

Functional Renormalization Group study of phase transitions in gauge theories

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TRIUMF

- Canada's National Laboratory for Particle and Nuclear Physics -

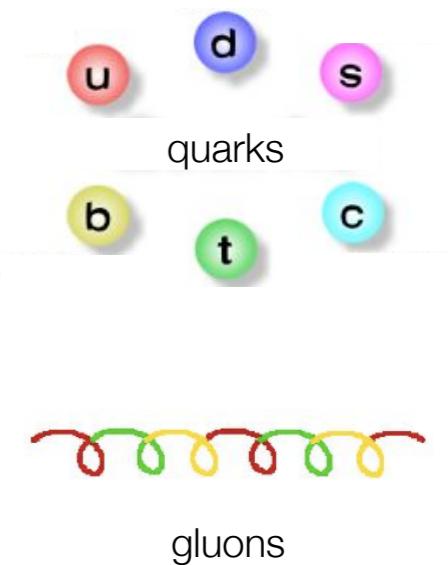
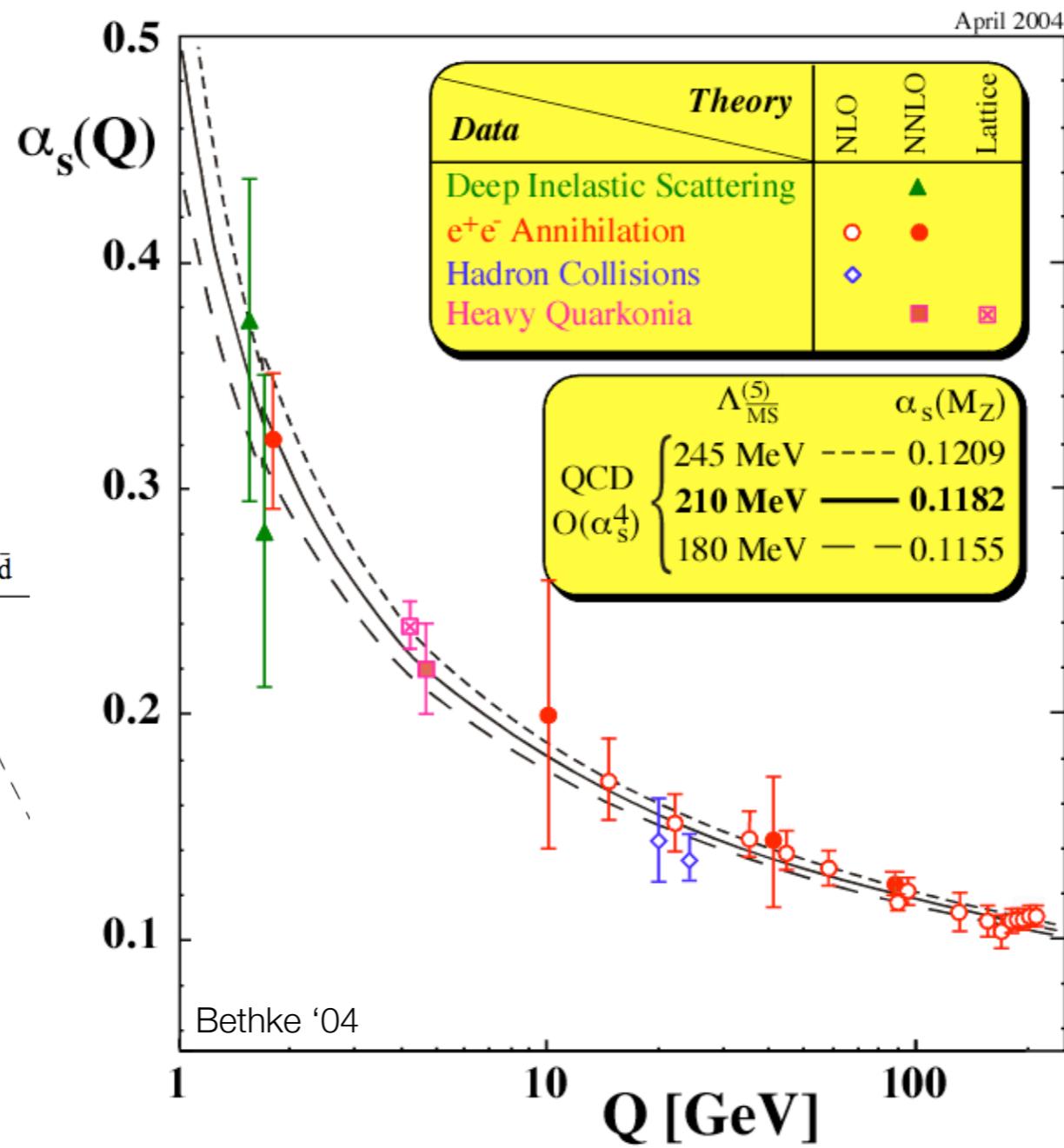
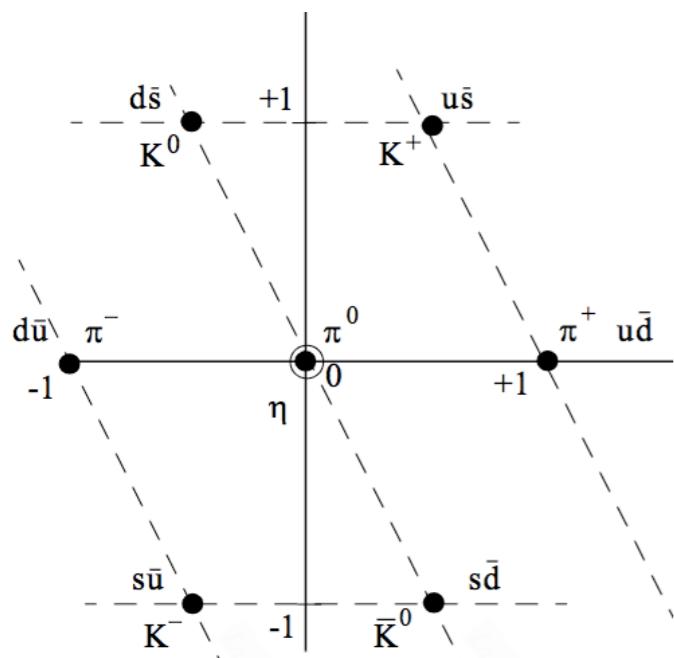
Heidelberg University, 15/12/2007

Outline

- Motivation
- Functional RG
- Chiral Phase Boundary of QCD
- (De-)Confinement Phase Transition in Yang-Mills Theory
- Conclusions and Outlook

