The isolated electron:
De Broglie's „hidden“ Thermodynamics, SU(2) Quantum Yang-Mills theory, and a strongly perturbed BPS Monopole

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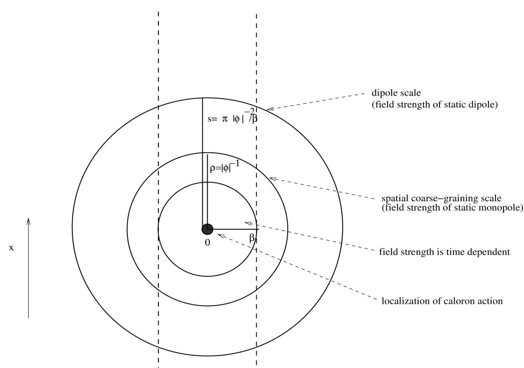

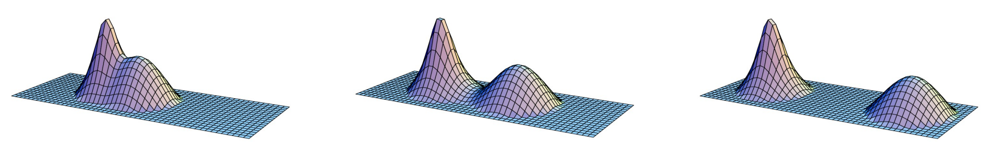

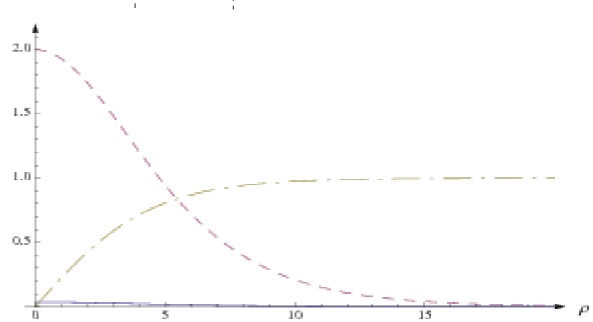

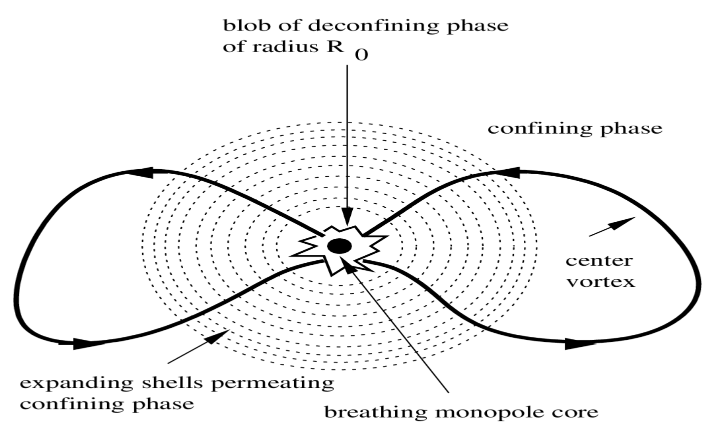

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- Louis de Broglie's arguments on existence of „thermodynamics of the isolated electron“

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  - deconfining, preconfining, and confining phases
  - thermal ground state and its excitations in deconfining phase
  - center-vortex loops in confining phase

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  - value of coupling e, caloron action
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- electron as self-intersecting, spatial center-vortex loop: size estimates

Louis de Broglie: reminder of derivation of wavelength λ, I

- consider particle at rest as a `clock´ with frequency ν_0 which extents through a large portion of space, that is, amplitude Ψ of oscillation is the same everywhere:

$$\boxed{\Psi = a_0 \sin(2\pi\nu_0 t_0)}$$

- by the rules of QM: rest energy equals quantum of action times ν_0:

$$\boxed{m_0 c^2 = h\nu_0}$$

- consider Lorentz-trafo in z direction:

$$\boxed{t_0 = \frac{t - \frac{\beta}{c} z}{\sqrt{1-\beta^2}} \qquad (\beta \equiv \frac{v}{c})}$$

$\Longrightarrow$ spatially “flat” oscillation becomes propagating wave with

$$\boxed{\nu = \frac{\nu_0}{\sqrt{1-\beta^2}}\,, \qquad v_p = \frac{c}{\beta}\,, \qquad \lambda = \frac{v_p}{\nu} = \frac{c\sqrt{1-\beta^2}}{\nu_0 \beta}}$$

Louis de Broglie: reminder of derivation of wavelength λ, II

- on the other hand: $$\frac{p}{h} = \frac{m_0 v}{h\sqrt{1-\beta^2}}$$

or upon substitution of $m_0 = \frac{h\nu_0}{c^2}$

$$\boxed{\frac{p}{h} = \frac{\nu_0 \beta}{c\sqrt{1-\beta^2}} = \lambda^{-1}}$$

- $\Longrightarrow$ existence of `clock´ frequency ν_0 essential in derivation of de Broglie wavelength λ !

