Nonperturbative approach to Yang-Mills thermodynamics

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outline

- motivation: failure of weak coupling expansions
- preview on phase diagram for SU(2)/SU(3)
- **deconfining phase:**
  – emerging, inert, adjoint scalar field
  – caloron interactions: pure gauge
  – thermal quasiparticle excitations
  – evolution of effective (electric) coupling
- **preconfining phase:**
  – emerging, inert, complex scalar field
  – monopole interactions: pure gauge
  – thermal quasiparticle excitations
  – evolution of effective (magnetic) coupling

- temperature dependence of thermodynamical quantities
- **confining phase:**
  – emerging complex scalar field
  – estimate for density of (spin-1/2) states (Hagedorn)
  – zero ground-state pressure and energy density
- summary

motivation

- **weak coupling expansions:**

  – magnetic gluons *weakly* screened (absence of sufficiently strong IR cutoff)$\Rightarrow$ break-down of PT at O(g^6)

  [Shuryak1979, Linde 1980,...]

  – resummations: T dependent UV divergences

  [Rebhan, Blaizot, Iancu, ... 1995-present]

  – effective theories: integrated-out hard modes ($p \sim T$) $\Rightarrow$ nonlocal and still IR unstable

  [Braaten&Pisarski 1988]

- **nonperturbative contributions:**

– Polyakov 1975:

IR problem resolved by correlating nontrivial topology (excl. in weak coupling expansions by ess. sing. of weight at $g = 0$)

– indeed: lattice sees spatial string tension $\sim T^2$

[Korthals-Altes, Hart&Philipsen, Engelhardt&Reinhardt& Langfeld, ... 1988-present]

– but: (incorrect) argument

excl. nontrivial-holonomy calorons

[Gross&Pisarski&Yaffe 1981]

preview on phase diagram

confining

ground state:

Cooper−pair condensate of single center−vortex loops, pressure precisely zero

excitations:

massless (single) and massive (self−intersecting) center−vortex loops (spin−1/2 fermions)

preconfining

ground state:

condensate of magnetic monopoles, collapsing center−vortex loops, negative pressure

excitations:

massive dual gauge modes

deconfining

ground state: interacting calorons and anticalorons, negative pressure

excitations: massless and massive gauge modes

power−like approach to Stefan Boltzmann limit

Hagedorn

2nd order like

intermezzo: SU(2) calorons

- **Harrington-Shepard solution ($Q = \pm 1$):**

$$A_\mu^C(\tau,\vec{x}) = \bar{\eta}_{a\mu\nu}\frac{\lambda^a}{2}\partial_\nu \ln \Pi(\tau,\vec{x}) \quad \text{where}$$

$$\Pi(\tau,\vec{x}) = \Pi(\tau,r) \equiv 1 + \frac{\pi\rho^2}{\beta r}\frac{\sinh\left(\frac{2\pi r}{\beta}\right)}{\cosh\left(\frac{2\pi r}{\beta}\right)-\cos\left(\frac{2\pi \tau}{\beta}\right)}\,.$$

– $F_{\mu\nu}[A^{C,A}] = (+,-)\tilde{F}_{\mu\nu}$ (BPS) $\Rightarrow$

$\theta_{\mu\nu}[A^{C,A}] \equiv 0\,,\;\; S[A^{C,A}] = \frac{8\pi^2}{g^2}$

– trivial holonomy: $\mathcal{P}_\infty = \mathbf{1} \;\;\Rightarrow$

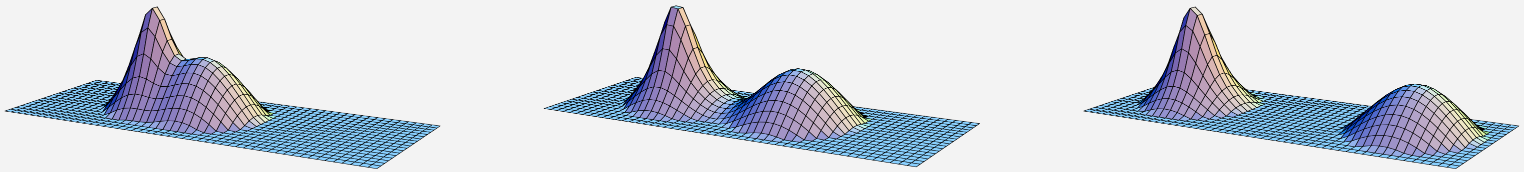
no finite-size (anti)monopole constituents

– stable under 1-loop quantum fluctuations

[Gross&Pisarski&Yaffe 1981]

▶ Lee-Lu-Kraan-van Baal solution ($Q = \pm 1$):

[Nahm 1981, Lee&Lu 1998, Kraan&van Baal 1998]



– $F_{\mu\nu}[A^{C,A}] = (+,-)\tilde{F}_{\mu\nu}$ (BPS) $\Rightarrow$ $S[A^{C,A}] = \frac{8\pi^2}{g^2}$

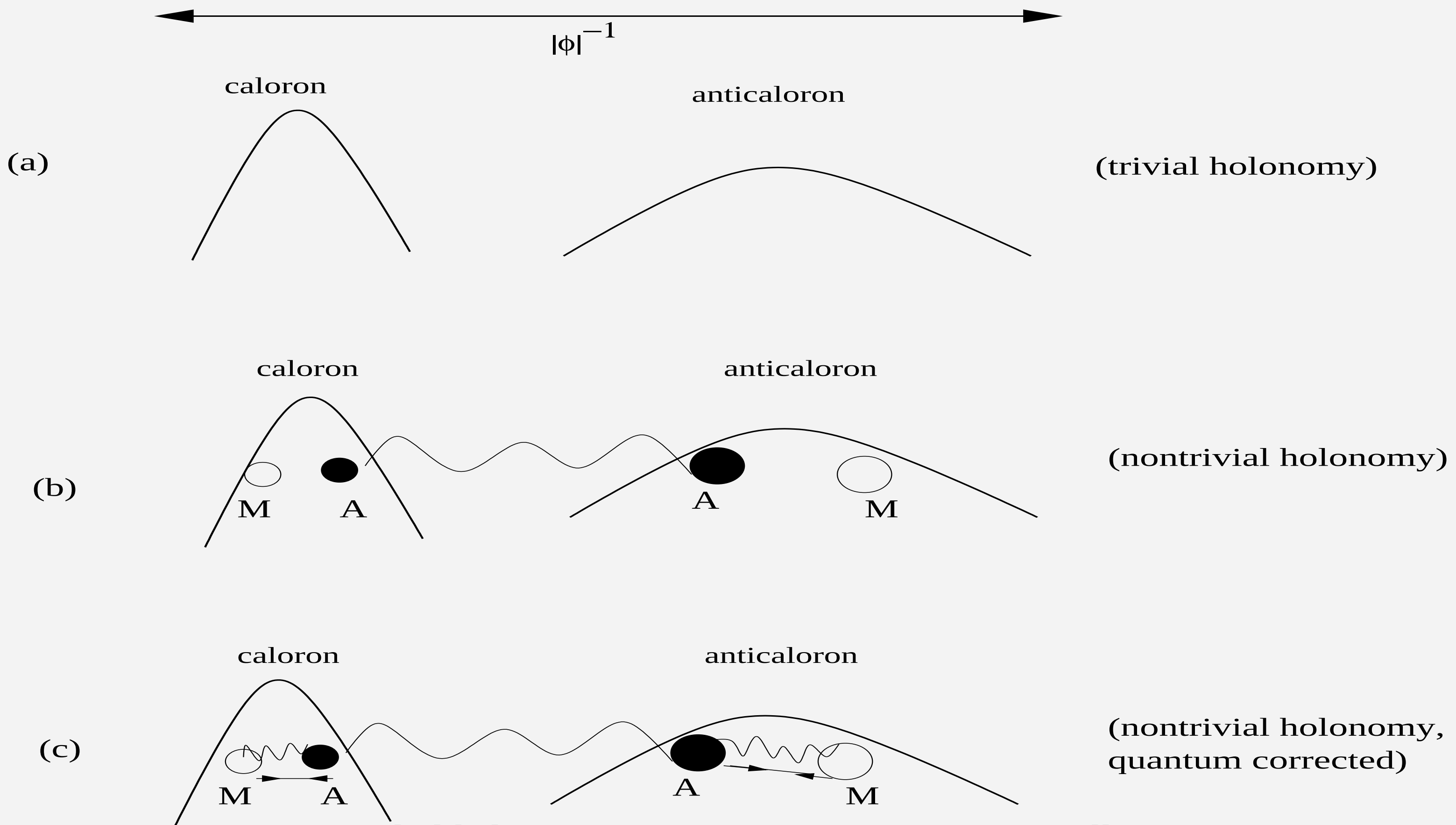
– nontrivial holonomy: $\mathcal{P}_\infty \notin \mathbf{Z}_2$ $\Rightarrow$
finite-size BPS (anti)monopole constituents

