SU(2) Yang-Mills thermodynamics

Seminar, condensed matter theory, 20 November 2014

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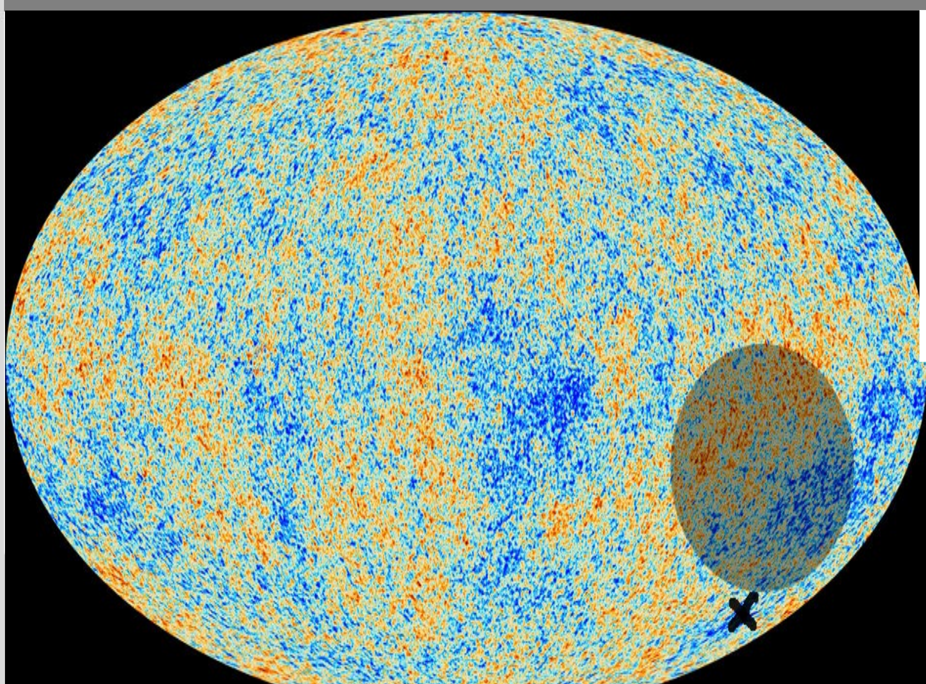
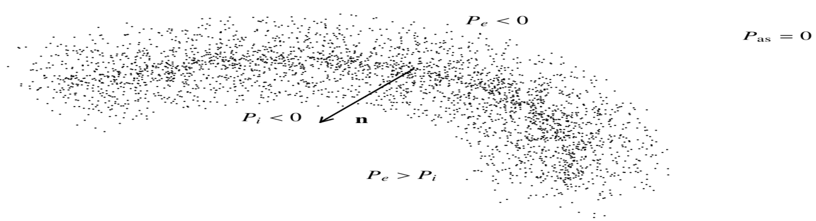
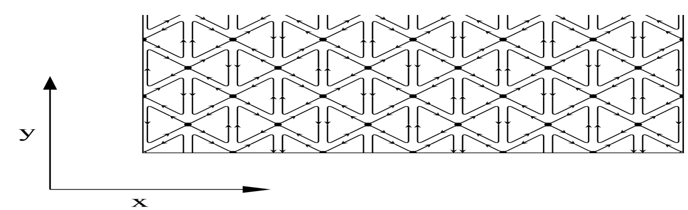






outline

- **motivation:** nonperturbative, analytical approach to YMTD

- **preview on phase structure**

- **essentials, thermal ground state:**
  coarse-graining over nonpropagating
  (anti-)calorons of winding number unity, effective action
- **adjoint Higgs mechanism:**
  massive vector modes and kinematic constraints (1),
  coupling, deconf.-preconf. phase boundary, (anti-)caloron action,
- **radiative corrections:**
  kinematic constraints (2), polarisation tensor of massless mode,
  longitudinal and transverse thermal dispersion, action of a(n)
  (anti)caloron, e-m dual interpretation
- **SU(2) postulate for photon propagation:**
  Yang-Mills scale or critical temperature
  (radio-frequency CMB observations)
- **CMB large-angle anomalies (WMAP, Planck):**
  possible explanation via SU(2) dispersion law,
  onset of dynamical breaking of statistical isotropy at redshift unity,
  SU(2) vector modes and cosmic neutrinos

outline, cntd.

- **Hagedorn transition: confining phase**

- **nonthermal behaviour:**
  Borel resummation of partition function

- **evolving center vortex loops (CVLs)**
  single CVLs, mass gap
  one-fold selfintersecting CVLs, emergent order

motivation

- Andrei Linde (1980):
  „Infrared Problem in the Thermodynamics of the Yang-Mills Gas“

  - soft magnetic sector screened weakly in perturbation theory (infrared instability)

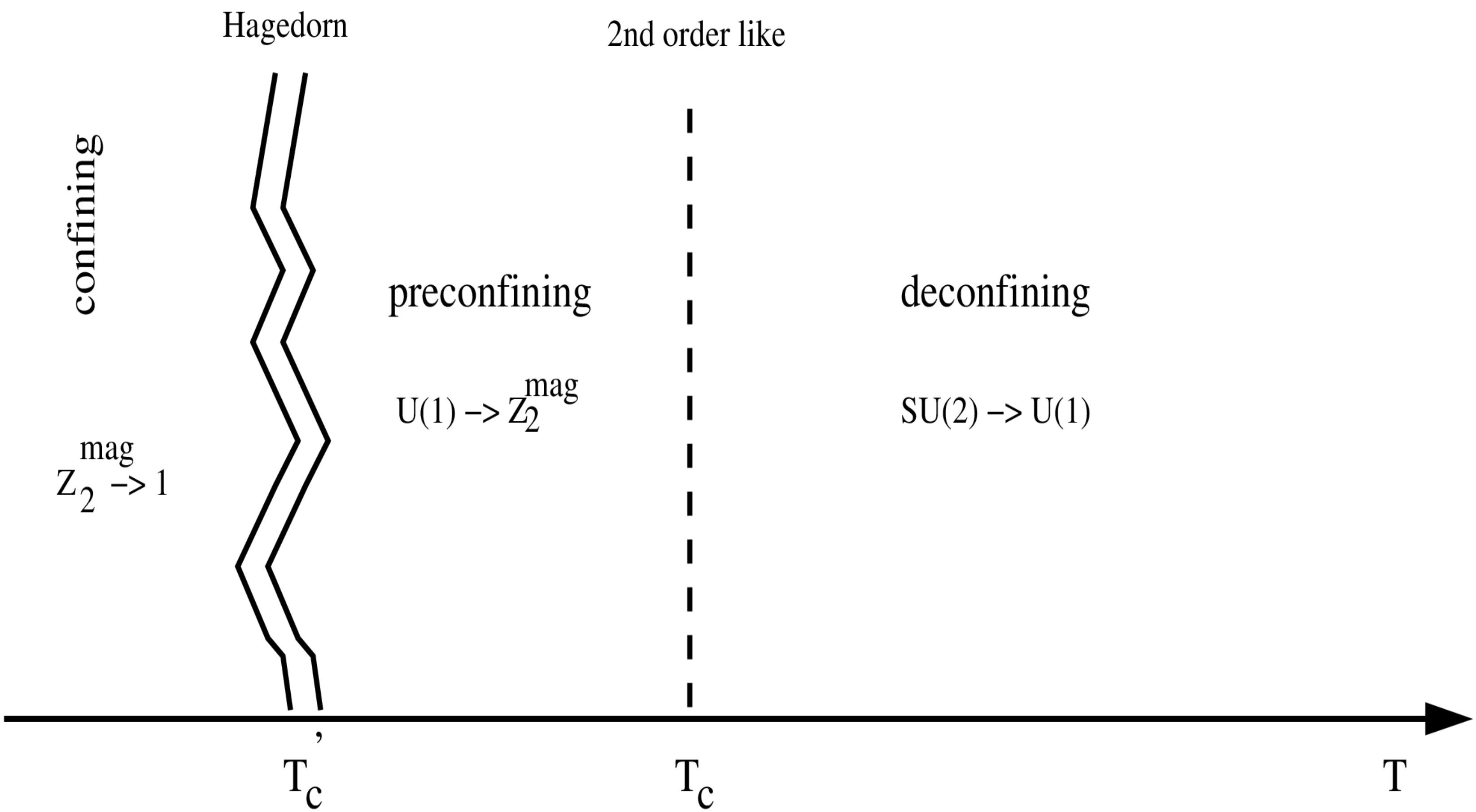
- - not even naive „convergence“ of series since kinetic and interaction energies comparable in this sector

- issue of finite-volume constraints in lattice gauge theory

  - correlations mediated by soft magnetic sector insufficiently probed by available lattice sizes

  - nonperturbative, lattice β function

preview on phase structure



nonperturbative Yang-Mills thermodynamics: SU(2)

[Herbst et Hofmann (2004), Hofmann (2005-2007), Giacosa et Hofmann (2006), Schwarz et al. (2007), Ludescher et Hofmann (2008), Falquez et al. (2010- 2011), Hofmann (2012)]

thermal ground state at high temperature:

- **Euclidean action:**

$$\boxed{S = \frac{\mathrm{tr}}{2}\int_0^\beta d\tau \int d^3x\, F_{\mu\nu}F_{\mu\nu}\,,}$$

$(\, \beta \equiv 1/T \,)$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$

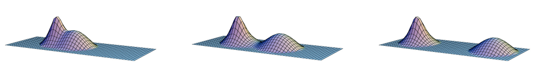
- **(anti)selfdual gauge fields:**

[Schafer et Shuryak (1996)]

$$\boxed{F_{\mu\nu}[A] = \pm\tilde{F}_{\mu\nu}[A] \;\Rightarrow\; \theta_{\mu\nu}[A] \equiv 0\,.}$$

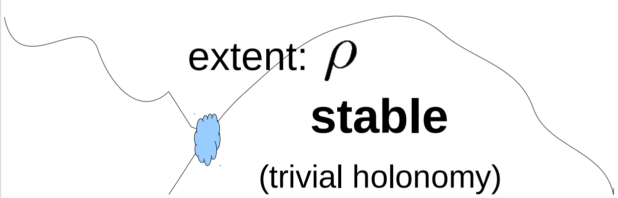
field configs. stabilized by winding: $S_3 \rightarrow SU(2) = S_3$

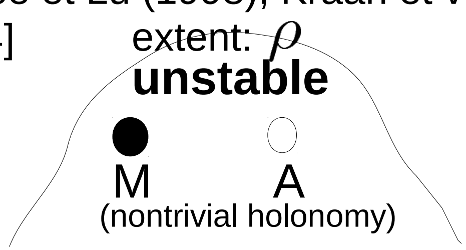
- **in particular:** (anti)calorons of winding number unity



[Harrington et Shepard (1977)

[Nahm (1981-84), Lee et Lu (1998), Kraan et v. Baal (1998), Diakonov et al. 2004]





spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field ϕ

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$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho\, t^a\, F_{\mu\nu}(\tau,\vec{0})\, \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\}\, F_{\mu\nu}(\tau,\vec{x})\, \left\{(\tau,\vec{x}),(\tau,\vec{0})\right\}$$

- unique, definition of (dimensionless) **family of phases**, where

$$\left\{(\tau,\vec{0}),(\tau,\vec{x})\right\} \equiv \mathcal{P}\exp\left[i\int_{(\tau,\vec{0})}^{(\tau,\vec{x})} dz_\mu\, A_\mu(z)\right]$$ and

$$\left\{(\tau,\vec{x}),(\tau,\vec{0})\right\} \equiv \left\{(\tau,\vec{0}),(\tau,\vec{x})\right\}^\dagger$$

- magnetic-magnetic correlations contribute only

- uniquely determined, annihilating operator: $D = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$

-$\{\hat{\phi}^a\}$ sharply dominated by cut-off for ρ integration, later!

spatial coarse-graining over (anti-)calorons: inert, adjoint scalar field

- no explicit β dependence in ϕ field dynamics (caloron action!)

