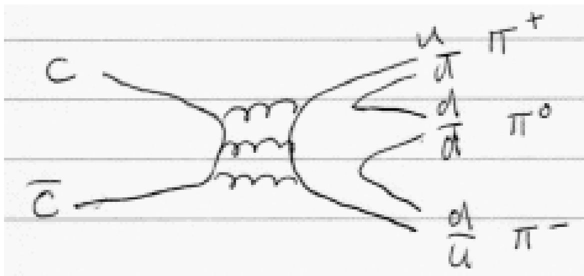
# back to strong interaction: determination of coupling constant $\alpha_s$

**1) decay width of ccbar and bbar states** $q^2 = (9\text{-}100)\ \mathrm{GeV}^2$

have seen that for heavy quarks one can define QCD potential $V(r) = -4/3\ \alpha_s/r + kr$

quarkonia: see Figure next page; e.g. J/ψ is triplet 1s state of ccbar in this potential

decay:




like triplet 1s ortho-positronium: e+e- -> 3γ

$$\Gamma(3\gamma) = 2(\pi^3 - 9)/9\pi\ \alpha^6 \cdot m$$

3 powers of alpha for 3 photons <-> 3 powers of $\text{alpha}_s$

for J/ψ remaining 3 powers: square of wave function at origin $(\Psi(0))^2 \infty 1/a^3$ with Bohr radius $a \infty 1/\alpha$

insert $4/3\ \alpha_s$ instead of $\alpha$ and use $m=1.6$ GeV -> $\alpha_s = 0.23$

similar for bbar decay: -> $\alpha_s = 0.18$

to eliminate dependence on $\Psi(0)$ take instead ratio

$$R_\mu = \Gamma_{\gamma\gamma\gamma} / \Gamma_{\mu\mu} = 10(\pi^2 - 9)/9\pi\ \alpha_s^3/\alpha^2$$

**2) R = σ (e+e- -> hadrons)/ σ (point) in continuum** (see slide 14 above) contains factor $1 + \alpha_s/\pi$ color-anticolor states of final state quarks can change into different color-anticolor state by exchange of one gluon (type antired-green etc.) $q^2 = (25\text{-}1000)\ \mathrm{GeV}^2$

