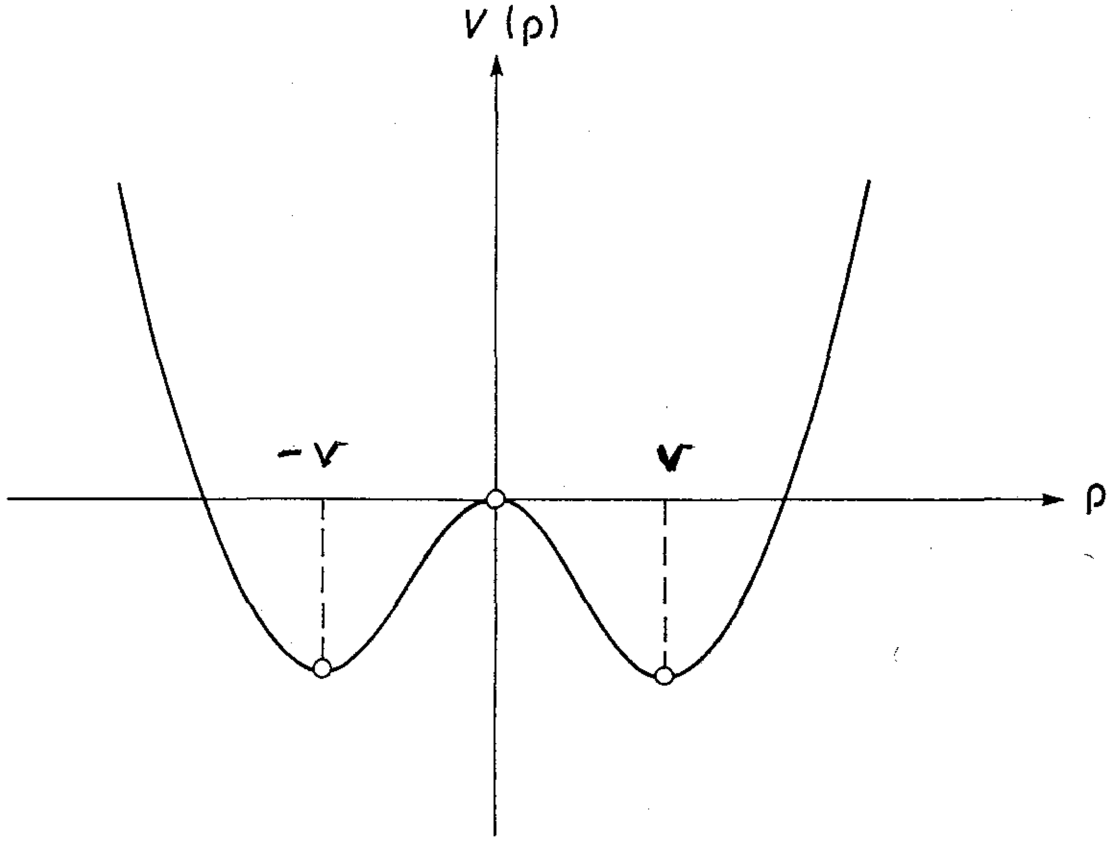
5.2 The Higgs field and spontanous symmetry breaking

We speak of spontanous symmetry breaking (SSB) if the equations governing the behaviour of a system have a symmetry which the ground state of the system does not have. We encounter this already in classical mechanics. Consider a point particle moving in one dimension, coordinate ϱ, in the following "Mexican hat" potential:

5 - 44a



$$V(\rho) = -\frac{1}{2}\mu^2\rho^2 + \frac{1}{4}\lambda\rho^4$$

$$\mu^2 > 0 \quad , \quad \lambda > 0 .$$

(5.46)

This potential has the reflection symmetry

$$V(\rho) = V(-\rho). \tag{5.47}$$

The equilibrium positions of the particle in this potential are found from the condition of vanishing force:

$$\frac{\partial V(\rho)}{\partial \rho} = 0 \;;$$

$\Longrightarrow$

$$\rho = 0 \quad , \quad \rho = \pm\, v = \pm\sqrt{\frac{\mu^2}{\lambda}} \;. \tag{5.48}$$

✓

Clearly, there are two stable equilibrium positions, classically. The particle is *either* at $\varrho = +v$ *or* at $\varrho = -v$. We have SSB!