Thermodynamics of isolated electron: Louis de Broglie

- from demand that entropy $$\boxed{S = \int \frac{dQ}{T}}$$

is a fundamental invariant under the Poincare group and from

$$\boxed{dQ = dQ_0 \sqrt{1-\beta^2}}$$

(considering an adiabatic variation of rest mass due to input of heat)

$$\Longrightarrow \quad \boxed{T = T_0 \sqrt{1-\beta^2}}$$

- increase of rest energy $Q_0 = m_0 c^2$ under boost can be decomposed into a **decrease** of internal heat to

$$Q = \sqrt{1-\beta^2}\, Q_0$$

and work A invested to boost the particle

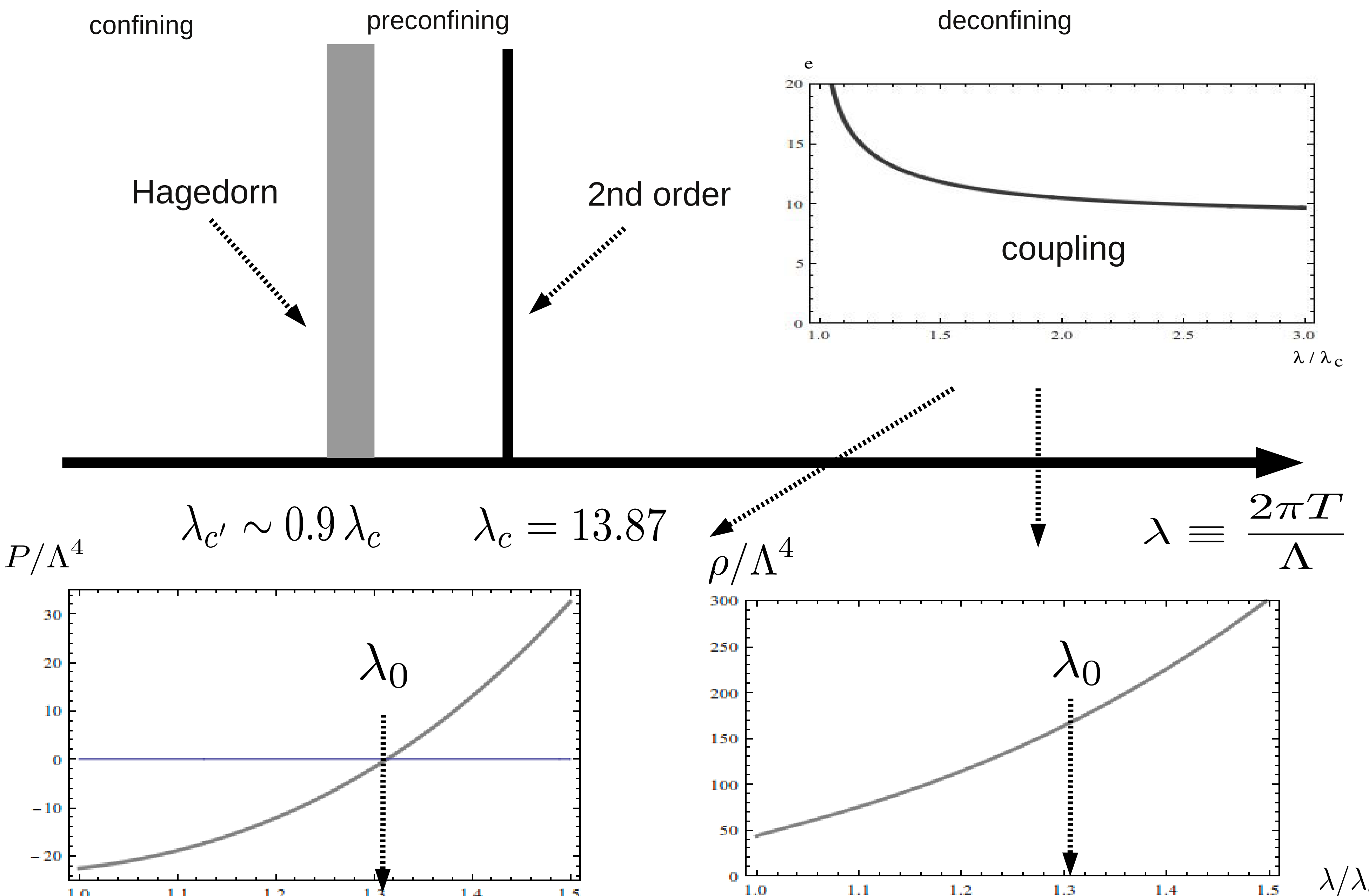
$$A = \frac{m_0 v^2}{\sqrt{1-\beta^2}}$$

$$\longrightarrow \quad \boxed{\frac{Q_0}{\sqrt{1-\beta^2}} = Q + A}$$

Particle with internal heat
can not be point-like.

