \right] + 2\lambda \right),$$

$$\rho(\lambda) = \Lambda^4 \left(\frac{2\lambda^4}{(2\pi)^6} \left[2\bar{\rho}(0) + 6\bar{\rho}(2a) \right] + 2\lambda \right),$$

where $\lambda \equiv 2\pi T/\Lambda$, $a \equiv m/(2T)$,
$\bar{P}(y) = \int_0^\infty \mathrm{d}x\, x^2 \log\left[1 - e^{-\sqrt{x^2+y^2}}\right]$,
$\bar{\rho}(y) = \int_0^\infty \mathrm{d}x\, x^2 \frac{\sqrt{x^2+y^2}}{\exp\left(\sqrt{x^2+y^2}\right)-1}$.

Thermal ground state estimate at high temperature

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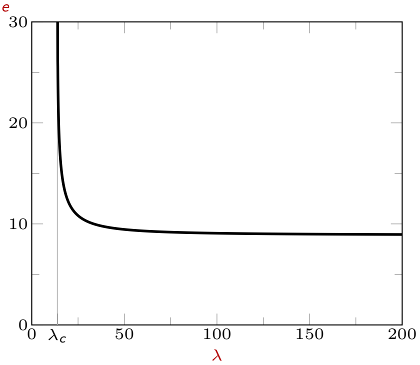
- Demand thermodynamical self-consistency at 1-loop level

$$\rho = T\frac{\mathrm{d}P}{\mathrm{d}T} - P$$

(Legendre transform) to obtain differential eq. for $e(\lambda)$

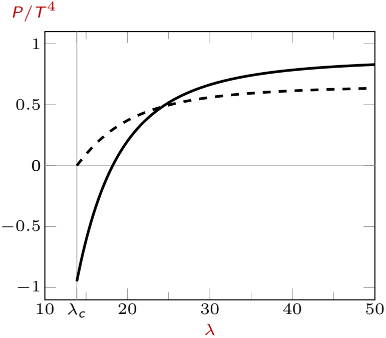
Thermal ground state estimate at high temperature

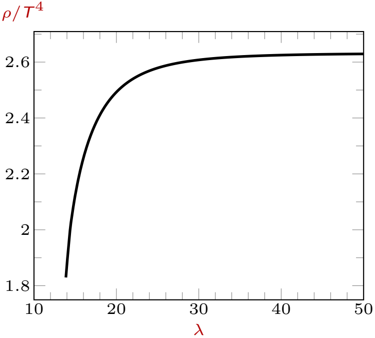
- $e \geq \sqrt{8}\pi$ in the deconfining phase
- Action of just-not-resolved calorons at $e = \sqrt{8}\pi$: $S = \hbar$
- $\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$ logarithmic pole of e



Thermal ground state estimate at high temperature

Pressure and energy density at one-loop level

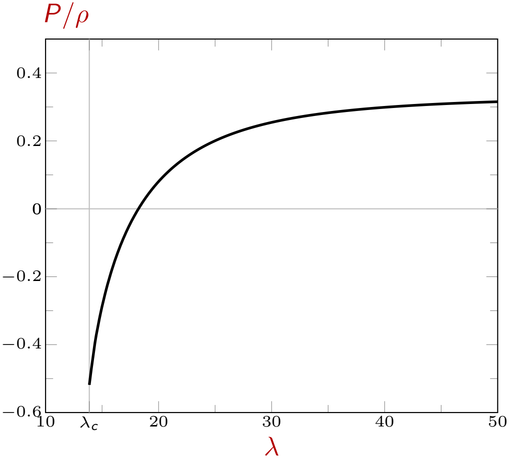




Dashed: only massive fields, ground state subtracted

Thermal ground state estimate at high temperature

Equation of state at one-loop level



Previous evidence for smallness of corrections

- $e \geq \sqrt{8}\pi$ in the deconfining phase $\rightarrow$ **large coupling**
- What controlls the loop expansion of radiative corrections?