Well known phenomena of every day life which are understood in terms of SSB are the spontanous magnetisation of a piece of iron and the condensation of water vapor.

The idea to use SSB in order to give mass to the bosons $W^{\pm}$ and Z was proposed by Weinberg and Salam in 1967 - 1968. Their proposal was based on work by Higgs, Englert & Brout, Kibble and others.

One introduces, in addition to the fields discussed in section 5.1, a <u>scalar field $\phi(x)$</u>.

This is taken as a complex, weak isospin doublet with hypercharge $y_H = 1/2$

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

$\phi_{1,2}(x)$ complex field.

(5.49)

$SU(2)_L$ transformations, global:

$$\phi(x) \longrightarrow \mathcal{U}\,\phi(x)$$

$$\mathcal{U} \in SU(2)$$

(5.50)

✓

$U(1)_Y$ global transformations:

$$\phi(x) \longrightarrow e^{i\chi y_H}\, \phi(x),$$
$$y_H = 1/2 . \qquad (5.51)$$

The Lagrange density is chosen as

$$\mathcal{L}_\phi(x) = \left(\partial_\mu \phi^\dagger(x)\right)\left(\partial^\mu \phi(x)\right) - V(\phi(x)) ,$$

$$V(\phi(x)) = -\mu^2\, \phi^\dagger(x)\phi(x) + \lambda\left(\phi^\dagger(x)\,\phi(x)\right)^2 ,$$

$$\mu^2 > 0 , \quad \lambda > 0 . \qquad (5.52)$$

✓

If we set for the "length" of the scalar field

$$+\sqrt{\phi^{+}(x)\,\phi(x)} = \frac{1}{\sqrt{2}}\,\rho(x) \qquad (5.53)$$

we get for $\rho(x)$ exactly the Mexican hat potential, eq. (5.46).

Now one learns in QFT that the lowest energy state, that is the <u>vacuum state</u>, corresponds to a constant field minimising $V(\phi)$. Clearly this corresponds to

$$\frac{1}{\sqrt{2}}\,\rho(x) = \sqrt{\phi^{+}(x)\,\phi(x)} = \frac{1}{\sqrt{2}}\,v\,,$$

$$v = \sqrt{\frac{\mu^2}{\lambda}}\,. \tag{5.54}$$

The most general constant field ϕ having this length can be parametrised by

$$\phi = \exp\left(i\,\frac{\tau_a}{2}\,\varphi_a\right)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

φ_a ($a = 1, 2, 3$) arbitrary, real,

$$\left(\sum_{a=1}^{3}\varphi_a^{2}\right)^{1/2} < 2\pi\,. \tag{5.55}$$

✓

Note that the Lagrangian, eq. (5.52), from which we started is $SU(2) \times U(1)$ symmetric. But none of the possible vacuum configurations, eq. (5.55), has this symmetry any more. We have SSB. Models of this type are indeed used in order to understand phenomena like spontanous magnetisation.

A completely new and exciting aspect shows up when we now couple our scalar field in an <u>$SU(2) \times U(1)$ gauge invariant</u> way to the particles which we had already in section 5.1. We know how to do this; we replace ordinary derivatives by covariant ones, that is we make a minimal substitution. We note that $\phi^{+}(x)\,\phi(x)$ is invariant, thus we get

$$\mathcal{L}_{\phi} \to \left(D_{\lambda}\phi(x)\right)^{+}\left(D^{\lambda}\phi(x)\right) - V(\phi(x)) \tag{5.56}$$

where

$$D_{\lambda}\phi(x) = \left(\partial_{\lambda} + ig W_{\lambda}^{a}\frac{\tau_a}{2} + ig' B_{\lambda} y_H\right)\phi(x). \tag{5.57}$$

This is not everything. We can also write down a gauge invariant <u>Yukawa coupling</u> term involving the electron, the neutrino and the Higgs fields:

$$\mathcal{L}_{Yuk} = -c_e \, \bar{e}_R \, \phi^\dagger \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + h.c.$$

Hyperchanges: $+1 \quad -1/2 \quad -1/2$

$$(5.58)$$

Note that this works because we <u>chose</u> y_H accordingly. In eq. (5.58) c_e is a complex Yukawa coupling constant.