– unstable under (1-loop) quantum fluctuations

[Diakonov et al. 2004]

deconfining phase

- emergent adjoint scalar ϕ^a: (general picture)



attraction extremely likely => negative pressure macroscopically
repulsion extremely unlikely => isolated and screened magnetic monopoles

- **phase of emergent adjoint scalar ϕ^a: (outline of derivation)**

– *unique* def. for kernel of diff. operator $\mathcal{D}$ containing ϕ^a's phase $\hat{\phi}^a(\tau)$:

$$\hat{\phi}^a(\tau) \sim \sum_{\text{HS (anti)caloron}} \mathrm{tr} \int d^3x \int d\rho \, \frac{\lambda^a}{2} \times$$

$$F_{\mu\nu}\left((\tau,0)\right) \left\{(\tau,0),(\tau,\vec{x})\right\} \times$$

$$F_{\mu\nu}\left((\tau,\vec{x})\right) \left\{(\tau,\vec{x}),(\tau,0)\right\} \, .$$

why unique:

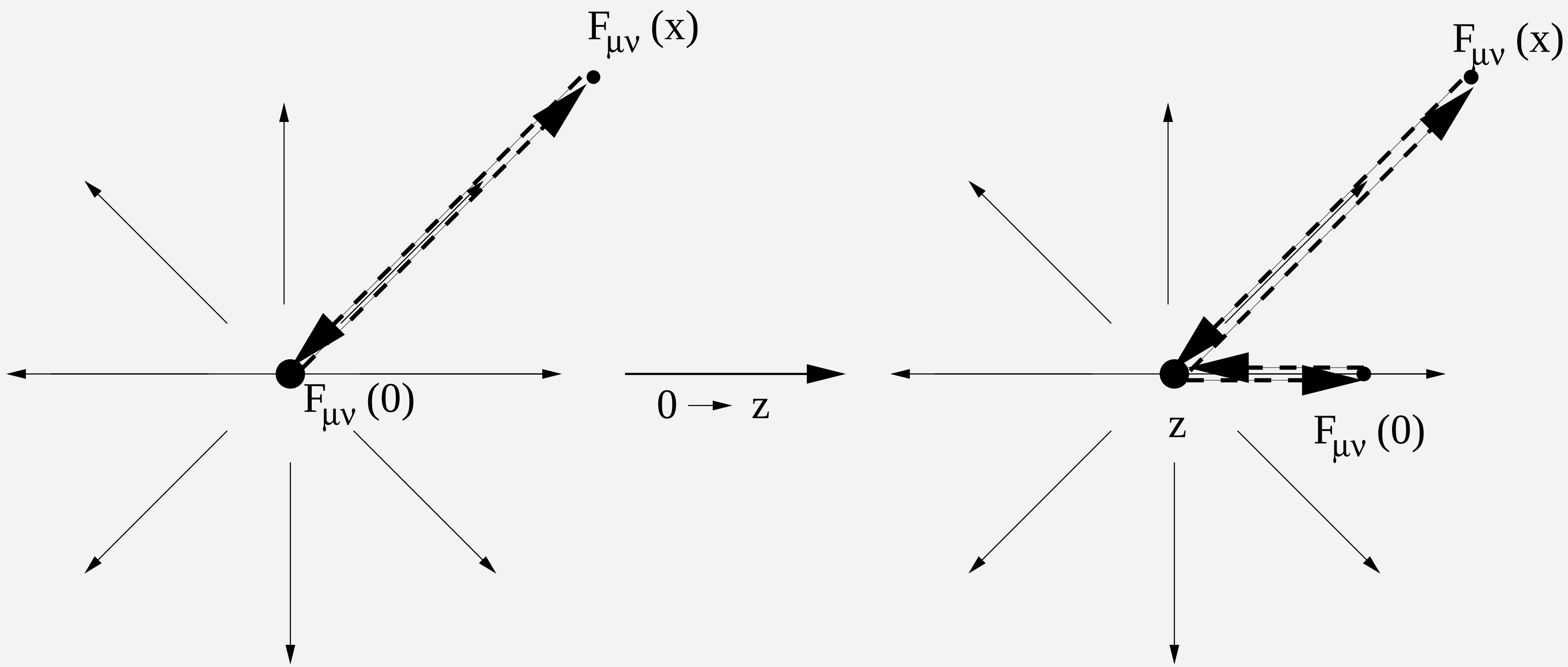
– periodic phase $\hat{\phi}^a$ does not know about dim. transm. $\Rightarrow$ saturated by (exact and stable) sol. to field equations (low. act. in top. sector, stability $\Leftrightarrow$ BPS and triv. hol.)

– local definition $\equiv 0$ (BPS)

– ratio of dim. quantities $\Rightarrow$ integr. over admissible moduli with flat measure ($d\rho$)

– no explicit dependence on T (action T indep.)

– shift invariance $0 \to \vec{z} \Rightarrow$ average already performed


$F_{\mu\nu}$ (x)
$F_{\mu\nu}$ (0)
0 → z
$F_{\mu\nu}$ (x)
z
$F_{\mu\nu}$ (0)

why unique (cntd):

– no higher n-point functions (dim. power counting)

– no higher top. charge than $Q = \pm 1$
(dim. power counting excludes average over extra dimensionful moduli)

– straight Wilson lines (absence of mass scale)

– nonadmissible moduli-averages:

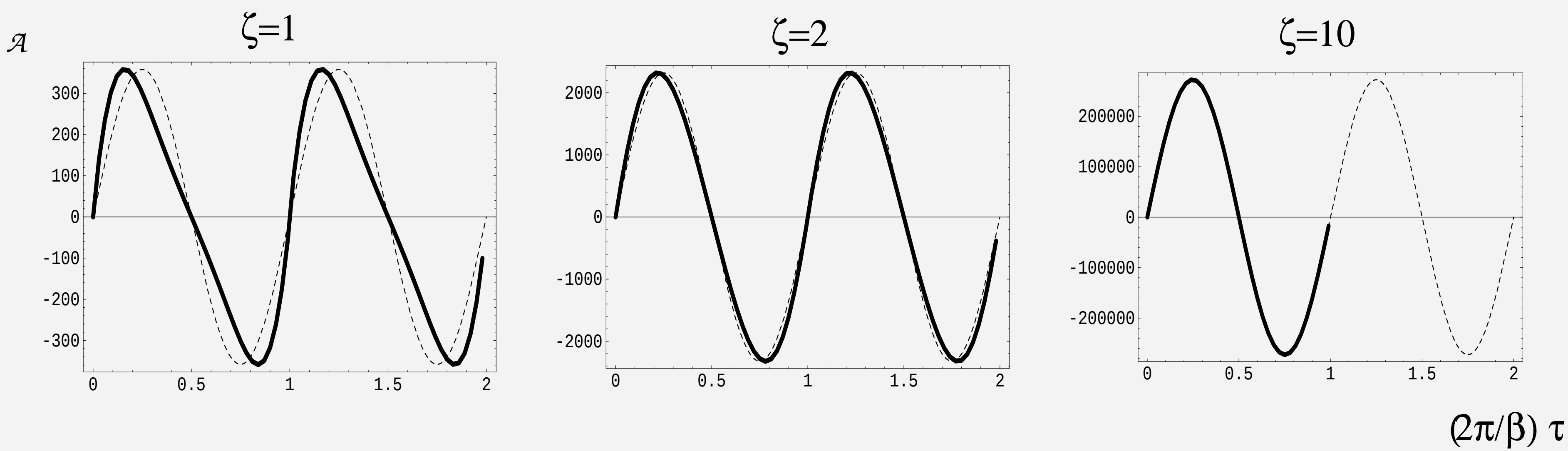
* global gauge rotations (gauge noninvariant object))

* temporal shifts (periodicity)

result of calculation:

– undetermined gauge orientation for
(anti)caloron contr. (angular reg.)