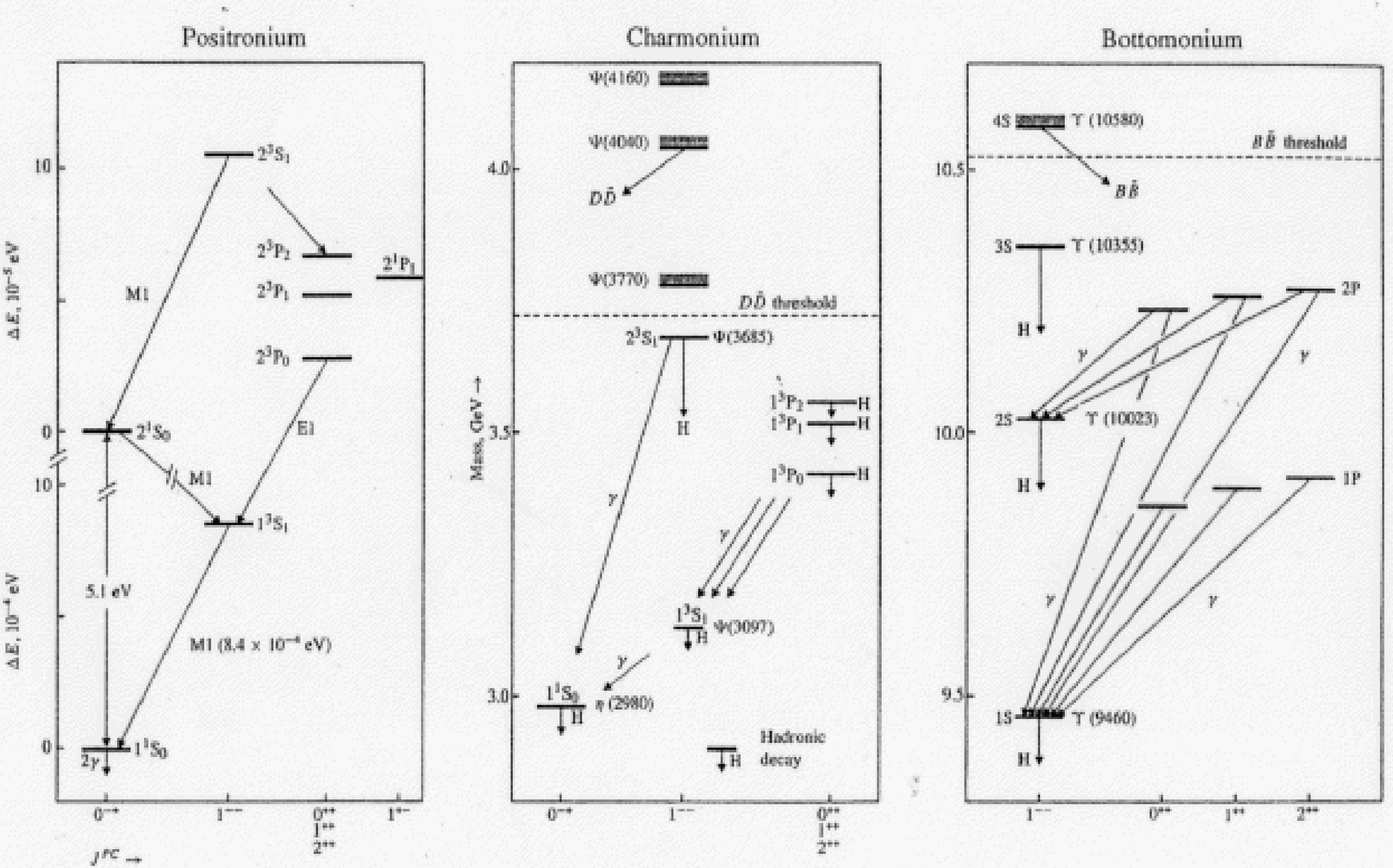



Fig. 4.8. The energy-level diagrams for positronium, charmonium and bottomonium. Note the changes in scale for positronium. Only states with $J^{PC} = 1^{--}$ can be accessed in $e^+e^-$ annihilation experiments. Note that the atomic physics convention is to label the lowest-lying P states of positronium as 2P, while for the charmonium and bottomonium states the nuclear physics nomenclature 1P is employed. The shading indicates broad states.

## 3) Deep Inelastic Scattering (DIS):

- scaling violations in structure functions $\quad q^2 = (25\text{-}100)\ \mathrm{GeV}^2$

lepton-proton DIS in terms of variables (x,y) (momentum and energy fraction of scattering quark in proton) described by structure functions $F_2(x,q^2)$ and $F_3(x,q^2)$

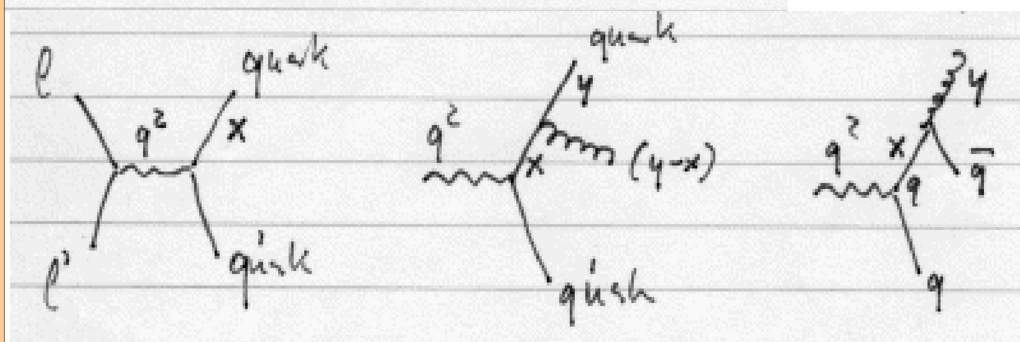
$$\frac{d^2\sigma(ep)}{dy\,dx} = \frac{4\pi\alpha^2}{q^4} \times s\, \frac{F_2^{ep}(x)}{x}\left[\frac{1+(1-y)^2}{2}\right]$$