But we can always make a *global* phase transformation of the right handed electron field e_R and rotate the phase of c_e away. That is, without loss of generality, we can suppose

$$c_e \text{ real}, \quad c_e \geqslant 0. \qquad (5.58a)$$

The complete Lagrange density for ν_e, e, W^a, B and ϕ is now

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}W^{\lambda\rho}\right) - \frac{1}{4}\,B_{\lambda\rho}B^{\lambda\rho} \\ & + \Big\{ (\bar{\nu}_{eL}, \bar{e}_L)\,\frac{i}{2}\gamma^\lambda D_\lambda \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} \\ & + \bar{e}_R\,\frac{i}{2}\gamma^\lambda D_\lambda e_R \\ & - c_e\,\bar{e}_R\,\phi^\dagger \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} \qquad + h.c. \Big\} \\ & + (D_\lambda\phi)^\dagger (D^\lambda\phi) - V(\phi) .\end{aligned}$$

(5.59)

This Lagrange density is $SU(2)\times U(1)$ gauge invariant.

The gauge transformations for the Higgs field act as follows:

SU(2):

$$\begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \longrightarrow \mathcal{U}(x) \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

$$\mathcal{U}(x) = \exp\left(i \frac{\tau_a}{2} \varphi_a(x) \right)$$

$\varphi_a(x)$ arbitrary real functions
$a = 1, 2, 3$

(5.60)

U(1)

$$\begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \to e^{i y_H \chi(x)} \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

$\chi(x)$ arbitrary real function.

(5.61)

It is easy to see that we can now for arbitrary Higgs field $\phi(x)$ make a <u>gauge transformation</u> to "rotate" $\phi(x)$ such that

$$\mathcal{U}(x)\,\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho(x) \end{pmatrix},$$

$$\rho(x) \geq 0. \tag{5.62}$$

In other words, we can require as a <u>gauge condition</u>

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho(x) \end{pmatrix}. \tag{5.63}$$

Strictly speaking, our considerations apply to the classical level. But in QFT the situation is very similar. The vacuum expection value of the Higgs (quantum) field is given by the field minimising the potential $V(\phi)$. With our gauge condition ~~and~~ and with eq. (5.55) we get

$$\langle \phi(x) \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix} ,$$

$$v = \sqrt{\frac{\mu^2}{\lambda}} \quad . \tag{5.64}$$

This vacuum expectation value of ϕ is clearly not invariant under the full $SU(2)_L \times U(1)_Y$ gauge group. But it is invariant under a $U(1)$ gauge group generated by $T_3 + Y$:

$$\exp\left[i\chi(x)(T_3+Y)\right]\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}$$

$$= \exp\left[i\chi(x)\left(\tfrac{1}{2}\tau_3+\tfrac{1}{2}\right)\right]\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix} \qquad (5.65)$$

$$=$$

since

$$\left(\tfrac{1}{2}\tau_3+\tfrac{1}{2}\right)\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix} = 0 . \qquad (5.66)$$

As one can check easily,
$T_3 + Y = Q$ is just the electromagnetic charge operator. We check this for our fermions:

	T_3	Y	Q
ν_{eL}	$1/2$	$-1/2$	0
e_L	$-1/2$	$-1/2$	-1
e_R	0	-1	-1

We conclude that the vacuum expectation value of the Higgs field breaks the $SU(2)_L \times U(1)_Y$ gauge symmetry but leaves the subgroup $U(1)$ generated by $Q = T_3 + Y$, the electromagnetic charge operator, invariant. This $\mathcal{U}(1)$ is clearly the electromagnetic gauge group which indeed should remain intact if we want to describe Nature correctly. One often writes

$$SU(2)_L \times \mathcal{U}(1)_Y \xrightarrow[SSB]{} \mathcal{U}_{em}(1) . \qquad (5.67)$$

The next steps in setting up the theory are straightforward.

Now one "shifts" the Higgs field by the vacuum expectation value, setting

$$\rho(x) = v + H(x),$$

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix}. \tag{5.68}$$

$H(x)$ is the physical Higgs quantum field which can be treated, for instance in perturbation theory, in the usual way.