- absorb β dependence of D into potential V

(BPS and EL yield: $\frac{\partial V(|\phi|^2)}{\partial|\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2}$ $\Rightarrow$

$$\boxed{V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}}$$

(Yang-Mills scale)

and

$$\boxed{|\phi| = \sqrt{\frac{\Lambda^3\beta}{2\pi}}}$$

- BPS equation:

$$\partial_\tau \phi = \pm 2i\,\Lambda^3\, t_3\, \phi^{-1} = \pm i\, V^{1/2}(\phi)$$

$\Rightarrow$ no **additive** ambiguity for V !

effective action (deconfining phase)

- $$\mathcal{L}_{\text{eff}}[a_\mu] = \text{tr}\left(\frac{1}{2}G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)$$

((i) perturbative renormalizability
(ii) ϕ 's inertness – no higher dim. operators to mediate 4-momentum transfer between ϕ and a_μ
(iii) gauge invariance)

- effective YM equation $D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]$ has ground-state solution:

$$a_\mu^{\text{gs}} = \mp\delta_{\mu 4}\frac{2\pi}{e\beta}\,t_3 \qquad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0)$$

$$\Longrightarrow \quad P_{gs} = -\rho_{gs} = -4\pi\Lambda^3 T\,.$$

interacting small-holonomy (anti)calorons (collapsing monopole-antimonopoel pairs)

adjoint Higgs (deconfining phase)

(SU(2) → U(1))

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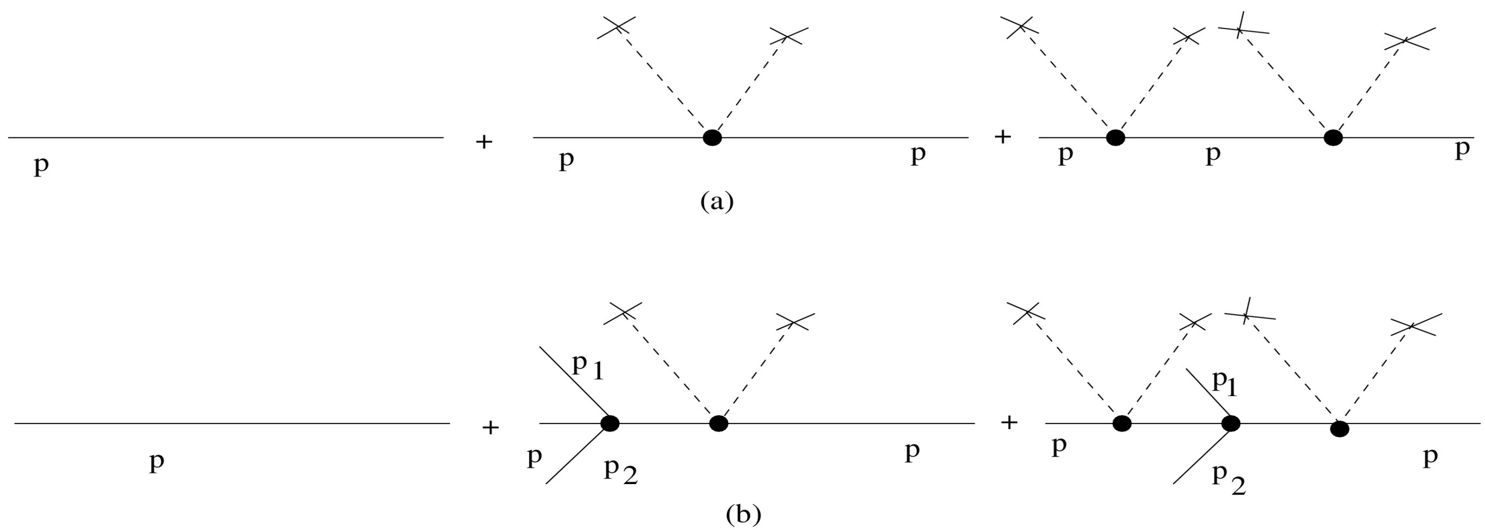
- from effective action:

$$\boxed{m_a^2 = -2e^2 \mathrm{tr}\,[\phi, t_a][\phi, t_a]} \xrightarrow{\text{unitary gauge}} \boxed{m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}\,,\ m_3 = 0}$$

- no momentum transfer to ϕ , **but this infinitely often**
  (Dyson series for mass generation):



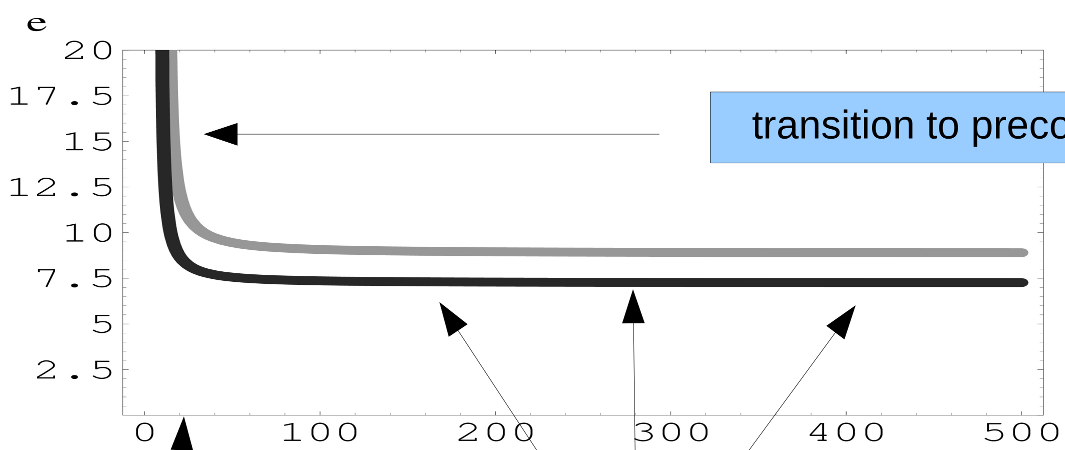
- no off-shell propagation of massive modes
  (otherwise: momentum transfer to ϕ !)

effective gauge coupling

- evolution of effective gauge coupling:

e

thermodynamical consistency



transition to preconfining phase

$\lambda \quad (\lambda \equiv \frac{2\pi T}{\Lambda}\,, \quad \lambda_c = 13.87)$

$$\lambda_c = \frac{2\pi T_c}{\Lambda} = 13.87$$

$$e = \sqrt{8}\pi$$

[Dolan et Jackiw (1974)]

coarse-graining dominated by $\rho \sim |\phi|^{-1}$

- restore $\hbar$

$$e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}\,.$$

$$S_{C/A} = \hbar\,.$$

[Brodsky et al. (2011);
Kaviani et Hofmann 2012,
Hofmann (2012,2013)]

in Euclidean, fundamental (classical) theory value of unit of action not set, only after comparison with effective theory, where $\hbar$ is determined to occur in certain powers at pointlike vertices do we know that $S_{C/A} = \hbar$

electric-magnetically dual interpretation:

- if SU(2) something to do with photons (later!) then **electric-magnetically dual** interpretation required: in units $c = \epsilon_0 = \mu_0 = k_B = 1$ fine-structure constant

$$\alpha = \frac{Q^2}{4\pi\hbar}\,,$$

for α to be unitless:

$$\boxed{Q \propto \frac{1}{e}\,.}$$

But: magnetic coupling in SU(2)

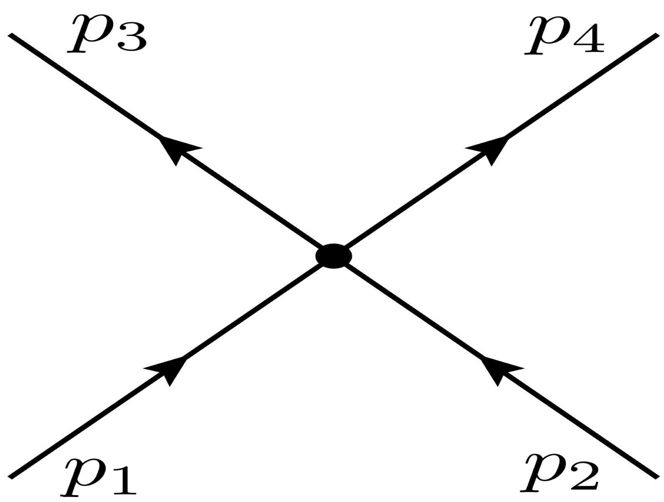
(charge of a magnetic monopole)

$$\boxed{g = \frac{4\pi}{e}\,.}$$

$\Rightarrow$ **U(1) part of** SU(2) to be interpreted in an **electric-magnetically dual way**.
(e.g., magnetic monopole $\longleftrightarrow$ electric monopole, etc.)

radiative corrections (deconfining phase)