– upon ρ integration with IR cutoff $\beta\xi$:
(amplitude for (anti)caloron contr. modulo phase shift
and undeterm. normalization [4 parameters])



$\Rightarrow$

– set of undetermined parameters spans entire kernel of

$$\mathcal{D} = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2$$

$\Rightarrow \mathcal{D}$ uniquely determined

– since for $\phi^a = |\phi|\hat{\phi}^a(\tau)$ we have $\bar{\theta}_{\mu\nu}[\phi] \equiv 0$

$\Rightarrow \hat{\phi}$ BPS

– for fixed global gauge we have

$$\partial_\tau \hat{\phi} = \pm \frac{2\pi i}{\beta} \lambda_3 \, \hat{\phi}$$

$$\Rightarrow \hat{\phi} = C \, \lambda_1 \, \exp\left(\pm \frac{2\pi i}{\beta} \lambda_3 (\tau - \tau_0)\right)$$

$\Rightarrow 4 \rightarrow 2$ parameters by BPS saturation

- **modulus of emergent adjoint scalar ϕ^a:**

– assume existence of a Yang-Mills scale Λ

– RHS of BPS equation for $\phi^a = |\phi|\hat{\phi}^a(\tau)$

* analytic in ϕ

* no explicit T dependence (resol. $|\phi|$ selfconsist.)

$\Rightarrow$ only viable possibility:

$\partial_\tau \phi = \pm i\,\Lambda^3\,\lambda_3\,\phi^{-1}$ where $\phi^{-1} \equiv \frac{\phi}{|\phi|^2}$.

$\Rightarrow |\phi| = \sqrt{\frac{\Lambda^3}{2\pi\,T}}$.

- **(coarse-grained) effective action and ground state:**

– RHS of BPS equation def. square root

of potential $V(|\phi|)$

– observation: $\frac{\partial^2_{|\phi|} V(|\phi|)}{|\phi|^2} \gg 1$ and $\frac{\partial^2_{|\phi|} V(|\phi|)}{T^2} \gg 1$

$\Rightarrow \phi$ fluctuates

neither quantum mechanically nor statistically (background)

– effective action:

$$S = \mathrm{tr} \int_0^\beta d\tau \int d^3x \left(\tfrac{1}{2} G_{\mu\nu} G_{\mu\nu} + D_\mu \phi D_\mu \phi + \Lambda^6 \phi^{-2}\right)$$

– solution to $D_\mu G_{\mu\nu} = 2ie[\phi, D_\nu \phi]$:

pure gauge $a_\mu^{bg} = \frac{\pi}{e} T \delta_{\mu 4} \lambda_3$

– since $G_{\mu\nu}[a^{bg}] = D_\nu[a^{bg}]\phi \equiv 0$

$\Rightarrow$ ground-state energy-density and pressure

$$\rho^{g.s} = 4\pi \Lambda^3 T = -P^{g.s} \neq 0$$

(holonomy-changing gluon exchanges & rad.cors. around const. (anti)monopoles imply

$\rho^{g.s} = P^{g.s} = 0 \rightarrow \rho^{g.s}, P^{g.s} \neq 0$)

thermal quasiparticle excitations:

rotation to unitary gauge $a_\mu^{bg} = 0$:

– gauge transformation singular but admissible (does not affect periodicity of fluct. δa_μ)

– but: $\mathrm{Pol}[a^{bg}] = -\mathbf{1} \stackrel{GT}{\rightarrow} \mathrm{Pol}[a^{bg}] = +\mathbf{1}$ and $\langle \mathrm{Pol} \rangle \neq 0$ (SU(2)$\longrightarrow$U(1)$\Rightarrow$ ex. massl. modes) $\Rightarrow$ deconfinement

adjoint Higgs mechanism:

– quasiparticle masses: $m_{W^\pm} = 4\,e^2|\phi| = 4\,e^2\frac{\Lambda_E^3}{2\pi T}$

– but: $m_\gamma = 0$

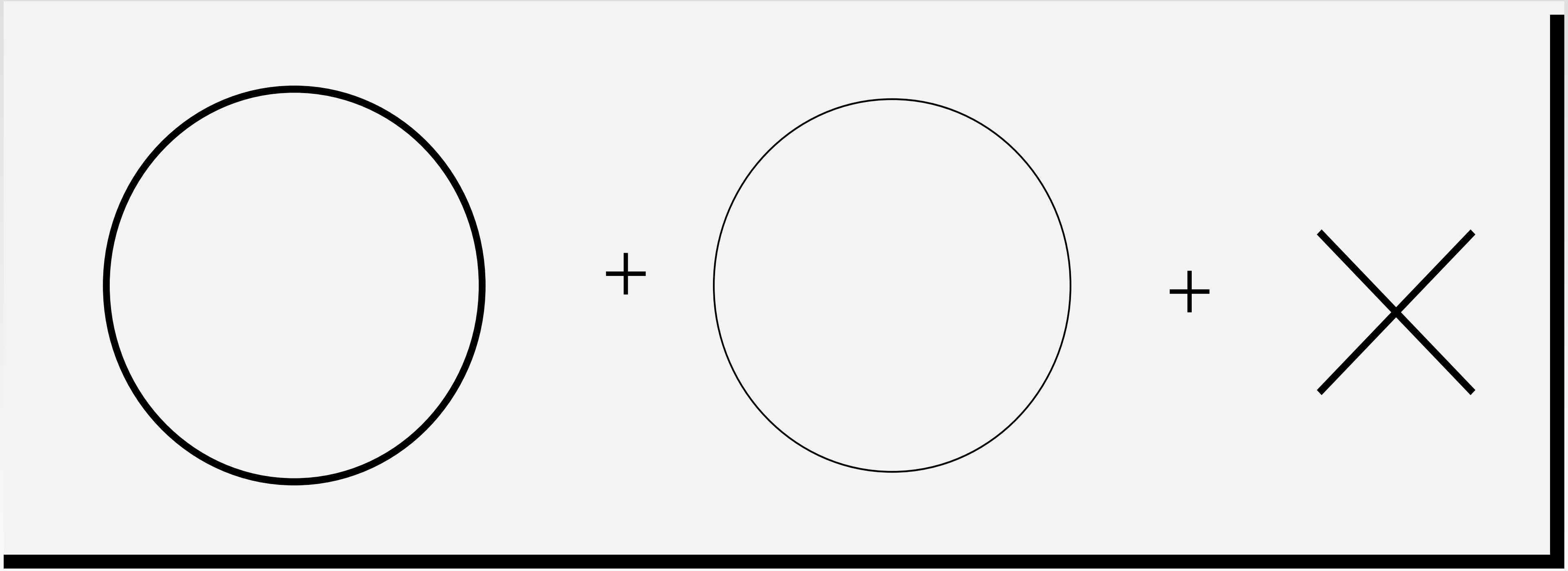
constraints on resolution of quantum fluctuations:

– in physical gauge (unitary-Coulomb): $|p^2 - m^2| \leq |\phi|^2$

$\Rightarrow$ loop exp. of therm. dyn. quant. almost trivial

– by far dominating (pressure P):

thermal part of 1-loop contr.+ ground state



evolution equation for e:

invariance of Legendre transformations:

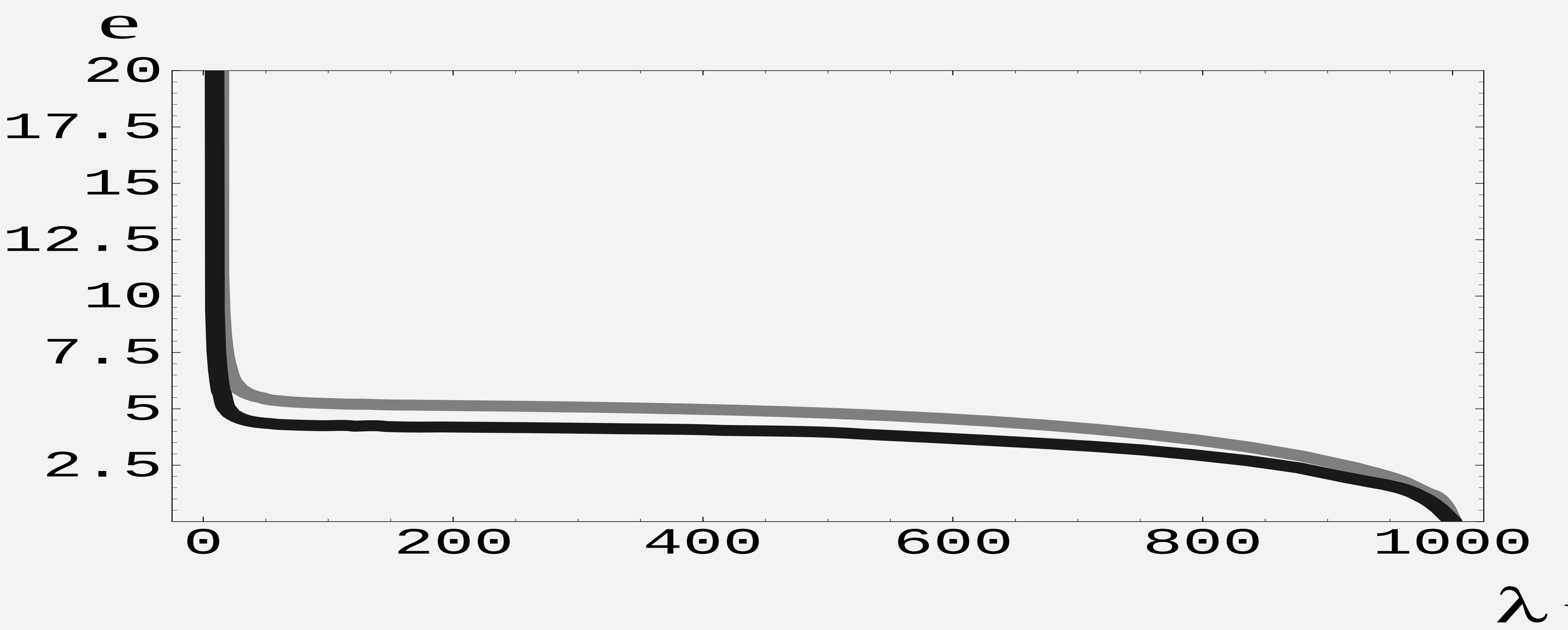
– $\partial_m P = 0 \Rightarrow$

$$\partial_a \lambda = -\frac{24\,\lambda^4\,a}{(2\pi)^6}\,D(2a)\ \ \text{where}$$

$$D(a) \equiv \int_0^\infty dx\,\frac{x^2}{\sqrt{x^2+a^2}}\,\frac{1}{\exp(\sqrt{x^2+a^2})-1}\,,$$

$$a \equiv \frac{m}{T}\,,\quad \lambda \equiv \frac{2\pi T}{\Lambda}\,.$$

– after inversion: $\lambda(a) \to a(\lambda) \Rightarrow$



– IR-UV decoupling

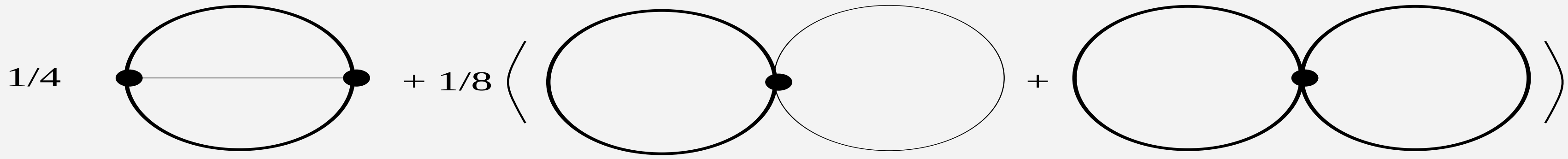
– logarithmic pole: $e \sim -\log(\lambda - \lambda_c)$

(total screening of isolated monopoles,

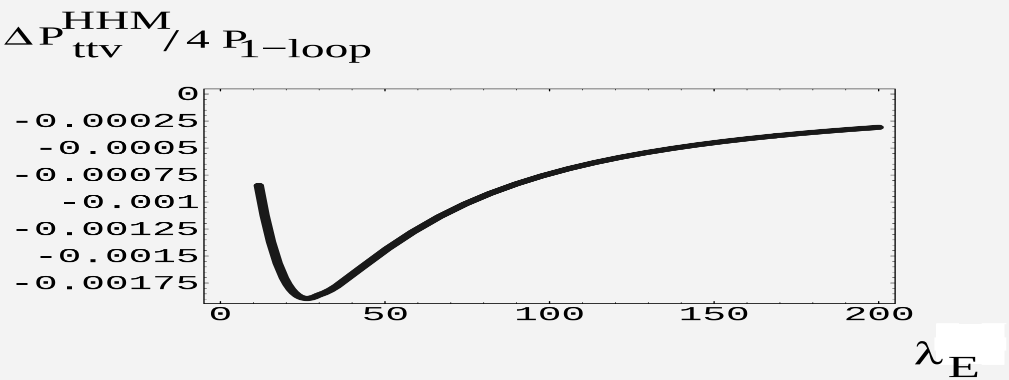
instability towards large holonomy, cond. of monop.)

two-loop corrections to P:

– dominating diagram: nonlocal



– relative correction:

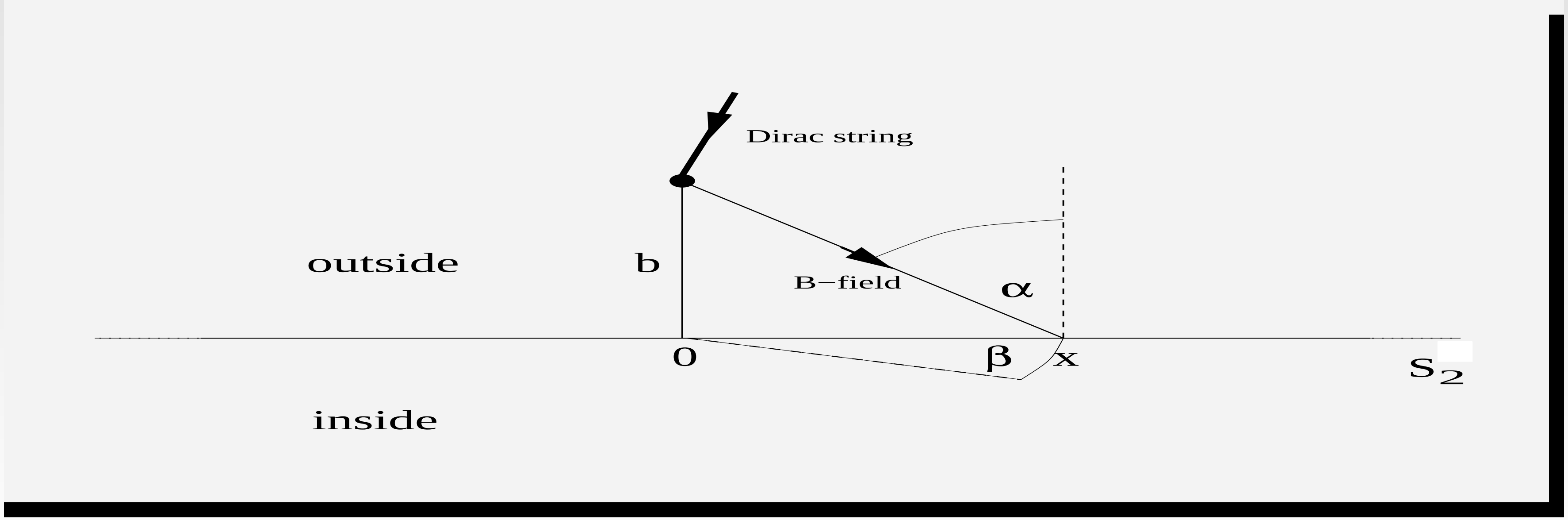


preconfi ning phase

- **inert, complex scalar field φ upon spatial coarse-graining:**

  φ's phase $\hat{\varphi}$:

  consider magnetic flux of a monopole-antimonopole system through $S^2_{R=\infty}$:
  (massless and each spatial momentum $= 0$)



