Z-pole observables

- mass M_Z
- total width Γ_Z
- partial widths Γ_{ff} for $Z^0 \to f\bar{f}$ with $f = e, \mu, \tau$, hadrons, c, b

Γ_{had} is partial width for final state hadrons

$\leftrightarrow \sigma_{had} = 12\pi \, \Gamma_{ee} \Gamma_{had} / M_Z^2 \Gamma_Z^2$

$$\Gamma_{ff} = N_c \frac{G_F M_Z^3}{24\pi\sqrt{2}} (g_V^{f2} + g_A^{f2})(1+\delta)$$

N_c : number of colors

δ : accounts for final state QED and QCD radiative effects

$\delta \simeq 1 + 3\alpha q_f^2/4\pi + n_f \alpha_s/\pi$

n_f = 1 quark, 0 leptons

Note: use Breit-Wigner formula

$$\sigma = \frac{4\pi \lambda\!\!\!^{-2}(2J+1)}{(2S+1)^2} \frac{\Gamma_{ee}\Gamma_Z/4}{[(E-E_0)^2 + \Gamma_Z^2/4]}$$

for $e^+e^- \to Z^0 \to$ anything

with E in cm, $E_0 = M_Z$, $\lambda\!\!\!^- = \frac{2}{E}$ de Broglie wavelength of colliding part., $S = 1/2$ (fermions)

$J = 1$ (Z^0 spin)

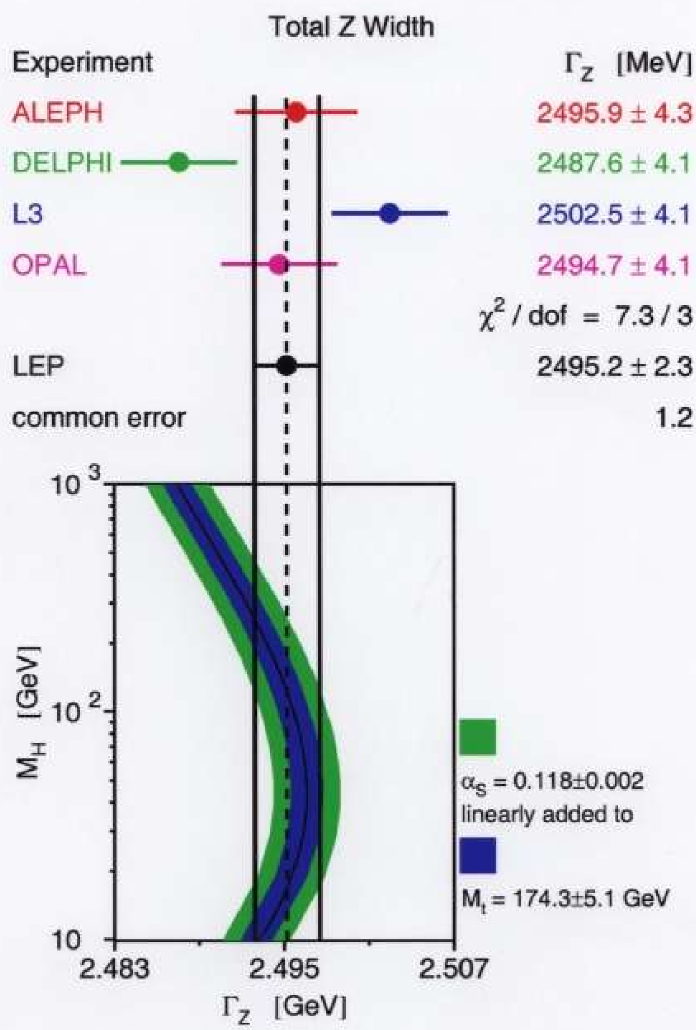
at resonance peak $\boxed{\sigma_{max}(e^+e^- \to Z^0 \to X) = \frac{12\pi}{M_Z^2}\left(\frac{\Gamma_{ee}}{\Gamma_Z}\right)}$

