5 Quantum Flavourdynamics, QFD

Quantum flavourdynamics is the theory of the electroweak interactions. The fundamental particles of QFD are shown in table 5.1.
The fermions, leptons and quarks, carry flavour quantum numbers. As we shall see shortly these are the quantum numbers of the electroweak gauge group $SU(2)_L \times U(1)_Y$, that is, of weak isospin and weak hypercharge. The gauge bosons γ, $W^{\pm}$, Z come into the game when this

Particles of QFD

				Charge
Leptons	ν_e	ν_μ	ν_τ	0
	e	μ	τ	-1
Quarks	u	c	t	$2/3$
	d	s	b	$-1/3$

Gauge bosons $\quad \gamma,\ W^\pm,\ Z$

Higgs boson $\quad H\ (?)$

group is made from a global symmetry group to a local one. This is, so far, completely analogous to what we found for QCD: The gluons are necessary to have local SU(3) - colour gauge invariance. In QCD we have eight massless gluons corresponding to the eight generators of the SU(3) group. For QFD we have as gauge group $SU(2)_L \times U(1)_Y$. Now SU(2) has three generators, U(1) has one generator. Thus, naively we expect $3 + 1 = 4$ massless

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vector bosons in QFD. In reality we find only one massless vector boson in Nature, the photon γ. The field which is supposed to be responsible for the miracle of giving mass — in the framework of a local gauge theory — to three vector bosons ($W^{\pm}$, Z) is the <u>Higgs field</u>. It turns out that this field also gives masses to the charged leptons and the quarks. It is also essential for the so called quark mixing in the

Cabibbo - Kobayashi - Maskawa (CKM) Matrix and the CP violation connected with it. Thus it is no surprise that one wants to discover this Higgs field and wants to investigate experimentally in detail its propertiès. One wants to find out if indeed the Higgs field exists and performs all the miracles it is supposed to do.

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On the side of the fermions the basic building blocks in QFD are not the Dirac field operators themselves but their <u>chiral components.</u> Let $\psi(x)$ be a Dirac field operator. We define the left and right chiral components as

$$\psi_L(x) = \frac{1}{2}(1-\gamma_5)\,\psi(x),$$

$$\psi_R(x) = \frac{1}{2}(1+\gamma_5)\,\psi(x). \tag{5.1}$$

Obviously we have

$$\psi(x) = \psi_L(x) + \psi_R(x). \tag{5.2}$$

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If $\psi(x)$ is a free Dirac field with mass m we have the Dirac equation

$$i\gamma^\mu \partial_\mu \psi(x) = m\,\psi(x). \tag{5.3}$$

This implies

$$i\gamma^\mu \partial_\mu \psi_L(x) = m\,\psi_R(x),$$

$$i\gamma^\mu \partial_\mu \psi_R(x) = m\,\psi_L(x). \tag{5.4}$$

Thus, for $m \neq 0$ the fields ψ_L and ψ_R are <u>not</u> solutions of the free Dirac equation. For $m = 0$, however, they are solutions of the free Dirac eq.

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In chapter 2.4 we have discussed the parity transformation for the Dirac field operator,

$$P: \quad \psi(x) \longrightarrow \psi'(x') = \gamma^0 \psi(x)$$

$$x' = \begin{pmatrix} t \\ -\vec{x} \end{pmatrix},$$

$$\mathcal{U}(P)\psi(x)\mathcal{U}^\dagger(P) = \gamma^0 \psi(x'). \tag{5.5}$$

For the chiral components this implies

$$\mathcal{U}(P)\,\psi_L(x)\,\mathcal{U}^\dagger(P) = \gamma^0\,\psi_R(x'),$$

$$\mathcal{U}(P)\,\psi_R(x)\,\mathcal{U}^\dagger(P) = \gamma^0\,\psi_L(x'). \tag{5.6}$$

Parity transforms left components into right ones and vice versa.

Let us discuss in more detail the case of the massless free Dirac field where both, $\psi_L(x)$ and $\psi_R(x)$ are solutions of the Dirac eq. for $m=0$.

$$i\gamma^\mu \partial_\mu \psi_L(x) = 0,$$

$$i\gamma^\mu \partial_\mu \psi_R(x) = 0. \tag{5.7}$$

We can now write down the expansions of ψ_L and ψ_R in terms of creation and annihilation operators as we did in section 2.4. There we used a basis in spin space corresponding to a fixed quantisation axis (3-axis after ~~rotation~~ rotation free Lorentz transf. to the rest system).