- momentum transfer in effective 4-vertex (unitary-Coulomb gauge):



s-channel: $\boxed{|(p_1+p_2)^2| \leq |\phi|^2}$

t-channel: $\boxed{|(p_1-p_3)^2| \leq |\phi|^2}$

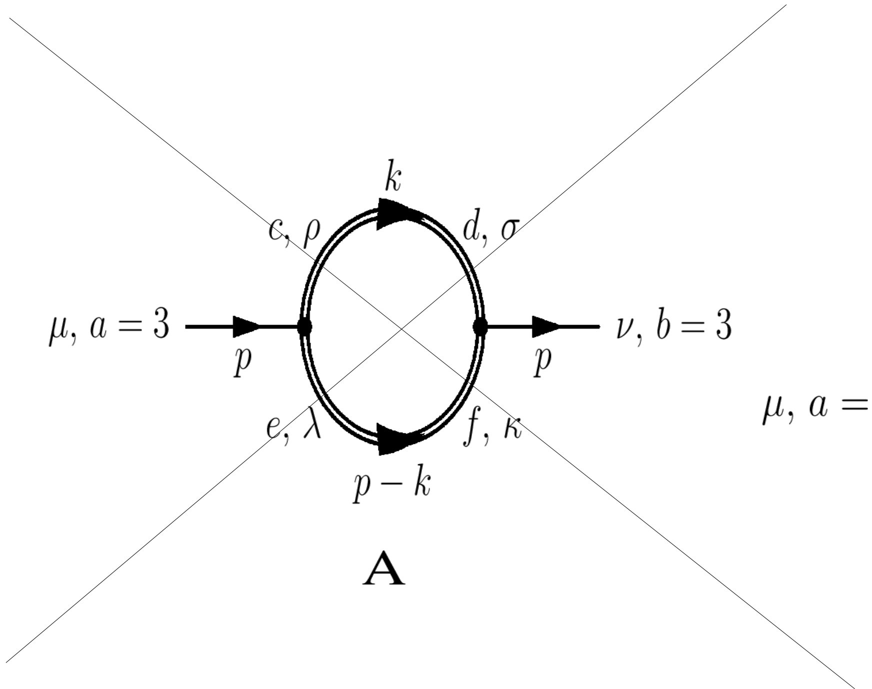
u-channel: $\boxed{|(p_1-p_4)^2| \leq |\phi|^2}$

- coherent average over all three Mandelstam channels $\rightarrow$ thermodynamical quanties: 2-loop/1-loop ($<10^{-3}$), 3-loop/1-loop ($<10^{-7}$), loop expansion into 1-PI diagrams probably terminates at finite order

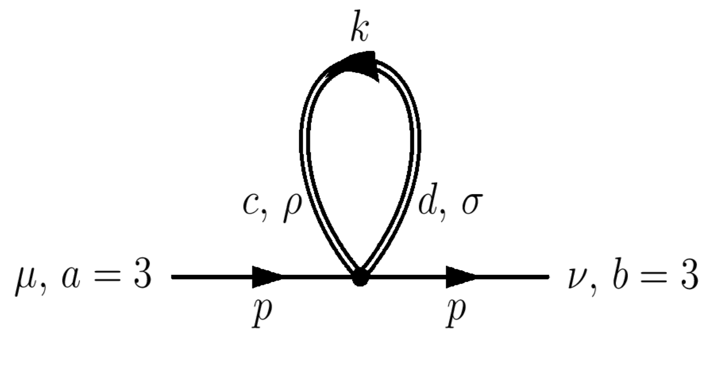
[RH, 2006]

radiative corrections (deconfining phase)

- polarization tensor of massless mode:



A



B

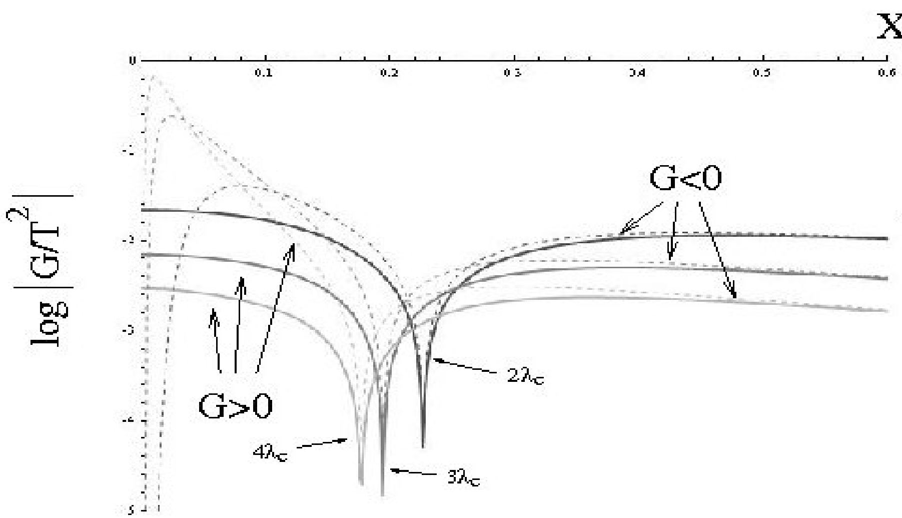
(excluded by kinematic constraints)

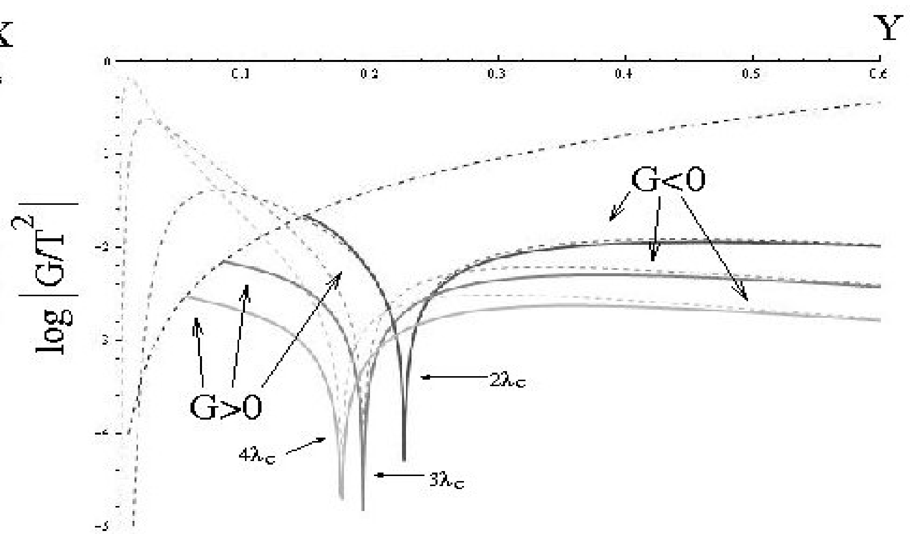
screening functions
G, F (transverse, longitud.)
as solutions of respective gap equations

radiative corrections (deconfining phase)

- transverse photons, screening function G :

[Schwarz et al. (2007), Ludescher & RH (2008)]



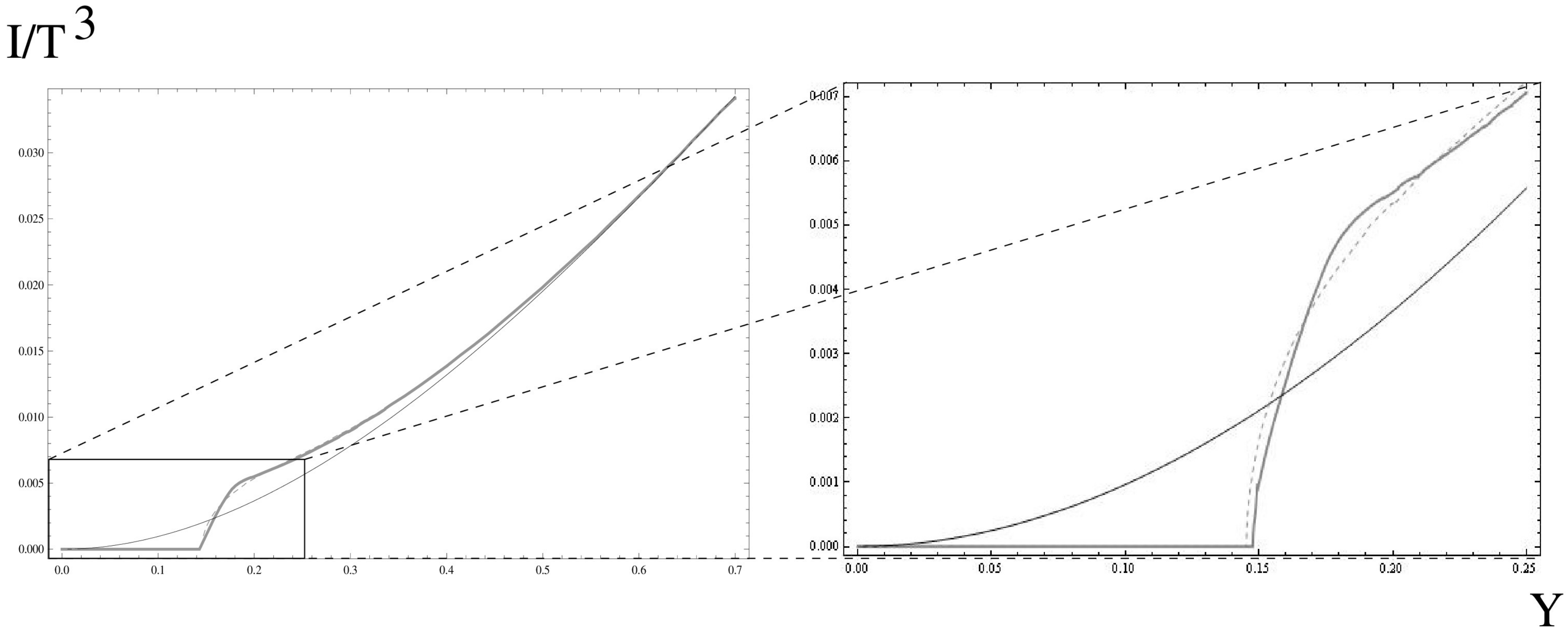


$$\left(X \equiv \frac{|\vec{p}|}{T}\right) \qquad \left(Y \equiv \frac{\omega}{T}\right)$$

$$Y^2 = X^2 + \frac{G}{T^2}$$

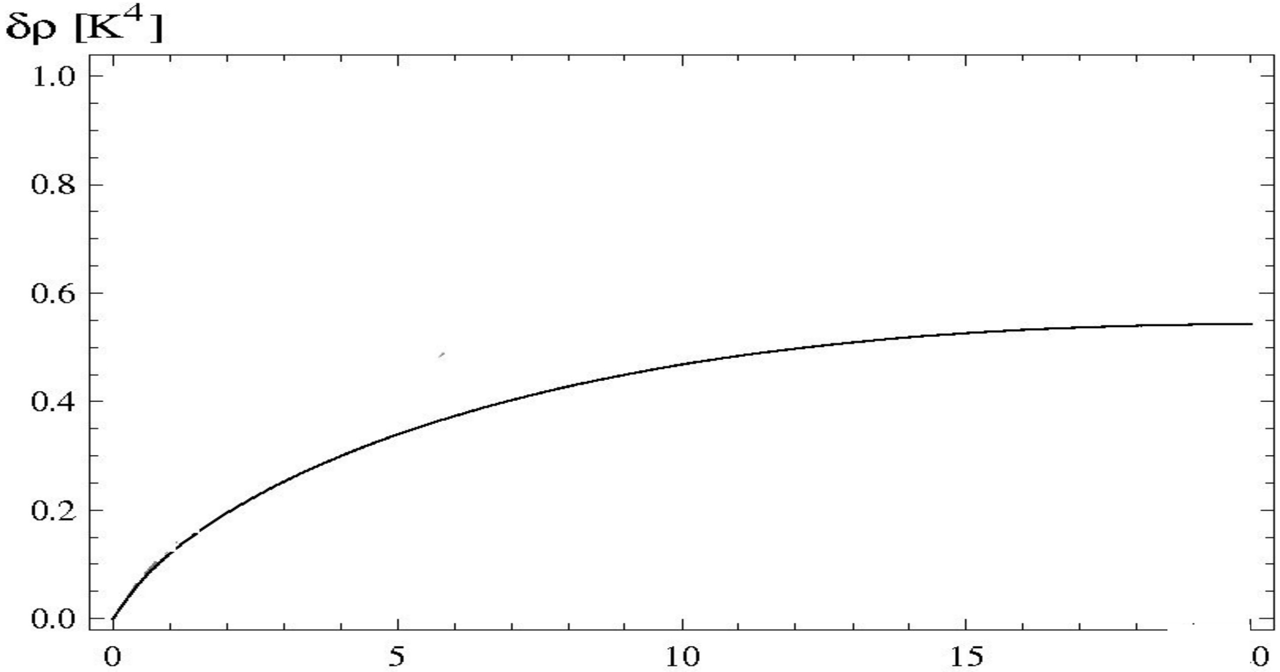
(spectral) radiative corrections (deconfining phase)