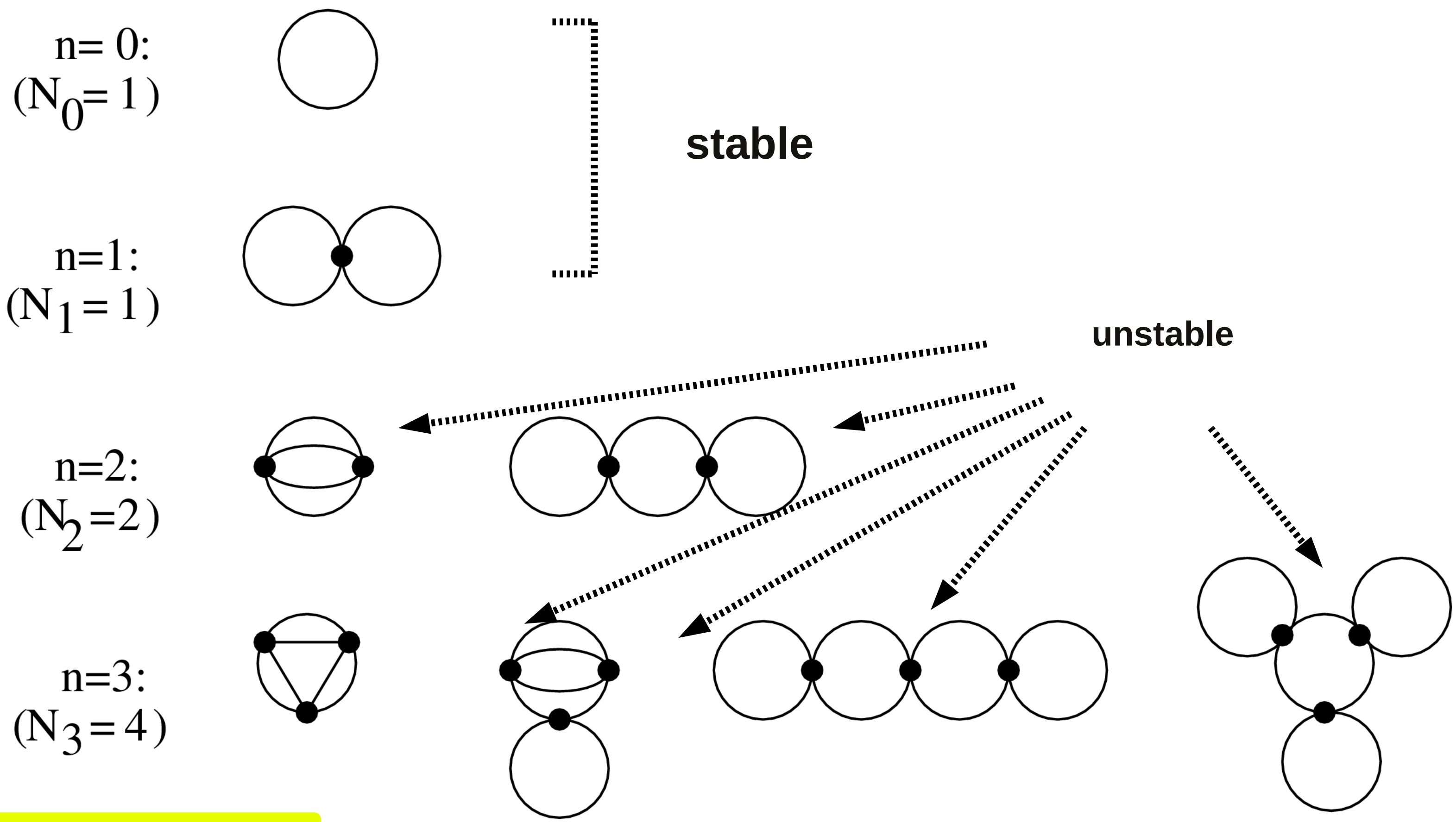
How does SU(2)
Yang-Mills thermodynamics
come in?