for $e^+e^- \to Z^0 \to$ hadrons

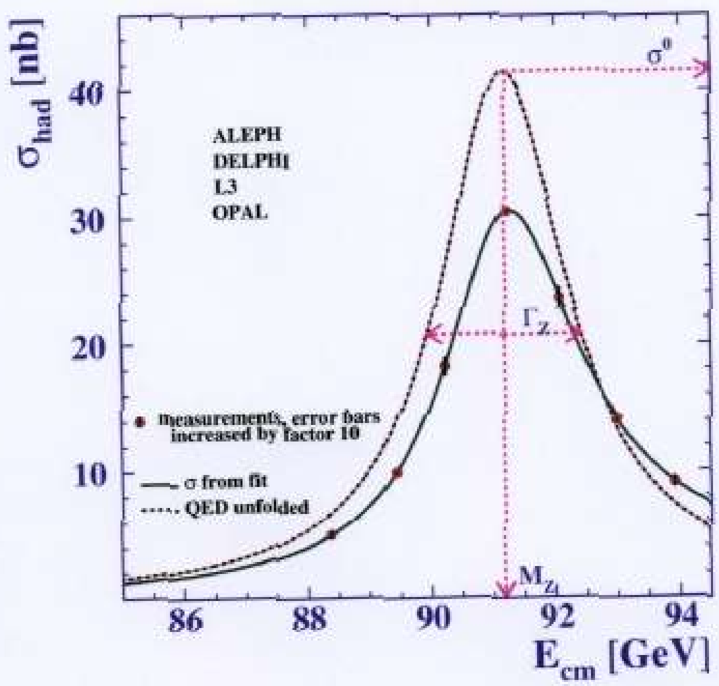
$$\sigma_{had} = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{had}}{\Gamma_Z^2}$$

etc ...

Total Width of Z



Hadronic cross section near Z-pole and QED deconvolution to get Z-parameters

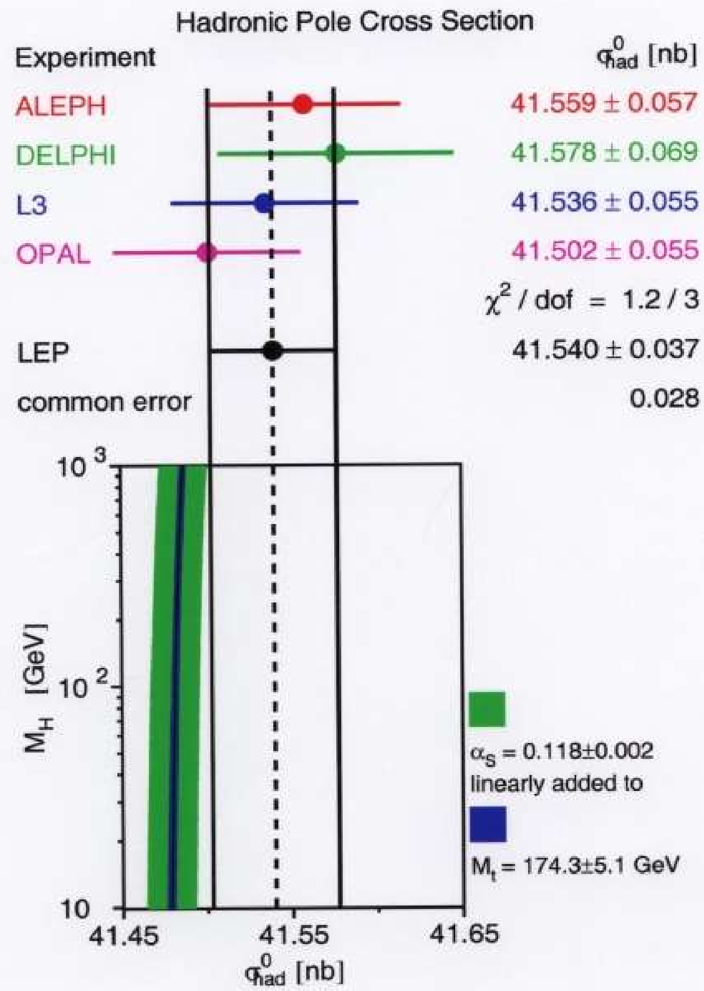


— measure σ(s)