Now the convenient basis is the so called <u>helicity basis</u> (helicity = spin component in momentum direction).
In detail one finds, always for $m=0$,

$$\psi_L(x) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left\{ e^{ipx}\, v_-(p)\, b_R^\dagger(\vec{p}) + e^{-ipx}\, u_-(p)\, a_L(\vec{p}) \right\}, \tag{5.8}$$

$$\psi_R(x) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left\{ e^{ipx}\, v_+(p)\, b_L^\dagger(\vec{p}) + e^{-ipx}\, u_+(p)\, a_R(\vec{p}) \right\}. \tag{5.9}$$

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Here the spinors $u_\pm$, $v_\pm$ are defined as follows:

$$u_\pm(p) = \sqrt{|\vec{p}|} \begin{pmatrix} \xi_{R,L}(\vec{p}) \\ \pm\, \xi_{R,L}(\vec{p}) \end{pmatrix}, \qquad (5.10)$$

$$v_\pm(p) = -\sqrt{|\vec{p}|} \begin{pmatrix} \pm\, \varepsilon\, \xi^*_{L,R}(\vec{p}) \\ \varepsilon\, \xi^*_{L,R}(\vec{p}) \end{pmatrix} \qquad (5.11)$$

and $\xi_{R,L}(\vec{p})$ are normalised eigenspinors of the spin operator in the direction of $\vec{p}$. With $\hat{p} = \vec{p}/|\vec{p}|$ we have

$$(\vec{\sigma}\hat{p})\, \xi_{R,L}(\vec{p}) = \pm\, \xi_{R,L}(\vec{p}),$$

$$\xi_R^\dagger(\vec{p})\, \xi_R(\vec{p}) = \xi_L^\dagger(\vec{p})\, \xi_L(\vec{p}) = 1. \qquad (5.12)$$

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These u and v spinors are eigenstates of γ_5:

$$\gamma_5\, u_\pm(p) = \pm\, u_\pm(p),$$

$$\gamma_5\, v_\pm(p) = \pm\, v_\pm(p).$$

(5.12a)

The creation and annihilation operators in eqs. (5.8), (5.9) satisfy the usual anticommutation relations

$$\{a_L(\vec{p}),\ a_L^\dagger(\vec{p}\,')\} = (2\pi)^3 2p^0\ \delta^3(\vec{p}-\vec{p}\,'),$$

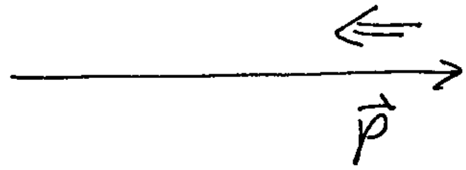
$$\{a_L(\vec{p}),\ a_R^\dagger(\vec{p}\,')\} = 0$$

etc.

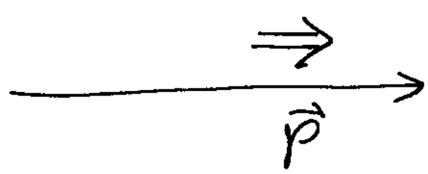
$$(5.13)$$

A simple analysis shows now the following for the states created by $a_L^\dagger(\vec{p}), \ldots, b_R^\dagger(\vec{p})$.

$a_L^{\dagger}(\vec{p})|0\rangle$ describes a zero mass particle with spin direction opposite to the momentum direction, that is, a left handed particle. (helicity $-1/2$)



$a_R^{\dagger}(\vec{p})|0\rangle$: spin and momentum directions parallel: right handed particle (helicity $+1/2$)



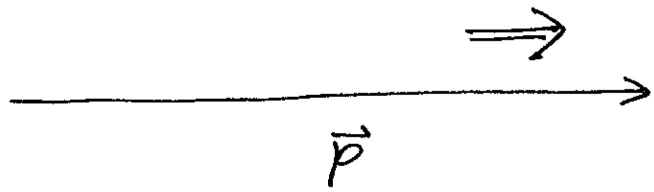
$b_L^\dagger(\vec{p})|0\rangle$ left handed antiparticle (helicity $-1/2$)

$b_R^\dagger(\vec{p})|0\rangle$ right handed antiparticle (helicity $+1/2$)

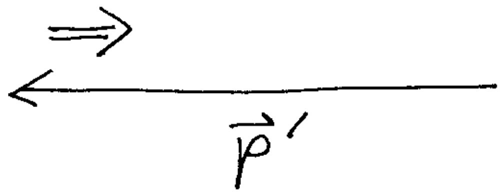
Note that the left handed (chiral) field $\psi_L(x)$ describes left handed particles and right handed antiparticles, see eq. (5.8).
The right handed (chiral) field $\psi_R(x)$ describes right handed particles and left handed antiparticles, see eq. (5.9).