Phase diagram of SU(2) Quantum Yang-Mills thermodynamics



Phase diagram of SU(2) Quantum Yang-Mills thermodynamics: Confining phase

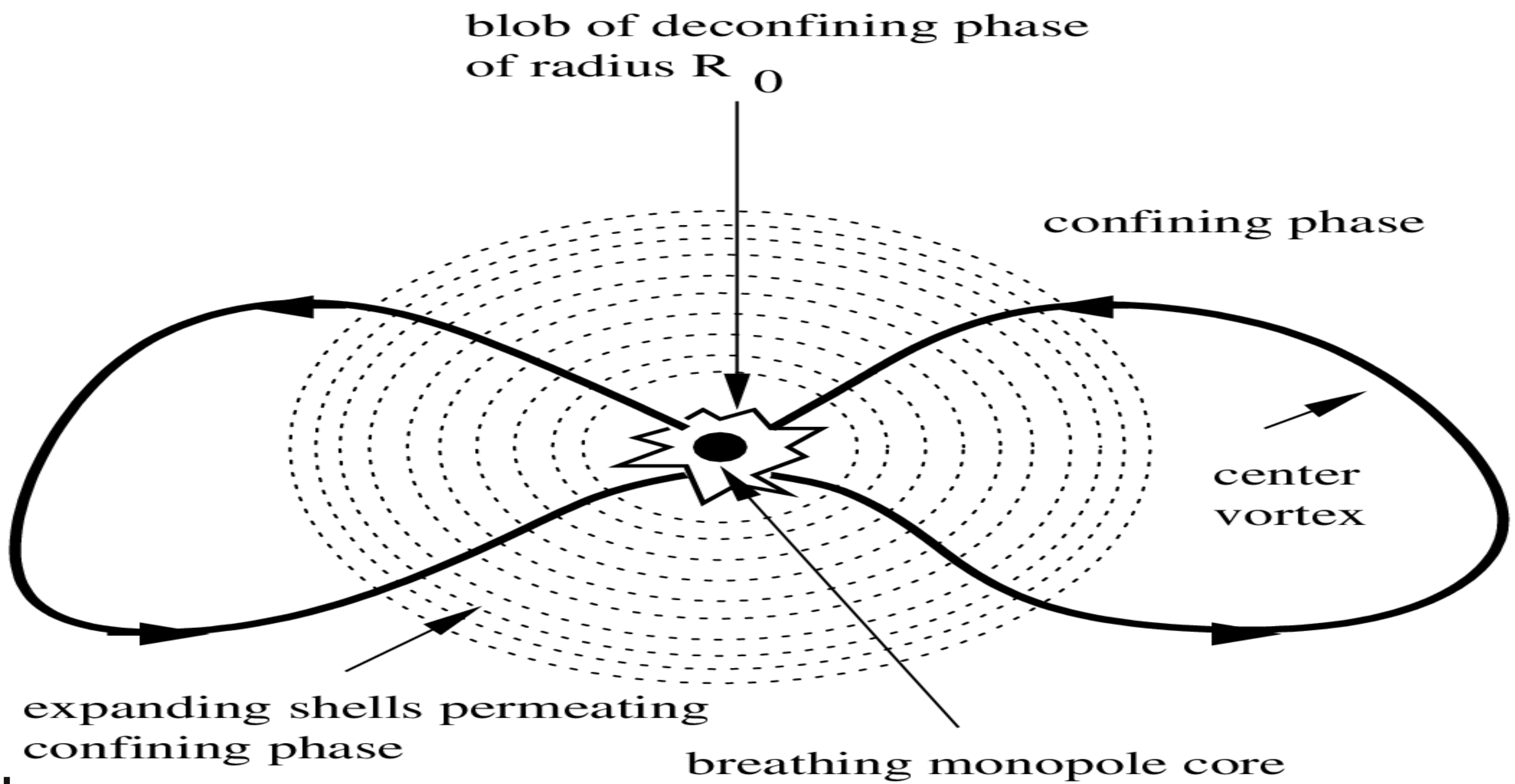
excitations: n-fold self-intersecting center-vortex loops



multiplicity $> n!$

[Bender & Wu, 1969]

Proposal for electron's anatomy



based on:

(i) electric-magnetically dual interpretation of U(1) charges in SU(2)

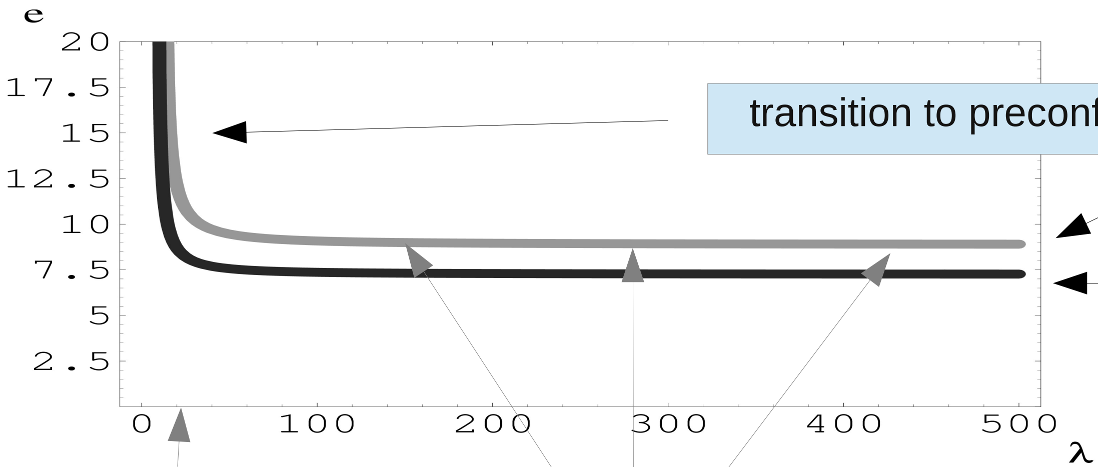
(ii) classical dynamics of strongly perturbed BPS monopole
[Fodor & Racz, 2004]