All we still have to do is to insert eq. (5.68) for the Higgs field in the expression for the Lagrange density, eq (5.59). An easy calculation gives

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\,\mathrm{Tr}\left(W_{\lambda\rho}W^{\lambda\rho}\right) - \frac{1}{4}\,B_{\lambda\rho}\,B^{\lambda\rho} \\
& + \left\{ \bar{\nu}_{eL}\frac{1}{2}i\gamma^{\lambda}\partial_{\lambda}\nu_{eL} + h.c. \right\} \\
& + \left\{ \bar{e}\,\frac{i}{2}\gamma^{\lambda}\partial_{\lambda}e + h.c. \right\} \\
& + W^{+}_{\lambda}\,W^{-\lambda}\,m_W^2\left(1 + \frac{H}{v}\right)^2 \\
& + \frac{1}{2}\,Z_{\lambda}\,Z^{\lambda}\,m_Z^2\left(1 + \frac{H}{v}\right)^2 \\
& - m_e\,\bar{e}e\left(1 + \frac{H}{v}\right) \\
& + \frac{1}{2}(\partial_{\lambda}H)(\partial^{\lambda}H) \\
& - \frac{1}{2}\,m_H^2\,H^2\left[1 + \frac{H}{v} + \frac{1}{4}\left(\frac{H}{v}\right)^2\right] \\
& + \mathcal{L}_{Int}\,.
\end{aligned} \tag{5.69}
$$

Here the field strength tensors $W_{\lambda\rho}$ and $B_{\lambda\rho}$ are to be expressed in terms of the potentials Z_μ and A_μ using eq. (5.38).

Furthermore, we have

$$m_W^2 = \frac{g^2 v^2}{4} = \frac{e^2 v^2}{4 \sin^2 \vartheta_W} ,$$

$$m_Z^2 = \frac{(g^2 + g'^2) v^2}{4} = \frac{e^2 v^2}{4 \sin^2 \vartheta_W \cos^2 \vartheta_W} ,$$

$$m_e = \kappa_e \frac{v}{\sqrt{2}} ,$$

$$m_H^2 = 2 \lambda v^2 . \tag{5.70}$$

The fermion - fermion - gauge boson interaction $\mathcal{L}_{Int}$ is as in eqs. (5.43), (5.44), Let us discuss these results.

The electron mass m_e arises from the Yukawa coupling, eq. (5.58), through the vacuum expectation value of the Higgs field.

$$\mathcal{L}_{Yuk}(x) = -c_e\,\bar{e}_R(x)\,\phi^\dagger(x)\begin{pmatrix}\nu_{eL}(x)\\ e_L(x)\end{pmatrix} + h.c.$$

$$\longrightarrow\ -c_e\,\bar{e}_R(x)\,.$$

$$\left(0\,,\ \frac{1}{\sqrt{2}}(v+H(x))\right)\begin{pmatrix}\nu_{eL}(x)\\ e_L(x)\end{pmatrix}$$

$$+\,h.c.$$

$$= -\frac{c_e}{\sqrt{2}}\left(v+H(x)\right)\bar{e}_R(x)\,e_L(x) + h.c.$$

$$= -\underbrace{c_e\frac{v}{\sqrt{2}}}_{=:\,m_e}\ \bar{e}(x)\,e(x)\left(1+\frac{H(x)}{v}\right). \qquad (5.71)$$

The vacuum Higgs field provides the mass for the electron. Correspondingly, the physical Higgs field couples to the electron with coupling constant m_e/v. As we shall see shortly, this generalises to all other fermions in the SM.
One of the aims of experimental Higgs physics will be to check this deep connection between mass and Higgs-fermion coupling.

The gauge bosons $W^{\pm}$ and Z get mass terms in eq. (5.69). These terms arise when we make in the kinetic term for the Higgs field:

$$(D_\lambda \phi(x))^\dagger (D^\lambda \phi(x))$$

the substitution, eq. (5.68). Again, the coupling of the physical Higgs field $H(x)$ to the bosons $W^{\pm}$, Z is, therefore, related in a precise way to m_W^2 and m_Z^2. The gauge potential A_λ gets no mass term, that is, the photon is massless as we want it.