- spectral distribution of energy density, massless mode –
  transverse propagation at $T = 2T_c$ in Rayleigh-Jeans regime



(integrated) radiative corrections (deconfining phase)

- difference between energy density of SU(2) and U(1), massless mode – transverse polarisations

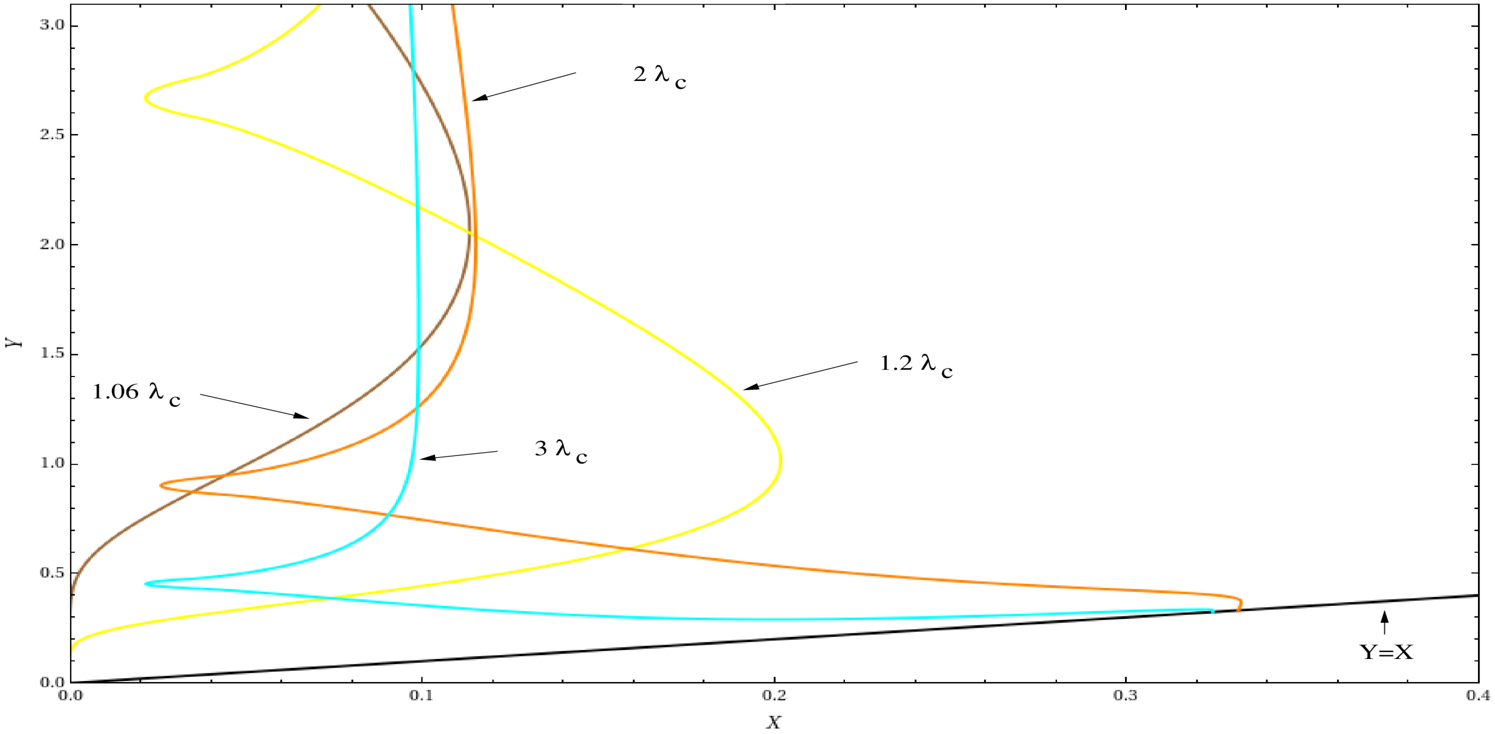


$T/T_c - 1$

(**positive** slope $\longleftrightarrow$ bias for **negative** temperature fluctuations, later!)

(spectral) radiative corrections (deconfining phase)

- low-momentum-support dispersion law, massless mode - longitudinal propagation



(charge-density waves: real-world magnetic modes, intergalactic magnetic fields [Falquez et al (2011)])

SU(2) postulate for photon propagation

- What is T_c ?

- strong increase of CMB line temperture below $\nu = 3$ GHz

$$T(\nu) = T_0 + T_R \left(\frac{\nu}{\nu_0}\right)^{\beta}$$

[Fixsen et al. (2009), Haslam et al. (1981), Reich et Reich (1986), Roger et al. (1999), Maeda et al. (1999)]

where: $T_0 = 2.725\,\mathrm{K}\,; \ \nu_0 = 1\,\mathrm{GHz}\,;$
$\beta = -2.62 \pm 0.04\,; T_R = (1.19 \pm 0.14)\,\mathrm{K}\,.$

(radio-frequency surveys of CMB yield line temperatures as:

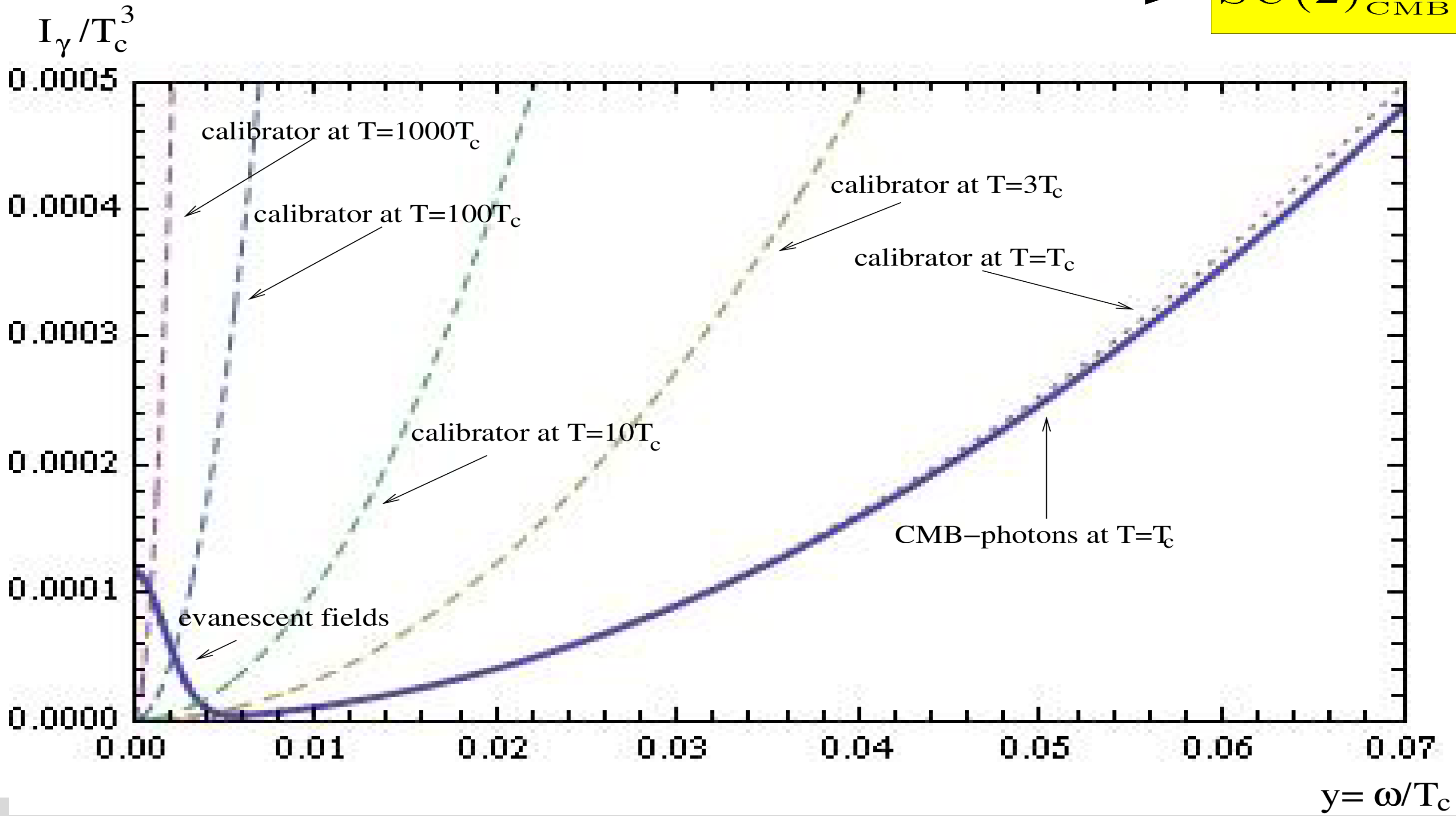
source	ν[GHz]	T[K]
Roger	0.022	21200 ± 5125
Maeda	0.045	4355 ± 520
Haslam	0.408	16.24 ± 3.4
Reich	1.42	3.213 ± 0.53
Arcade2	3.20	2.792 ± 0.010
Arcade2	3.41	2.771 ± 0.009 .