(iii) structure of deconfining thermal ground state, small-holonomy (anti)calorons,
dissociation of large-holonomy (anti)calorons **[Herbst & RH, 2004; RH, TQYM,2016]**

(i) electric-magnetically dual interpretation: deconfining-phase gauge coupling and caloron action

$$e$$ thermodynamical consistency

transition to preconfining phase



SU(2)

SU(3)

small-mass expansion, pressure
[Dolan et Jackiw (1974)]

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

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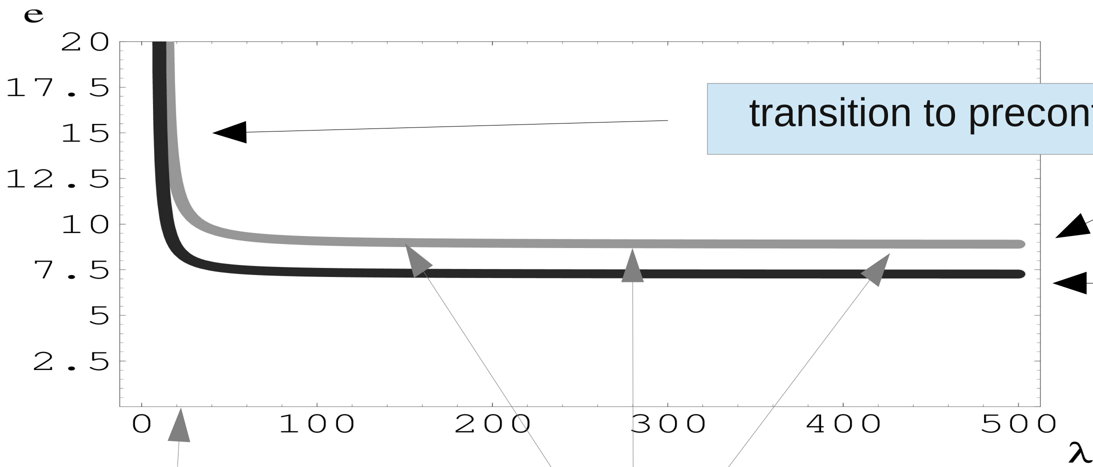

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$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

coarse-graining dominated by $\rho \sim |\phi|^{-1}$

- restore $\hbar$

$$e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}\,.$$

$$S_{C/A} = \hbar\,.$$

[Brodsky et al. (2011); Kaviani & RH 2012; RH 2012,2013]

(i) electric-magnetically dual interpretation: argument involving QED fine-structure constant

if SU(2) something to do with photons **[RH (2005), Grandou & RH (2015), etc]**
then **electric-magnetically dual** interpretation required:
in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\boxed{\alpha = \frac{Q^2}{4\pi\hbar}\,,}$$

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for α to be unitless:

$$\boxed{Q \propto \frac{1}{e}\,.}$$

$(e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}\,.)$

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But: magnetic coupling in SU(2)

$$\boxed{g = \frac{4\pi}{e}\,.}$$

$\Rightarrow$ SU(2) to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole, etc.)

(ii) classical dynamics of strongly perturbed BPS monopole

- hyperboloidal conformal transformation of the original field equations for the profiles