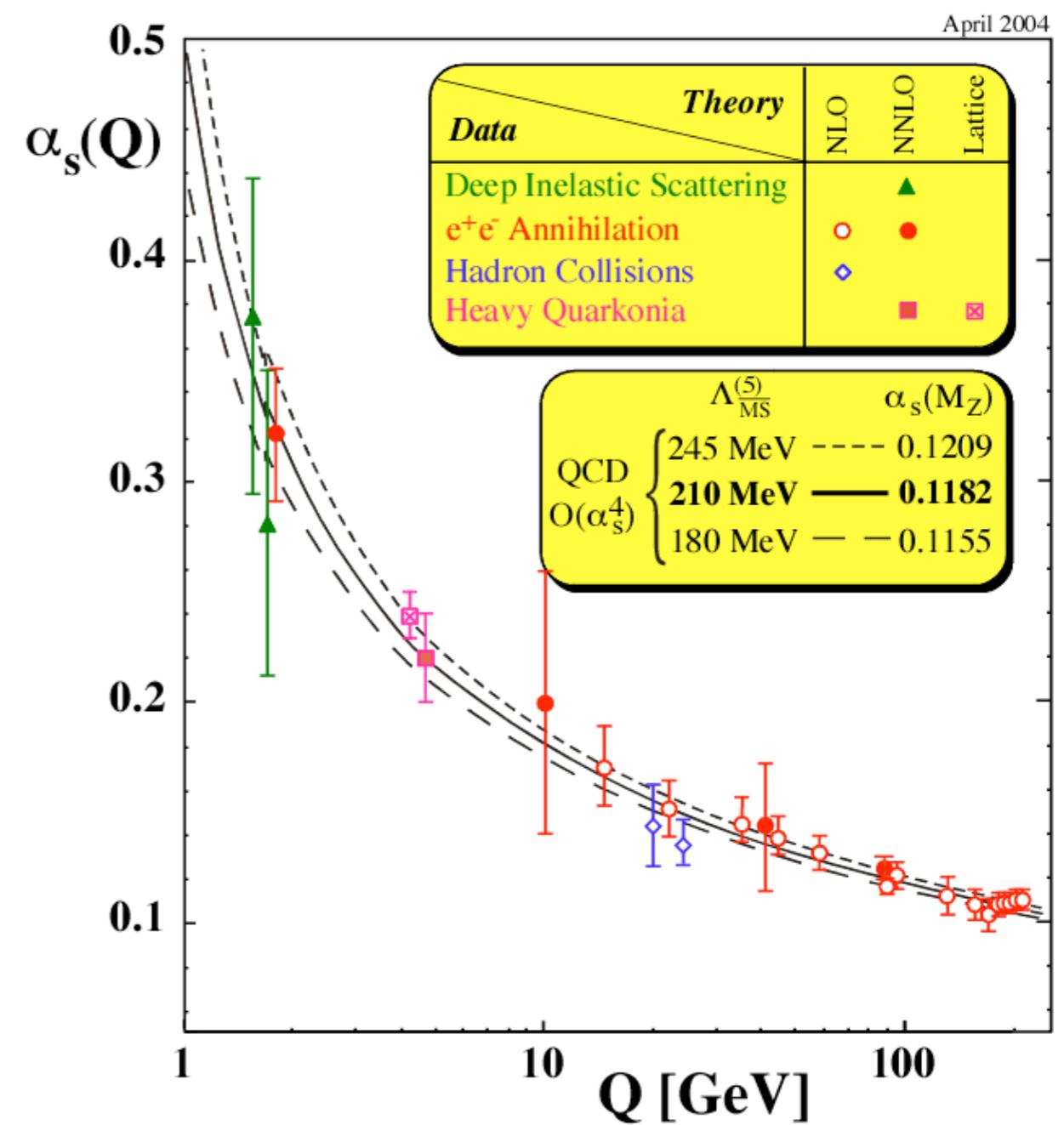
Motivation

Challenges in QCD



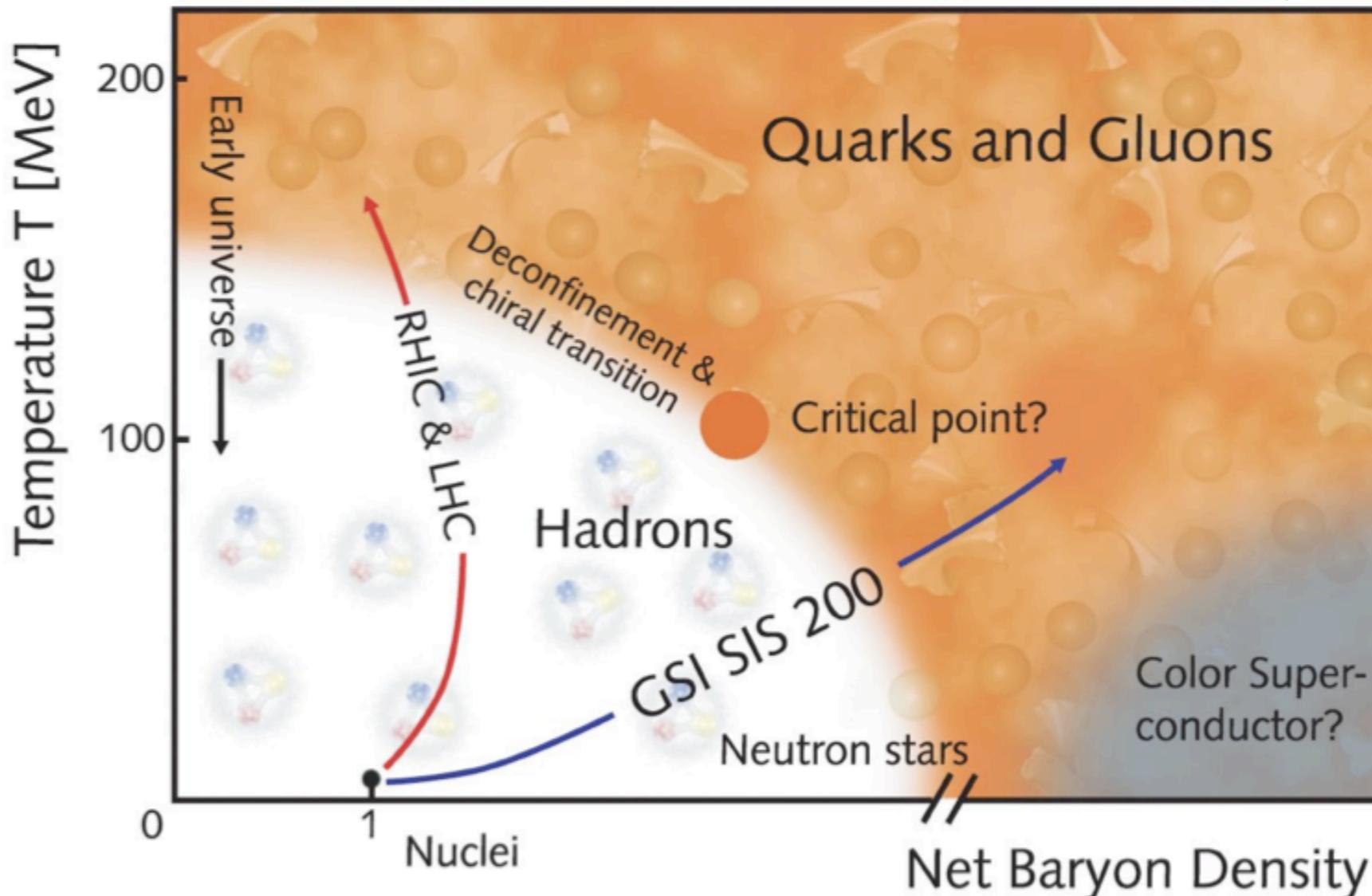
Challenges in QCD

- **Asymptotic freedom** at high momenta (Gross & Wilczek '73, Politzer '73)
- running coupling exhibits Landau pole at **small momenta**
→ pQCD fails
- Understanding of QCD in the mid-momentum regime is needed to study **confinement & chiral symmetry** breaking



QCD phase diagram

FAIR, www.gsi.de



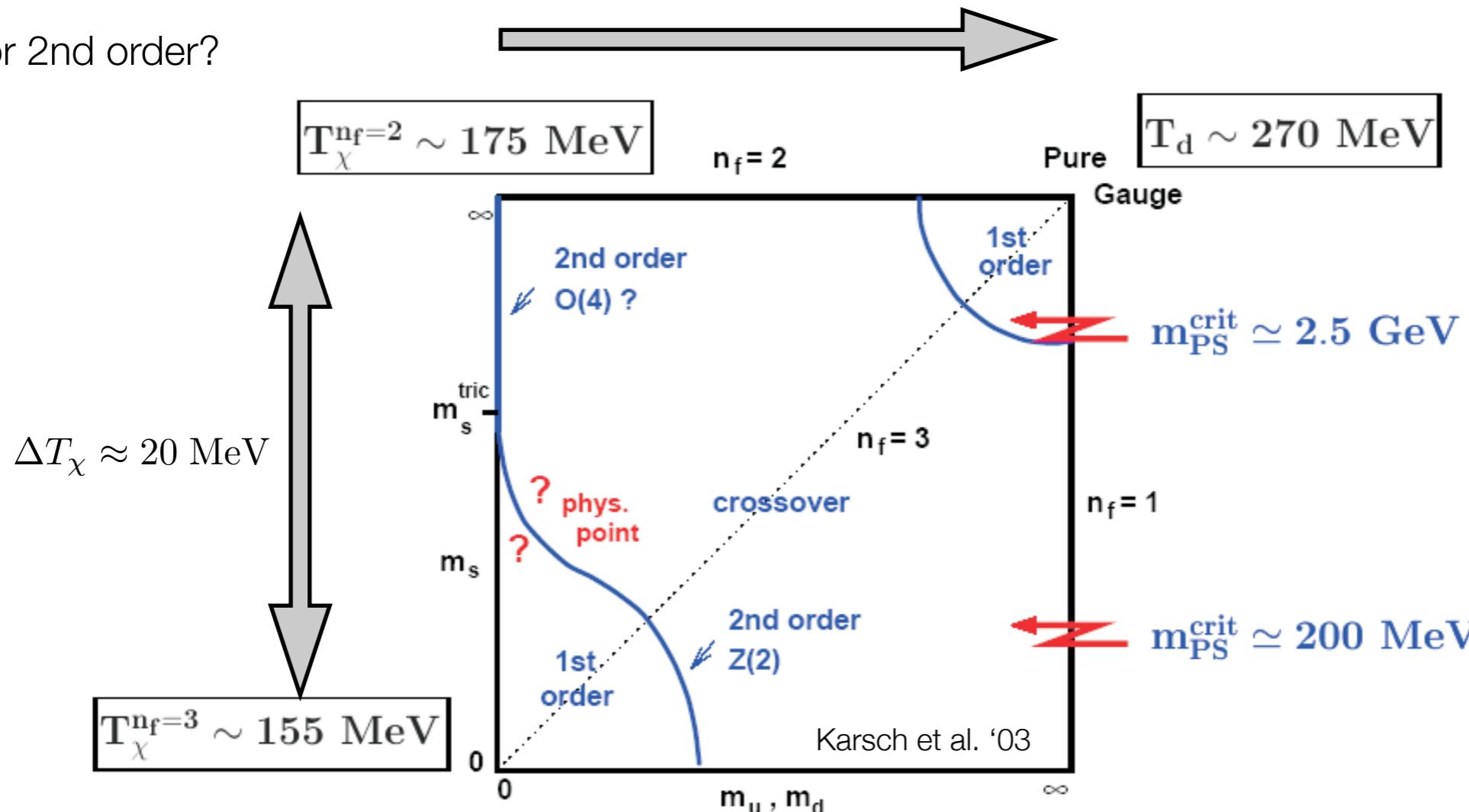
perturbation theory fails:

- not convergent even for very high temperatures: strongly interacting QGP
- phase transitions: long-range fluctuations are important

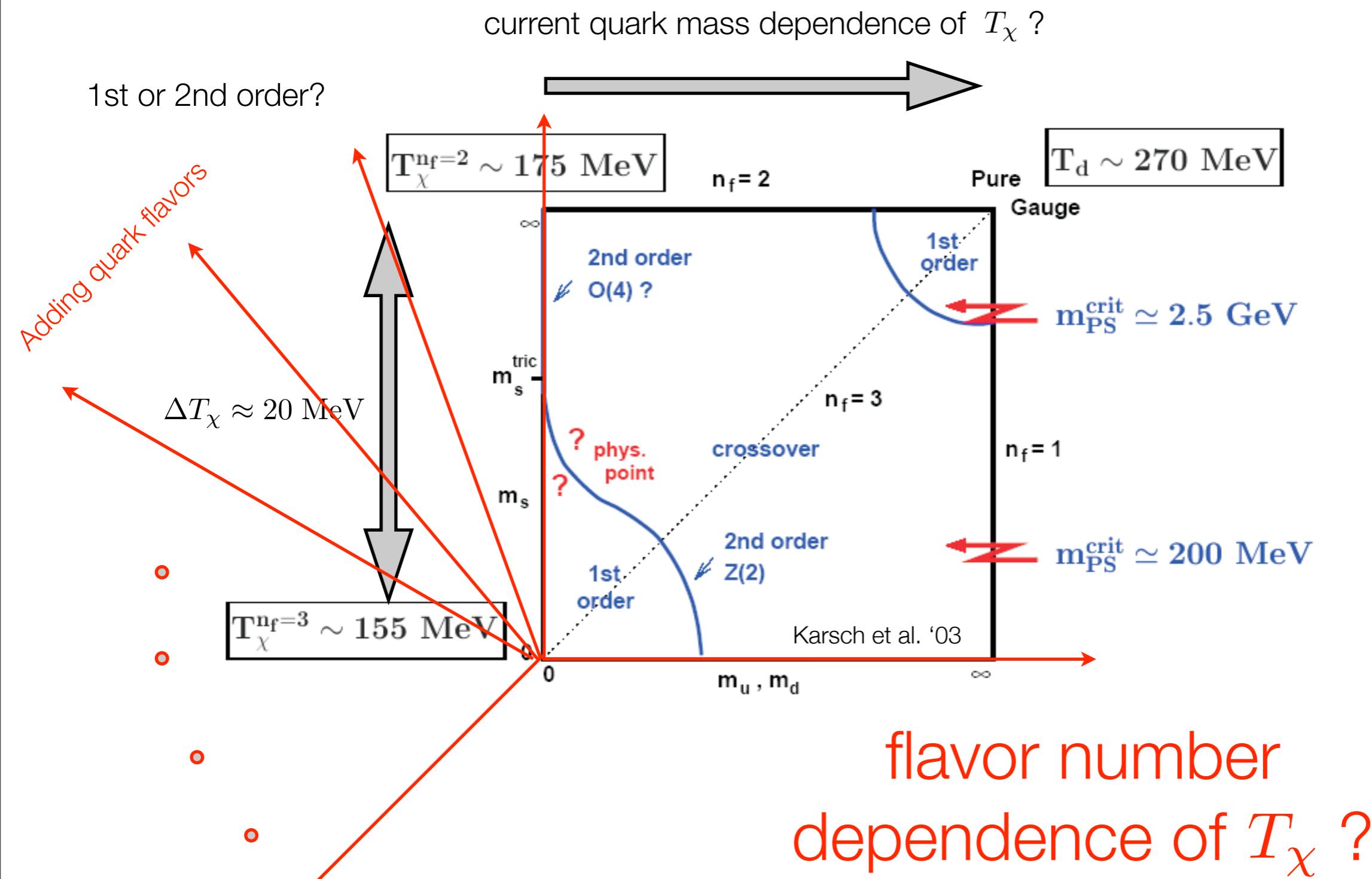
QCD at finite temperature

current quark mass dependence of T_χ ?