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}\left(u(x)+\bar{u}(x)\right) + \frac{1}{9}\left(d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right)$$

$$\frac{du(x,q^2)}{d(\ln q^2)}\,dx = \frac{\alpha_s(q^2)}{2\pi}\left(\int_{y=x}^{y=1} u(y,q^2)\, \underbrace{P_{QQ}\left(\frac{x}{y}\right)}_{\text{splitting function}} \frac{dy}{y}\right)dx$$

quark retains fraction $\left(\frac{x}{y}\right)$

**$q^2$ evolution of quark distributions** (Altarelli-Parisi, DGLAP):
**scaling** that is valid for point-like constituents (quarks) **violated** because quark with initial mom. fraction y can radiate a gluon with momentum (y-x) before quark with mom franction x/y scatters
process described by splitting function $P_{QQ}$



**generally structure functions and $\alpha_s(q^2)$ are obtained together**

**- sum rules:** (see e.g. Perkins ch. 5.9)

to obtain results independent of gluon distribution of nucleon use e.g. Gross- Llewellyn Smith sum rule for DIS of neutrinos off nucleons

cross section described in terms of two function $F_2$ (see above) and $F_3$

$$\frac{F_2^{\nu N}(x)}{x} = u(x) + d(x) + \bar{u}(x) + \bar{d}(x)$$

$$\frac{F_3^{\nu N}(x)}{x} = u(x) + d(x) - \bar{u}(x) - \bar{d}(x) \quad \text{valence quarks}$$

$$\int_0^1 dx \left( F_3^{\nu p}(x, q^2) + F_3^{\bar{\nu} p}(x, q^2) \right) = 3 \quad \text{'free parton' model or } q^2 \to \infty$$

$$\text{for finite } q^2 : \; 3\left(1 - \left(\frac{\alpha_s}{\pi} + 3.58 \frac{\alpha_s^2}{\pi^2} + 19.0 \frac{\alpha_s^3}{\pi^3}\right)\right) - \Delta HT$$

neutrino
scatters off d and ubar
antineutrino
scatters off dbar and u
via W exchange

GLS sum rule

**data by CCFR – NuTeV (1995) in neutrino and antineutrino scattering at $q^2 = 3\ GeV^2$**

**4) Event-shapes in e+e- -> hadrons**

essentially 3-jet vs 2-jet cross section (see above), measures gluon radiation probability and scales with $\alpha_s$ $q^2 = (100 - 20\,000)\ \mathrm{GeV}^2$

**5) jets in DIS e.g. at HERA and high energy hadron collisions**

advantage over e+e-: scale can be set by $q^2$, i.e. can be varied over a wide range in single experiment at one beam energy

disadvantage: depend on parton distributions in proton

HERA: $q^2 = (100\text{-}4000)\ \mathrm{GeV}^2$ data from H1 and ZEUS

highest $q^2$ sofar from Tevatron $(40\text{-}250\ \mathrm{GeV})^2$ in D0 and CDF

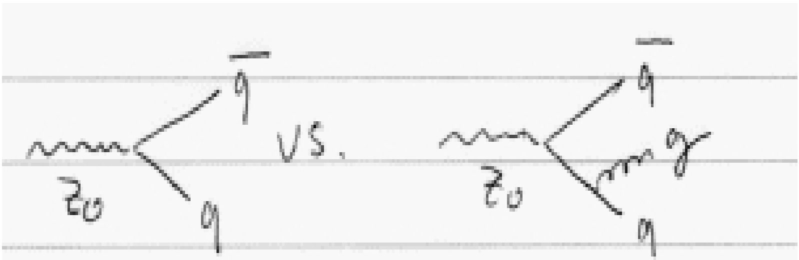
**6) τ-decay** $\tau \to \nu_\tau + \text{hadrons}$ $q^2 \approx m_\tau^2$