Constraints [RH (2016)]

- Massive fields propagate strictly thermally (on-shell), $p^2 = m^2$ due to inertness of ϕ
- Massless field's deviation from on-shell condition restricted by $|p^2| \leq |\phi|^2$ (fixes three-vertex resolution)
- Four-vertex resolution constrained by $|\phi|$
  $\Rightarrow\ \Gamma_{[4]} = \frac{1}{3}\left(\Gamma_{[4]}|_s + \Gamma_{[4]}|_t + \Gamma_{[4]}|_u\right)$,
  where one demands
  s-channel: $|(p_1 + p_2)^2| \leq |\phi|^2$
  t-channel: $|(p_3 - p_1)^2| \leq |\phi|^2$
  u-channel: $|(p_2 - p_3)^2| \leq |\phi|^2$

Previous evidence for smallness of corrections

- 2-loop corrections of order 10^{-2} compared to 1-loop
- All-loop order resummation of propagators must be performed
- Ratio of independent loop momenta $\tilde{K}$ and vertex constraints K for bubble diagrams of genus g given by

$$\frac{\tilde{K}}{K} \leq \frac{4}{5}\left(1+\frac{1}{V_4}(1-2g)\right), \quad V_4\text{: number of 4-vertices}$$
$$\frac{\tilde{K}}{K} \leq \frac{2}{3}\left(1+\frac{2}{V_3}(1-2g)\right), \quad V_3\text{: number of 3-vertices}$$

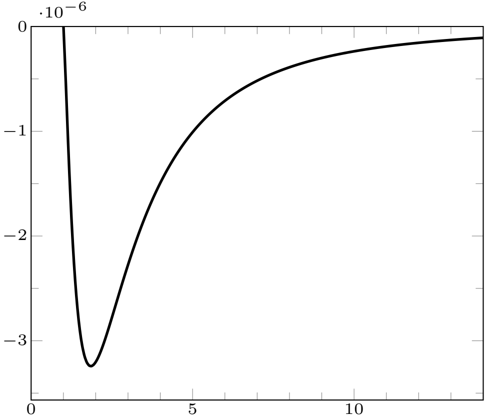
- Conjecture: termination at finite 2PI loop order, when intersection of the compact regions defined by the constraints becomes empty
- 2PI here refers to bubble diagrams that become 1PI contributions to the polarisation tensor upon cutting any single line

[Herbst, RH, Rohrer (2004)], [Schwarz, RH, Giacosa (2006)], [RH (2006)]

Previous evidence for smallness of corrections

From now on: restricted to massive sector

$\Delta P|_{2-\text{loop}}/P|_{1-\text{loop}}$



λ/λ_c

Sign constraints in the massive sector

Observation [Krasowski, RH (2014)]

For $p^2 = q^2 = m^2$:

$$|(p \pm q)^2| \leq |\phi|^2 \quad \Rightarrow \quad \mathrm{sgn}(p^0) = \mp \mathrm{sgn}(q^0) \tag{1}$$

Programme:

- Construct all 2PI bubble diagrams in the massive sector (only 4-vertices)
- Each 4-vertex constraint excludes certain sign-configurations by virtue of (1)
- For all possible combinations of vertex constraints calculate R:

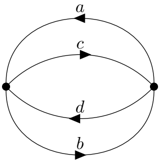
Measure R for constraining power:

$$R \equiv \frac{\text{number of non-excluded sign configurations}}{\text{number of a priori possible sign configurations}}$$

Sign constraints in the massive sector

Exclusion tables: Example

$\emptyset$			
a	b	c	d
ab	ac	ad	



Sign constraints in the massive sector

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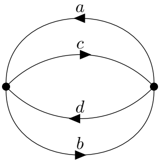

$$s = |(a+d)^2| = |(b+c)^2| \leq |\phi|^2 \;\Rightarrow \mathrm{sgn}(a_0) = -\mathrm{sgn}(d_0),\, \mathrm{sgn}(b_0) = -\mathrm{sgn}(c_0)$$

$$t = |(a-b)^2| = |(c-d)^2| \leq |\phi|^2 \;\Rightarrow \mathrm{sgn}(a_0) = \mathrm{sgn}(b_0),\, \mathrm{sgn}(c_0) = \mathrm{sgn}(d_0)$$

$$u = |(a-c)^2| = |(d-b)^2| \leq |\phi|^2 \;\Rightarrow \mathrm{sgn}(a_0) = \mathrm{sgn}(c_0),\, \mathrm{sgn}(d_0) = \mathrm{sgn}(b_0)$$