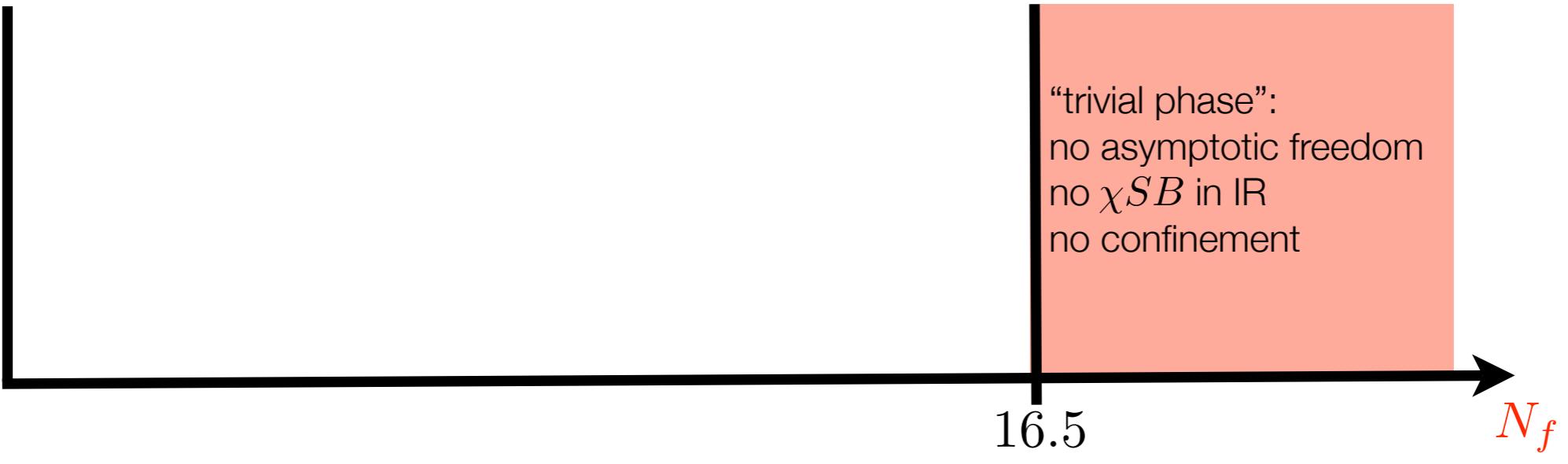
1st or 2nd order?



QCD at finite temperature



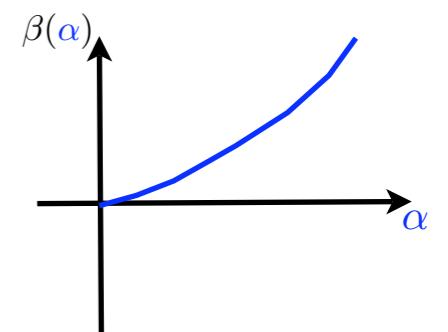
Many flavor QCD



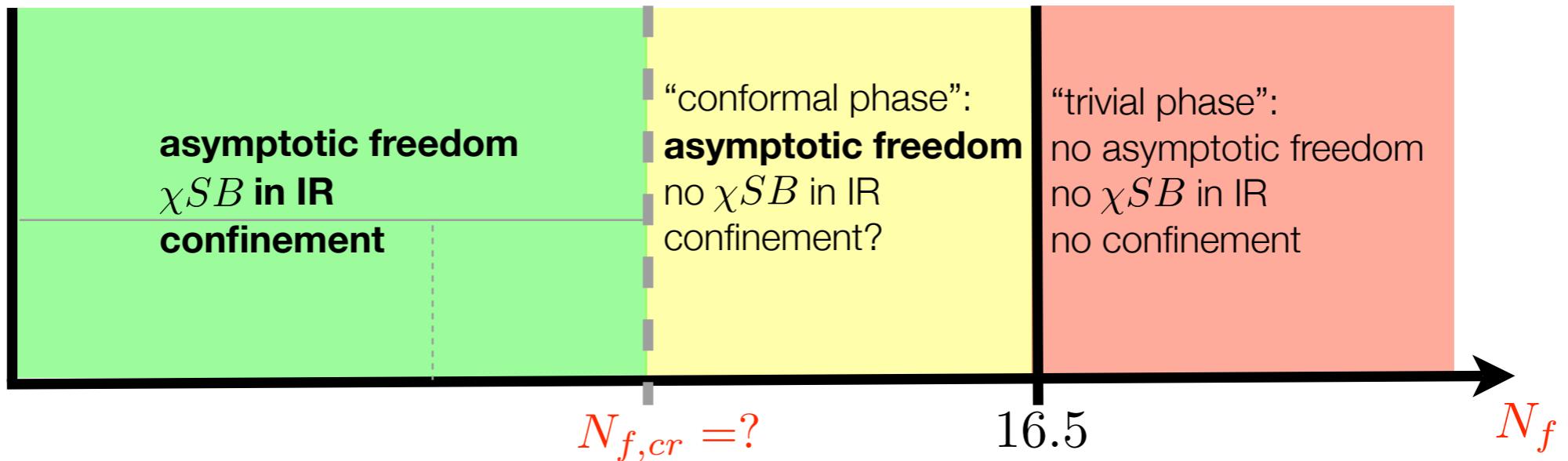
- one-loop β -function

$$\partial_t \alpha \equiv \beta(\alpha) = - \underbrace{\frac{1}{6\pi} (11N_c - 2N_f)}_{b_1} \alpha^2$$

- $b_1 < 0 \implies N_f > \frac{11}{2} N_c \stackrel{N_c=3}{=} 16.5$ (QCD is NOT asymptotically free)



Many flavor QCD



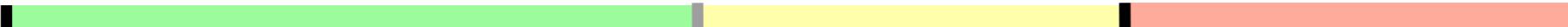
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- $b_1 < 0 \implies N_f > \frac{11}{2} N_c \stackrel{N_c=3}{=} 16.5$ (QCD is **NOT** asymptotically free)

- $b_1 > 0$: QCD is asymptotically free

Many flavor QCD



$N_{f,cr} = ?$

- one-
 - $b_1 < 0$
 - $b_1 > 0$: QCD is asymptotically free
- $N_{f,cr} = 12$ (Appelquist et al. '96)

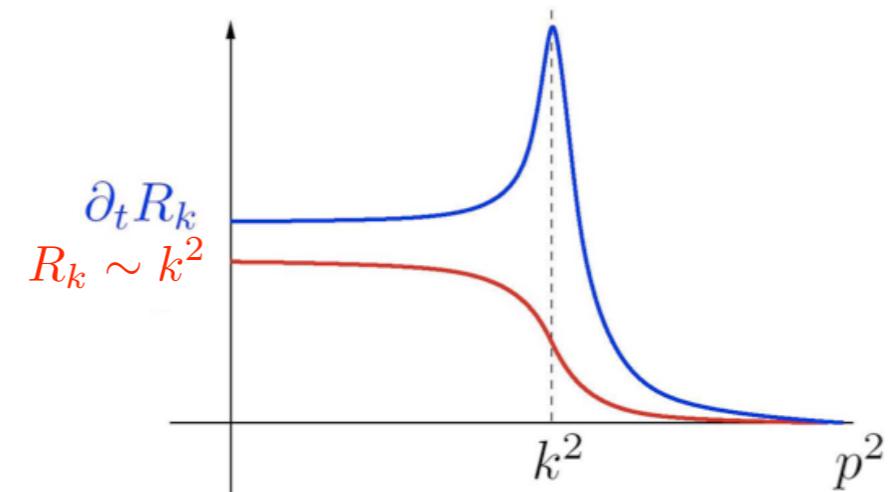
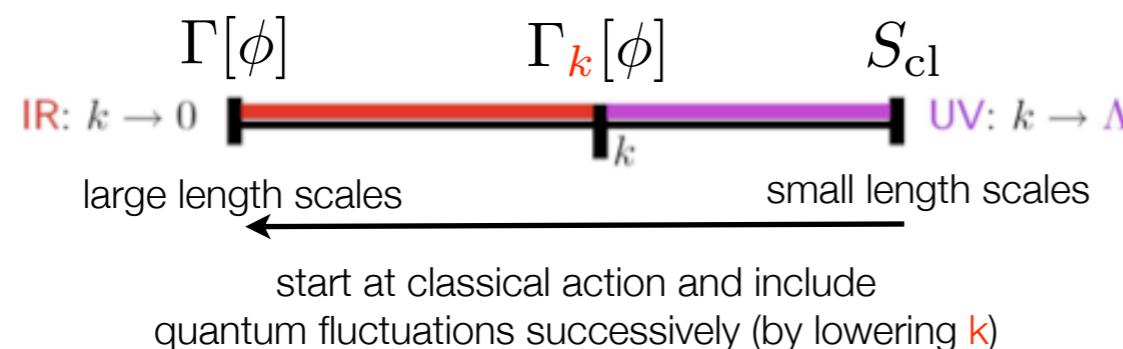
$N_{f,cr} = 8$ (Brown et al. '92)

$N_{f,cr} = 10$ (Kogut et al. '92)

$N_{f,cr}$ from Functional RG : [this talk](#)

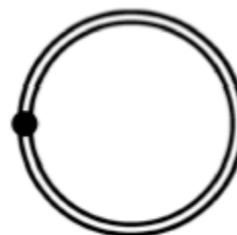
Functional RG

Functional Renormalization Group - Flow equation



- RG flow equation for the effective action: (C. Wetterich '93)

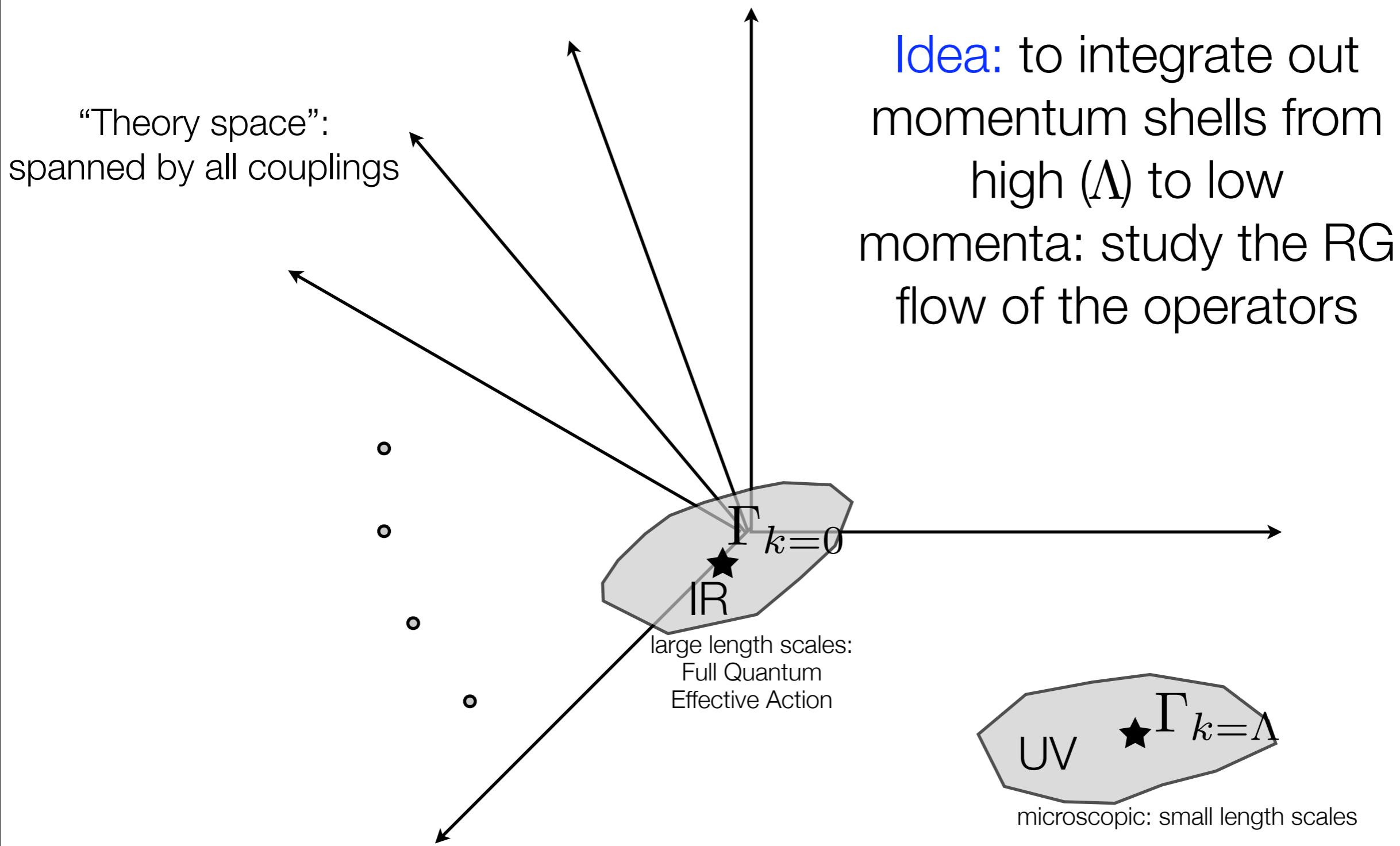
$$k \partial_k \Gamma_k \equiv \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} = \frac{1}{2}$$