For a Dirac field with mass $m \neq 0$ we can, of course, always define the left and right chiral fields as we have done in eq. (5.1). But there is no longer a strict correspondence of chirality and helicity as it is for $m = 0$.

In fact, for a particle with $m \neq 0$ the helicity is not a Lorentz-invariant concept. To see this, suppose that we have a particle with mass m and positive helicity and momentum $\vec{p}$.



Now we can enter a very fast rocket and overtake the particle such that seen from the rocket it moves in opposite direction. But the spin still points in the same direction.



Seen from the observer in the rocket the helicity of the particle is negative.

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5.1 The gauge group of QFD

To start the discussion let us consider a fictitious world where we have as fermions only the electron e and one neutrino ν_e. We know today that both e and ν_e have mass. But their masses are small – at least on the scale of LHC energies – and we shall make in a first step the approximation that e and ν_e are massless. We have then four chiral fields:

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$\nu_{eL}(x) \qquad \nu_{eR}(x)$

$e_L(x) \qquad e_R(x)$

For a long time people thought that the right chiral neutrino field did not exist in Nature. This changed with the discovery of neutrino oscillations. In this chapter we shall ignore these phenomena and consider only the three fields

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}, \qquad e_R(x)$$

The Lagrange density for these free massless fields is

$$\mathcal{L}_0(x) =$$

$$\left(\bar{\nu}_{e_L}(x),\ \bar{e}_L(x)\right)\left(\frac{i}{2}\gamma^\lambda \overrightarrow{\partial}_\lambda - \frac{i}{2}\gamma^\lambda \overleftarrow{\partial}_\lambda\right)\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}$$

$$+\ \bar{e}_R(x)\left(\frac{i}{2}\gamma^\lambda \overrightarrow{\partial}_\lambda - \frac{i}{2}\gamma^\lambda \overleftarrow{\partial}_\lambda\right) e_R(x) .$$

(5.14)

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We see that there is a complete symmetry - at this stage - between the left handed neutrino and electron fields. Indeed, we can make a $SU(2)$ transformation of these fields and we find invariance of $\mathcal{L}_0$.

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \longrightarrow \mathcal{U} \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}$$

$$\mathcal{U} \in SU(2) \qquad (5.15)$$

$\Longrightarrow$

$$\mathcal{L}_0(x) \longrightarrow \mathcal{L}_0(x). \qquad (5.16)$$

This $SU(2)$ group is the <u>weak isospin group</u>. We shall sometimes denote it by $SU(2)_L$ since it acts non trivially only on left chiral fields.

Now we can again bring into play the point of view of H. Weyl. If ν_{e_L} and e_L are not distinguishable at this stage we should be free to choose the axes in this two dimensional space as we want at each space-time point. We know from our discussion of QCD that we can write down a theory which has <u>local</u> SU(2) gauge invariance:

$$\begin{pmatrix} \nu_{e_L}(x) \\ e_L(x) \end{pmatrix} \longrightarrow \mathcal{U}(x) \begin{pmatrix} \nu_{e_L}(x) \\ e_L(x) \end{pmatrix} \tag{5.17}$$

$\mathcal{U}(x) \in SU(2)$ for each x.

We have to introduce three vector fields since $SU(2)$ has three generators:

$$W_\lambda^a(x) \qquad a = 1, 2, 3 . \tag{5.18}$$

The corresponding field strengths are

$$W_{\lambda\rho}^a(x) = \partial_\lambda W_\rho^a(x) - \partial_\rho W_\lambda^a(x)$$
$$- g\, \varepsilon_{abc}\, W_\lambda^b(x)\, W_\rho^c(x), \tag{5.19}$$

where g is the $SU(2)$ gauge coupling constant. As for QCD it is convenient to introduce the potential and field-strength matrices:

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$$W_\lambda(x) = W_\lambda^a(x)\,\frac{\tau_a}{2}\,,$$

$$W_{\lambda\rho}(x) = W_{\lambda\rho}^a(x)\,\frac{\tau_a}{2}\,. \qquad (5.20)$$

Here τ_a $(a = 1,2,3)$ are the Pauli matrices.