Sign constraints in the massive sector

Exclusion tables: Example

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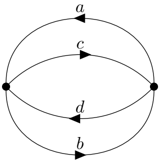

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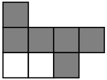
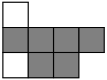
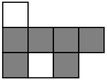
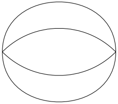
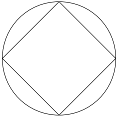
s :  t :  u : 

$$R = \frac{2+2+2+1+1+1+1+1+1}{3^2 \cdot 2^{4-1}} = 1/6$$

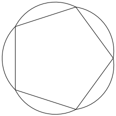
Sign constraints in the massive sector



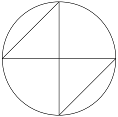
$R = \frac{1}{6} = 0.1667$



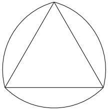
$R = \frac{1}{72} = 0.0139$



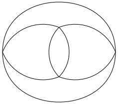
$R = \frac{17}{3888} = 0.0044$



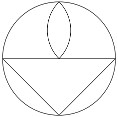
$R = \frac{13}{3888} = 0.0033$



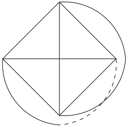
$R = \frac{5}{108} = 0.0463$



$R = \frac{1}{81} = 0.0123$



$R = \frac{7}{1944} = 0.0036$



$R = \frac{1}{324} = 0.0031$

- No termination at 6-loop level despite $R \to 0$

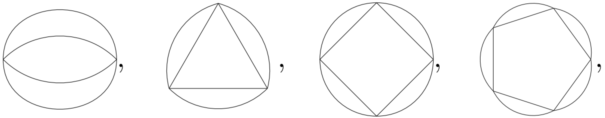
Sign constraints in the massive sector

Non-termination of the loop expansion

By induction:

The class $\mathcal{C}$ of dihedral symmetry 2PI bubble diagrams enjoys $R > 0$ at all loop orders. (A configuration of vertex constraints can be found, such that all constraints are equivalent.)
This *falsifies* the conjecture about termination of 2PI expansion.

$\mathcal{C} = \Big\{$



$, \quad \dots \Big\}$

Question:

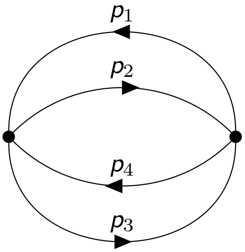
Does the hierarchical suppression by loop orders observed up to 2-loop persist at 3-loop level?

The 2PI three-loop diagram

...in dimensionless variables

$$\Delta P|_{\text{3-loop}} = i\frac{\Lambda^4}{48\lambda^2} e^4 \frac{1}{(2\pi)^6} \sum_{\text{signs}} \int_{\text{constrained}} \mathrm{d}\theta_1 \mathrm{d}\varphi_1 \mathrm{d}r_1 \mathrm{d}r_2 \mathrm{d}\theta_3 \sin\theta_1 \sin\theta_3$$
$$\times \sum_{\{r_3\}} r_1^2 r_2^2 r_3^2 P(\{p_i\}) \frac{n_B'(|\vec{p}_1|) n_B'(|\vec{p}_2|) n_B'(|\vec{p}_3|) n_B'(|\vec{p}_4|)}{8|p_1^0 p_2^0 p_3^0 p_4^0|}$$

- r_i, θ_i, φ_i independent spherical variables of spatial momenta, integration constrained
- Sum over r_3 from on-shellness constraint of p_4 after using momentum conservation $p_4 = p_2 + p_3 - p_1$
- $P(\{p_i\})$ polynomial in scalar products of four-momenta
- $n_B'(|\vec{p}|) \equiv n_B(2\pi\sqrt{|\vec{p}|^2 + m^2}/\lambda^{3/2})$



The 2PI three-loop diagram

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$$\times \sum_{\{r_3\}} r_1^2 r_2^2 r_3^2 P(\{p_i\}) \frac{n_B'(|\vec{p}_1|) n_B'(|\vec{p}_2|) n_B'(|\vec{p}_3|) n_B'(|\vec{p}_4|)}{8|p_1^0 p_2^0 p_3^0 p_4^0|}$$

