E.6 Classical solutions of Dirac's equation

Show that

$$u_s(p)\, e^{-ipx} \qquad (s = \pm 1/2)$$

and

$$v_s(p)\, e^{ipx} \qquad (s = \pm 1/2)$$

are solutions of the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\, \psi(x) = 0 .$$

Here $u_s(p)$ and $v_s(p)$ are given on p. 2-43 and p. 2-45 of the lectures, respectively.

Show that

$$\sum_{s=\pm 1/2} u_s(p)\, \bar{u}_s(p) = \not{p} + m .$$

✓

E.7 Use the anticommutation relations for $a_r(\vec{p})$, $a_r^\dagger(\vec{p})$, $b_r(\vec{p})$, $b_r^\dagger(\vec{p})$ to show that

$$\{ \psi_\alpha(\vec{x},t), \bar{\psi}_\beta(\vec{y},t) \} =$$

$$\gamma^0_{\alpha\beta} \, \delta^3(\vec{x}-\vec{y})$$

$(1 \leq \alpha, \beta \leq 4)$.

✓

E.8 Current and Charge

The fourdimensional current density operator for electrons and positrons is given by

$$j^\mu(x) = -e : \bar{\psi}(x)\gamma^\mu \psi(x):$$

Normal ordering for Dirac fields is defined analogously to the scalar field case except that for each exchange of two Fermi operators an extra minus sign is added.

Thus

$$: b_r^\dagger(\vec{p})\, a_s(\vec{p}\,') : = b_r^\dagger(\vec{p})\, a_s(\vec{p}\,')$$

$$: a_s(\vec{p}\,') \, b_r^\dagger(\vec{p}) : \; = - \, b_r^\dagger(\vec{p}) \, a_s(\vec{p}\,')$$

etc.

The charge operator Q is defined by

$$Q = \int d^3x \; j^0(\vec{x}, t)$$

a) Express Q in terms of annihilation and creation operators and show that Q is independent of t.

b) Calculate the following matrix elements

$\langle 0 | Q | 0 \rangle$,

$\langle e^-(\vec{p}\,', s') | Q | e^-(\vec{p}, s) \rangle$,

$\langle e^+(\vec{p}\,', s') | Q | e^+(\vec{p}, s) \rangle$.