The Lagrange density having local $SU(2)$ invariance is now easily written down. With the $SU(2)$-covariant derivative

$$D_\lambda \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} = \left(\partial_\lambda + ig\,W_\lambda(x)\right) \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \qquad (5.20a)$$

we get

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$$\mathcal{L}(x) = -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}(x)\,W^{\lambda\rho}(x)\right)$$

$$+\left\{\left(\bar{\nu}_{e_L}(x), \bar{e}_L(x)\right)\frac{i}{2}\gamma^\lambda D_\lambda \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix}\right.$$

$$\left. + h.c. \right\}$$

$$+ \left\{ \bar{e}_R(x)\frac{i}{2}\gamma^\lambda \partial_\lambda e_R(x) + h.c. \right\}. \tag{5.21}$$

Invariance transformation:

$$W_\lambda(x) \to \mathcal{U}(x) W_\lambda(x) \mathcal{U}^\dagger(x) - \frac{i}{g}\mathcal{U}(x)\partial_\lambda \mathcal{U}^\dagger(x),$$

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \to \mathcal{U}(x)\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix},$$

$$e_R(x) \to e_R(x),$$

$$\mathcal{U}(x) \in SU(2). \tag{5.22}$$

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The next thing to note is that the Lagrange density, eq. (5.21), still has two global phase invariances

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \longrightarrow e^{i\varphi'} \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix},$$

$$e_R(x) \longrightarrow e^{i\varphi}\, e_R(x), \qquad (5.23)$$

where φ and φ' are real constants We could be tempted to "gauge" both these phase invariances. But this does not work if we want to describe Nature correctly. We shall only "gauge" a particular combination of the two phase

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transformations, the so called <u>weak hypercharge</u> transformations. To write all this down we arrange our three spinor fields into one common spinor with three components:

$$\psi(x) = \begin{pmatrix} \nu_{e_L}(x) \\ e_L(x) \\ e_R(x) \end{pmatrix} . \qquad (5.24)$$

The Hypercharge transformations are:

$$\psi(x) \longrightarrow e^{i\chi Y}\,\psi(x) \qquad (5.25)$$

where

$$Y = \begin{pmatrix} y_L & 0 & 0 \\ 0 & y_L & 0 \\ 0 & 0 & y_R \end{pmatrix} \qquad (5.26)$$

is the hypercharge generator and χ the phase parameter.

We assign hypercharge y_L to the left handed SU(2) doublet ν_{e_L}, e_L and y_R to the right handed SU(2) singlet e_R. The precise values for y_L and y_R are left open for the moment. Gauging of these hypercharge transformations works as in QED. We have to introduce one abelian vector field $B_\lambda(x)$. The corresponding field strength tensor is

$$B_{\lambda\rho}(x) = \partial_\lambda B_\rho(x) - \partial_\rho B_\lambda(x). \tag{5.27}$$

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Of course, there is also a corresponding coupling parameter g'. The complete $SU(2)\times U(1)$ covariant derivative of the spinor field $\psi(x)$ is now

$$D_\lambda \psi(x) = \left(\partial_\lambda + ig\, W^a_\lambda(x) T_a + ig' B_\lambda(x) Y\right)\psi(x), \tag{5.28}$$

where

$$T_a = \left(\begin{array}{c|c} \frac{1}{2}\tau_a & 0 \\ \hline 0 & 0 \end{array}\right)$$

$$a = 1, 2, 3 . \tag{5.29}$$

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The Lagrange density which is invariant under local $SU(2)_L \times U(1)_Y$ transformations is

$$\mathcal{L}(x) = -\frac{1}{2} \, Tr \left(W_{\lambda\rho}(x) \, W^{\lambda\rho}(x) \right)$$

$$- \frac{1}{4} \, B_{\lambda\rho}(x) \, B^{\lambda\rho}(x)$$

$$+ \left\{ \overline{\psi}(x) \, \frac{i}{2} \, \gamma^{\lambda} D_{\lambda} \psi(x) + h.c. \right\}.$$

(5.30)