- Of the 9 vertex constraint configurations (*ss*, *tt*, *uu*, *st*, *ts*, *su*,*us*, *tu*, *ut*) only two are inequivalent (diagonal and off-diagonal):

$$\Delta P|_{\text{3-loop}} = \frac{1}{3}\Delta P|_{\text{3-loop},ss} + \frac{2}{3}\Delta P|_{\text{3-loop},st}$$

- Constraint in the diagonal case:

$$|(p_1 + p_4)^2| = |(p_2 + p_3)|^2 \leq 1$$

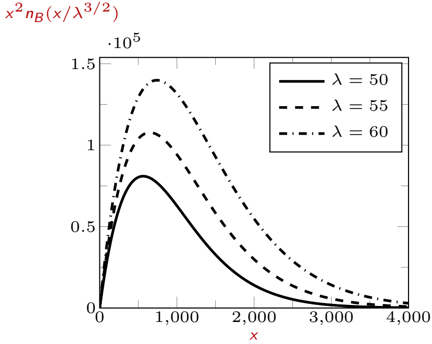
- Constaints in the off-diagonal case:

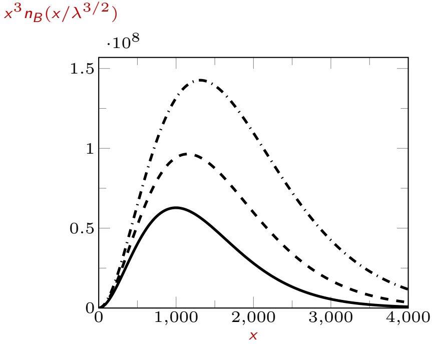
$$|(p_1 + p_4)^2| = |(p_2 + p_3)|^2 \leq 1, \qquad |(p_1 - p_2)^2| = |(p_3 - p_4)|^2 \leq 1$$

The 2PI three-loop diagram

Need for a high-T expression

- Monte Carlo integration is possible due to Bose functions
- Problem: Integrand's maximum likely gets shifted to large radial variables $\propto \lambda^{3/2}$
- This statement is independent of the exact power law in $P(\{p_i\})$:





- Work in dimensionless variables: 4-momentum components $a^\mu, b^\mu \to a^\mu/|\phi|, b^\mu/|\phi|$ and $m \to m/|\phi|$
- Introduce variational parameter z to get an equation $|(a+b)^2| \leq 1 \quad \to \quad (a+b)^2 = -z, \quad z \in [-1,1]$
- Consider limit $\lambda \gg \lambda_c$, $|\vec{a}|, |\vec{b}| \gg m = 2\sqrt{8}\pi$, then, at fixed $|\vec{a}|$:
  1. $\cos \sphericalangle \vec{a}\vec{b} \equiv \cos\alpha \geq -1 \quad \Rightarrow \quad z \in [0,1]$
  2. $\cos\alpha = \frac{z+2m^2-2\sqrt{\mathbf{a}^2+m^2}\sqrt{\mathbf{b}^2+m^2}}{2|\mathbf{a}||\mathbf{b}|}$, maximised by $z=1$
  3. Difference of solutions of $|\vec{b}|$ maximised by $\cos\alpha = -1$ and $z=1$

$$|\mathbf{b}|_{\max/\min} = |\mathbf{a}| \left(1 + \frac{z}{2m^2} \pm \frac{1}{2}\sqrt{4\frac{z}{m^2} + \frac{z^2}{m^4} + \frac{4z}{|\mathbf{a}|^2} + \frac{z^2}{|\mathbf{a}|^2 m^2}}\right)$$

- Variation of z defines band in $|\vec{b}|$ around $|\vec{a}|$ and band in α around π.