The physical Higgs boson $H(x)$, finally, corresponds to an electrically neutral particle of mass m_H. Its trilinear and quadrilinear self-couplings are fixed.

Looking at our Lagrange density, eq. (5.69), we see that, indeed, the Higgs field, together with SSB, has performed all the miracles which we wanted.

5.3 The extension of QFD to more fermions and the effective low energy coupling

To include all known fermions into our scheme is straightforward. For the time being we neglect again neutrino masses. Then we have the following chiral fields and their quantum numbers. For the quarks d'_L, s'_L and b'_L we write the fields with prime, since we shall see that there is "quark mixing". The $T_3 = -1/2$ components d'_L, s'_L, b'_L

will turn out to be linear combinations of quark fields with definite mass. This will lead to the CKM matrix.

			T	T_3	Y	Q
ν_{eL}	$\nu_{\mu L}$	$\nu_{\tau L}$	1/2	1/2	− 1/2	0
e_L	μ_L	τ_L	1/2	− 1/2	− 1/2	− 1
e_R	μ_R	τ_R	0	0	− 1	− 1
u_L	c_L	t_L	1/2	1/2	1/6	2/3
d'_L	s'_L	b'_L	1/2	− 1/2	1/6	− 1/3
u_R	c_R	t_R	0	0	2/3	2/3
d_R	s_R	b_R	0	0	− 1/3	− 1/3

The flavour quantum numbers of the leptons and quarks.

T : weak isospin, T_3 third component,

Y : hypercharge, Q el. charge

We arrange now all fermion fields in a big spinor

$$\psi = \begin{pmatrix} \nu_{eL} \\ e_L \\ \vdots \\ \vdots \\ b_R \end{pmatrix} \qquad (5.72)$$

Let T_a $(a = 1, 2, 3)$ and Y be the corresponding generators of $SU(2)_L \times U(1)_Y$. The covariant derivative of ψ is then

$$D_\lambda \psi = (\partial_\lambda + ig W_\lambda^a T_a + ig' B_\lambda Y)\psi .$$

(5.73)

The general Yukawa coupling term can be written as

$$\mathcal{L}_{Yuk} = \bar{\psi}(x)\left(\phi_1(x)C_1 + \phi_2(x)C_2\right)\psi(x) + h.c. \tag{5.74}$$

where $C_{1,2}$ are two, in general complex, matrices. Of course, we require $\mathcal{L}_{Yuk}$ to be $SU(2)_L \times U(1)_Y$ invariant. We shall see what this implies for $C_{1,2}$ in the next section.

Putting now everything together, our Lagrange density for QFD is

$$\mathcal{L} = -\frac{1}{2} \mathrm{Tr}\left(W_{\lambda\rho} W^{\lambda\rho}\right) - \frac{1}{4} B_{\lambda\rho} B^{\lambda\rho}$$

$$+ \left\{ \bar{\psi} \frac{i}{2} D_\lambda \psi + h.c. \right\}$$

$$+ \mathcal{L}_{Yuk}$$

$$+ \left(D_\lambda \phi\right)^\dagger \left(D^\lambda \phi\right) - V(\phi) .$$

(5.75)

After SSB, that is, with the substitution eq. (5.68) we get

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2} \operatorname{Tr}\left(W_{\lambda\rho} W^{\lambda\rho}\right) \\
& - \frac{1}{4} B_{\lambda\rho} B^{\lambda\rho} \\
& + W_\lambda^{+} W^{-\lambda} m_W^2 \left(1 + \frac{H}{v}\right)^2 \\
& + \frac{1}{2} Z_\lambda Z^\lambda m_Z^2 \left(1 + \frac{H}{v}\right)^2 \\
& + \left\{ \bar{\psi} \frac{i}{2} \gamma^\lambda D_\lambda \psi + h.c. \right\} \\
& - \bar{\psi} M \psi \left(1 + \frac{H}{v}\right) \\
& + \frac{1}{2} \partial_\lambda H \, \partial^\lambda H - \frac{1}{2} m_H^2 H^2 \Big[1 \\
& \qquad + \frac{H}{v} + \frac{1}{4}\left(\frac{H}{v}\right)^2 \Big] .
\end{aligned}
\tag{5.76}
$$