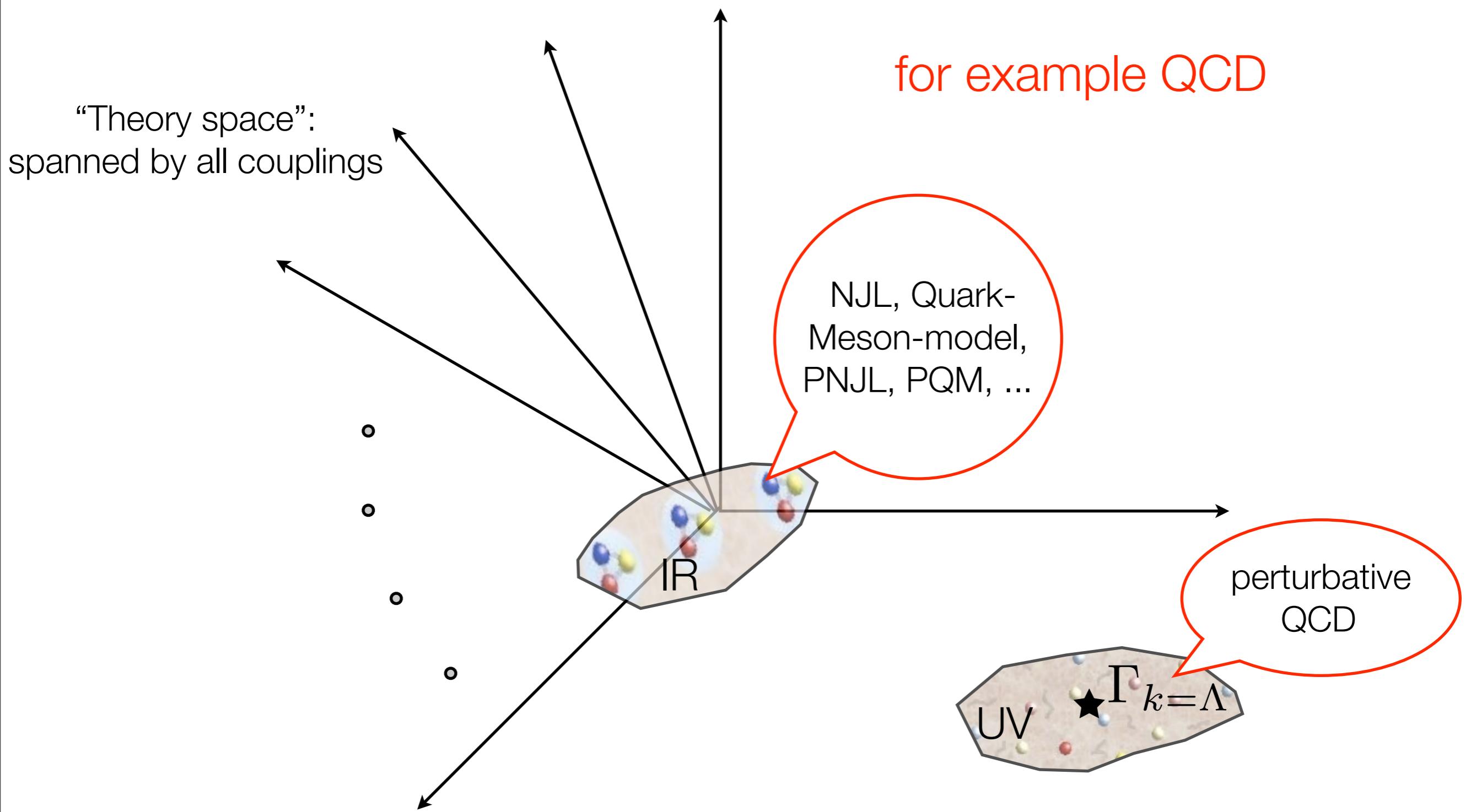
- chiral symmetry is preserved: $R_k = R_k(i\partial)$

- gauge symmetry  modified Ward-Takahashi identities
(Reuter & Wetterich '97; Freire, Litim, Pawłowski '00)