To see the physical processes described by the Lagrange density eq. (5.30) we consider the fermion - fermion - gauge boson coupling given by it:

$$\mathcal{L}_{Int}(x) = -\bar{\psi}\gamma^\lambda (g W_\lambda^a T_a + g' B_\lambda Y)\psi$$

$$= -\frac{g}{\sqrt{2}}\left(W_\lambda^+ \bar{\nu}_{eL}\gamma^\lambda e_L + W_\lambda^- \bar{e}_L \gamma^\lambda \nu_{eL}\right)$$

$$- \frac{1}{2}\left(g W_\lambda^3 + 2y_L g' B_\lambda\right) \bar{\nu}_{eL}\gamma^\lambda \nu_{eL}$$

$$+ \frac{1}{2}\left(g W_\lambda^3 - 2y_L g' B_\lambda\right) \bar{e}_L \gamma^\lambda e_L$$

$$- y_R g' B_\lambda \bar{e}_R \gamma^\lambda e_R ,$$

where (5.31)

$$W_\lambda^\pm = \frac{1}{\sqrt{2}}\left(W_\lambda^1 \mp i W_\lambda^2\right). \tag{5.32}$$

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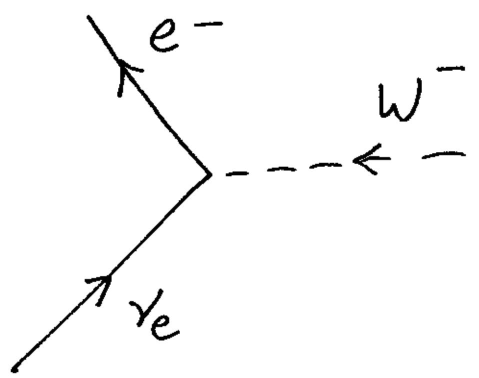
The terms involving $W_\lambda^{\pm}$ look already quite promising for <u>weak interactions</u>. They describe the transition of an electron to a neutrino and vice versa, with emission or absorption of a W boson.

The coupling has the correct V-A structure observed experimentally since only the left chiral fields occur there.



$$-i\frac{g}{2\sqrt{2}}\gamma^\lambda(1-\gamma_5)$$

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Our theory should also include the <u>electromagnetic coupling term</u> where only the electron enters:

$$\mathcal{L}'_{em}(x) = -e\, A_\lambda(x)\left(-\bar{e}(x)\gamma^\lambda e(x)\right)$$

$$= -e\, A_\lambda(x)\Big(-\bar{e}_L(x)\gamma^\lambda e_L(x)$$

$$- \bar{e}_R(x)\gamma^\lambda e_R(x)\Big). \tag{5.33}$$

Here e is the positron charge, $e > 0$.
The neutrino does not couple here since it has zero charge and is an elementary field. The point is now to <u>arrange</u> the coupling, eq. (5.31), such that eq. (5.33) is correctly included.

As free parameters we have y_L, y_R and g' but since only the products $g' y_L$ and $g' y_R$ enter in eq. (5.31) there are only two independent parameters. By convention we can set, therefore,

$$y_L = -\frac{1}{2} \qquad (5.34)$$

Now we consider the neutral vector fields W_λ^3 and B_λ of our theory. We see from eq. (5.31) that the neutrino current $\bar{\nu}_{eL} \gamma^\lambda \nu_{eL}$ couples only to a certain linear combination of W_λ^3 and B_λ. The corresponding normalised field is

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$$Z_\lambda = \frac{1}{\sqrt{g^2+g'^2}} \left(g W_\lambda^3 - g' B_\lambda \right) .$$

(5.35)

The neutrino does not couple to the photon. Thus Z_λ can have no photon component. The good candidate for the electromagnetic vector potential A_λ is, therefore, the linear combination of W_λ^3 and B_λ which is <u>orthogonal</u> to Z_λ, eq. (5.35):

$$A_\lambda = \frac{1}{\sqrt{g^2+g'^2}} \left(g' W_\lambda^3 + g B_\lambda \right) .$$

(5.36)