--- extract $\sigma_{ew}(s)$ unfolding for radiative QED (and QCD) corrections

Hadronic Decay Cross Section and Width of Z

$$\sigma_{had} = 12\,\pi \Gamma_{ee} \Gamma_{had} / \mathrm{M}_Z^2\ \Gamma_Z^2$$



Asymmetries : $\longleftrightarrow \sin^2\vartheta_W^{eff} = \frac{1}{2}\left(1 - \frac{g_V^{eff}}{g_A^{eff}}\right)$

- forward-backward asymmetries $e^+e^- \to e^+e^-$

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

σ_F cross section for e^- to travel forward with respect to e^-

first observed at PETRA and PEP, TRISTAN; complicated expression in terms of $g_A^e, g_V^e, g_A^f, g_V^f$

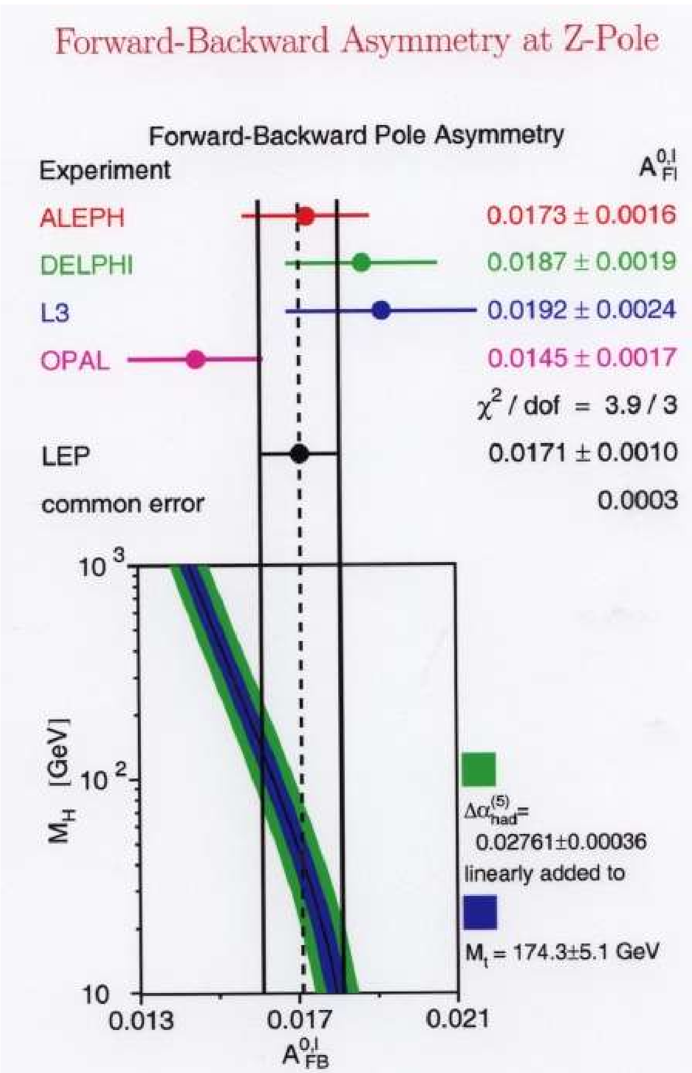
- for $s \ll M_Z^2$ $\quad A_{FB} = -\frac{3 g_A^2}{4\sqrt{2}} \frac{G_F \cdot s}{\alpha}$

at small s depends only on g_A, very small dependence on g_V and $\sin^2\vartheta_W$