$$H(r,t) = h(r,t)/r + H_\infty$$

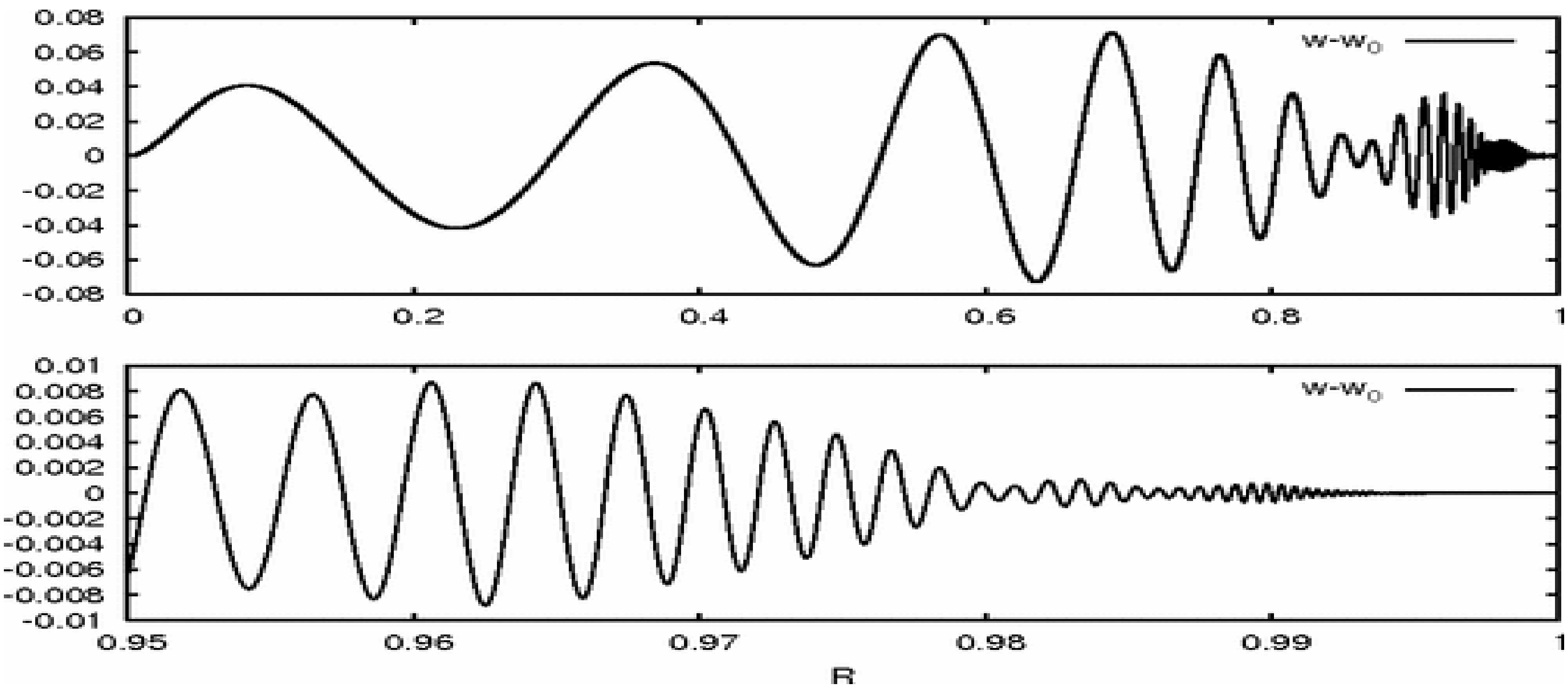
and

$$w(r,t)$$

of the adjoint Higgs and off-Cartan gauge fields fields, respectively

[Fodor & Racz, 2004]

- under a spherically symmetric initial pulse in energy density:
  - high-frequency oscillations in w forming expanding shells



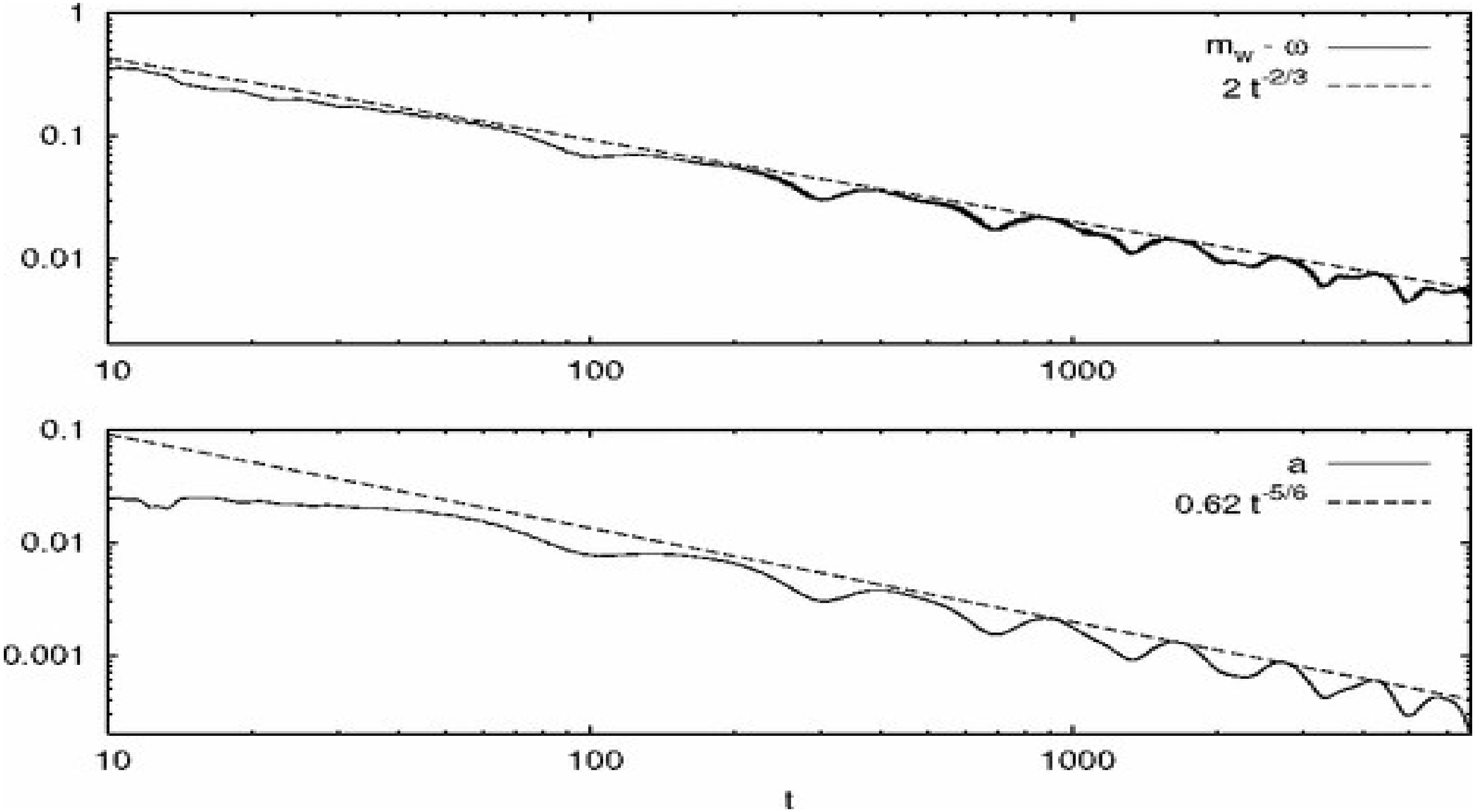
(ii) classical dynamics of strongly perturbed BPS monopole