$R_\tau = \Gamma(\tau \to \nu_\tau + \text{hadrons}) / \Gamma(\tau \to \nu_\tau\ \ \ e)$

**7) most precise data on $\alpha_s$ :**

QCD corrections to hadronic width of $Z^0$

measured very precisely at LEP




**8) data compared to lattice spectroscopy**

e.g. Υ(1S-1P) or Υ(1S-2S) or ψ (1S-1P)

| Process | $Q$ [GeV] | $\alpha_S(Q^2)$ | $\alpha_S(M_Z^2)$ | Theory |
|---|---|---|---|---|
| DIS [BjSR] | 1.58 | $0.375^{+0.062}_{-0.081}$ | $0.122^{+0.005}_{-0.009}$ | NNLO |
| DIS [GLS] | 1.73 | $0.26^{+0.04}_{-0.05}$ | $0.108^{+0.006}_{-0.009}$ | NNLO |
| DIS [$\nu$] | 5.0 | $0.193^{+0.019}_{-0.018}$ | $0.111 \pm 0.006$ | NLO |
| DIS [$\mu$] | 7.1 | $0.180 \pm 0.014$ | $0.113 \pm 0.005$ | NLO |
| DIS [jets] | 22.1 | $0.148 \pm 0.016$ | $0.118 \pm 0.009$ | NLO |
| $\tau$ [$R_{had}$] | 1.77 | $0.33 \pm 0.03$ | $0.118 \pm 0.004$ | NNLO |
| $Q\bar{Q}$ [1P,2S–1S] | 3.6 | $0.219 \pm 0.007$ | $0.115 \pm 0.002$ | LGT(NR) |
| $Q\bar{Q}$ [1P–1S] | 5.0 | $0.190^{+0.014}_{-0.011}$ | $0.112 \pm 0.004$ | LGT(W) |
| $Q\bar{Q}$ [decay] | 10.0 | $0.167^{+0.015}_{-0.011}$ | $0.113^{+0.007}_{-0.005}$ | NLO |
| $e^+e^-$ [$\sigma_{had}$] | 31.6 | $0.163 \pm 0.022$ | $0.133 \pm 0.015$ | NNLO |
| $e^+e^-$ [shapes] | 10.5 | $0.165 \pm 0.018$ | $0.113 \pm 0.006$ | NLO |
| $e^+e^-$ [shapes] | 35.0 | $0.140 \pm 0.020$ | $0.119 \pm 0.014$ | NLO |
| $e^+e^-$ [shapes] | 58.0 | $0.130 \pm 0.008$ | $0.122 \pm 0.007$ | NLO |
| $e^+e^-$ [shapes] | 58.0 | $0.132 \pm 0.008$ | $0.124 \pm 0.007$ | resum. |
| $e^+e^-$ [frag.] | 22–91 | | $0.126 \pm 0.009$ | NLO |
| $Z^0$ [$\Gamma_{had}$] | 91.2 | $0.123 \pm 0.005$ | $0.123 \pm 0.005$ | NNLO |
| $Z^0$ [shapes] | 91.2 | $0.118 \pm 0.006$ | $0.118 \pm 0.006$ | NLO |
| $Z^0$ [shapes] | 91.2 | $0.122 \pm 0.005$ | $0.122 \pm 0.005$ | resum. |
| $e^+e^-$ [shapes] | 133 | $0.107 \pm 0.008$ | $0.113 \pm 0.009$ | resum. |
| $p\bar{p}, pp \to \gamma + X$ | 4.0 | $0.206^{+0.045}_{-0.032}$ | $0.112^{+0.012}_{-0.010}$ | NLO |
| $p\bar{p} \to b\bar{b}X$ | 20.0 | $0.145^{+0.018}_{-0.019}$ | $0.113 \pm 0.011$ | NLO |
| $p\bar{p} \to W +$ jets | 80.6 | $0.123 \pm 0.025$ | $0.121 \pm 0.024$ | NNLO |

there are very many possibilities
to extract $\alpha_s$ from data
the trick always is to find
process that can be calculated
in perturbation theory (up to say
NLO or NNLO) without
any nonperturbative effects
being the limiting factor
in terms of error