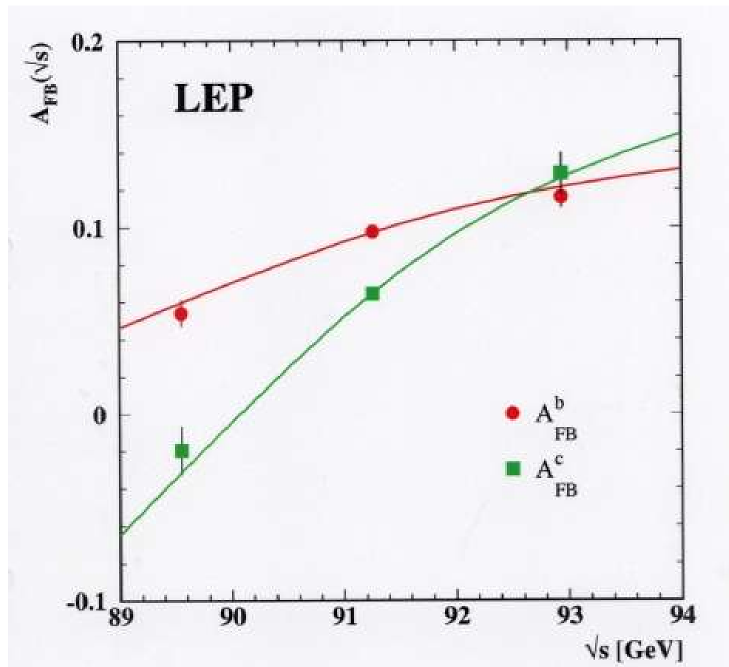
- for $s \simeq M_Z^2$ "pole asymmetry", for $P_e = 0$

$$A_{FB}^{0,f} = \frac{3}{4} A_e A_f \qquad f = e, \mu, \tau, b, c, s \text{ and } q \text{ for hadronic charge asymmetry}$$

smallness of $\frac{g_V}{g_A}$ for e, μ, τ $\simeq 0.08$ makes A_f very sensitive to vacuum pol. $\delta A_{e,\mu,\tau} \simeq -8\,\delta(\sin^2\vartheta_W^{eff})$



and for heavy quarks:



expect at Z-pole

$$A_{FB}^{0c} = \frac{3}{4} A_e A_c = \frac{3}{4} A_e A_u \simeq 0.075$$

$$A_{FB}^{0b} = \frac{3}{4} A_e A_b = \frac{3}{4} A_e A_d \simeq 0.105$$

left-right coupling constant asymmetry A_f

$$A_f = \frac{2 g_A^f g_V^f}{g_A^{f2} + g_V^{f2}} = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}$$

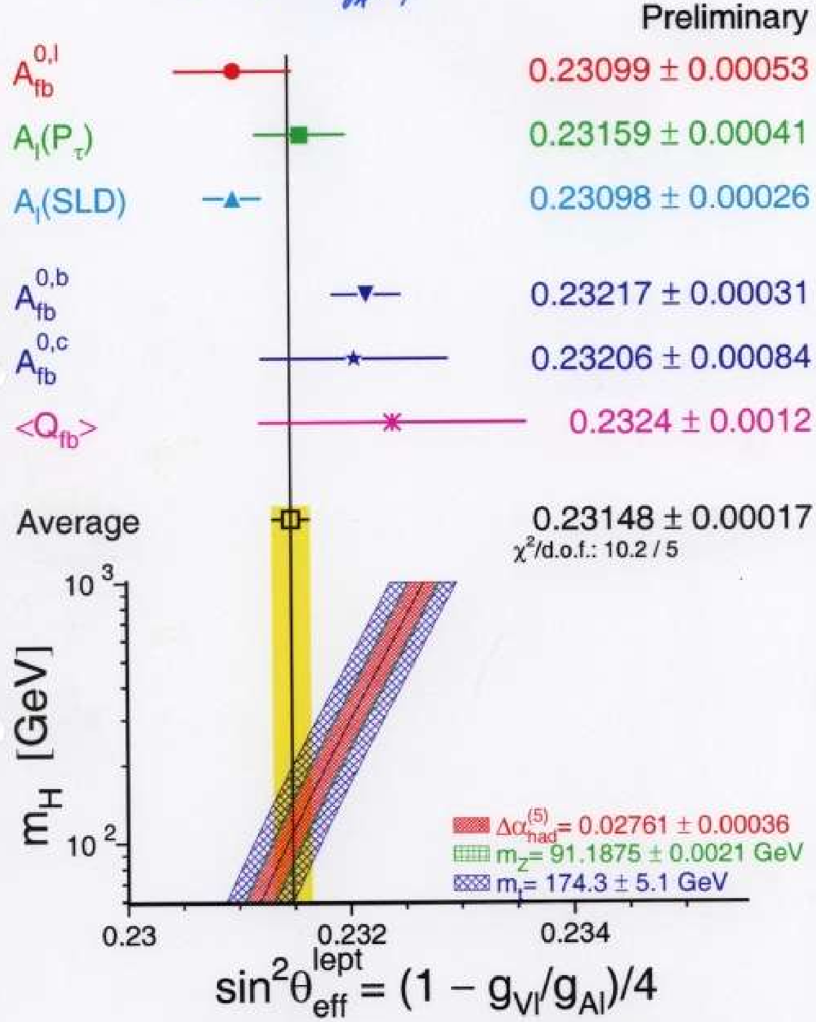
- Z-pole left-right asymmetry $e^+e^- \to f\bar{f}$ $f \neq e$