Functional Renormalization Group - Features

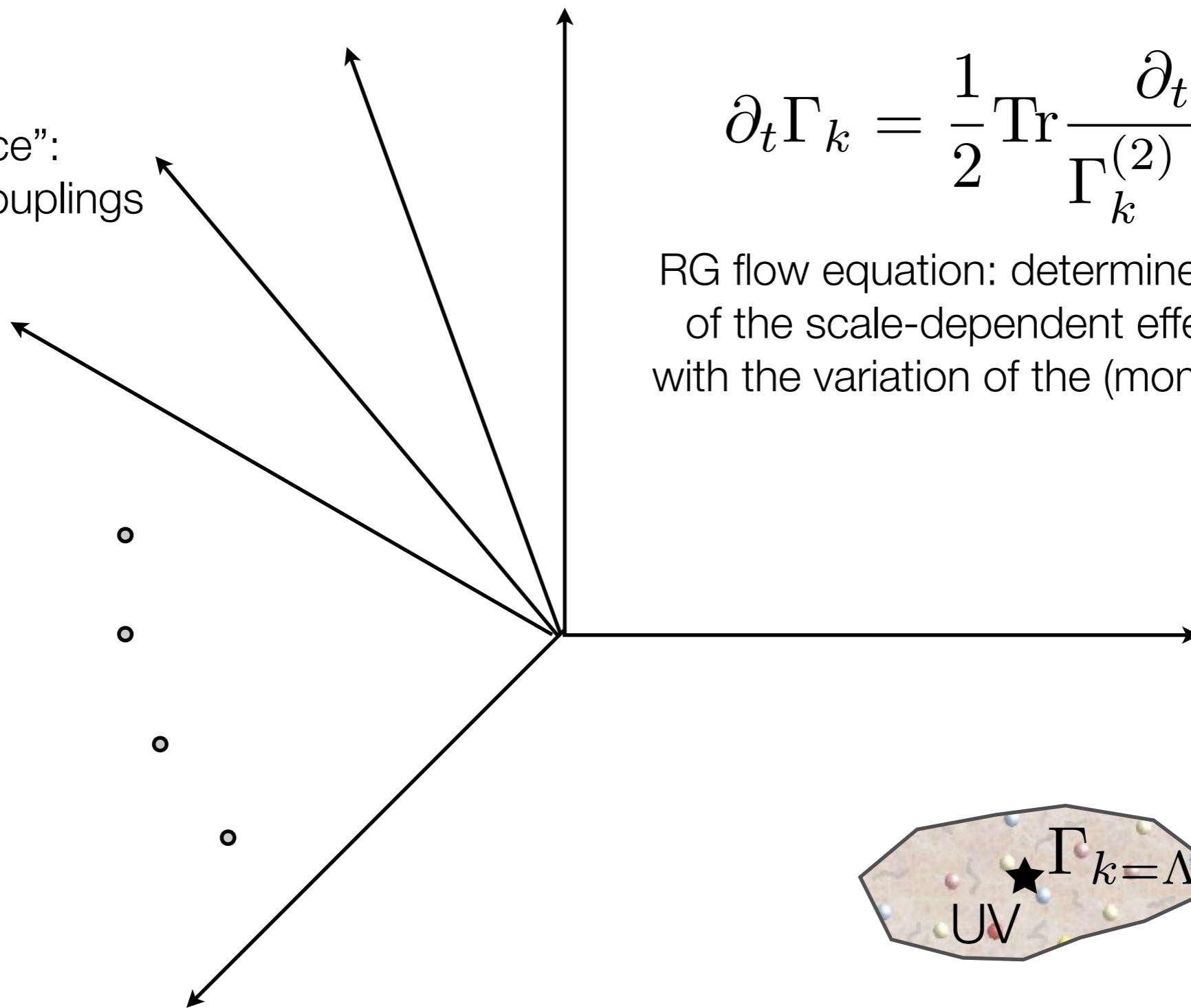


Functional Renormalization Group - Features



Functional Renormalization Group - Features

“Theory space”: spanned by all couplings

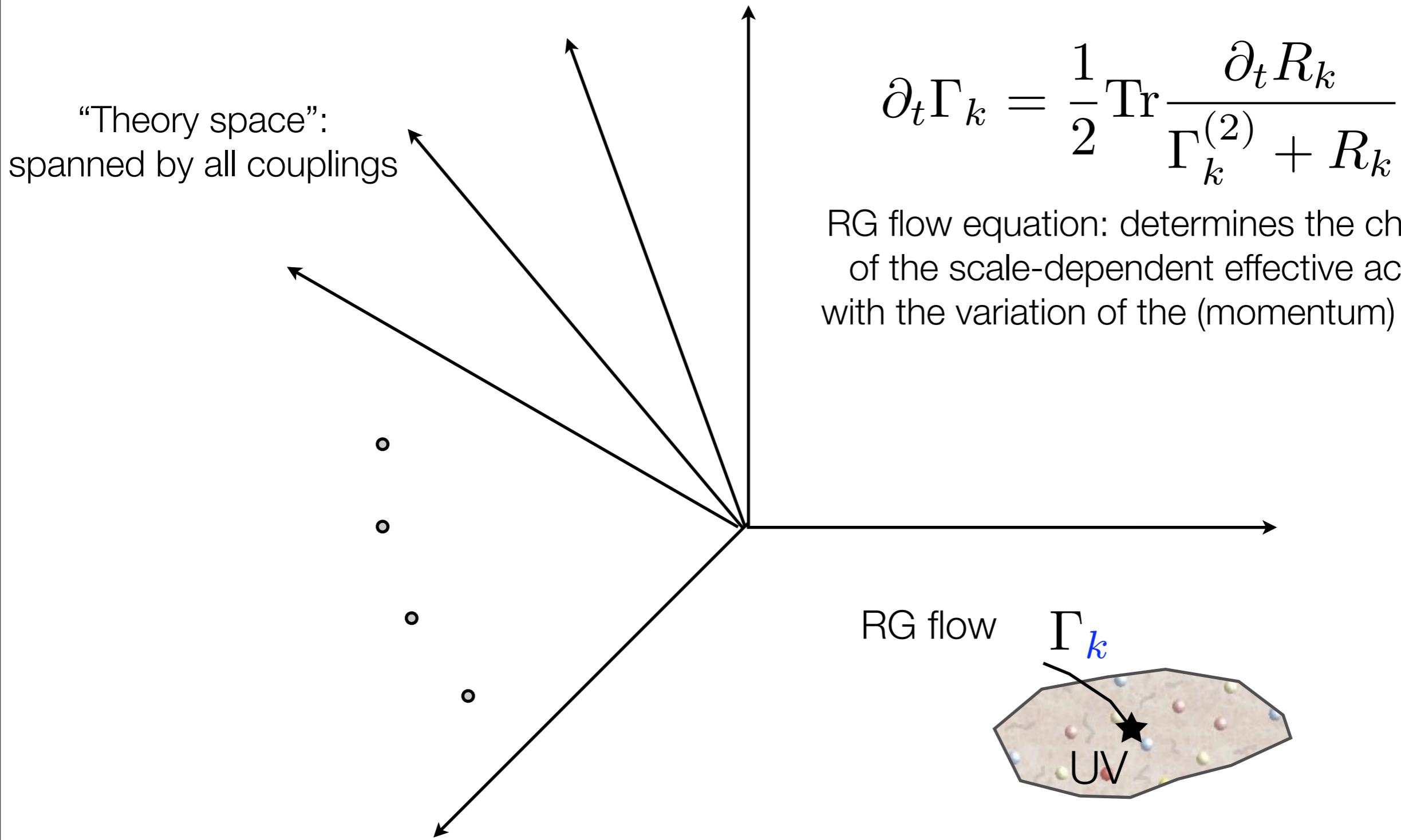


$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

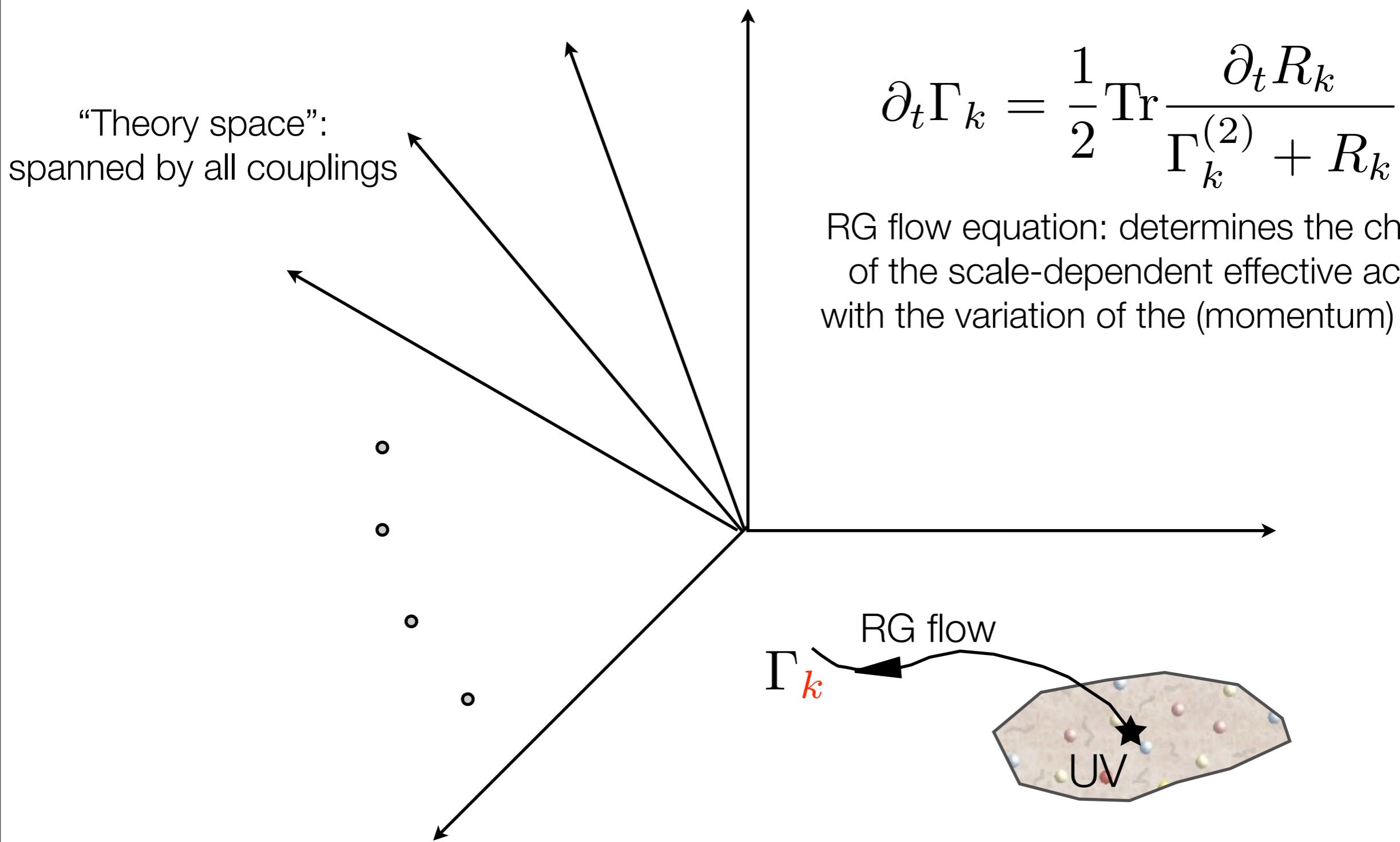
RG flow equation: determines the change
of the scale-dependent effective action
with the variation of the (momentum) scale



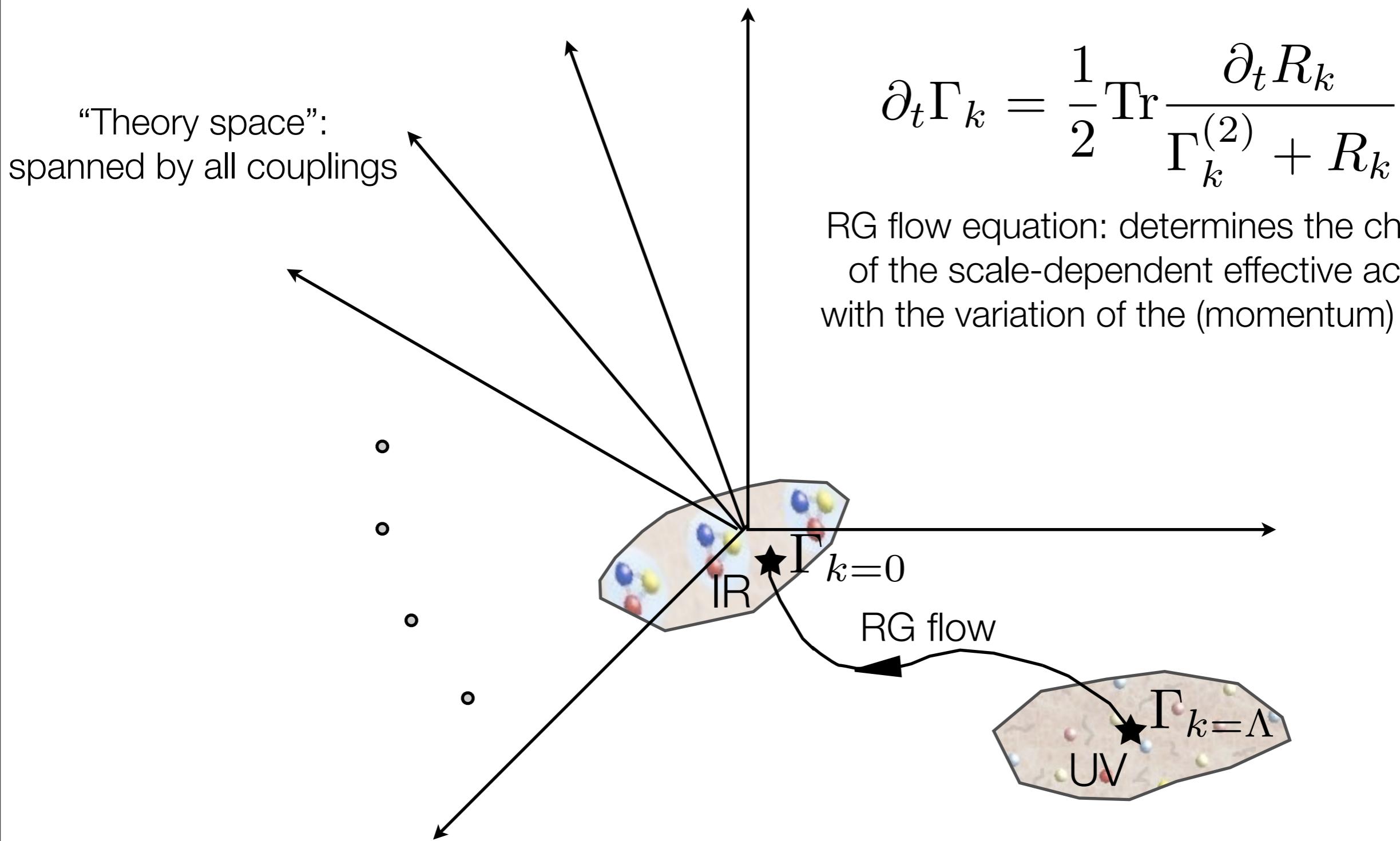
Functional Renormalization Group - Features



Functional Renormalization Group - Features



Functional Renormalization Group - Features



Chiral phase boundary of QCD

Aspects of the NJL model

- classical action of the NJL model:

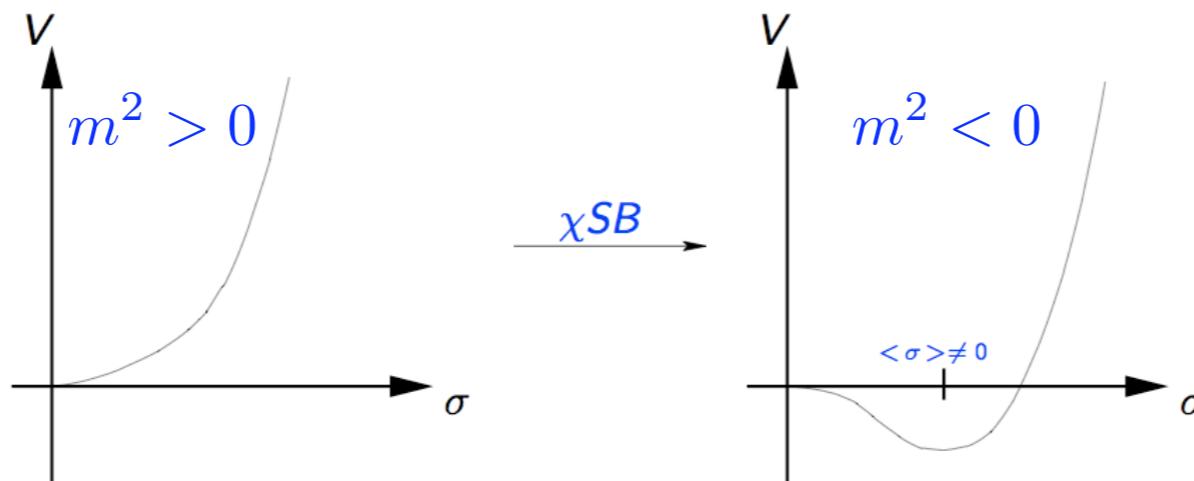
$$S = \int_x \{ \bar{\psi} i\partial^\mu \psi + \bar{\lambda}_\sigma [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] \}$$

- spontaneous symmetry breaking if quark condensate is non-vanishing: $\langle \bar{\psi}\psi \rangle \neq 0$

- bosonization of the NJL model yields $(\sigma = -2\bar{\lambda}_\sigma \bar{\psi}\psi, \pi = -2\bar{\lambda}_\sigma \bar{\psi}\gamma_5\psi)$ Hubbard-Stratonovich transformation

$$S = \int_x \left\{ \bar{\psi} i\partial^\mu \psi + \bar{\psi} (\sigma + i\gamma_5 \pi) \psi - \frac{1}{\bar{\lambda}_\sigma} (\sigma^2 + \pi^2) \right\}$$

→ $\bar{\lambda}_\sigma$ is inverse proportional to the scalar mass parameter, $m^2 \propto \frac{1}{\bar{\lambda}_\sigma}$



