5.4 The mass matrix and the Cabibbo-Kobayashi-Maskawa matrix

In this section we shall study the Yukawa coupling of the fermions with the Higgs field, see eq. (5.74). After SSB this term leads to the fermion masses and it is also responsible for the quark mixing in the charged weak current.

We start by looking at the detailed form of $\mathcal{L}_{Yuk}$ which is compatible with $SU(2)_L \times U(1)_Y$ invariance.

Consider an arbitrary $SU(2)_L$ doublet of fermion fields

$$\begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix}$$

Our Higgs field also is a $SU(2)_L$ doublet. We can, therefore, construct two singlet expressions of the type Higgs field × fermion field:

$$\phi^{\dagger} \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} = \phi_1^{\dagger} \psi_{1L} + \phi_2^{\dagger} \psi_{2L} \,, \tag{5.83}$$

$$\phi^{T} \varepsilon \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix} = \phi_1 \psi_{2L} - \phi_2 \psi_{1L} \,. \tag{5.84}$$

✓

Such singlet expressions can now be combined with a singlet fermion field to give a $SU(2)_L$-invariant Yukawa coupling term. We have then, in addition, to check if the term is also $U(1)_Y$ invariant. A simple exercise shows that the most general $SU(2)_L \times U(1)_Y$ invariant Yukawa term in the SM has the following form

$$\mathcal{L}_{Yuk} = -(\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R)\, C_\ell \begin{pmatrix} \phi^+ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \\ \phi^+ \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \end{pmatrix}$$

$$+ (\bar{u}_R, \bar{c}_R, \bar{t}_R)\, C_q' \begin{pmatrix} \phi^T \varepsilon \begin{pmatrix} u_L \\ d_L' \end{pmatrix} \\ \phi^T \varepsilon \begin{pmatrix} c_L \\ s_L' \end{pmatrix} \\ \phi^T \varepsilon \begin{pmatrix} t_L \\ b_L' \end{pmatrix} \end{pmatrix}$$

$$- (\bar{d}_R', \bar{s}_R', \bar{b}_R')\, C_q \begin{pmatrix} \phi^+ \begin{pmatrix} u_L \\ d_L' \end{pmatrix} \\ \phi^+ \begin{pmatrix} c_L \\ s_L' \end{pmatrix} \\ \phi^+ \begin{pmatrix} t_L \\ b_L' \end{pmatrix} \end{pmatrix}$$

$$+ h.c. \qquad \checkmark \ (5.85)$$

In eq. (5.85) the matrices C_e, C_q' and C_q are in principle arbitrary complex 3×3 matrices. But not all such matrices lead to different physical theories. We have already seen in our simplified theory in section 5.2 where we considered only e and ν_e that the phase of the parameter c_e was arbitrary. Thus we could impose ~~the~~ the condition $c_e \geqslant 0$, see eq. (5.58a). Here, in eq. (5.85), we have, as we shall see, more freedom in field redefinitions which do not change the physics.

We can, for instance, make a change of basis for the right handed lepton fields carrying the same quantum numbers

$$\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \longrightarrow \mathcal{U}_1 \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \tag{5.86}$$

where $\mathcal{U}_1$ is an arbitrary unitary constant matrix, $\mathcal{U}_1 \in \mathcal{U}(3)$:

$$\mathcal{U}_1^\dagger \mathcal{U}_1 = \mathbb{1} . \tag{5.87}$$

All terms in the SM Lagrangian, eq. (5.75) remain the same except the Yukawa term where we have to make the replacement

$$C_e \longrightarrow \mathcal{U}_1^\dagger C_e . \tag{5.88}$$

In a similar way we can make replacements of the lepton doublets, the quark doublets and the quark singlets if they carry the same quantum numbers.

$$\begin{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \end{pmatrix} \longrightarrow V_1 \begin{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \\ \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \end{pmatrix} ,$$

$$V_1 \in \mathcal{U}(3) . \tag{5.89}$$

Analogous for the quark doublets with $V_2 \in \mathcal{U}(3)$.