$$A_{LR}^0 = \left(\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \right) = A_e$$ using left or right circularly polarized beams

very precisely measured by SLD at SLC

- extremely sensitive to $\sin^2\vartheta_W$; PRL 87 (1997) 2075

$\sin^2\theta_W^{eff} = \frac{1}{4}\left(1 - \frac{g_V^{eff}}{g_A^{eff}}\right)$

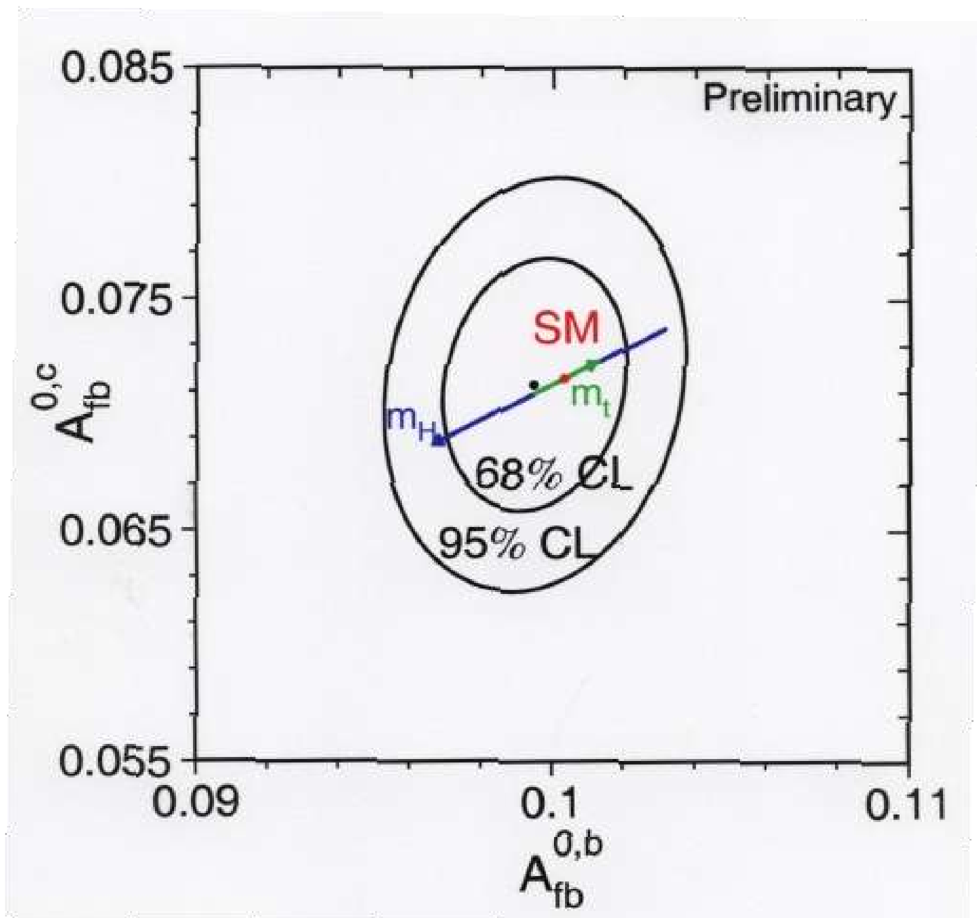


large sensitivity to m_H!