)

evanescent low-frequency modes

- bump from evanescent modes ($\omega < m_\gamma$),
  m_γ photon Meissner mass (condensation of electric monopoles)
- T_c very close to present CMB temperature T_0 (onset of dec.-prec. PT)
  [Hofmann (2009)]

$\longrightarrow$ $\boxed{SU(2)_{CMB}}$



Yang-Mills scale of $SU(2)_{CMB}$:

$$T_c = \frac{13.87}{2\pi}\Lambda_{\mathrm{CMB}} = 2.725\,\mathrm{Kelvin} \sim 2 \times 10^{-4}\,\mathrm{eV}$$

some CMB large-angle anomalies: WMAP and Planck

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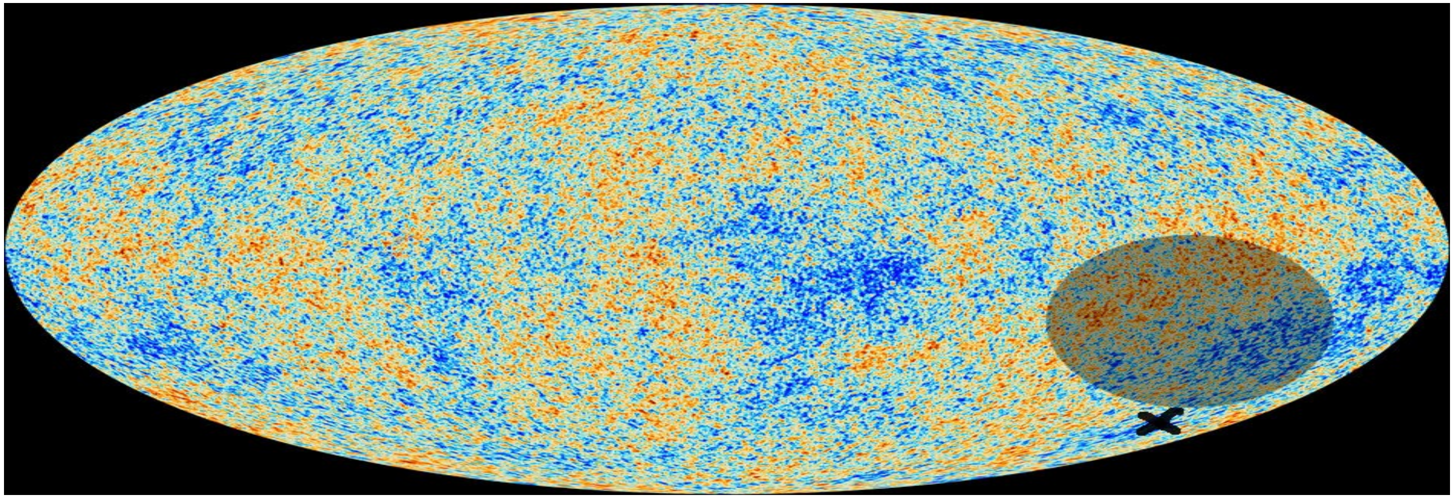
- dipolar power asymmetry (extends from $l = 2, \cdots, 600$ in blocks of $\Delta l = 100$)
  [Hansen et al. (2009), Ade et al. (2013), etc.]

- low variance on ecliptic North, associated with l=2,3
  [Monteserin et al. (2008), Cruz et al., (2011), Ade et al. (2013), etc.]

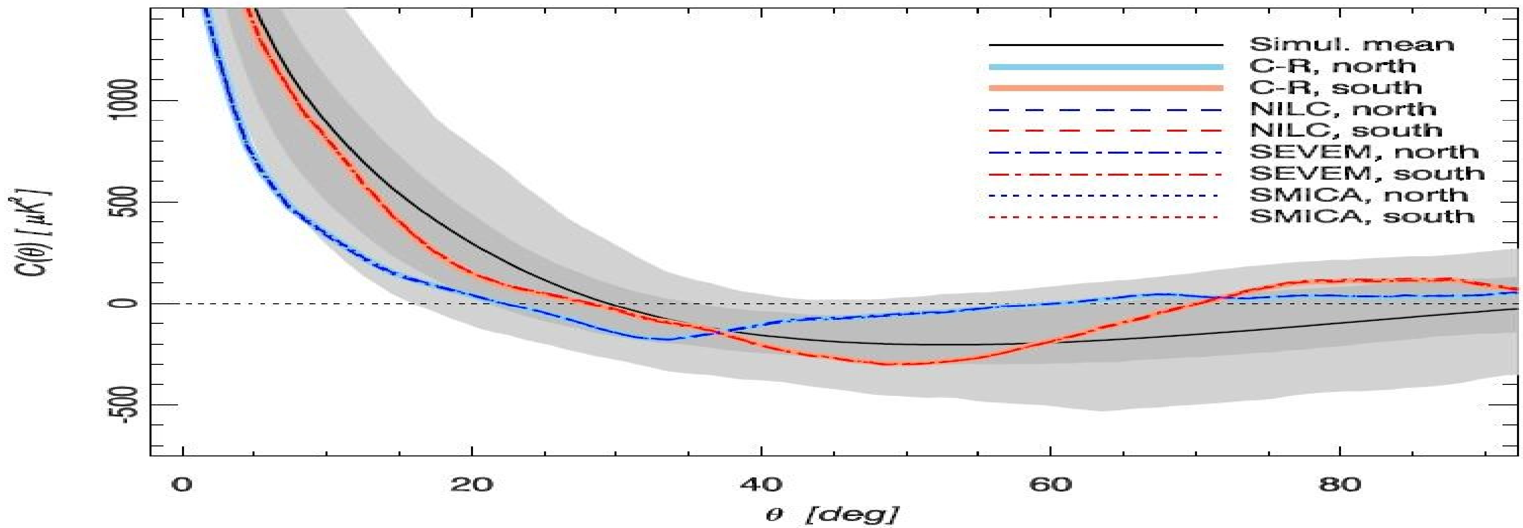
- anomalous alignment of l=2,3 (3°-9°)
  [Tegmark et al. (2003), de Oliveira-Costa et al. (2004), Ade et al. (2013), etc.
  (estimator of axis: maximum of angular momentum dispersion),
  Copi et al. (2004), Schwarz et al. (2004), Bielewicz et al. (2005,2009), Copi et al. (2010), etc.
  (multipole vector decomposition)]

- cold spot (-73μK@4°; -20μK@10°; l,b=207.8°,-56.3°)
  [Viela et al. (2004), Cruz et al. 2005, Rudnick et al. (2007), Ade et al. (2013), etc.]
- hemispherical asymmetry
  (for l=2-40 max. larger power on hemisphere l,b=237°,-20°)
  [Eriksen et al. (2004), Hansen et al. (2004), Park (2004), Ade et al. (2013), etc.]
- mirror parity violation (plane of max. antisymmetry: l,b=262°,-14°)
  [Finelli et al.(2012); Ben-David et al. (2012), etc.]
- **suppression of $\langle TT\rangle(\theta) \equiv C(\theta)$ for $\theta \geq 60^\circ$ on ecliptic North**
  [Spergel et al. (2003), Copi et al. (2004,2007,2009,2010), Ade et al. (2013), etc.]

cold spot



TT suppression on ecliptic North



successful phenomenological attempt at explanation: multiplicative, dipolar modulation model

[Gordon et al. (2005), Eriksen et al. (2007), Hoftuft et al. (2009), Ade et al. (2013)]

$$\vec{d}(\vec{n}) = (1 + A\vec{p}\cdot\vec{n})\vec{s}_{\rm iso} + \vec{n}$$

dipole amplitude — dipole direction — instrumental noise — isotropic CMB sky

maximum likelihood at: $A \sim 0.07;\ \ l_p \sim 220^\circ; b_p \sim -21^\circ$

- robust against change of foreground treatment and experiment (WMAP,Planck)

- comparison with CMB cold spot: $l_{cs} \sim 207.8^\circ; b_{cs} \sim -56.3^\circ$

$$\Rightarrow \angle \vec{p}, \vec{e}_{cs} \sim 36^\circ$$

dynamical breaking of statistical isotropy:

- integrated blackbody anomaly due to SU(2)$_{\mathrm{CMB}}$:

- $\boxed{\delta\rho(T) \equiv \rho_{\mathrm{SU(2)_{CMB}}} - \rho_{\mathrm{U(1)}}}$
- $T = \bar{T}(t) + \delta T(t,\vec{x})$
- SU(2)$_{\mathrm{CMB}}$ bias factor $F(\bar{T},\delta T)$ for δT in phys. voxel volume $\Delta V \sim \frac{(2\pi a)^3}{k_s^3}$. (Silk cutoff)

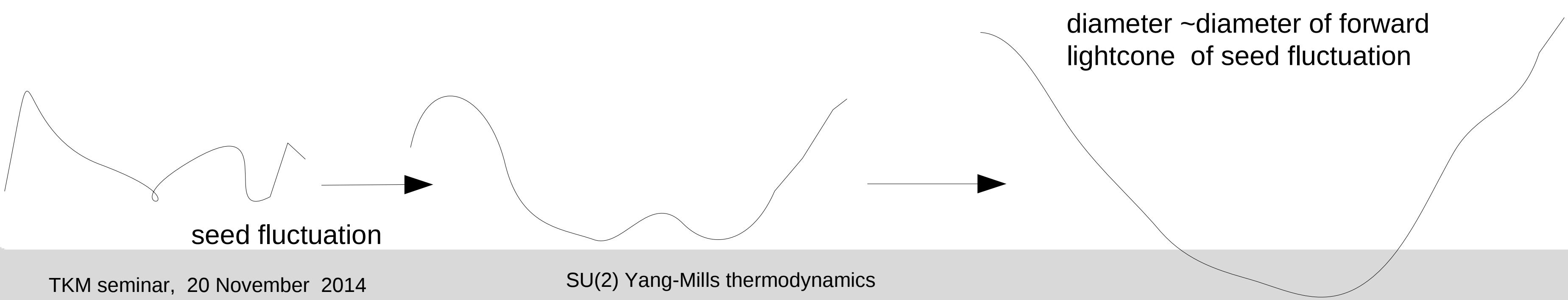
$$\boxed{F(\bar{T},\delta T) = \frac{P_{\mathrm{SU(2)}}}{P_{\mathrm{U(1)}}}}$$

where

$$\boxed{P = \frac{\exp(-\rho\Delta V/\bar{T})}{\int_{T_0}^{\infty} dT\, \exp(-\rho\Delta V/\bar{T})}}$$

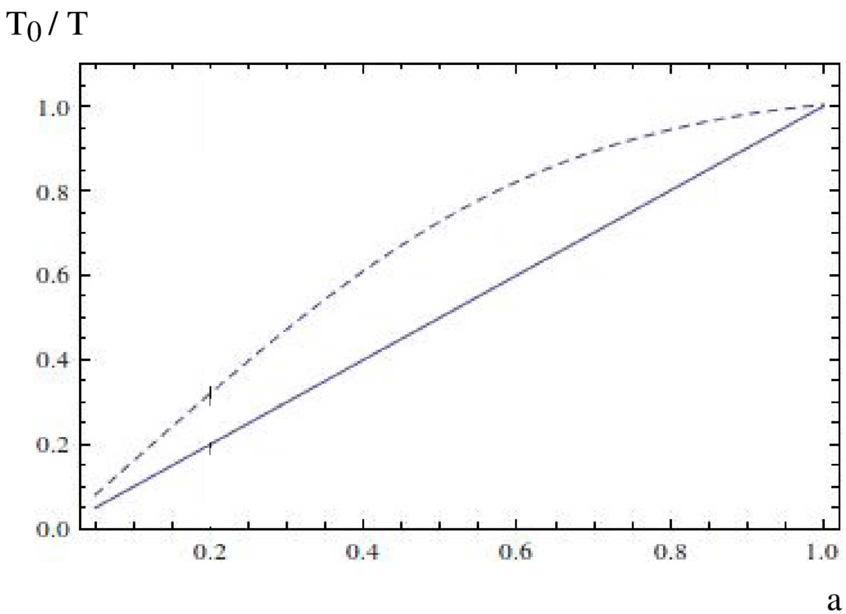
Since slope of $\delta\rho$ positive $\Longrightarrow$ negative δT favoured!