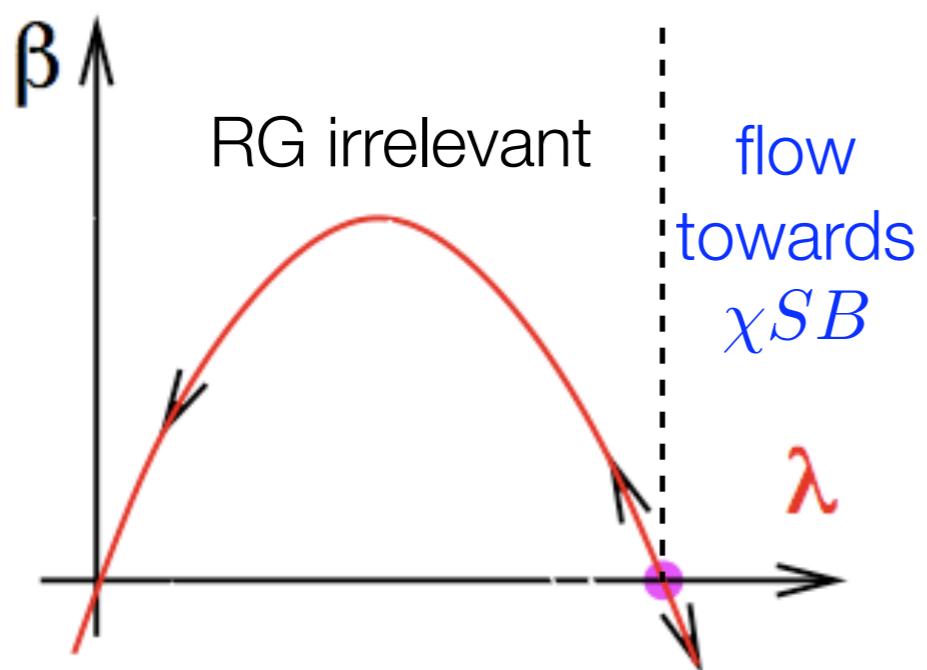
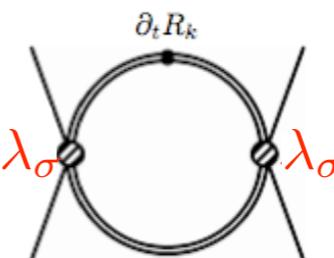
RG flow of the NJL model

- effective action of the NJL model:

$$\Gamma_k = \int_x \left\{ \bar{\psi} i \not{\partial} \psi + \frac{\lambda_\sigma}{2k^2} [(\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2] \right\}$$

- RG flow of the four-fermion coupling:

$$k \partial_k \lambda_\sigma = 2\lambda_\sigma - N_c \lambda_\sigma$$



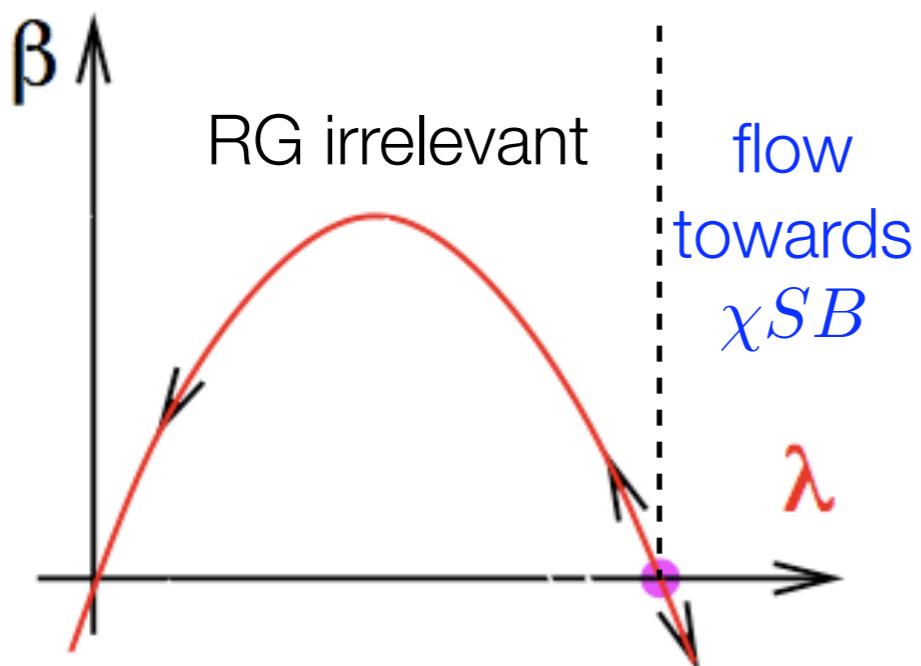
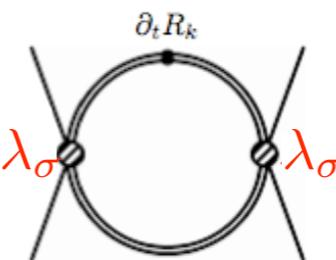
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Problem: QCD starting point
is $\lambda_\sigma = 0$ for $\Lambda \gg \Lambda_{\text{QCD}}$
→ $\lambda_\sigma(\Lambda)$ must be “tuned by hand”

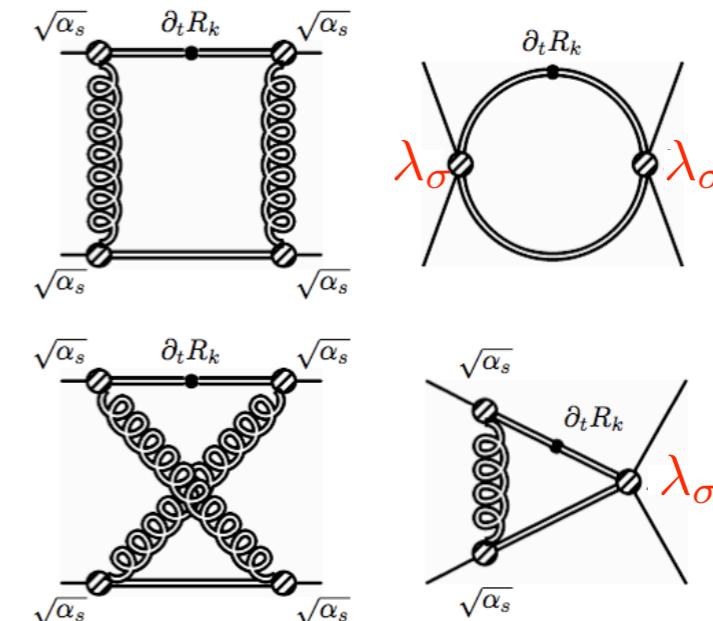
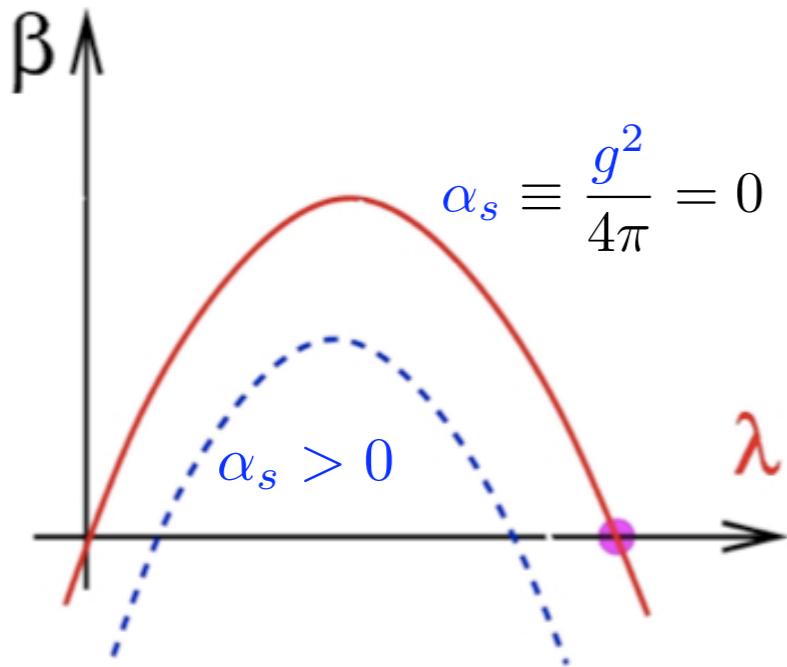
Flow of four-fermion interactions: adding gluons

- effective action:

$$\Gamma_k = \int_x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} (i\partial + \bar{g} A) \psi + \frac{\lambda_\sigma}{2k^2} [(\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2] \right\}$$

- flow equation for the four-fermion coupling:

$$k \partial_k \lambda_\sigma = 2\lambda_\sigma - \frac{N_c}{4\pi^2} \lambda_\sigma^2 - \frac{3}{8\pi^2} \frac{N_c^2 - 1}{N_c} g^2 \lambda_\sigma - \frac{9}{256\pi^2} \frac{3N_c^2 - 8}{N_c} g^4$$



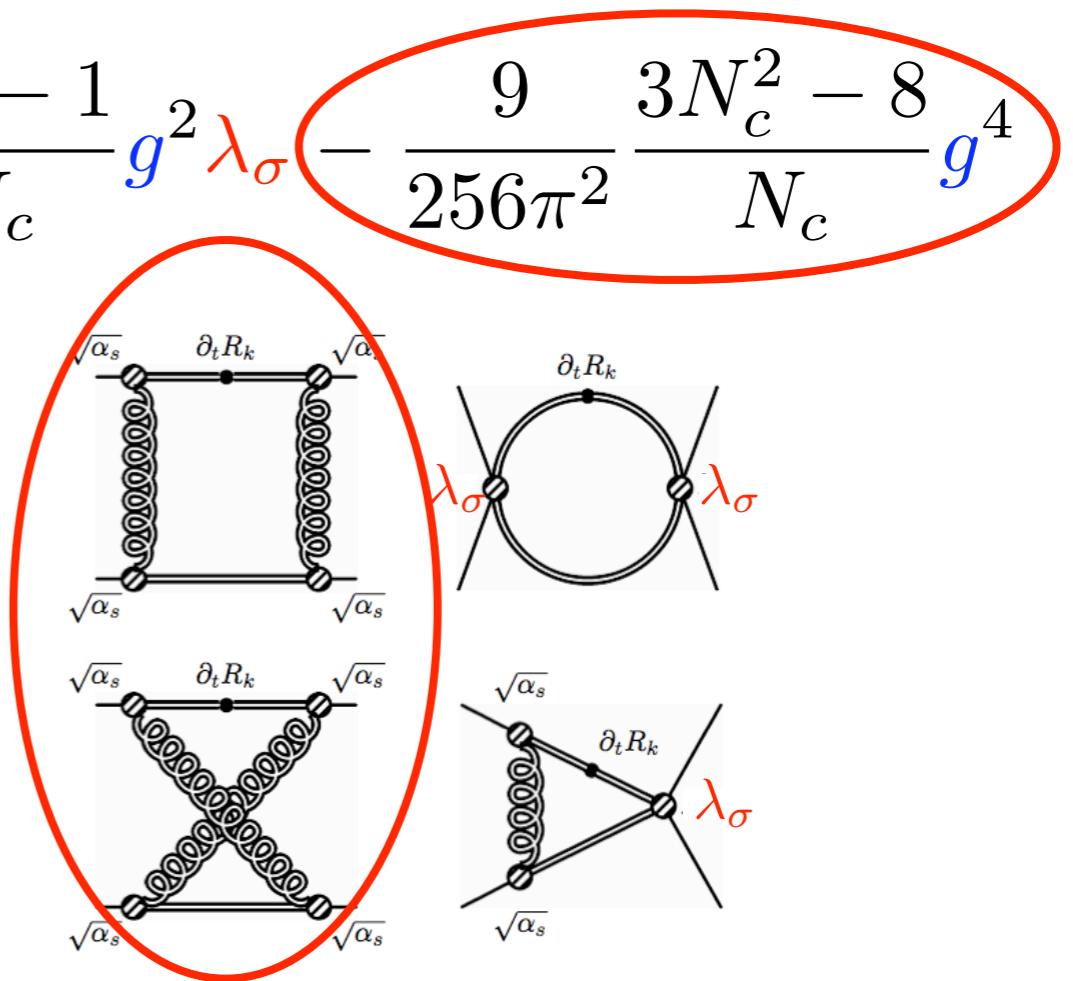
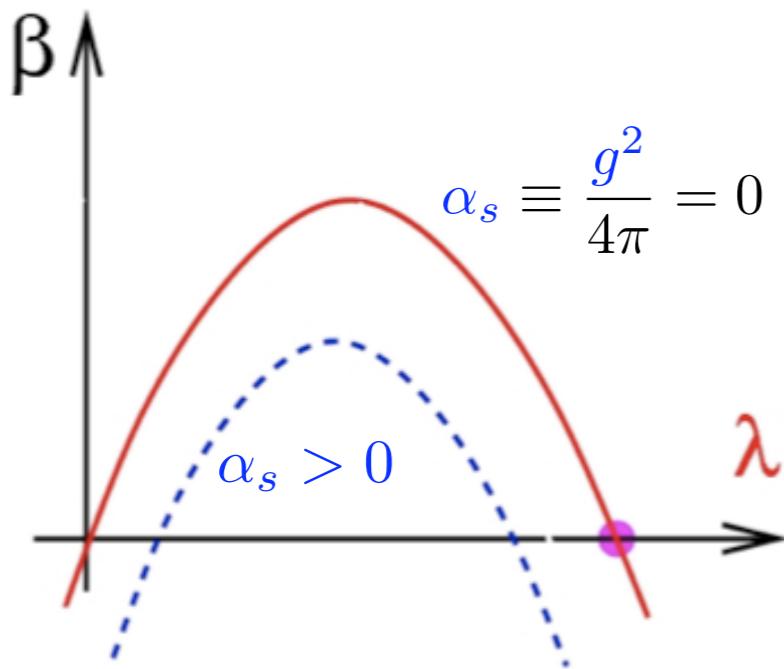
RG flow of the NJL model: adding gluons