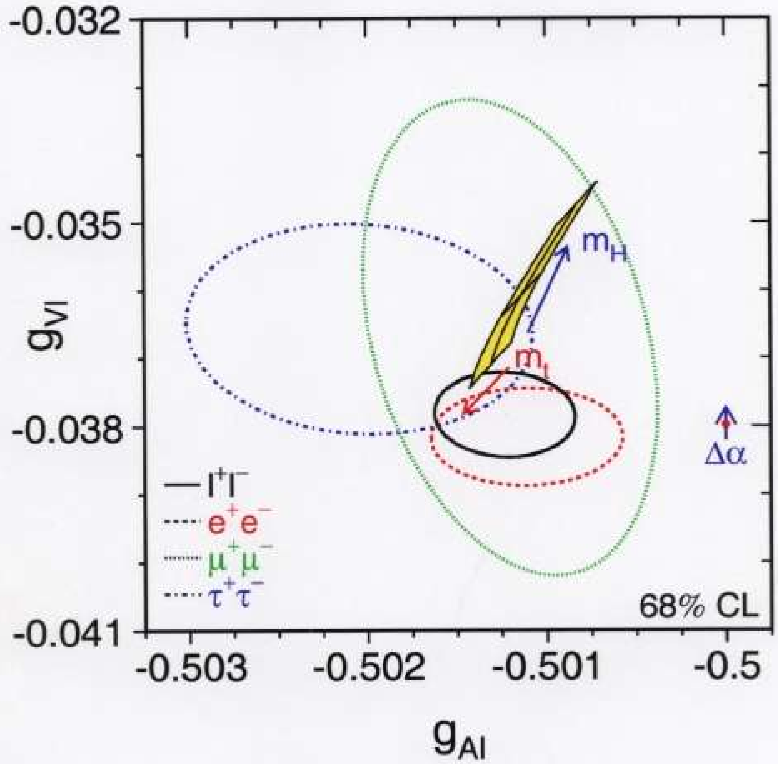
Correlations of various Z-pole observables test:

- consistency of standard model
- lepton universality
- sensitivity to various radiative corrections
- α_s, m_t, m_H

forward-backward asymmetries of charm vs beauty quarks in final state



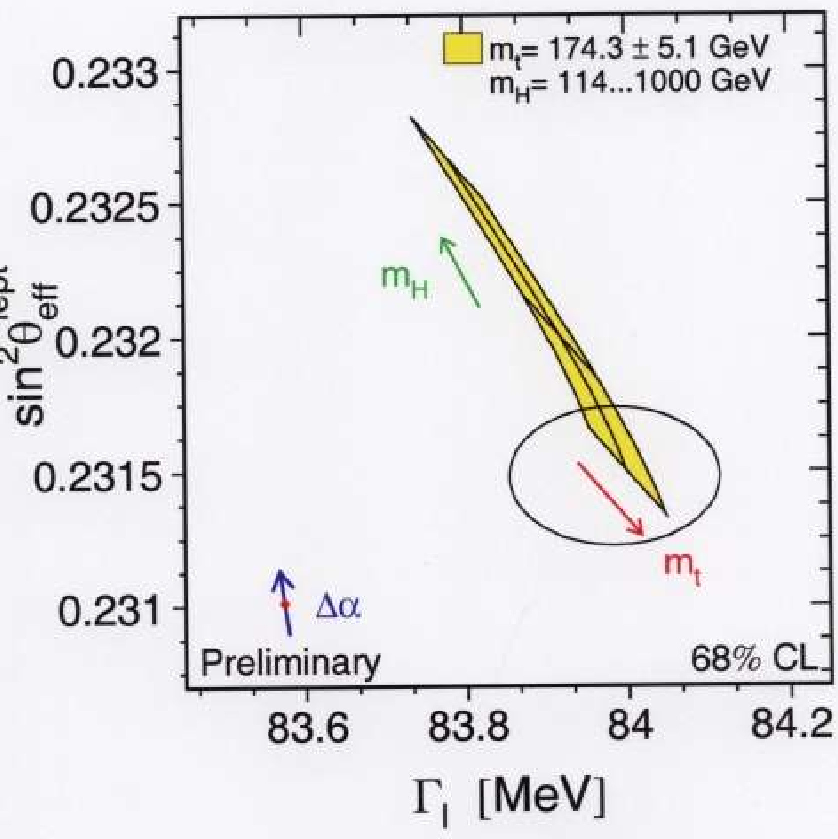
Vector and Axial Vector Coupling of Leptons



from A_{FB} and partial widths

- test for lepton universality
- agreement w. standard model and sensitivity to m_t, m_H