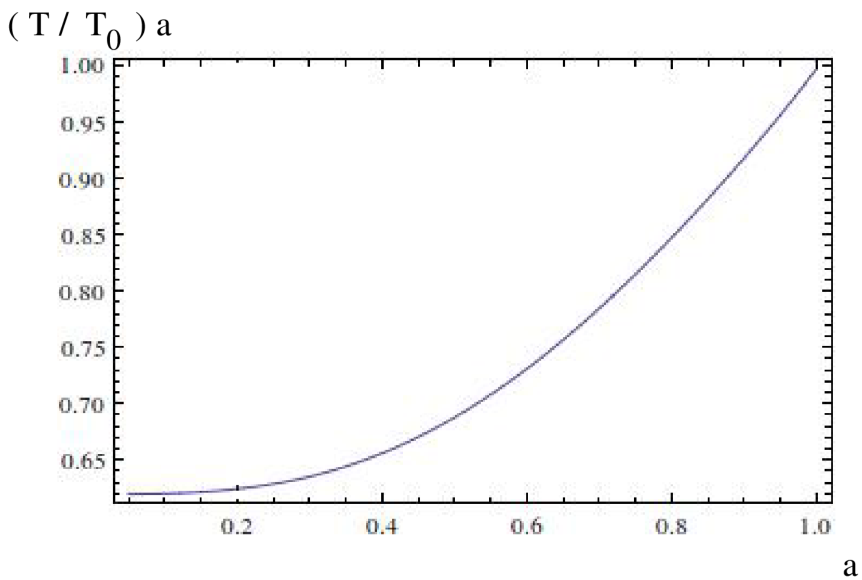
(dynamical breaking of statistical isotropy, RH Nature Phys. 2013)



Violation of conformal scaling $T \propto a^{-1}$

(a **scale factor in FRLW cosmology,** $a = 1$ **today)** [RH, 2014]





$$\Longrightarrow \quad T = 0.62\, a^{-1} \times T_0\,, \qquad \left(a \leq \frac{1}{10}\right)$$

$\Longrightarrow$ discrepancy between redshifts for instantaneous reionisation extracted by observation of Gunn-Peterson trough in quasar spectra and from CMB TT angular spectrum (Planck) repealed

(only tree-level properties of effective SU(2), thermal ground state + massive modes and photon, are required!)

cosmic neutrinos:

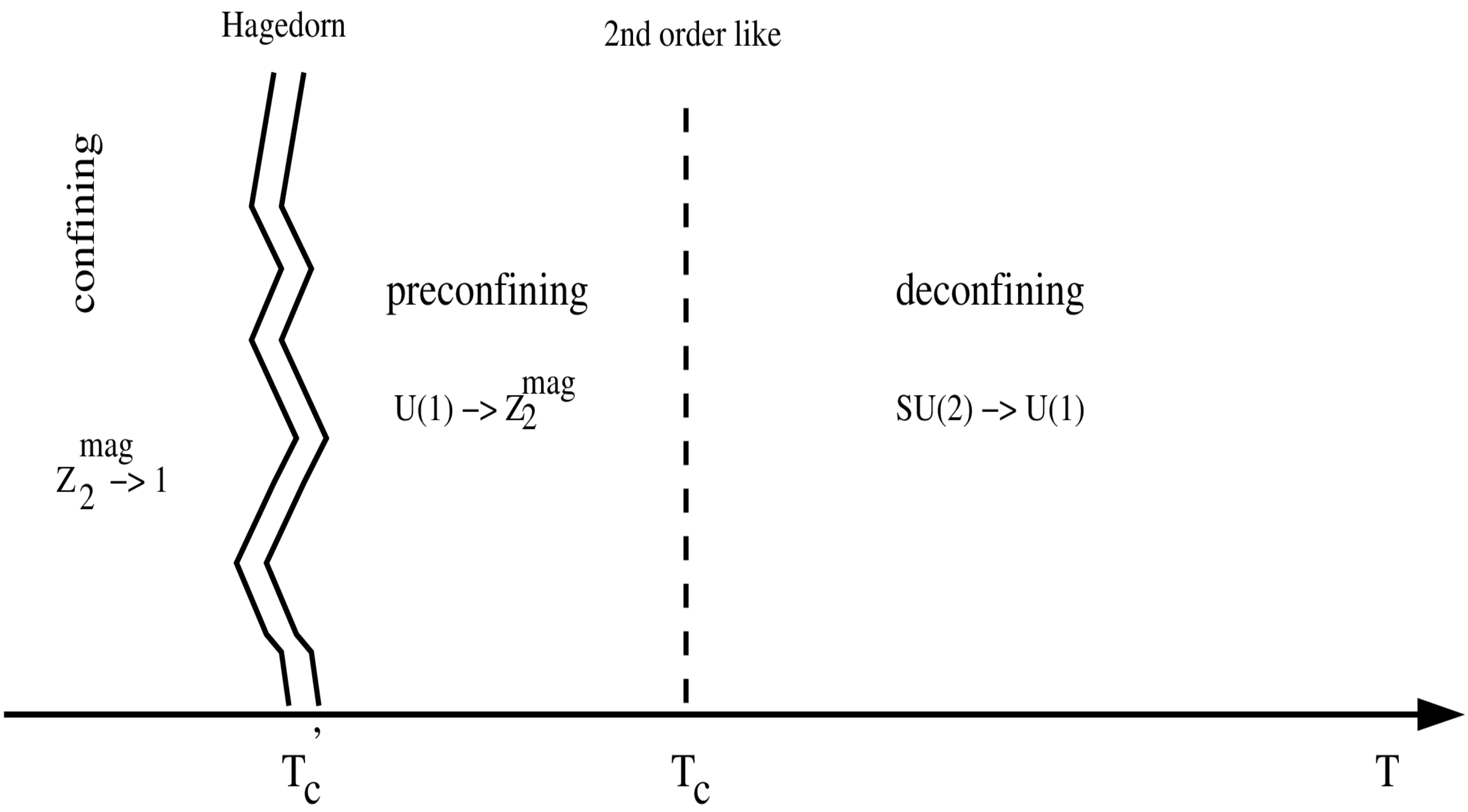
$\Longrightarrow$ to have $N_{\rm eff} = 3.36$ (Planck) today, cosmic neutrinos could be **thermal quasi-particles** whose temperature **coincides** with the CMB temperasture and whose universal **mass is induced** by interaction with the CMB,

$$m_\nu = \xi T \qquad (*)$$

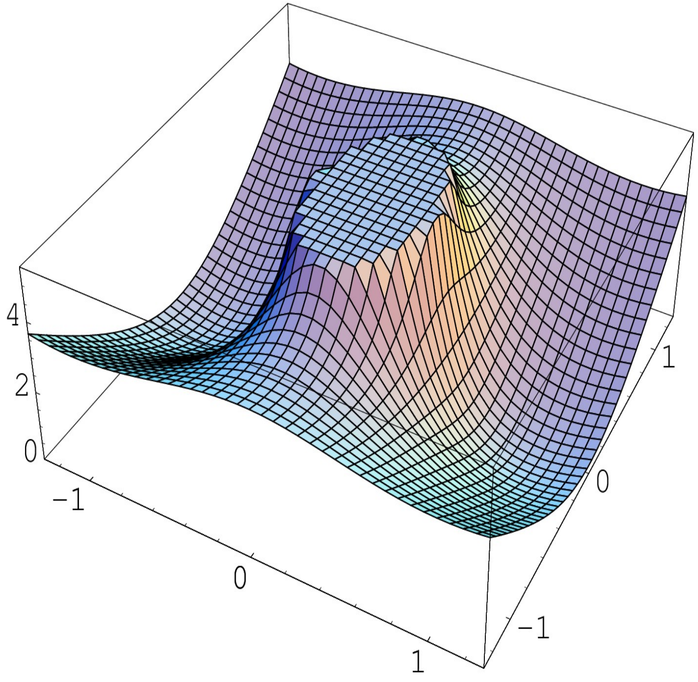
with $\xi = 3.973$ [RH, 2014]

$\Longrightarrow$ **(*) is expected if neutrinos are single center-vortex loops in the confining phase of (other) SU(2) Yang-Mills theories**