The 2PI three-loop diagram

Properties of the constraints

Limits of the widths $\Delta|\vec{b}|$ and $\Delta\alpha$ and mean values $|\bar{b}|$, $\bar{\alpha}$ of the allowed bands of $|\vec{b}|$ and α for $|\vec{a}| \gg m$

$$|\bar{b}| = |\vec{a}|\left(1 + \frac{1}{m^2}\right) \approx 1.002|\vec{a}|$$

$$\Delta|\vec{b}| = |\vec{a}|\sqrt{\frac{4}{m^2} + \frac{1}{m^4}} \approx 0.112|\vec{a}|$$

$$\sin(\bar{\alpha}) = \sin\left(\frac{\pi}{2} + \arccos\left(\frac{1}{2|\vec{a}|^2} - 1\right)\right) \approx \frac{1}{2|\vec{a}|}$$

$$\Delta\alpha = \pi - \arccos\left(\frac{1}{2|\vec{a}|^2} - 1\right) \approx \frac{1}{|\vec{a}|}$$

The 2PI three-loop diagram

Properties of the constraints

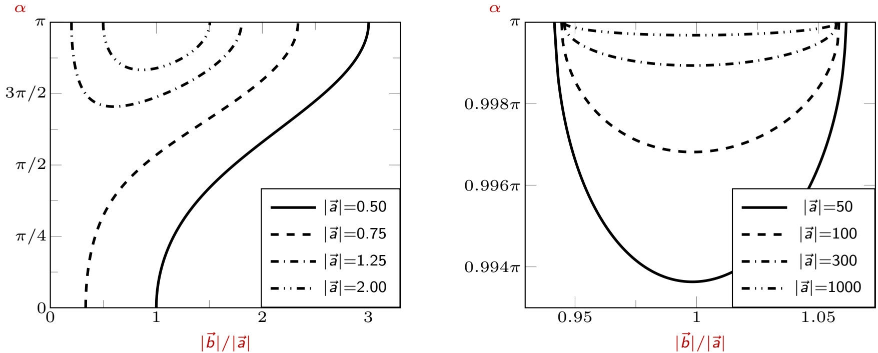


Figure: Visualisation of $|(a+b)^2| = |2m^2 - 2\sqrt{\vec{a}^2 + m^2}\sqrt{\vec{b}^2 + m^2} - 2|\vec{a}||\vec{b}|\cos\alpha)| = 1$ at fixed $|\vec{a}|$ and $\lambda \gg \lambda_c$ (saturated constraint)

The 2PI three-loop diagram

High temperature limits

Recall:

$$\Delta P|_{\text{3-loop}} = i\frac{\Lambda^4}{48\lambda^2}e^4\frac{1}{(2\pi)^6}\sum_{\text{signs}}\int_{\text{constrained}} \mathrm{d}\theta_1\mathrm{d}\varphi_1\mathrm{d}r_1\mathrm{d}r_2\mathrm{d}\theta_3 \sin\theta_1 \sin\theta_3$$

$$\times \sum_{\{r_3\}} r_1^2 r_2^2 r_3^2 P(\{p_i\})\frac{n_B'(|\vec{p}_1|)n_B'(|\vec{p}_2|)n_B'(|\vec{p}_3|)n_B'(|\vec{p}_4|)}{8|p_1^0 p_2^0 p_3^0 p_4^0|}$$

- With the constraint $|(p_1+p_4)^2| = |(p_2+p_3)^2| \leq 1$, one can reduce to one independent scalar product at high temperatures:

$$p_1 \cdot p_2 = \pm\sqrt{r_1^2+m^2}\sqrt{r_2^2+m^2} - r_1 r_2 \cos\theta_1$$

- Sum over r_3 becomes trivial, angles can be explicitly integrated or replaced by mean value times width as a function of r_2
- Analytic expressions are obtained

The 2PI three-loop diagram

High-temperature limits

- For $\lambda \gg \lambda_c$, masses normalised by $|\phi|$:

$$\frac{1}{3}\Delta P|_{3-\text{loop,ss}} = i\Lambda^4\lambda^{13}e^4\frac{1}{15}\frac{1}{(2\pi)^{15}}\frac{1}{m^8}\left(1+\frac{1}{4m^2}\right)\left(\int_0^\infty \mathrm{d}y\, y^4 n_B^2(y)\right)^2$$

$$= i\Lambda^4\lambda^{13}\frac{1}{3375}\frac{1}{(2\pi)^{15}}\frac{1}{m^4}\left(1+\frac{1}{4m^2}\right)\left(\pi^4 - 90\zeta(5)\right)^2$$