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This is the occasion to define the <u>weak mixing angle</u> ϑ_W by

$$\sin\vartheta_W = \frac{g'}{\sqrt{g^2+g'^2}} ,$$

$$\cos\vartheta_W = \frac{g}{\sqrt{g^2+g'^2}} . \tag{5.37}$$

By convention we can require $g \geqslant 0$ and $g' \geqslant 0$ and, correspondingly $0 \leq \vartheta_W \leq \pi/2$. With this we have

$$\begin{aligned} Z_\lambda &= \cos\vartheta_W \, W_\lambda^3 - \sin\vartheta_W \, B_\lambda , \\ A_\lambda &= \sin\vartheta_W \, W_\lambda^3 + \cos\vartheta_W \, B_\lambda . \end{aligned} \tag{5.38}$$

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Rewriting the interaction Lagrange density, eq (5.31), in terms of Z_λ and A_λ we find

$$\mathcal{L}_{Int} = -\frac{g}{\sqrt{2}}\left(W_\lambda^+ \bar{\nu}_{eL} \gamma^\lambda e_L + h.c. \right)$$

$$- \sqrt{g^2+g'^2}\, Z_\lambda \Big\{ \frac{1}{2} \bar{\nu}_{eL} \gamma^\lambda \nu_{eL}$$

$$- \frac{1}{2} \bar{e}_L \gamma^\lambda e_L$$

$$- \sin^2\vartheta_W \left(- \bar{e}_L \gamma^\lambda e_L + y_R \bar{e}_R \gamma^\lambda e_R \right) \Big\}$$

$$- \frac{g g'}{\sqrt{g^2+g'^2}} A_\lambda \left(- \bar{e}_L \gamma^\lambda e_L + y_R \bar{e}_L \gamma^\lambda e_R \right).$$

(5.39)

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Now it is obvious how to get the correct electromagnetic coupling, eq. (5.33). We set

$$y_R = -1 \,, \tag{5.40}$$

$$\frac{g g'}{\sqrt{g^2 + g'^2}} = e \,. \tag{5.41}$$

With this we get from eq. (5.37)

$$e = g \sin\vartheta_W = g' \cos\vartheta_W = \sqrt{g^2+g'^2} \cos\vartheta_W \sin\vartheta_W \,. \tag{5.42}$$

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The Lagrange density, eq. (5.31), describing the coupling of our fermions ν_e, e to the gauge bosons gets the final form

$$\mathcal{L}_{Int} = -e \Big\{ A_\lambda J^\lambda_{em} + \frac{1}{\sqrt{2}\,\sin\vartheta_W} \left(W^+_\lambda \bar{\nu}_{eL} \gamma^\lambda e_L + h.c. \right) + \frac{1}{\sin\vartheta_W \cos\vartheta_W} Z_\lambda J^\lambda_{NC} \Big\}, \quad (5.43)$$

where the electromagnetic and weak neutral currents are:

$$J^\lambda_{em} = -\bar{e}_L \gamma^\lambda e_L - \bar{e}_R \gamma^\lambda e_R = -\bar{e} \gamma^\lambda e,$$

$$J^\lambda_{NC} = \frac{1}{2} \bar{\nu}_{eL} \gamma^\lambda \nu_{eL} - \frac{1}{2} \bar{e}_L \gamma^\lambda e_L - \sin^2\vartheta_W J^\lambda_{em}. \quad (5.44)$$

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We see that our gauge theory based on the group $SU(2)_L \times U_Y(1)$ gives us a coupling structure which looks promising for describing the electroweak interactions. The charged weak current $\bar{\nu}_{eL} \gamma^\lambda e_L$:

$$\bar{\nu}_{eL} \gamma^\lambda e_L = \bar{\nu}_e \gamma^\lambda \frac{1}{2}(1-\gamma_5) e \tag{5.45}$$

has the V-A structure observed in Nature. The electromagnetic current J^λ_{em} is a pure vector current, V. The weak neutral current J^λ_{NC} is a certain mixture of V and A. There is a unified description

of these couplings but not a real unification of weak and electromagnetic interactions:
The gauge group $SU(2)_L \times U(1)_Y$ is not simple, thus we have two independent constants, g and g', describing the coupling. Also, we had to choose the hypercharge assignment for e_R, that is the constant y_R, by hand such that the correct electromagnetic coupling came out. One would like to have a physical principle dictating $y_R = -1$.

Finally there is still a major problem to solve. Our theory, so far, contains four massless gauge bosons whereas in Nature we have only the photon as massless boson. In the next section we shall discuss the mechanism which is supposed to give mass to the bosons $W^{\pm}$ and Z, the so called <u>Higgs-mechanism</u>.

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