$$\begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \longrightarrow \mathcal{U}_2 \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix},$$

$$\mathcal{U}_2 \in \mathcal{U}(3); \tag{5.90}$$

$$\begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix} \longrightarrow \mathcal{U}_3 \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}$$

$$\mathcal{U}_3 \in \mathcal{U}(3). \tag{5.91}$$

✓

For the matrices C_e, C_q', C_q this implies the following transformations

$$C_e \longrightarrow \mathcal{U}_1^+ C_e V_1 ,$$

$$C_q' \longrightarrow \mathcal{U}_2^+ C_q' V_2 ,$$

$$C_q \longrightarrow \mathcal{U}_3^+ C_q V_2 . \tag{5.92}$$

Note that the <u>same</u> quark doublets stand to the left of C_q' and C_q in eq. (5.85). Therefore, also the <u>same</u> matrix V_2 must be used for the transformations of C_q' and C_q in eq. (5.92).

✓

Let us first have a look at the matrix C_e for the leptons. The matrix $C_e C_e^\dagger$ is hermitean, positive semi definite, and transforms under eq. (5.92) as

$$C_e C_e^\dagger \longrightarrow \mathcal{U}_1^\dagger C_e V_1 V_1^\dagger C_e^\dagger \mathcal{U}_1 = \mathcal{U}_1^\dagger C_e C_e^\dagger \mathcal{U}_1 . \tag{5.93}$$

Thus, we can choose an appropriate $\mathcal{U}_1$ to achieve diagonal form for $C_e C_e^\dagger$ with non negative diagonal elements:

$$C_e C_e^\dagger = \begin{pmatrix} c_e^2 & 0 & 0 \\ 0 & c_\mu^2 & 0 \\ 0 & 0 & c_\tau^2 \end{pmatrix},$$

$$c_e , c_\mu , c_\tau \geqslant 0 . \tag{5.94}$$

✓

From eq. (5.94) we see that $\mathcal{C}_e$ must then be of the form

$$\mathcal{C}_e = \begin{pmatrix} c_e & 0 & 0 \\ 0 & c_\mu & 0 \\ 0 & 0 & c_\tau \end{pmatrix} W ,$$

$$W \in \mathcal{U}(3). \tag{5.95}$$

Choosing now $V_1 = W^\dagger$ in the transformation, eq (5.92), we see that we can bring $\mathcal{C}_e$ to diagonal form with non negative diagonal elements:

✓

$$C_\ell = \begin{pmatrix} c_e & 0 & 0 \\ 0 & c_\mu & 0 \\ 0 & 0 & c_\tau \end{pmatrix},$$

$$c_e, c_\mu, c_\tau \geq 0 . \qquad (5.96)$$

For the quarks we can transform the matrix C_q' (eq. (5.85)) in exactly the same way to diagonal form. That is, we can require

$$C_q' = \begin{pmatrix} c_u & 0 & 0 \\ 0 & c_c & 0 \\ 0 & 0 & c_t \end{pmatrix}$$

$$c_u, c_c, c_t \geq 0 . \qquad (5.97)$$

✓

But, turning now to $\mathcal{C}_q$, we see that things are different, since V_2 in eq. (5.92) is no longer free for us. We used it already to diagonalise $\mathcal{C}_q'$. We still have $\mathcal{U}_3$ at our disposal and we can use this to achieve for $\mathcal{C}_q$ a form as in eq. (5.95) for $\mathcal{C}_e$. We have then

$$\mathcal{C}_q = \begin{pmatrix} c_d & 0 & 0 \\ 0 & c_s & 0 \\ 0 & 0 & c_b \end{pmatrix} V^{\dagger} ,$$

$$c_d , c_s , c_b \geq 0 ,$$

$$V \in \mathcal{U}(3) . \tag{5.98}$$

✓

Choosing now $\mathcal{U}_3 = V^+$ in eq. (5.92) we get for C_2

$$C_2 = V \begin{pmatrix} c_d & 0 & 0 \\ 0 & c_s & 0 \\ 0 & 0 & c_b \end{pmatrix} V^+ . \tag{5.99}$$

We shall require this as standard form for C_2. The matrix $V \in \mathcal{U}(3)$ which occurs here is the famous <u>CKM (Cabibbo-Kobayashi-Maskawa) matrix</u>. We summarise our findings thus far.