- a localized **breathing state** appears in association with the energy density within the monopole core whose frequency ω_0 approaches the mass

$$m_w = eH_\infty$$

of the two off-Cartan modes in a power-like way in time

[Fodor & Racz, 2004]



(ii) classical dynamics of strongly perturbed BPS monopole

There is a generic circular frequency

$$\boxed{\omega_0 = 2\pi\nu_0 = e\,H_\infty}$$

associated with core breathing of the (initially perturbed) BPS monopole, that is, a **clock in de Broglie's sense**.

(iii) structure of deconfining thermal ground state

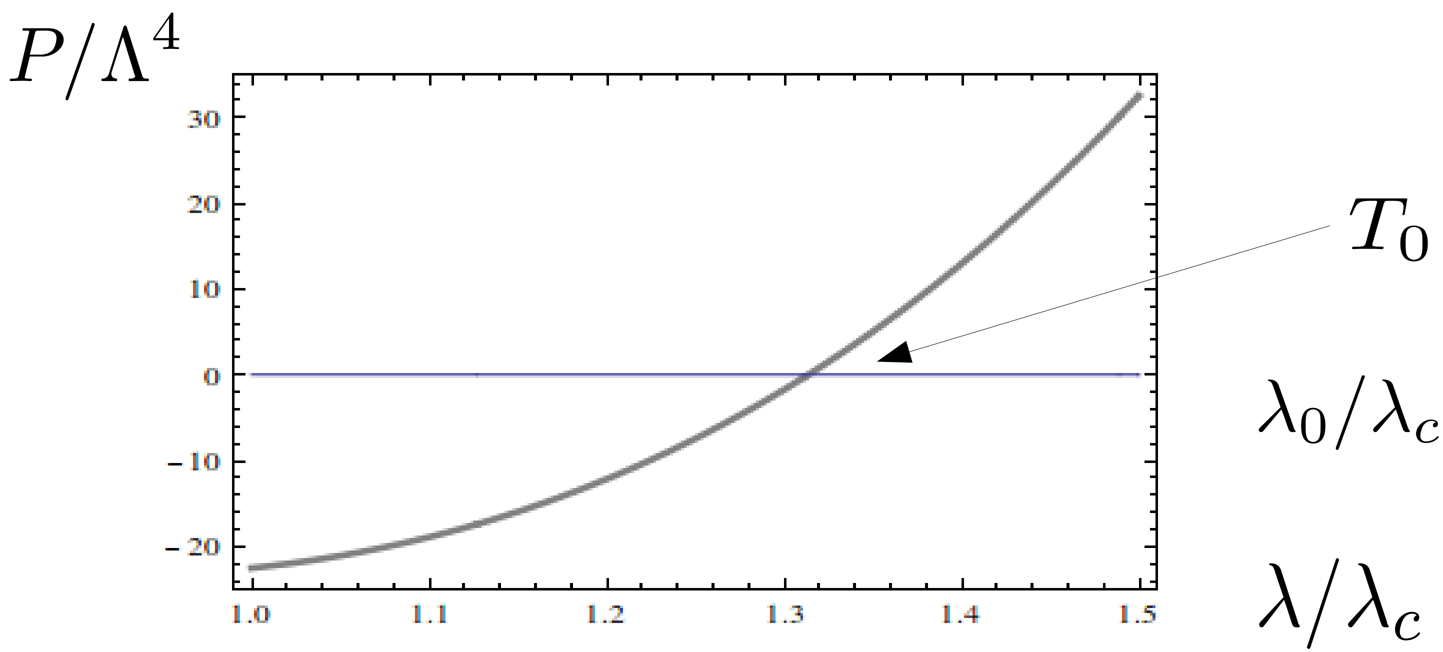
- Only (anti)calorons of scale $\rho \sim |\phi|^{-1}$ contribute to emergent a priori estimate of thermal ground state $\Longrightarrow$ coupling e relevant in describing caloron dissociation and ensuing monopole- antimonopole