Table from Ellis, Sterling, Webber

for QCD running coupling constant with $n_f = 3$, $n_b = 3$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{7}{4\pi}\alpha_s(\mu^2)\ln\left(\frac{q^2}{\mu^2}\right)}$$

since gluon carry color charge
spread out color charge $\rightarrow$
antishielding
$\alpha_s$ decreases w. incr. $q^2$

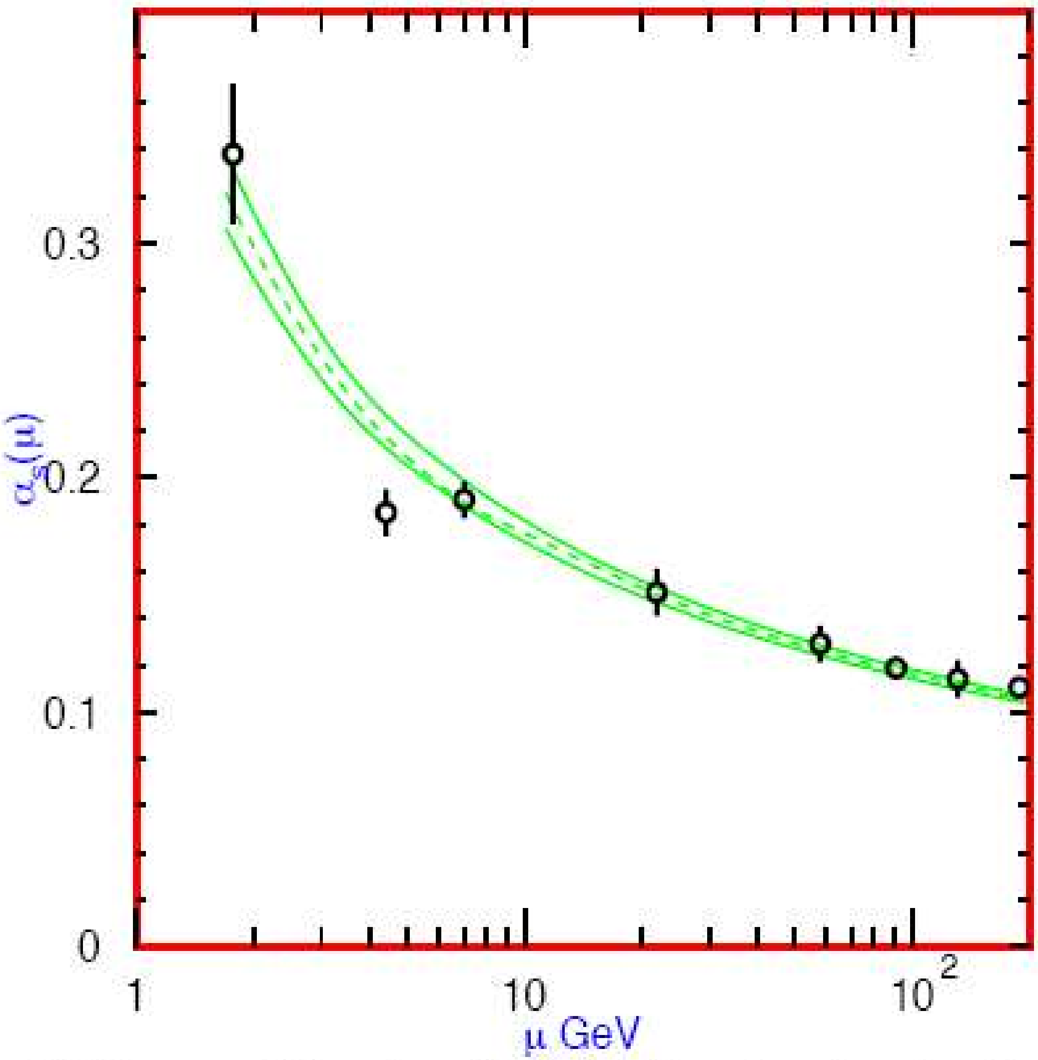



**Figure 9.2:** Summary of the values of $\alpha_s(\mu)$ at the values of $\mu$ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing $\mu$. The data are, in increasing order of $\mu$, $\tau$ width, $\Upsilon$ decays, deep inelastic scattering, $e^+e^-$ event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV, $Z$ width, and $e^+e^-$ event shapes at 135 and 189 GeV.