– after thermal average:

$$\lim_{e\to\infty} \bar{F}_{\pm,\mathrm{th}}(\delta) = \pm\frac{\delta}{\pi}\,, \qquad (0 \le \delta \le \pi)\,.$$

– identification: $\frac{\delta}{\pi} = \frac{\tau}{\beta}$

– $\hat{\varphi}$ satisfies

$$\partial_\tau^2 \hat{\varphi} + \left(\frac{2\pi}{\beta}\right)^2 \hat{\varphi} = 0\,.$$

φ's modulus:

– BPS sat., exist. of YM-scale, analyticity of $\sqrt{V(\varphi)}$

$$\Rightarrow \partial_\tau \varphi = \pm i \frac{\Lambda_M^3\,\varphi}{|\varphi|^2} = \pm i \frac{\Lambda_M^3}{\bar{\varphi}}.$$

pure gauge and ground state:

– φ inert

– effective action:

$$S = \int_0^\beta d\tau \int d^3x \, \left[\frac{1}{4}\, G^D_{\mu\nu} G^D_{\mu\nu} + \frac{1}{2}\overline{\mathcal{D}_\mu\varphi}\mathcal{D}_\mu\varphi + \frac{1}{2}\frac{\Lambda_M^6}{\bar{\varphi}\varphi}\right]$$

– pure gauge solution to $\partial_\mu G^D_{\mu\nu} = ig\left[\overline{\mathcal{D}_\nu\varphi}\varphi - \bar{\varphi}\mathcal{D}_\nu\varphi\right]$:

$$a_\mu^{D,bg} = \pm\delta_{\mu 4}\frac{2\pi}{g\beta}$$

– Polakov loop inert upon rot. to unitary gauge

$\Rightarrow$ test-charge but no total confinement

– ground-state pressure and energy density:

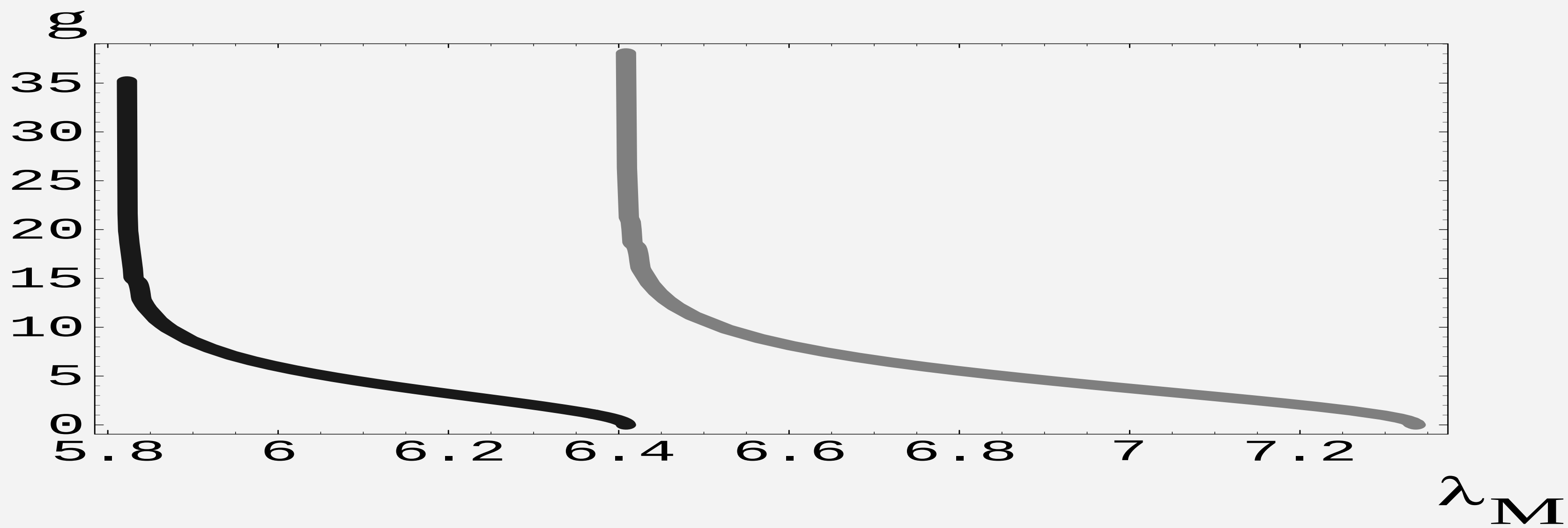
$$\rho^{gs} = \pi\,\Lambda_M^3\,T = -P^{gs}$$

quasiparticle excitations and evolution equation:

– abelian (dual) Higgs mechanism

– inv. of Legendre trafos:

$$\partial_a \lambda_M = -\frac{12}{(2\pi)^6}\,\lambda_M^4\,a\,D(a) \Rightarrow$$

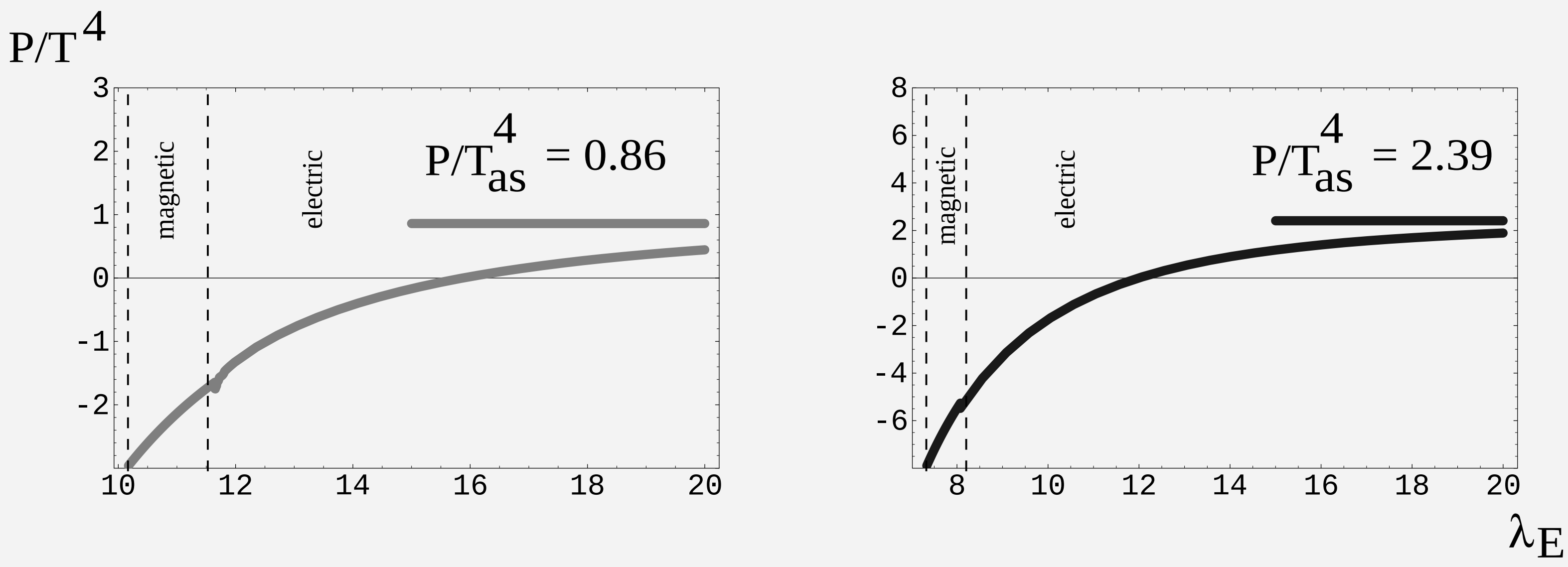


thermodynamical quantities

Scales Λ and Λ_M related by continuity of pressure across deconf.-preconf. phase boundary !