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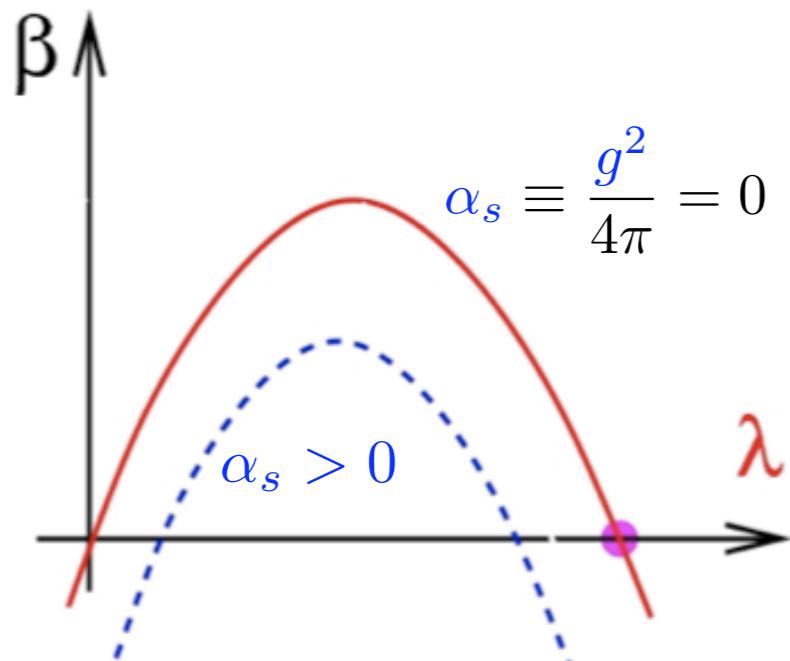
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fixed points vanish, if α_s becomes large enough

→ there exists a “critical” value for α_s above which we have χSB

→ we can have $\lambda_\sigma(\Lambda) = 0$ and still have χSB in the IR

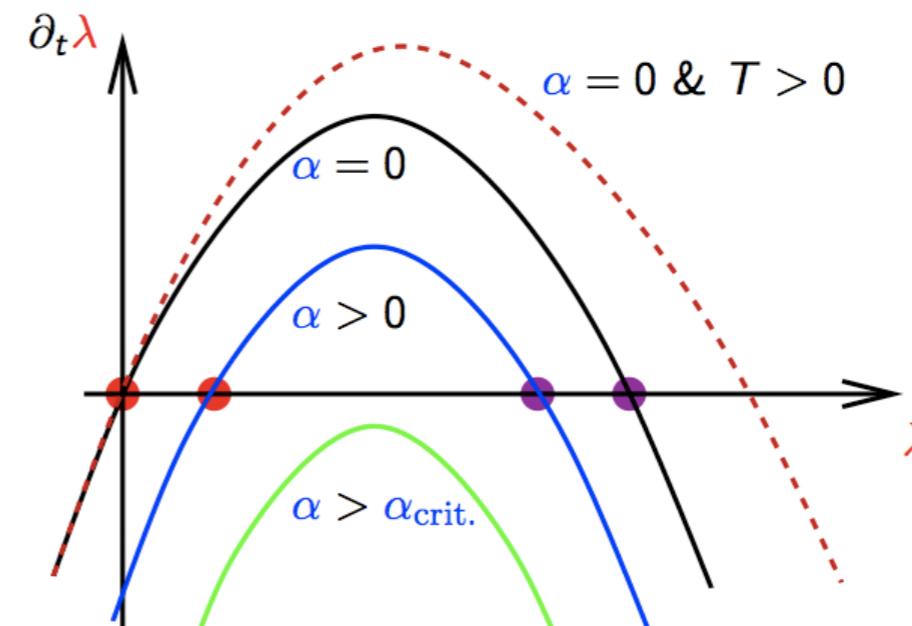
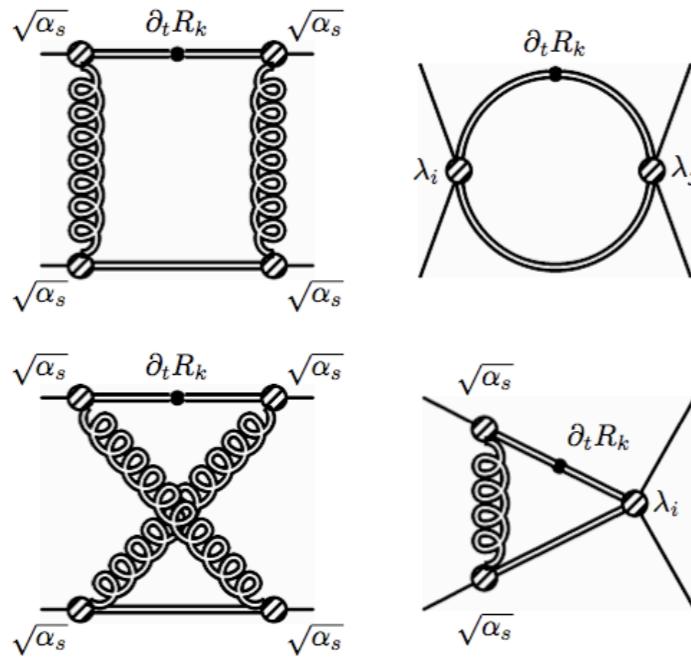
RG flow for the chiral QCD sector

- effective action:

$$\begin{aligned}\Gamma_k &= \Gamma_k^{\text{gauge}} + \int_x \bar{\psi} (iZ_\psi \partial + Z_1 \bar{g} A) \psi + \frac{1}{2} \left[\frac{\lambda_-}{k^2} (V - A) + \frac{\lambda_+}{k^2} (V + A) \right. \\ &\quad \left. + \frac{\lambda_\sigma}{k^2} (S - P) + \frac{\lambda_{VA}}{k^2} [2(V - A)^{\text{adj}} + (1/N_c)(V - A)] \right]\end{aligned}$$

- no Fierz-ambiguity
- initial values for four-fermion interactions : $\lim_{\Lambda \rightarrow \infty} \lambda_i = 0$
- truncation checks: momentum dependencies, regulator dependencies, higher order interactions (H. Gies, J. Jaeckel, C. Wetterich '04)

“Criticality” at zero and finite temperature

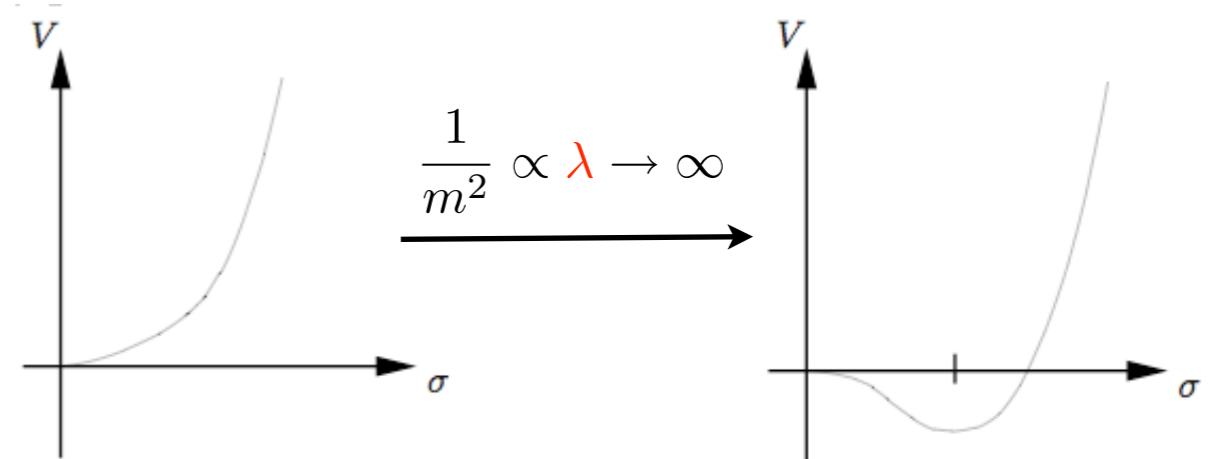


- critical gauge coupling α_{cr} :