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available on the PDG WWW pages (URL: `http://pdg.lbl.gov/`) November 17, 2006 13:11

In this review, we will use only the new result [166] of $\alpha_s(M_Z) = 0.1170 \pm 0.0012$, which is consistent with the value $\alpha_s(M_Z) = 0.121 \pm 0.003$ used in the last version of this review [167].

166. Q. Mason *et al.*, Phys. Rev. Lett. **95**, 052002 (2005).
167. C. T. H. Davies *et al.*, Phys. Rev. Lett. **92**, 022001 (2004).

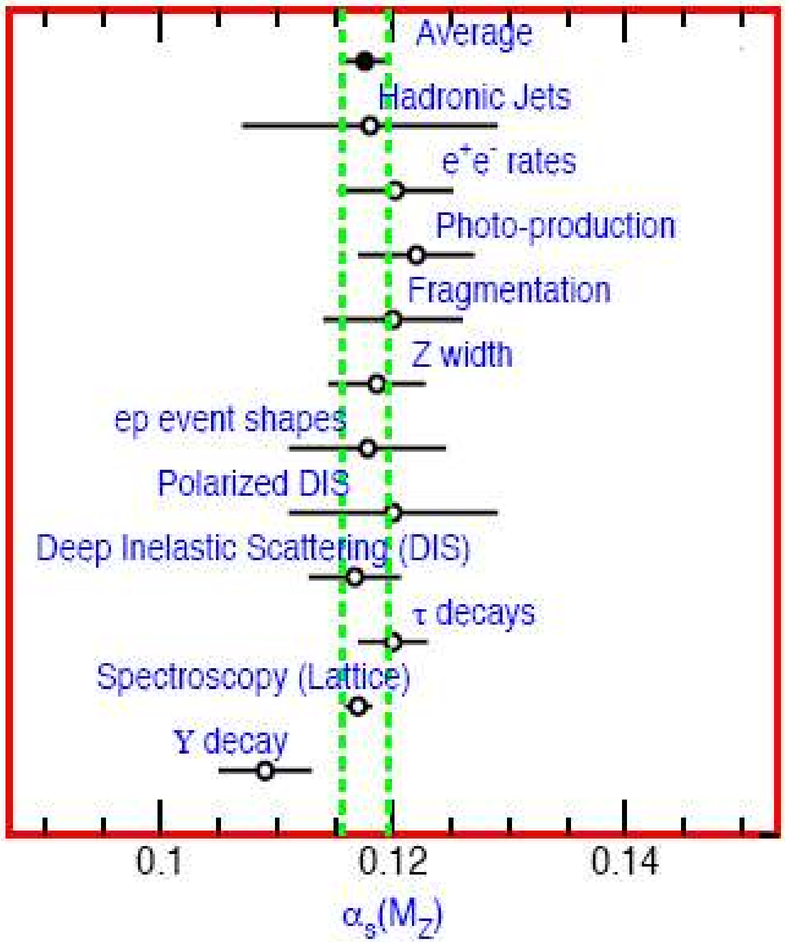


**Figure 9.1:** Summary of the value of $\alpha_s(M_Z)$ from various processes. The values shown indicate the process and the measured value of $\alpha_s$ extrapolated to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties. The average quoted in this report which comes from these measurements is also shown. See text for discussion of errors.