- **pressure (infrared sensitive):**

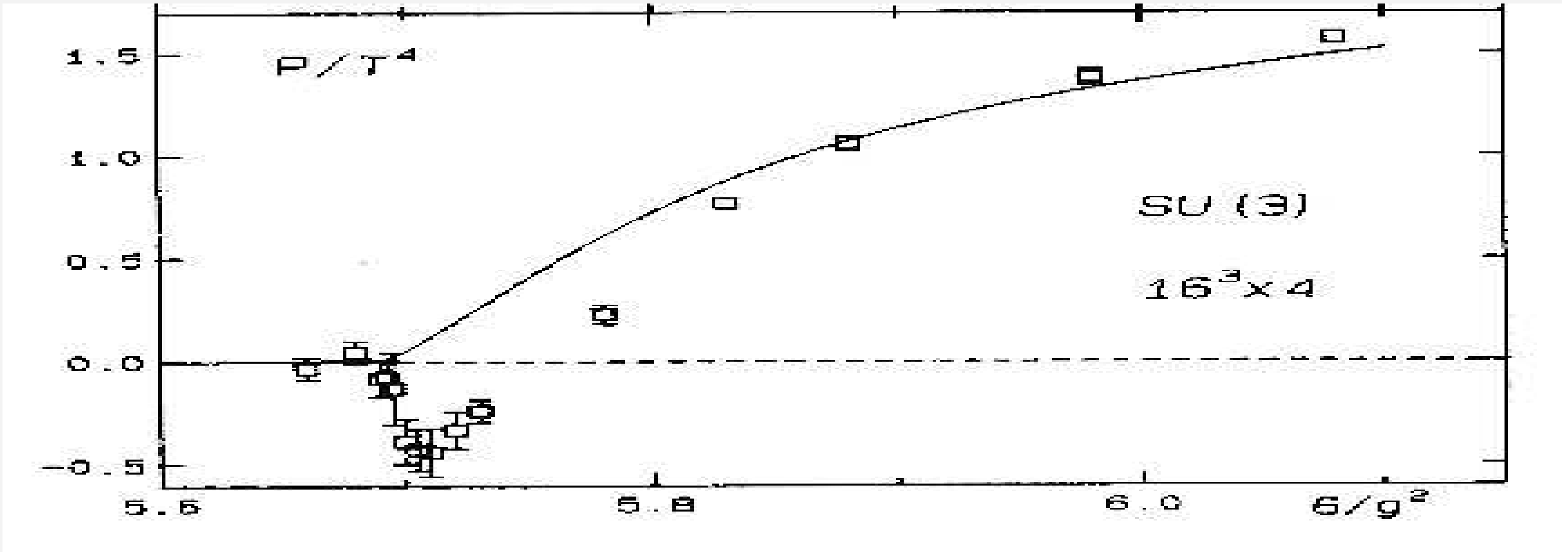
SU(2) SU(3)



- **pressure [lattice]:**

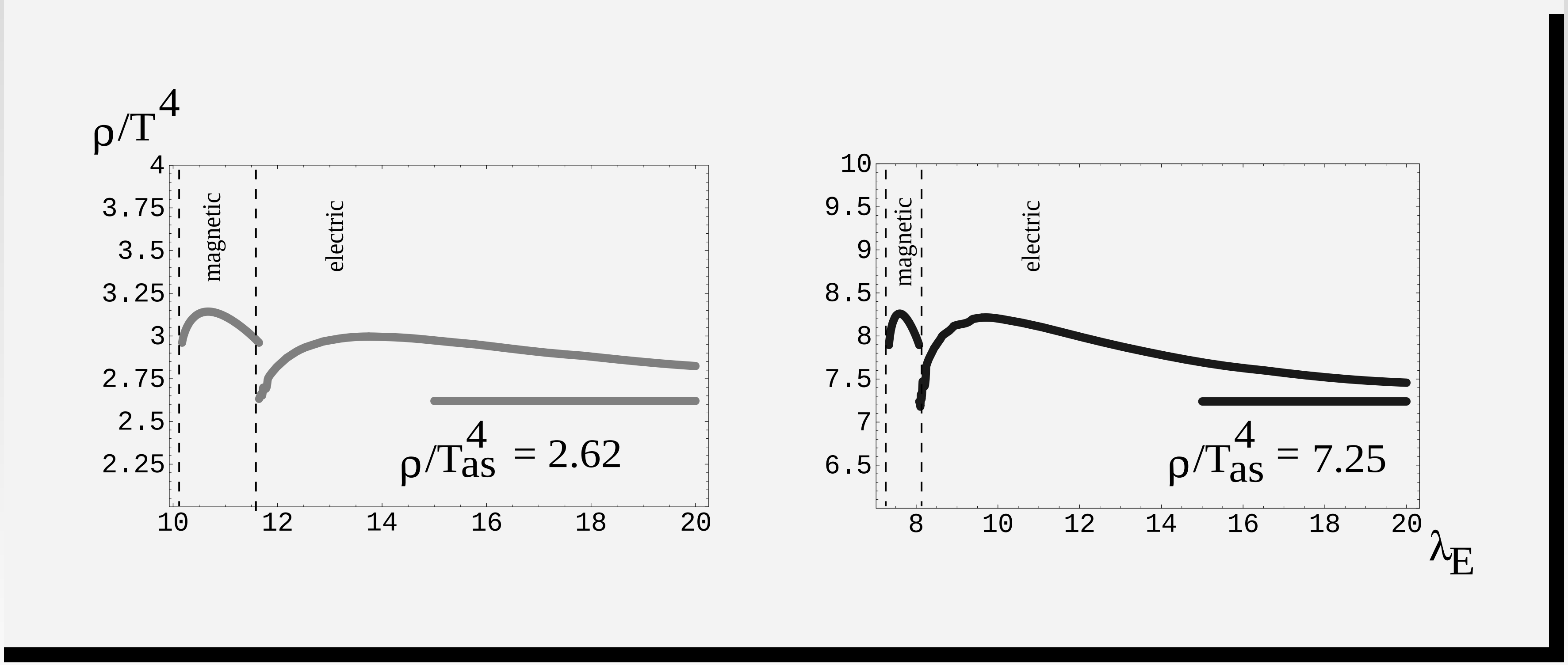
differential method [Brown1988,Deng1988]:

SU(3)



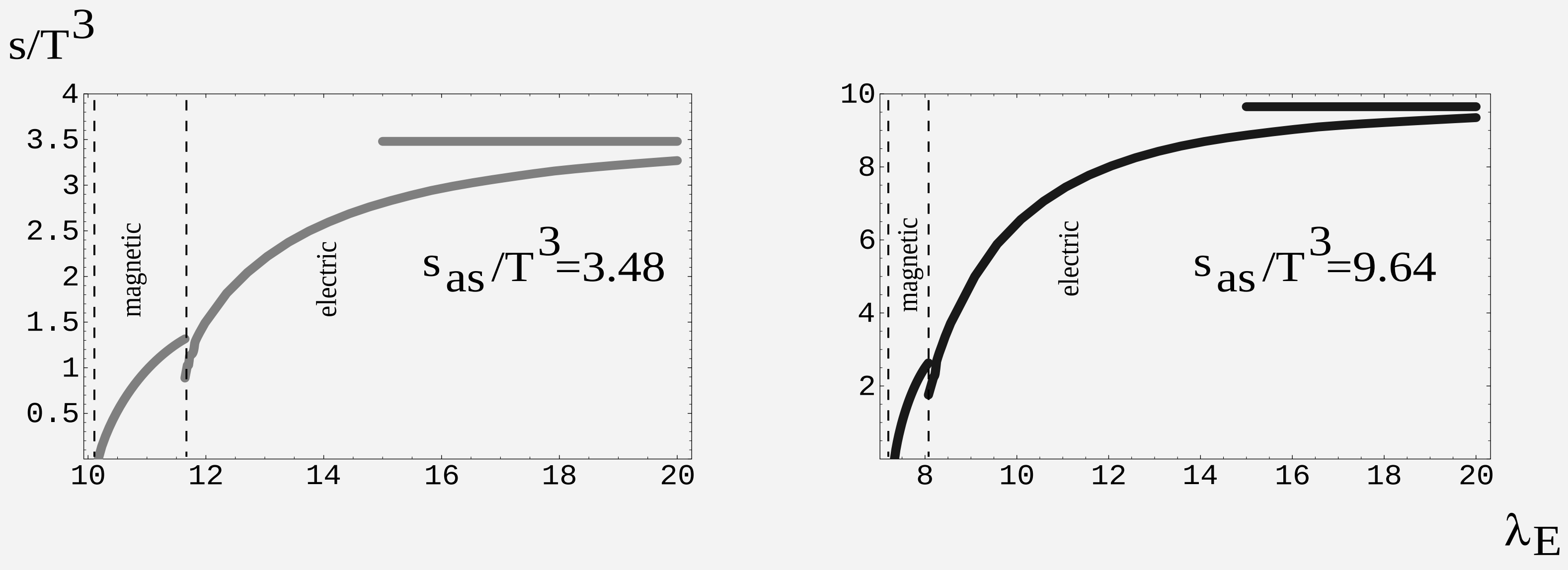
- **energy density (infrared sensitive):**