Here

$$M = -\left(C_2 + C_2^{+}\right)\frac{v}{\sqrt{2}} \; . \qquad (5.77)$$

is the general mass matrix of the fermions. The covariant derivative term of the ψ field leads, as before, to the fermion - gauge boson coupling.

$$\bar{\psi}\frac{1}{2}i\gamma^{\lambda}D_{\lambda}\psi + h.c. =$$

$$\left(\bar{\psi}\frac{i}{2}\gamma^{\lambda}\partial_{\lambda}\psi + h.c.\right) + \mathcal{L}_{Int} \; , \qquad (5.78)$$

$$\mathcal{L}_{Int} = -e\Big\{A_{\lambda}J^{\lambda}_{em} + \frac{1}{\sin\vartheta_W\cos\vartheta_W}Z_{\lambda}J^{\lambda}_{NC} + \frac{1}{\sqrt{2}\sin\vartheta_W}\left(W^{+}_{\lambda}J^{\lambda}_{cc} + W^{-}_{\lambda}J^{\lambda +}_{cc}\right)\Big\}. \qquad (5.79)$$

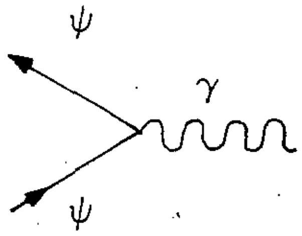
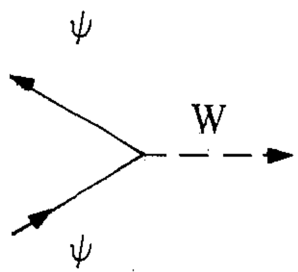
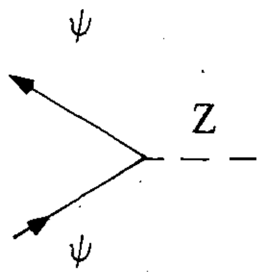
The currents are

$$J^\lambda_{em} = \bar{\psi}\gamma^\lambda (T_3 + Y)\psi$$
$$= \bar{\psi}\gamma^\lambda Q \psi$$
$$(Q = T_3 + Y), \qquad (5.80)$$

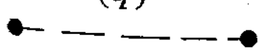
$$J^\lambda_{NC} = \bar{\psi}\gamma^\lambda T_3 \psi - \sin^2\vartheta_W \, J^\lambda_{em}, \qquad (5.81)$$

$$J^\lambda_{CC} = \bar{\psi}\gamma^\lambda (T_1 + i T_2)\psi. \qquad (5.82)$$

The diagrammatic terms for the couplings of $\mathcal{L}_{Int}$ are as follows:

Diagrammteil	analytischer Ausdruck
	$-\mathrm{i}\, e\, \mathbf{Q}\, \gamma^{\lambda}$
	$-\mathrm{i}\,\frac{e}{\sqrt{2}\sin\vartheta_W}\,(\mathbf{T}_1 + \mathrm{i}\,\mathbf{T}_2)\,\gamma^{\lambda}$
	$-\mathrm{i}\,\frac{e}{\sin\vartheta_W \cos\vartheta_W}\,(\mathbf{T}_3 - \sin^2\vartheta_W\,\mathbf{Q})\,\gamma^{\lambda}$

Fermion - gauge boson vertices in QFD

Diagrammteil	analytischer Ausdruck
$\gamma(q)$ 	$\dfrac{-ig_{\rho\lambda}}{q^2}$
$W(q)$ 	$\dfrac{-i\left(g_{\rho\lambda}-\dfrac{q_\rho q_\lambda}{m_W^2}\right)}{q^2-m_W^2} \xrightarrow[q^2 \ll m_W^2]{} \dfrac{ig_{\rho\lambda}}{m_W^2}$
$Z(q)$ 	$\dfrac{-i\left(g_{\rho\lambda}-\dfrac{q_\rho q_\lambda}{m_Z^2}\right)}{q^2-m_Z^2} \xrightarrow[q^2 \ll m_Z^2]{} \dfrac{ig_{\rho\lambda}}{m_Z^2}$

Gauge boson propagators
in QFD