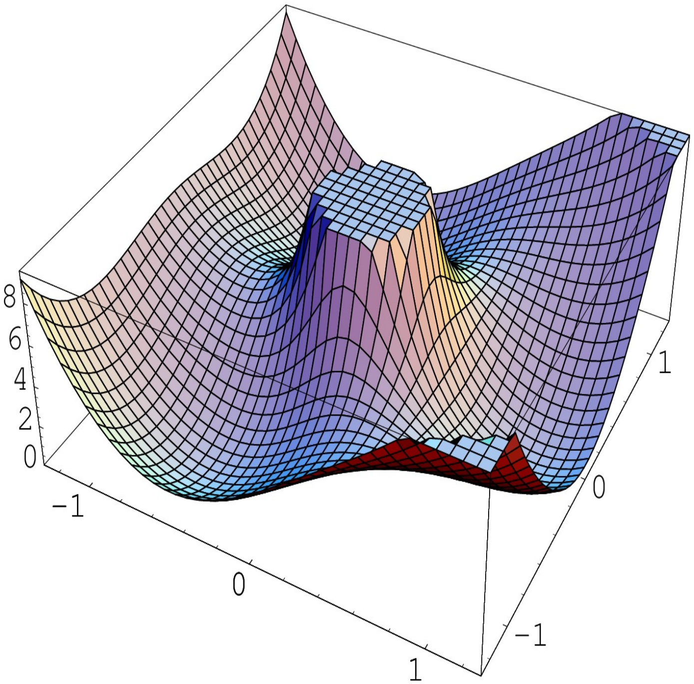
phase structure



Hagedorn transition



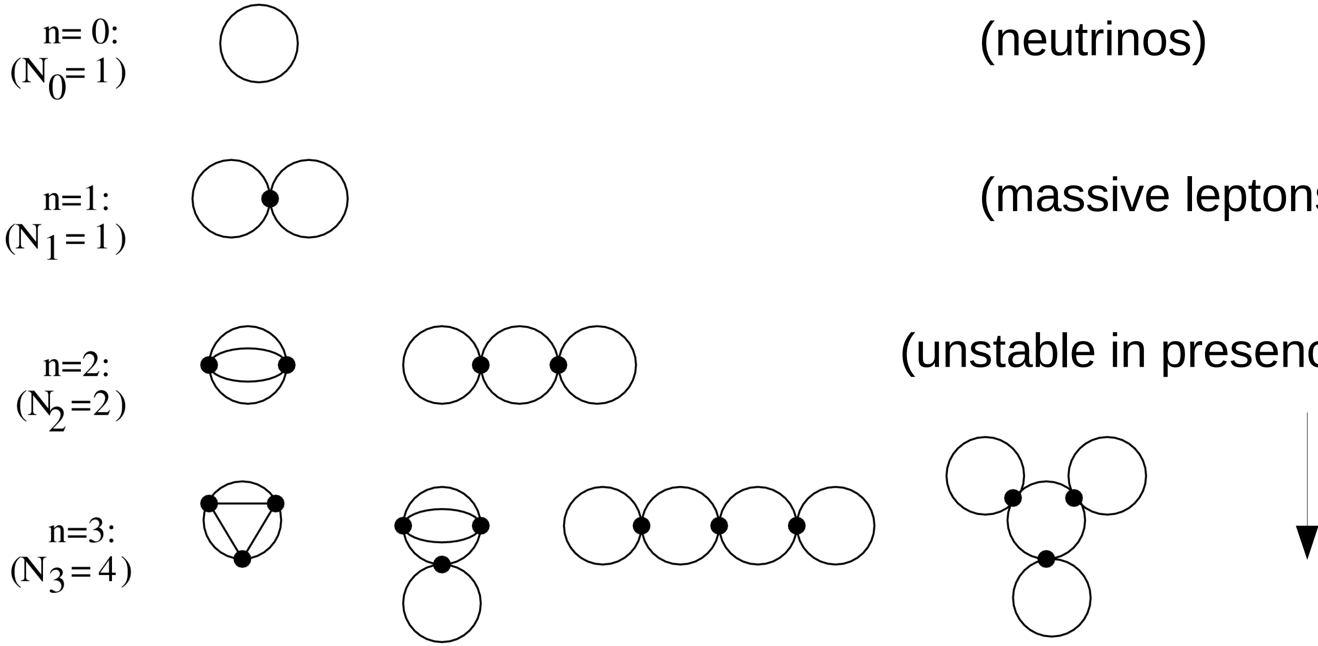
SU(2)



SU(3)

- at T_c' no propagating gauge modes, decay of ground state described by local magnetic center jumps ('t Hooft loop relaxes to minimum of potential by creating units of center flux) [Scheffler, Stamatescu, RH 2007; RH 2007]

spectrum and pressure, confining phase



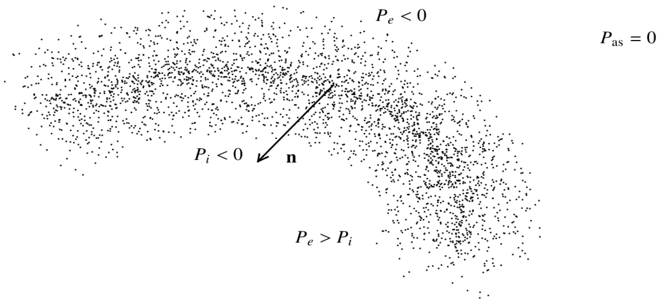
- multipicity = number of bubble diagrams in $\lambda\phi^4$ theory
- Borel resummation of thermodynamical pressure [Bender & Wu, 1969, 1971]

$$P_{\rm as} \sim \frac{\Lambda^4}{2\pi^2}\hat{\beta}^{-4}\left(\frac{7\pi^4}{180} + \sqrt{2\pi}\,\hat{\beta}^{\frac{3}{2}}\sum_{l=0}^{L} a_l \sum_{n\geq 1}(32\lambda)^n\, n!\, n^{\frac{3}{2}+l}\right)\,,(\hat{\beta}\equiv\frac{\Lambda}{T}\,,\lambda\equiv\exp(-\hat{\beta}))\,.$$

$\Longrightarrow$ analytic continuation and inverse Borel
pressure is not thermodynamic but contains imaginary
contamination which outgrows real part with increasing „temperature“

[RH 2007]

evolving n=0 and n=1 center vortex loops
(J. Moosmann and RH, 2008)

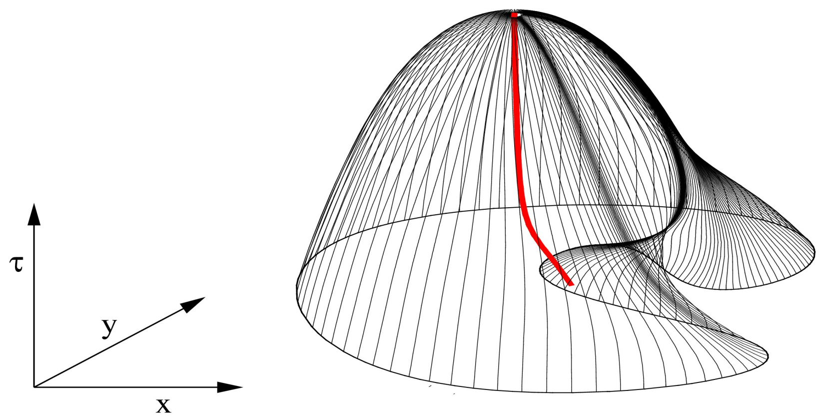


- shrinking of vortex loop by 1-dim analog of Ricci flow
  (curve shrinking equation): $$\partial_\tau \vec{x} = \frac{1}{\sigma}\,\partial_s^2 \vec{x}\,,$$

  where s arc length, σ string tension, and τ (dimensionless) flow parameter
- upon re-scaling $\hat{x} \equiv \sqrt{\sigma}\vec{x}, \xi = \sqrt{\sigma} s \;\Rightarrow$

$$\partial_\tau \hat{x} = \partial_\xi^2 \hat{x}$$

evolving n=0 center vortex loops



n=0 CVL shrinks into round point after finite „time“ $\tau = T$

[Gage & Hamilton 1886, Grayson 1987]

- consider an ensemble of n=0 CVLs of same area A_0
- factorise effective „action“ into a conformal and nonconformal part
  $S = F_c \times F_{nc}$ in accord with 2D Euclidean point symmetry
  and demand conformal invariance for curve length $L \to \infty$, e.g.

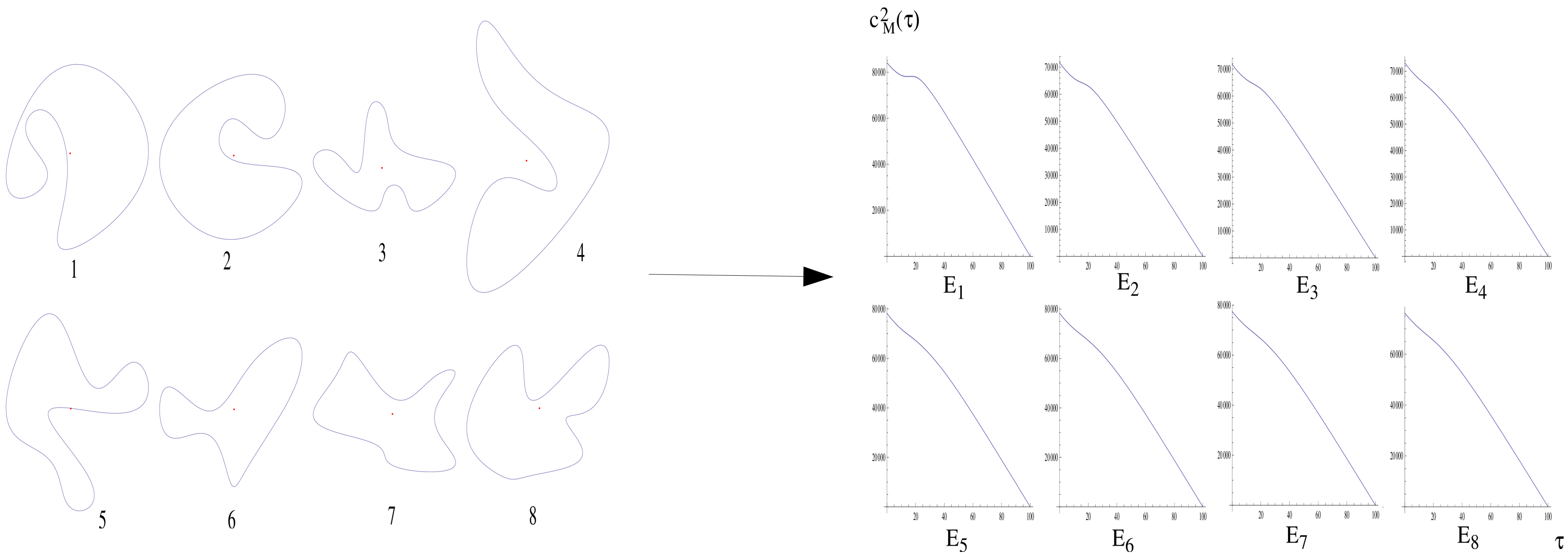
$$S = \frac{L(\tau)^2}{A(\tau)} \left(1 + \frac{c(\tau)}{L(\tau)}\right)$$

isoperimetric ratio

- determine $c(\tau)$ from $\frac{d}{d\tau} Z = 0$ under curve shrinking

evolving n=0 center vortex loops, cntd.