$$\approx \boxed{i\Lambda^4\lambda^{13}\cdot 5.2968\cdot 10^{-20}}$$

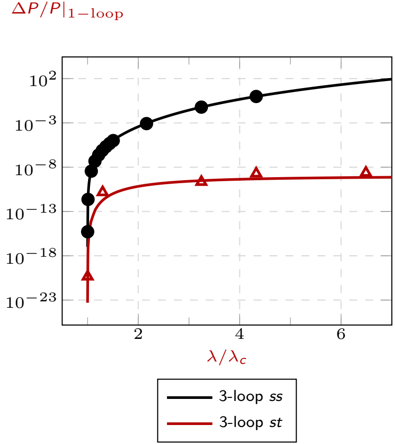
$$\frac{2}{3}\Delta P|_{3-\text{loop},st} = i\frac{\Lambda^4}{\lambda^2}e^4 C\int \mathrm{d}r_2 \frac{r_2^3}{(r_2^2+m^2)^2} n_B^4\left(2\pi\sqrt{\frac{r_2^2+m^2}{\lambda^3}}\right)$$

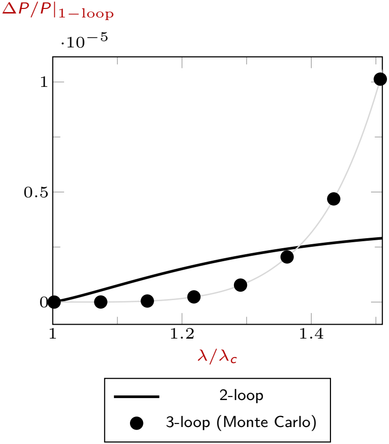
$$\approx \boxed{i\Lambda^4\lambda^4\cdot 2.2011\cdot 10^{-12}}$$

- Higher constrained channels *st* are smaller in hierarchy by λ^9
- Still, a contribution $\propto \lambda^{13}$ seems catastrophic, since $P|_{1-\text{loop}} \propto \lambda^4$

The 2PI three-loop diagram

Results and comparison to $P|_{1-\text{loop}}$

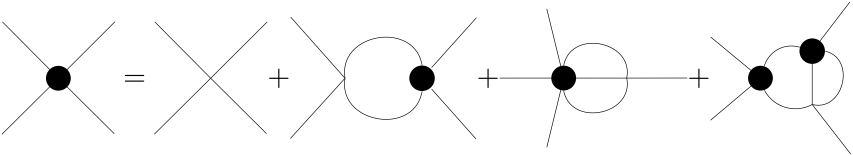




- $|\Delta P|_{3-\text{loop},ss}| \gg |\Delta P|_{3-\text{loop},st}|$ even near λ_c
- $|\Delta P|_{3-\text{loop}}| \ll |\Delta P|_{2-\text{loop}}|$ near λ_c

Resummation of diagrams of dihedral symmetry

Dyson-Schwinger equation of the four-vertex:



Assumptions

- Tree-level propagators sufficiently close to resummed propagators
- Resummed 4-vertex has the same tensorial structure, i.e. is rescaled by scalar form factor $f(\lambda; s, t, u)$
- Truncate after second diagram on the RHS

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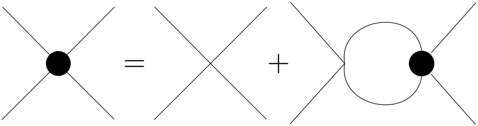

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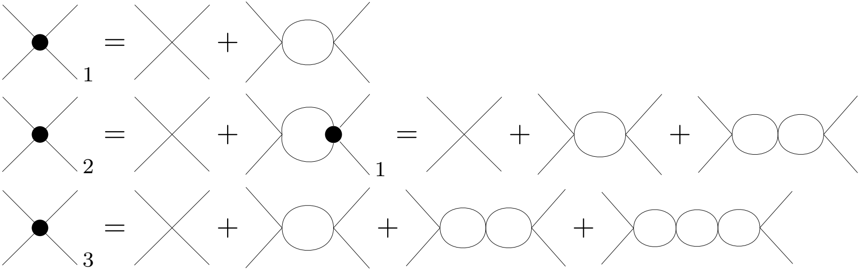
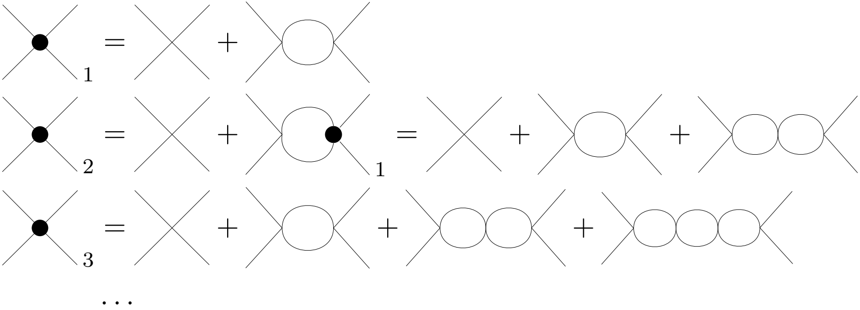
Iterative, truncated DSE:

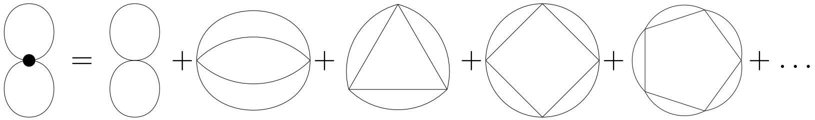


...

Iterative, truncated DSE:



Upon closing legs class $\mathcal{C}$ appears (dihedral group symmetry):



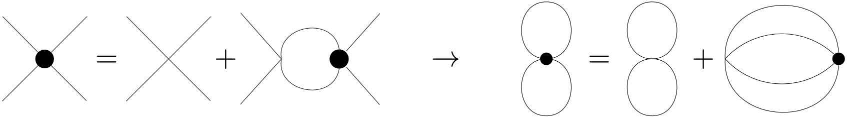
Resummation of diagrams of dihedral symmetry

Rescaling at high temperatures

At high temperatures $\lambda \gg \lambda_c$ constraints suppress s, t, and u dependence of $f(\lambda; s, t, u) \to f(\lambda, 0) \equiv f(\lambda)$, because

$$|s|, |t|, |u| \leq |\phi|^2 \propto \frac{1}{T}$$

Factoring $f(\lambda)$ out of the integrations, the DSE implies



$$\Rightarrow f(\lambda)|_{\lambda \gg \lambda_c} = \frac{\Delta P|_{2-\mathrm{loop}}}{\Delta P|_{2-\mathrm{loop}} - \Delta P|_{3-\mathrm{loop}}} \approx -0.94 \cdot 10^{15} i \lambda^{-11.6}$$

Resummation of diagrams of dihedral symmetry

Rescaling at high temperatures

$$f(\lambda)|_{\lambda \gg \lambda_c} = \frac{\Delta P|_{2-\text{loop}}}{\Delta P|_{2-\text{loop}} - \Delta P|_{3-\text{loop}}} \approx -0.94 \cdot 10^{15} i \lambda^{-11.6} \quad (2)$$

Regarding 3-loop and 2-loop diagram with resummed vertices yields

$$f(\lambda)^2 \Delta P|_{3-\text{loop,TL}} \approx -4.7 \cdot 10^{10} i \Lambda^4 \lambda^{-10.2}$$
$$f(\lambda) \Delta P|_{2-\text{loop,TL}} \approx 4.7 \cdot 10^{10} i \Lambda^4 \lambda^{-10.2}$$

- Contributions are now well-bounded and even cancel at leading order in λ
- Postulating (2) to persist down to λ_c implies $f(\lambda)|_{\lambda \gtrsim \lambda_c} \approx 1$

Summary

Conclusions

- Radiative corrections to the thermal ground state pressure do not terminate at finite 2PI order as was previously conjectured
- Even worse: Except close to λ_c, they can not be considered small at fixed orders (λ^{13} behaviour even at 3-loop level)
- Way out: Formal resummation of an infinite class of diagrams yields a well-bounded and smooth high-temperature extension
- Imaginary small corrections interpreted as non-thermal

Outlook

- Higher loop order diagrams outside the class $\mathcal{C}$ controlled by constraints or more resummation necessary?
- Inclusion of the massless quasi-particles

Detailed reference: https://arxiv.org/abs/1703.07398