SU(2) SU(3)



- **entropy density (infrared safe):** $sT = P + \rho$

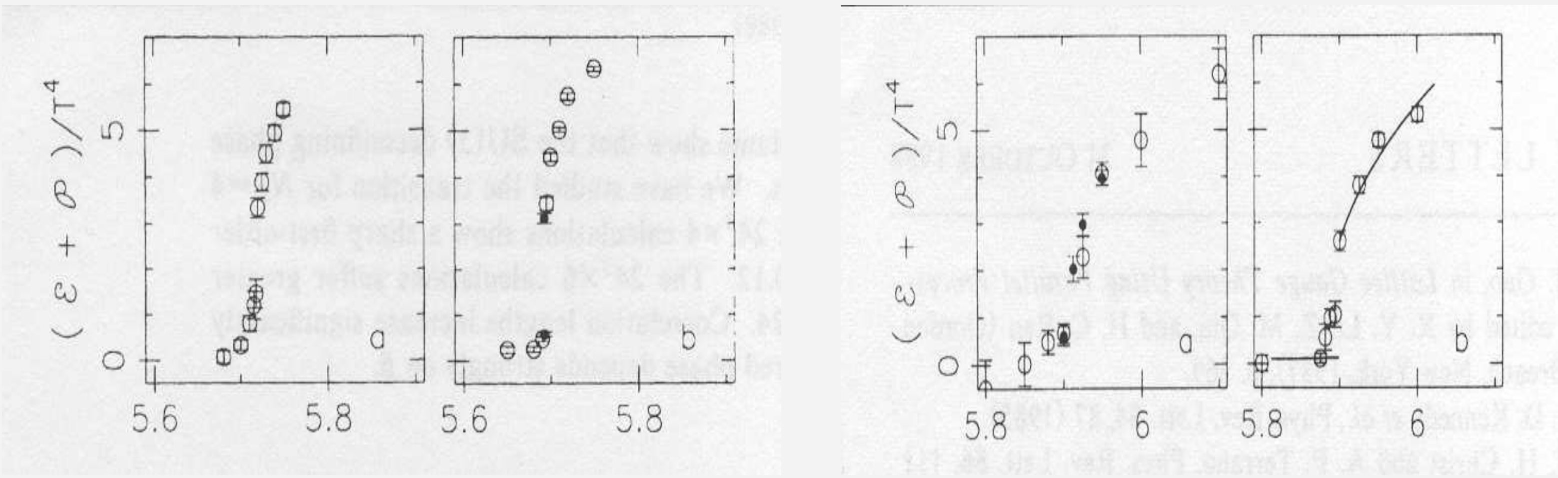
SU(2) SU(3)



entropy density [lattice]:

differential method [Brown1988,Deng1988]:

SU(3)



(a) (b) (c) (d)

confining phase

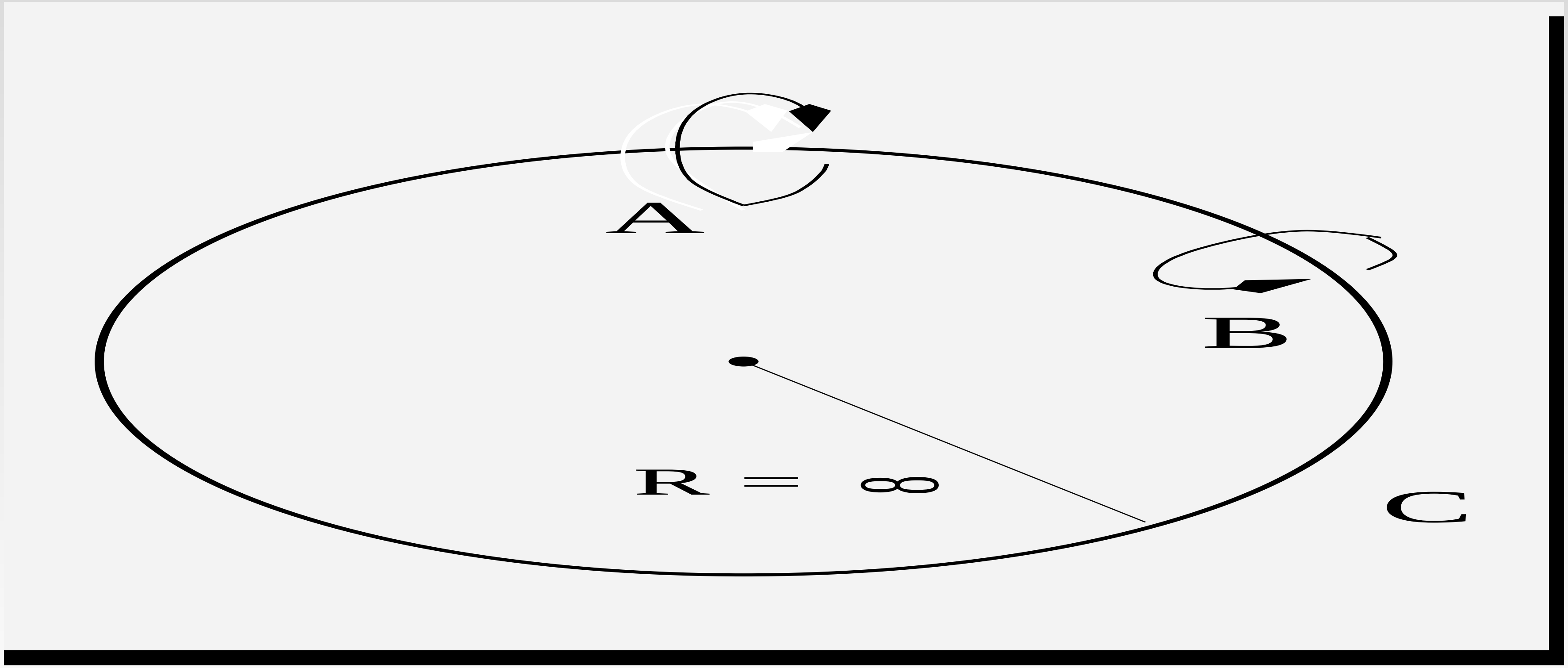
- **emerging complex scalar field Φ:**
  - inside preconfining ($g < \infty$) phase center-vortex loops collapse:

    $$P_v(r) = -\frac{1}{2}\,\frac{\Lambda_M^3\beta}{2\pi}\,\frac{1}{g^2\,r^2}$$

  - for $g \longrightarrow \infty$ single center-vortex loops zero pressure away from their (infinitely thin) cores
    $\Rightarrow$ stable, massless spin-1/2 particles created
  - phase transition charact. by center jumps of order-parameter ('t Hooft loop)

confining phase

- **Φ's phase:**
  - consider $\Gamma_{\frac{\Phi}{|\Phi|}}(x) \equiv \lim_{g\to\infty} \exp[ig \oint_{C(x)} dz_\mu \, (a^D)^\mu]$ where



- **Φ's phase (cntd):**

  – center flux through C:

  $\lim_{g\to\infty,\vec{p}\to 0}\ F_{\pm,0;\mathrm{th}} \propto 0, \pm 1$

  (discrete parameter for Φ's phase)

  – for SU(2): identification of ± 1

  – for SU(3): $0, \pm 1$ describe different, degenerate minima

  – potential: no propagating gauge modes $\Rightarrow$ minima at zero energy density

  – symmetry fixes (generic) potential to be

- **Φ's potential:**

– for SU(2):

$$V_C = \overline{v_C}\, v_C \equiv \overline{\left(\frac{\Lambda_C^3}{\Phi} - \Lambda_C\, \Phi\right)} \left(\frac{\Lambda_C^3}{\Phi} - \Lambda_C\, \Phi\right)$$

– for SU(3):

$$V_C = \overline{v_C}\, v_C \equiv \overline{\left(\frac{\Lambda_C^3}{\Phi} - \Phi^2\right)} \left(\frac{\Lambda_C^3}{\Phi} - \Phi^2\right)$$

▶ quantum fluctuations?