✓

Redefinition of fields allows the following standard forms for C_e, C_q', C_q

$$C_e = \begin{pmatrix} c_e & 0 & 0 \\ 0 & c_\mu & 0 \\ 0 & 0 & c_\tau \end{pmatrix}$$

$$C_q' = \begin{pmatrix} c_u & 0 & 0 \\ 0 & c_c & 0 \\ 0 & 0 & c_t \end{pmatrix}$$

$$C_q = V \begin{pmatrix} c_d & 0 & 0 \\ 0 & c_s & 0 \\ 0 & 0 & c_b \end{pmatrix} V^\dagger$$

$$c_e, c_\mu, \ldots, c_b \geq 0$$

V CKM - Matrix (5.100)

$$\boldsymbol{V} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}$$

$$\equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad (5.101)$$

✓

The CKM matrix $V \in \mathcal{U}(3)$.

That is, it satisfies

$$V^\dagger V = V V^\dagger = \mathbb{1}_3 , \qquad (5.101a)$$

or in components

$$V^*_{ji} V_{jk} = \delta_{ik} ,$$
$$V_{ij} V^*_{kj} = \delta_{ik} . \qquad (5.101b)$$

The rows and the columns of V must each be a set of orthonormal unit vectors. The <u>unitarity condition</u> for the CKM matrix, eq. (5.101a), plays a central role in the SM theory as well as in the experimental analyses.

With our considerations so far we have not exhausted all freedom to make redefinitions of the type given in eq. (5.92). We still can make certain phase transformation which leave the general forms, eq. (5.100), invariant. Setting in eq. (5.92)

$$V_2 = \mathcal{U}_2 = \mathcal{U}_3 = \mathcal{U}_\varphi = \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix} \tag{5.102}$$

we see that C_2' of eq. (5.100) is invariant and the CKM matrix V is transformed into $V' = \mathcal{U}_\varphi^\dagger V$.

Clearly, the form of $\mathcal{C}_2$, eq. (5.100), is also unchanged if we multiply V with a diagonal phase matrix $\mathcal{U}_\chi \in \mathcal{U}(3)$ from the right,

$$\mathcal{U}_\chi = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix} . \tag{5.103}$$

Thus we can use the transformations

$$V \longrightarrow \mathcal{U}_\varphi^\dagger \, V \, \mathcal{U}_\chi , \tag{5.104}$$

with $\mathcal{U}_\varphi$ and $\mathcal{U}_\chi \in \mathcal{U}(3)$, in order to bring the CKM matrix to a standard form.

✓

We have in principle 6 free phases in eq. (5.104): $\varphi_1, \varphi_2, \varphi_3, \chi_1, \chi_2, \chi_3$. But it is easy to see that only the <u>phase differences</u> matter and there are five independent ones. Thus, we can impose five phase conditions on the matrix elements of V. Various forms for such conditions have been proposed and used in the literature. Nowadays the most frequently used conditions are as follows

$$
\begin{aligned}
V_{11} &\geq 0\,, \\
V_{12} &\geq 0\,, \\
V_{23} &\geq 0\,, \\
V_{33} &\geq 0\,, \\
V_{11}\,V_{22} - V_{12}\,V_{21} &\geq 0\,.
\end{aligned}
\tag{5.105}
$$

Now the task is to construct the most general matrix $V \in \mathcal{U}(3)$ respecting eq. (5.105). The result is shown in eq. (5.106).

✓

$$\mathbf{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} = \cos\vartheta_{ij} \quad , \quad s_{ij} = \sin\vartheta_{ij}$$

$$0 \le \vartheta_{ij} \le \pi/2 \quad , \quad 0 \le \delta_{13} < 2\pi \tag{5.106}$$

Note that we find that V is parametrised by three angles $\vartheta_{12}, \vartheta_{23}, \vartheta_{13}$ and one phase δ_{13}. As we will see, CP violation in the standard model can only occur if various conditions are fulfilled, in particular, that $\delta_{13} \neq 0, \pi$.

The parametrisation (5.106) was proposed in L.L. Chau & W.Y. Keung, PRL 53, 1802 (1984).