Correlation $\sin^2\vartheta_{eff}^{\ell}$ and $\Gamma_{\ell\ell}$: purely leptonic measurements



• only vacuum pol of γ

↑ sensitivity to error in $\alpha(m_Z)$

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
STANFORD LINEAR ACCELERATOR CENTER

CERN-PH-EP/2005-041
SLAC-R-774
hep-ex/0509008
7 September 2005

Precision Electroweak Measurements on the Z Resonance

The ALEPH, DELPHI, L3, OPAL, SLD Collaborations,[1]
the LEP Electroweak Working Group,[2]
the SLD Electroweak and Heavy Flavour Groups

Accepted for publication in *Physics Reports*

Updated: 20 February 2006

	Measurement with Total Error	Systematic Error	Standard Model High-Q^2 Fit	Pull
$\Delta\alpha^{(5)}_{\mathrm{had}}(m^2_{\mathrm{Z}})$ [59]	0.02758 ± 0.00035	0.00034	0.02767 ± 0.00035	0.3
m_{Z} [GeV]	91.1875 ± 0.0021	$^{(a)}0.0017$	91.1874 ± 0.0021	0.1
Γ_{Z} [GeV]	2.4952 ± 0.0023	$^{(a)}0.0012$	2.4965 ± 0.0015	0.6
σ^0_{had} [nb]	41.540 ± 0.037	$^{(a)}0.028$	41.481 ± 0.014	1.6
R^0_ℓ	20.767 ± 0.025	$^{(a)}0.007$	20.739 ± 0.018	1.1
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	$^{(a)}0.0003$	0.01642 ± 0.00024	0.8
+ correlation matrix Table 2.13				
$\mathcal{A}_\ell$ (P_τ)	0.1465 ± 0.0033	0.0015	0.1480 ± 0.0011	0.5
$\mathcal{A}_\ell$ (SLD)	0.1513 ± 0.0021	0.0011	0.1480 ± 0.0011	1.6
R^0_{b}	0.21629 ± 0.00066	0.00050	0.21562 ± 0.00013	1.0
R^0_{c}	0.1721 ± 0.0030	0.0019	0.1723 ± 0.0001	0.1
$A^{0,\mathrm{b}}_{\mathrm{FB}}$	0.0992 ± 0.0016	0.0007	0.1037 ± 0.0008	2.8
$A^{0,\mathrm{c}}_{\mathrm{FB}}$	0.0707 ± 0.0035	0.0017	0.0742 ± 0.0006	1.0
$\mathcal{A}_{\mathrm{b}}$	0.923 ± 0.020	0.013	0.9346 ± 0.0001	0.6
$\mathcal{A}_{\mathrm{c}}$	0.670 ± 0.027	0.015	0.6683 ± 0.0005	0.1
+ correlation matrix Table 5.11				
$\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$ ($Q^{\mathrm{had}}_{\mathrm{FB}}$)	0.2324 ± 0.0012	0.0010	0.23140 ± 0.00014	0.8
m_{t} [GeV] (Run-I [212])	178.0 ± 4.3	3.3	178.5 ± 3.9	0.1
m_{W} [GeV]	80.425 ± 0.034		80.389 ± 0.019	1.1
Γ_{W} [GeV]	2.133 ± 0.069		2.093 ± 0.002	0.6
+ correlation given in Section 8.3.2				

Table 8.4: Summary of measurements included in the analyses of the five SM input parameters. The top 15 results are included in the Z-pole and the high-Q^2 fit, while the bottom three results are only used in the high-Q^2 fit. The total errors in column 2 include the systematic errors listed in column 3. Although the systematic errors include both correlated and uncorrelated sources, the determination of the systematic part of each error is approximate. The SM results in column 4 and the pulls (absolute value of the difference between measurement and fit in units of the total measurement error, see Figure 8.14) in column 5 are derived from the SM analysis of all 18 results, including also the correlations between results presented in Chapter 7.1. The direct measurements of m_{W} and Γ_{W} used here are preliminary.
$^{(a)}$Only common systematic errors are indicated.