– Φ relaxes to one minimum of $\Phi = \pm\Lambda_C$ (SU(2))
$\Phi = \Lambda_C \exp[\frac{2\pi ik}{3}]\,,\ (k = 0, 1, 2)\,,$ (SU(3))

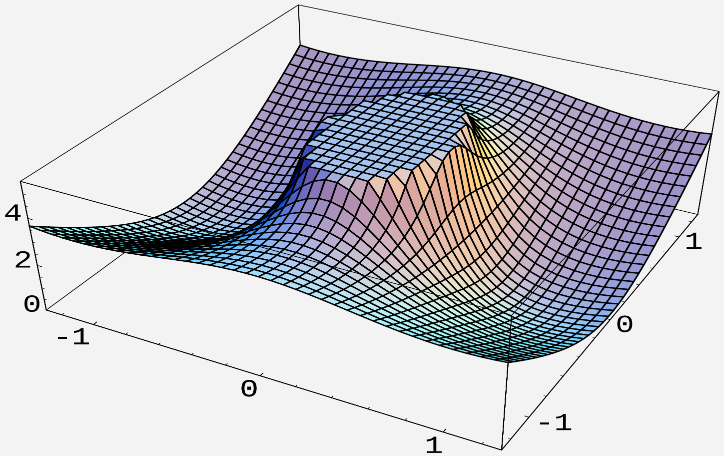
– observation: with $\Phi \equiv |\Phi| \exp\left[i\frac{\theta}{\Lambda}\right]$

$$\left.\frac{\partial_\theta^2 V_C(\Phi)}{|\Phi|^2}\right|_{\Phi_{\text{min}}} = \left.\frac{\partial_{|\Phi|}^2 V_C(\Phi)}{|\Phi|^2}\right|_{\Phi_{\text{min}}}$$

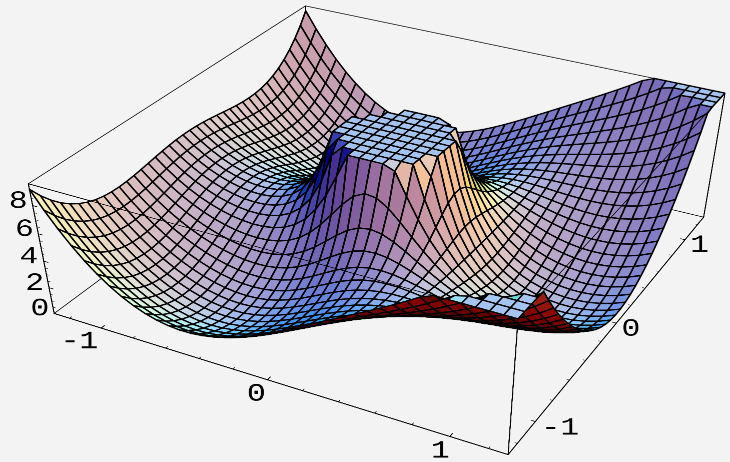
$$= \begin{cases} 8 & (\text{SU}(2)) \\ 18 & (\text{SU}(3)) \end{cases} .$$

$\Rightarrow$ quantum fluctuations ABSENT!

Φ's potential (cntd):



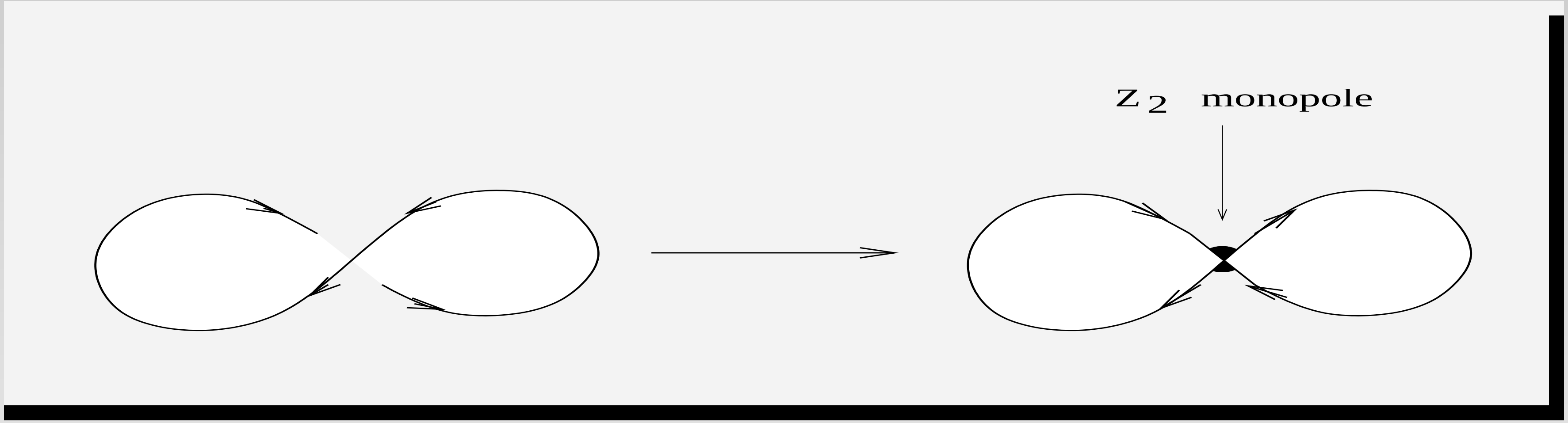
SU(2)



SU(3)

- **massless and massive spin-1/2 particles:**

– mass and charge generation by twisting:



[Reinhardt 2002]

– estimate for density of states:

$$\tilde{\rho}(E) \;>\; \frac{\sqrt{8\pi}}{3\Lambda_C}\exp[\left(\frac{E}{\Lambda_C}-1\right)\log\left(\frac{E}{\Lambda_C}-1\right)]\times \left(\log\left(\frac{E}{\Lambda_C}-1\right)+1\right)$$

summary

- motivation for a nonperturbative approach
- three instead of two phases for SU(2)/SU(3) Yang-Mills theory (phase structure unclear for SU(N) with N > 3 !)
- derivation of inert scalar fields for de – and preconfining phase upon coarse-graining over interacting topological defects
- quasiparticle spectrum and constraints for residual quantum fluctuations in physical gauge
- evolution of effective gauge couplings
- ground-state decay into massless and massive spin-1/2 particles in preconfining-confining transition: Hagedorn

- no ground-state pressure and energy density in confining phase
- order parameter:

  Polyakov loop $\longrightarrow$
  elec. $Z_{2/3}$ degenerate (deconfining phase),
  elec. $Z_{2/3}$ unique (preconfining phase),
  zero (confining phase)

  dual order parameter:

  't Hooft loop $\longrightarrow$
  zero (deconfining phase),
  mag. $Z_{2/3}$ break. embedded in $\mathrm{U}_D(1)$ or $\mathrm{U}_D(1)^2$ break. (preconfining phase),
  brok. mag. $Z_{2/3}$ only (local) symmetry (confining phase)

Thank you !