[Herbst & RH 2004; RH 2016]

- If monopole liberated by dissociation of a large-holonomy caloron in deconfining phase at temperature T_0 then

$$\boxed{H_\infty(T_0) = \pi T_0}$$

[Diakonov et al 2004]



$$T_0 = \lambda_0 \frac{\Lambda}{2\pi}$$

$$\lambda_0/\lambda_c = 1.32$$

(iii) structure of deconfining thermal ground state

We have: [Diakonov et al 2004]

$$e(1.32) = 12.96 \qquad \text{(A)}$$

$$\rho/\rho^{\rm gs}(1.32) = 8.31 \qquad \text{(B)}$$

with: $$\rho^{\rm gs}(T_0) = 4\pi\Lambda^3 T_0 = 32\frac{H_\infty^4(T_0)}{\lambda_0^3}$$

We assume static BPS monopole of mass

$$m_{\rm mon} = \frac{8\pi^2}{e^2(T_0/T_c)}H_\infty(T_0) = \frac{8\pi^2}{12.96^2}H_\infty(T_0)$$

to be immersed into blob of deconfining phase of radius R_0 at T_0 .

Size estimate for self-intersection region:

Thus, from (A) and (B) we have:

$$\boxed{\begin{aligned} m_0 &= e\,H_\infty(T_0) = 12.96\,H_\infty(T_0) = m_{\rm mon} + E_0 \\ &= \frac{8\pi^2}{12.96^2} H_\infty(T_0) + \frac{4}{3}\pi R_0^3\,\rho(T_0) \\ &= H_\infty(T_0)\left(\frac{8\pi^2}{12.96^2} + 8.31 \times \frac{128\pi}{3}\left(\frac{R_0}{18.31}\right)^3 H_\infty^3(T_0)\right) \end{aligned}}$$

$$\boxed{\Longleftrightarrow \qquad R_0 = 4.10\,H_\infty^{-1}(T_0)}$$

Comparing this to the Compton wave length $\lambda_C = m_0^{-1} = 2.426 \times 10^{-12}$ m,

$$\Longrightarrow \quad \boxed{R_0 \sim 53.14\,\lambda_C \sim 1.29 \times 10^{-10}\,\mathrm{m}}$$

[RH, 2017]

(~ 4.12 core size of monopole)

hierarchy between Yang-Mills scale (~ T_c) and electron mass m_0 and Coulomb self-energy:

from $H_\infty(T_0) \Longrightarrow$

$$\boxed{\Lambda = \frac{1}{118.6}\, m_0}$$

$$\Longrightarrow \boxed{T_c = 13.87/(2\pi \times 118.6)\, m_0 = 0.019\, m_0 = 9.49 \text{ eV}}$$

For comparison: average electron temperature at ITER envisaged to be

$$\boxed{\bar{T}_e = 8.8 \text{ eV !!}}$$

Coulomb self-energy correction:

$$\Delta m_0 = \int_{R_0}^{\infty} dr\, \frac{r^2}{r^4} = R_0^{-1} = \frac{1}{4.10} H_\infty(T_0) \ll 12.96\, H_\infty = m_0 \quad \text{or} \quad \boxed{m_0 \sim 53.14\, \Delta m_0\,.}$$

Summary and outlook

- brief review of Louis de Broglie's argument on electron's inherent thermodynamics
- phase structure of SU(2) YMTD, deconfining phase, confining phase
- electron as a one-fold self-intersecting center-vortex loop
- strongly perturbed BPS monopole
- structure of deconfining thermal ground state
- size estimates
- Coulomb self-energy and relation between Yang-Mills scale and crit. Temp.
- open questions:
  - spherically symmetric perturbation of BPS monopole generalizable to other perturbations
  - spatial extent of loops (depending on environmentally provided resolution?)
  - Schrödinger wave mechanics facilitated by small core sizes vs. R_0
  - What happens under acceleration? (radiation, radiation reaction,...)