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^0 [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5

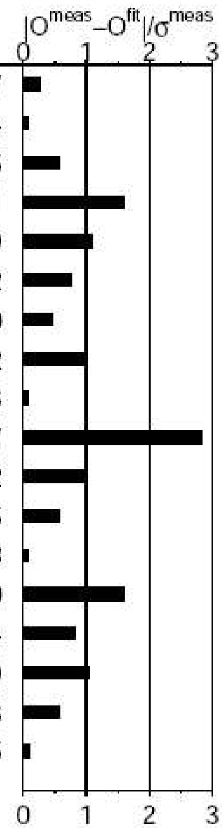


Figure 8.14: Comparison of the measurements with the expectation of the SM, calculated for the five SM input parameter values in the minimum of the global χ^2 of the fit. Also shown is the pull of each measurement, where pull is defined as the difference of measurement and expectation in units of the measurement uncertainty. The direct measurements of m_W and Γ_W used here are preliminary.

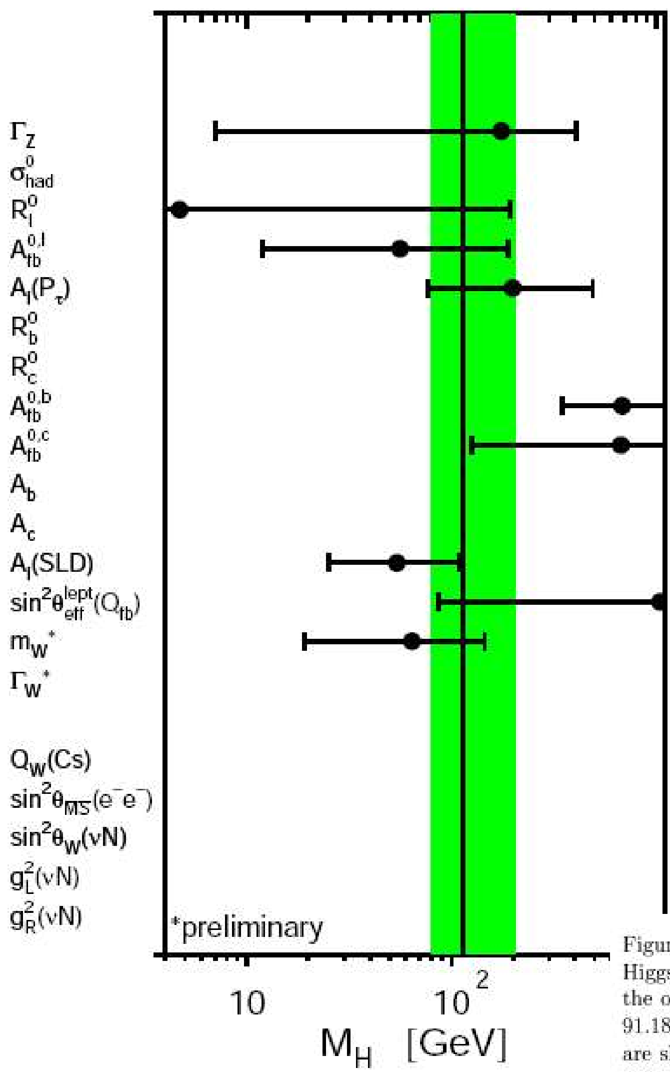


Figure 8.15: Constraints on the mass of the Higgs boson from each pseudo-observable. The Higgs-boson mass and its 68% CL uncertainty is obtained from a five-parameter SM fit to the observable, constraining $\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = 0.02758 \pm 0.00035$, $\alpha_S(m_Z^2) = 0.118 \pm 0.003$, $m_Z = 91.1875 \pm 0.0021$ GeV and Tevatron Run-I $m_t = 178.0 \pm 4.3$ GeV. Only significant constraints are shown. Because of these four common constraints the resulting Higgs-boson mass values cannot be combined. The shaded band denotes the overall constraint on the mass of the Higgs boson derived from all pseudo-observables reported in Table 8.3. The direct measurements of m_W and Γ_W used in that analysis are preliminary.