✓

The CKM matrix in the form (5.106) is the starting point for Wolfenstein's parametrisation.
Here we define 4 new parameters $\lambda, A, \varrho, \eta$ instead of the angles $\vartheta_{12}, \vartheta_{23}, \vartheta_{13}$ and the phase δ_{13} by setting:

$$\lambda = s_{12},$$

$$A = \frac{s_{23}}{(s_{12})^2},$$

$$\varrho - i\eta = \frac{s_{13}\, e^{-i\delta_{13}}}{s_{12}\, s_{23}}. \qquad (5.107)$$

✓

The reverse transformation is

$$s_{12} = \lambda \,,$$
$$s_{23} = \lambda^2 A \,,$$
$$s_{13}\, e^{-i\delta_{13}} = \lambda^3 A\,(\rho - i\eta) \,.$$

(5.108)

As we will see $\lambda = \sin\vartheta_{12}$ is, to a good approximation, equal to the sine of the Cabbibo angle. Thus $\lambda \approx 0.22$ is a relatively small number compared to 1 and it makes sense to expand the matrix V in powers of λ:

$$c_{12} = \sqrt{1 - s_{12}^2} = \sqrt{1 - \lambda^2}$$

$$= 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + O(\lambda^6)$$

etc.

(5.109)

In this way we find for V the following form:

$$c_{12} = 1 - \frac{1}{2}\lambda^2 + O(\lambda^4),$$
$$c_{23} = 1 + O(\lambda^4),$$
$$c_{13} = 1 + O(\lambda^6),$$

(5.110)

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} + O(\lambda^4)$$

$$\lambda = s_{12}\,, \quad A = s_{23}/(s_{12})^2\,, \quad \rho - i\eta = \frac{s_{13}\, e^{-i\delta_{13}}}{s_{12}\, s_{23}} \tag{5.111}$$

Today this form of the CKM matrix is frequently used in phenomenological analyses.

Now we go back to the standard form for our matrices C_ℓ, C_q' and C_q, eq. (5.100). We insert these matrices in $\mathcal{L}_{Yuk}$, eq. (5.85) and go to the unitary gauge. That is, we replace $\phi(x)$ by its expression in the unitary gauge (see eq. (5.68))

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\,(v + H(x)) \end{pmatrix}. \qquad (5.112)$$

✓

We get then for $\mathcal{L}_{Yuk}$

$$\mathcal{L}_{Yuk} = -\left\{ (\bar{e}, \bar{\mu}, \bar{\tau}) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \right.$$

$$+ (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$\left. + (\bar{d}', \bar{s}', \bar{b}') V \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} V^\dagger \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \right\}$$

$$\left(1 + \frac{H}{v}\right) \tag{5.113}$$

Here we set

$$m_e = c_e \frac{v}{\sqrt{2}} \;, \; - - - - \;, \; m_b = c_b \frac{v}{\sqrt{2}} \;. \tag{5.114}$$

We see that SSB produces mass terms and couplings to the physical Higgs field $H(x)$ proportional to these masses. This is the extension of the result which we found for the simplified model in section 5.2 to the general case. We see that in (5.113) the mass terms for the charged leptons and the up-type quarks are already in standard form since we chose the matrices C_ℓ and C_q' appropriately. For the down-type quarks we can achieve the standard form for the mass terms by defining new fields, the "mass eigenfields":

$$\begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix} = V^{\dagger} \begin{pmatrix} d'(x) \\ s'(x) \\ b'(x) \end{pmatrix} . \tag{5.115}$$

Here the fields $d'(x), s'(x), b'(x)$ are, as we recall, the weak isospin partners with $T_3 = -1/2$ to the mass eigenfields with $T_3 = +1/2$, that is, to $u(x), c(x), t(x)$. In terms of the new fields d, s, b we get for $\mathcal{L}_{Yuk}$ the following:

In the unitary gauge:

$$\mathcal{L}_{Yuk} = \Big\{ -(\bar{e},\bar{\mu},\bar{\tau}) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$-(\bar{u},\bar{c},\bar{t}) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$-(\bar{d},\bar{s},\bar{b}) \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \Big\}$$

$$\cdot \left(1 + \frac{H}{v}\right)$$

$$m_e = c_e \frac{v}{\sqrt{2}}, \; - - - - - , \; m_b = c_b \frac{v}{\sqrt{2}} \qquad (5.116)$$

