E.4 Matrix elements

Evaluate the following matrix elements for a scalar field operator $\phi(x)$

(a) $\langle 0 | \phi(x) | \vec{k} \rangle$,

(b) $\langle \vec{k} | \phi(x) | 0 \rangle$,

(c) $\langle \vec{k}' | : \phi(x) \phi(x) : | \vec{k} \rangle$.

Here $: \; :$ means the normal ordering prescription, that is, inside $: \; :$ products of creation and annihilation operators are ordered such that all creation operators stand to the left of all annihilation operators.

(d) Show that

$$\langle \vec{k}' | : \phi(x) \phi(x) : | \vec{k} \rangle$$

$$= 2 \langle \vec{k}' | \phi(x) | 0 \rangle \langle 0 | \phi(x) | \vec{k} \rangle .$$

E. 5 Scattering on a potential

Calculate the S-matrix element for scattering of a scalar particle on a potential in lowest order. The interaction Lagrange density is

$$\mathcal{L}'(x) = -\frac{1}{2} V(\vec{x}) : \phi(x)\, \phi(x) : .$$

Choose for the potential the following functions:

(a) $$V(\vec{x}) = V_0 \frac{1}{(2\pi \ell^2)^{3/2}} e^{-\frac{1}{2}\frac{\vec{x}^2}{\ell^2}},$$

(b) $$V(\vec{x}) = V_0 \int d^3x' \frac{\rho(\vec{x}')}{4\pi |\vec{x} - \vec{x}'|},$$

which implies

$$\Delta V(\vec{x}) = - V_0\, \rho(\vec{x}).$$

Here, $\rho(\vec{x})$ is a density function satisfying

$$\int d^3x \, \rho(\vec{x}) = 1,$$

$$\rho(\vec{x}) \geqslant 0.$$