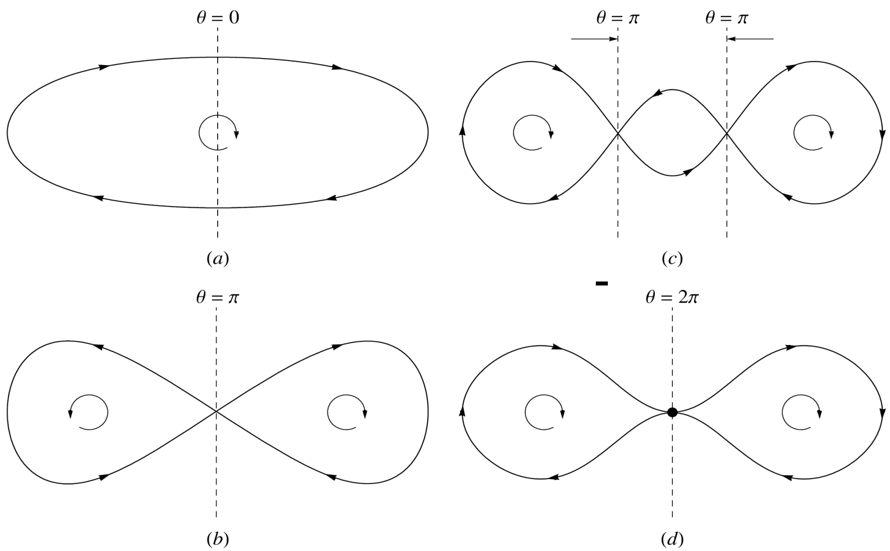
- $c(\tau)$ acts as order parameter for restoration of conformal symmetry when limit of round points is reached



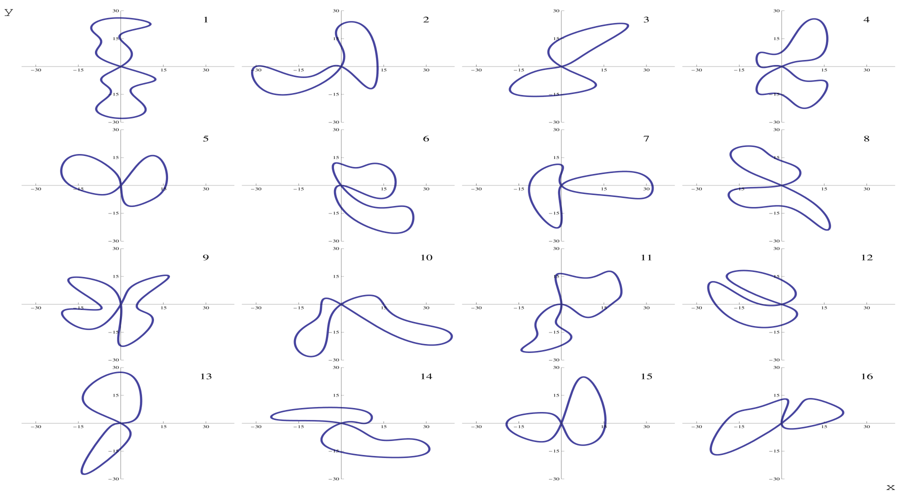
- even in sense of an ensemble average, n=0 CVLs disappear from spectrum at **finite resolution** (mass gap)

evolving n=1 center vortex loops

- topological transition from n=0 to n=1 by twisting and pinching



- ensemble of n=1 CVLs



evolving n=1 center vortex loops, cntd.

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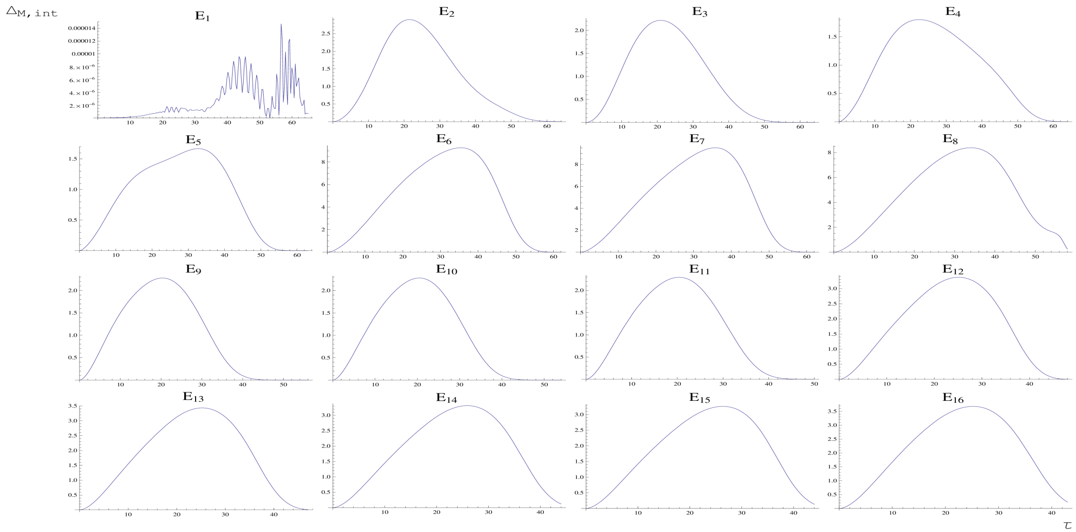
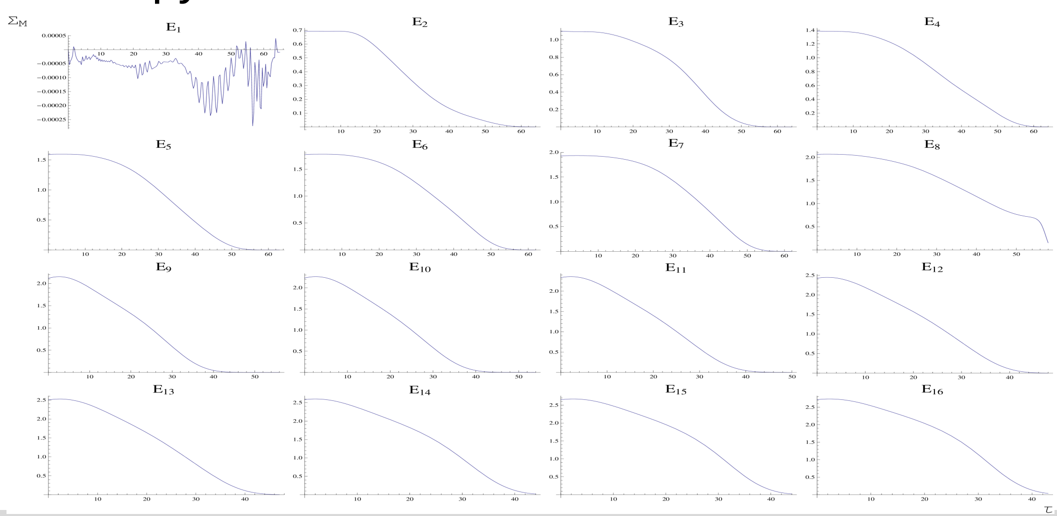
- evolution of variance of position of intersection point

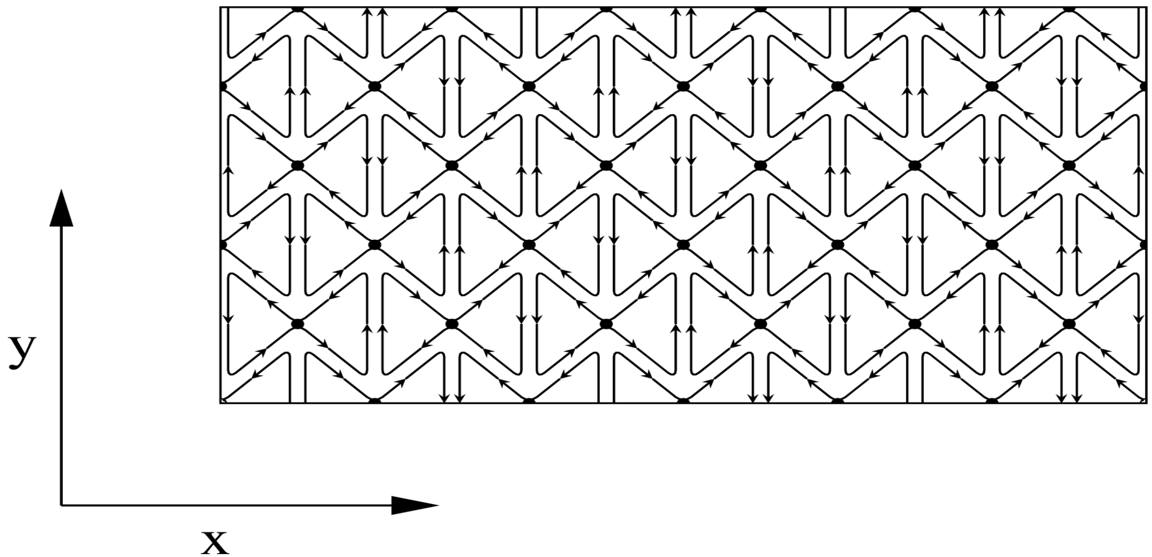


- evolution of entropy



evolving n=1 center vortex loops, cntd.

- possible configuration of planar electrons in high Tc superconducting state?



Summary

- SU(2) thermodynamics nonperturbatively:
  caloron, thermal ground state, adjoint Higgs mechanism, caloron action=$\hbar$

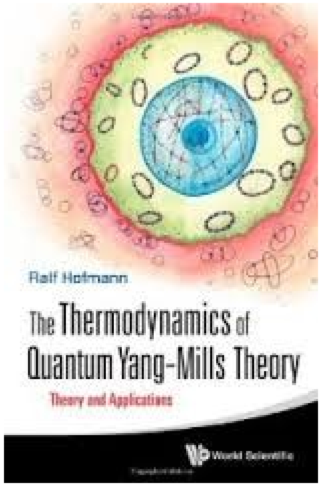
- blackbody anomaly:
  thermal photon dispersion, critical temperature for dec.-prec. PT from low-frequency spectral anomaly (Arcade2, terrestial radio-frequency CMB observations), longitudinal (magnetic) charge density waves

- CMB large-angle anomalies:
  Yang-Mills favours **negative temperature fluctuations**, semiquantitative model, cosmic neutrinos and relativistic vector modes

- confining phase: nonthermal Hagedorn transition, n=0,1 as stable solitons,
  curve shrinking as a renormalisation group flow, mass gap for n=0,
  collapse of ensemble onto one member n=1 (entropy vanishes at finite resolution)

Thank you.

more info in:



(World Scientific, 2011)

and:

- R. Hofmann, " Relic photon temperature versus redshift and the cosmic neutrino background", arxiv: 1407.1266 [physics, hep-th], subm. Ann. d. Physik
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- N. Krasowski and R. Hofmann, "Evidence for absence of one-loop photon-photon scattering in a thermal, deconfining SU(2) Yang-Mills plasma", Annals of Physics **347**, 287-308 (2014); arxiv:1301.4716 [physics, hep-th]
- R. Hofmann and D. Kaviani, "The quantum of action and finiteness of Radiative corrections: Deconfining SU(2) Yang-Mills thermodynamics", Quantum Matter **1**, 41-52 (2012)