The transformation to the mass eigenfields, eq. (5.115), ~~h~~as, of course, also consequences for the other terms of $\mathcal{L}_{QFD}$; eqs (5.76), (5.79). We have to look at the fermion - fermion - gauge boson coupling $\mathcal{L}_{Int}$, eq. (5.79) which contains the electromagnetic, the weak neutral and the weak charged currents. We have written the general expressions for these currents in eqs. (5.80) - (5.82). Substituting there the neutrino, the lepton and the quark fields from p. 5-66 we get for J^{λ}_{em} :

$$
\begin{aligned}
J^\lambda_{em} = &-(\bar{e}, \bar{\mu}, \bar{\tau})\, \gamma^\lambda \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \\
&+ \frac{2}{3} (\bar{u}, \bar{c}, \bar{t})\, \gamma^\lambda \begin{pmatrix} u \\ c \\ t \end{pmatrix} \\
&- \frac{1}{3} (\bar{d}', \bar{s}', \bar{b}')\, \gamma^\lambda \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}
\end{aligned}
\qquad (5.117)
$$

Now we have to replace d', s', b' by the mass eigenfields d, s, b according to eq. (5.115).

$$(\bar{d}', \bar{s}', \bar{b}')\, \gamma^\lambda \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

$$= (\bar{d}, \bar{s}, \bar{b})\, V^\dagger \gamma^\lambda V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= (\bar{d}, \bar{s}, \bar{b})\, \gamma^\lambda \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \qquad (5.118)$$

The CKM matrix V drops out here since it is unitary and the final answer for J^λ_{em} is

$$J^{\lambda}_{em} = -(\bar{e}, \bar{\mu}, \bar{\tau})\gamma^{\lambda}\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$+\frac{2}{3}(\bar{u}, \bar{c}, \bar{t})\gamma^{\lambda}\begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

$$-\frac{1}{3}(\bar{d}, \bar{s}, \bar{b})\gamma^{\lambda}\begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

(5.119)

In a completely analogous way the CKM matrix V drops out when we express J^{λ}_{NC} in terms of the mass eigenfields.

$$
\begin{aligned}
J^{\lambda}_{NC} = {} & (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\,\gamma^\lambda \frac{1}{2}\,\frac{1-\gamma_5}{2} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \\
& + (\bar{e}, \bar{\mu}, \bar{\tau})\,\gamma^\lambda \left(-\frac{1}{2}\,\frac{1-\gamma_5}{2} + \sin^2\vartheta_W \right) \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \\
& + (\bar{u}, \bar{c}, \bar{t})\,\gamma^\lambda \left(\frac{1}{2}\,\frac{1-\gamma_5}{2} - \frac{2}{3}\sin^2\vartheta_W \right) \begin{pmatrix} u \\ c \\ t \end{pmatrix} \\
& + (\bar{d}, \bar{s}, \bar{b})\,\gamma^\lambda \left(-\frac{1}{2}\,\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\vartheta_W \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix} .
\end{aligned}
\tag{5.120}
$$

But V does remain when we express the charged weak current in terms of mass eigenfields.

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = \mathbf{V}^{\dagger} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

$$J^{\lambda}_{CC} = (\bar{\nu}_{e_L}, \bar{\nu}_{\mu_L}, \bar{\nu}_{\tau_L}) \gamma^{\lambda} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

$$+ \underbrace{(\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^{\lambda} \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}}$$

$$= (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^{\lambda} \mathbf{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

(5.121)

To summarise: we have seen that the Yukawa coupling term gives rise to the lepton and quark masses after SSB.
The weak isospin partners of the mass eigen fields u, c, t are fields d', s', b' which are not mass eigenfields. The down type mass eigenfields d, s, b are related to d', s', b' by a unitary matrix V, the CKM matrix.
This matrix shows up and can therefore be measured in the coupling of the quarks to the $W^{\pm}$ bosons.