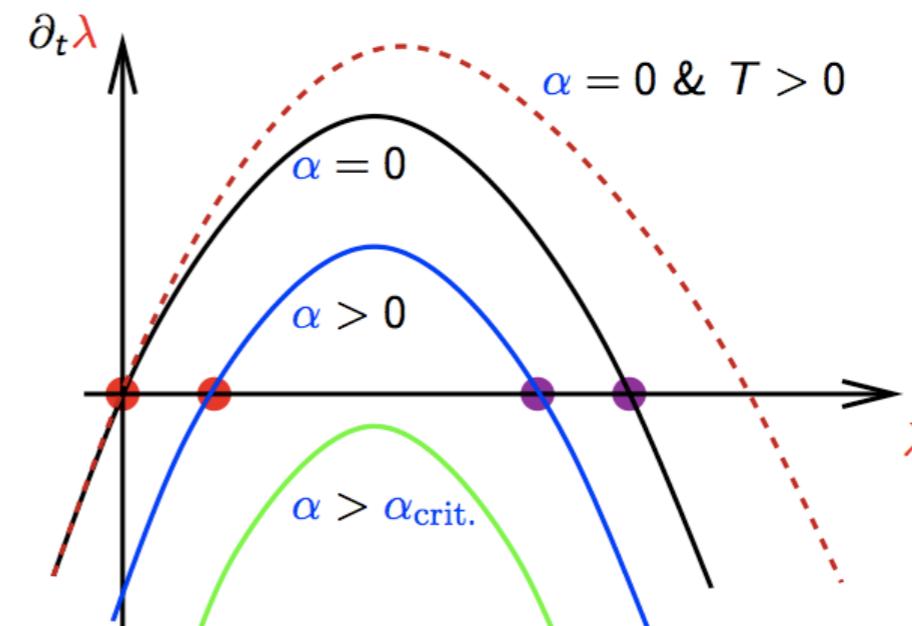
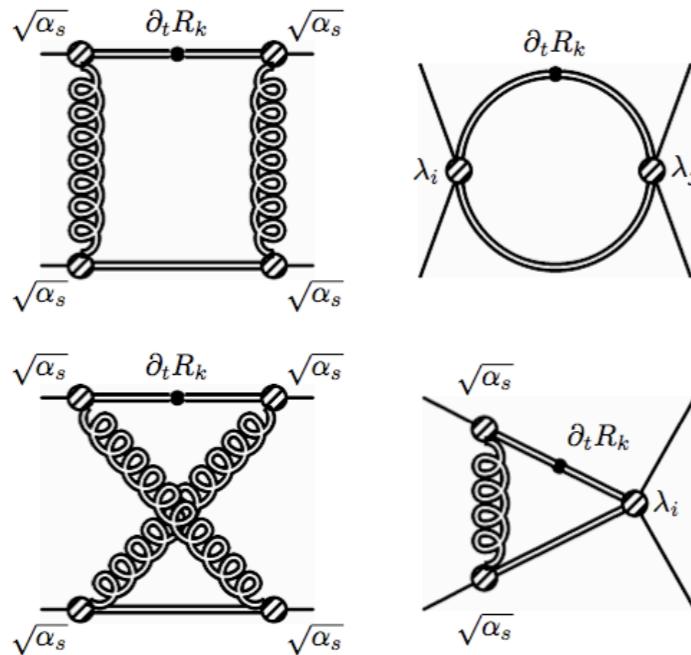
if $\alpha_s > \alpha_{cr}$ \rightarrow no fixed points $\rightarrow \chi SB$

- at zero temperature: (H. Gies, J. Jaeckel '05)

$$\alpha_{cr} \approx 0.85$$



“Criticality” at zero and finite temperature



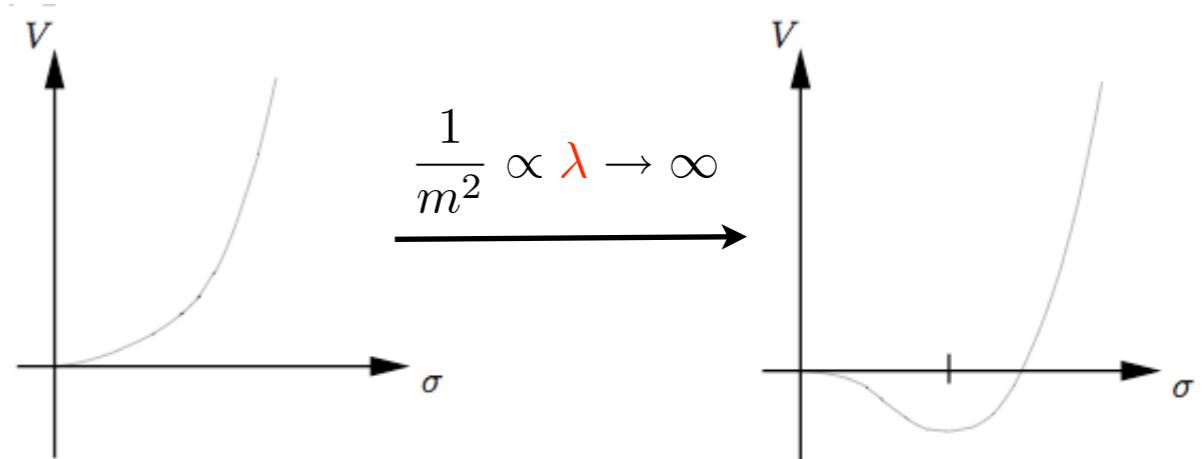
- critical gauge coupling α_{cr} :

if $\alpha_s > \alpha_{cr}$ \rightarrow no fixed points $\rightarrow \chi SB$

- at finite temperature: (JB, H. Gies '05)

$$\alpha_{cr}(T/k) > \alpha_{cr}(T = 0)$$

quarks acquire a thermal mass



RG flow of gluodynamics: running gauge coupling

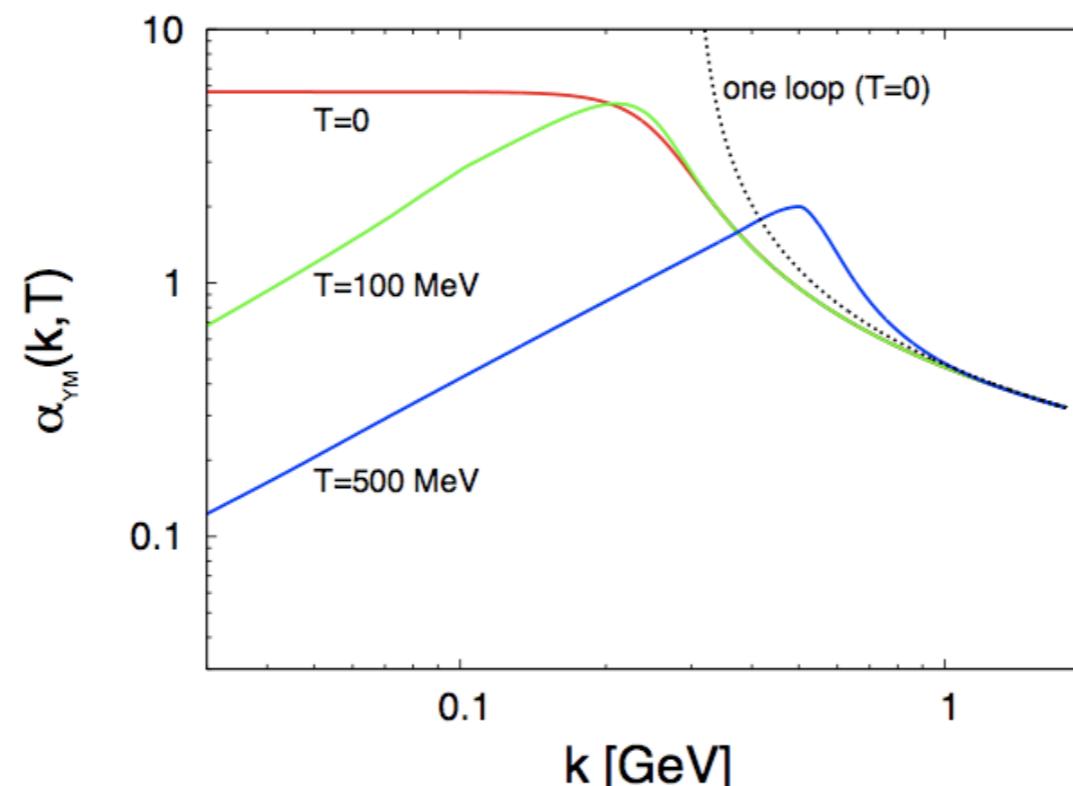
- operator expansion with background field method (Reuter & Wetterich '94; Freire, Litim, Pawłowski '00)
- ansatz for the gauge sector:

$$\Gamma_k = \int_x W_k(\vartheta) + \Gamma^{\text{gf}} + \Gamma^{\text{gh}} + \bar{\psi}(iD + M_{\bar{\psi}\psi})\psi + \Gamma_k^{\text{q-int}}[\bar{\psi}, \psi]$$

with $W_k(\vartheta) = Z_A F_{\mu\nu}^a F_{\mu\nu}^a + w_2 (F_{\mu\nu}^a F_{\mu\nu}^a)^2 + w_3 (F_{\mu\nu}^a F_{\mu\nu}^a)^3 + \dots$

- flow: $\partial_t Z_A \curvearrowright \partial_t w_2 \curvearrowright \partial_t w_3 \curvearrowright \partial_t w_4 \curvearrowright \dots$ (H. Gies '02)
- running coupling: $g^2 = Z_A^{-1} \bar{g}^2$ (Abbott '82: background gauge)

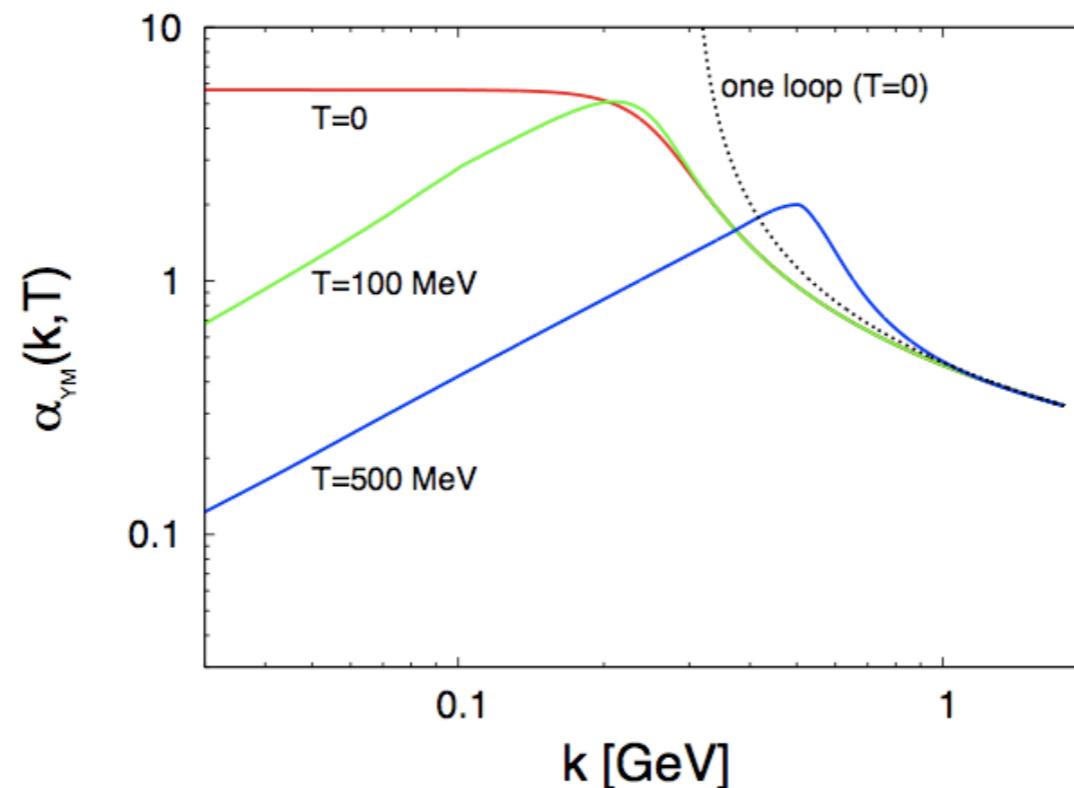
RG flow of gluodynamics



- IR fixed point at zero temperature (H. Gies '02)

cf. vertex expansion in Landau-gauge QCD: SDE: v. Smekal et al. '97, Fischer et al. '02; RG: Pawłowski et al. '03, Fischer&Gies '04;

RG flow of gluodynamics

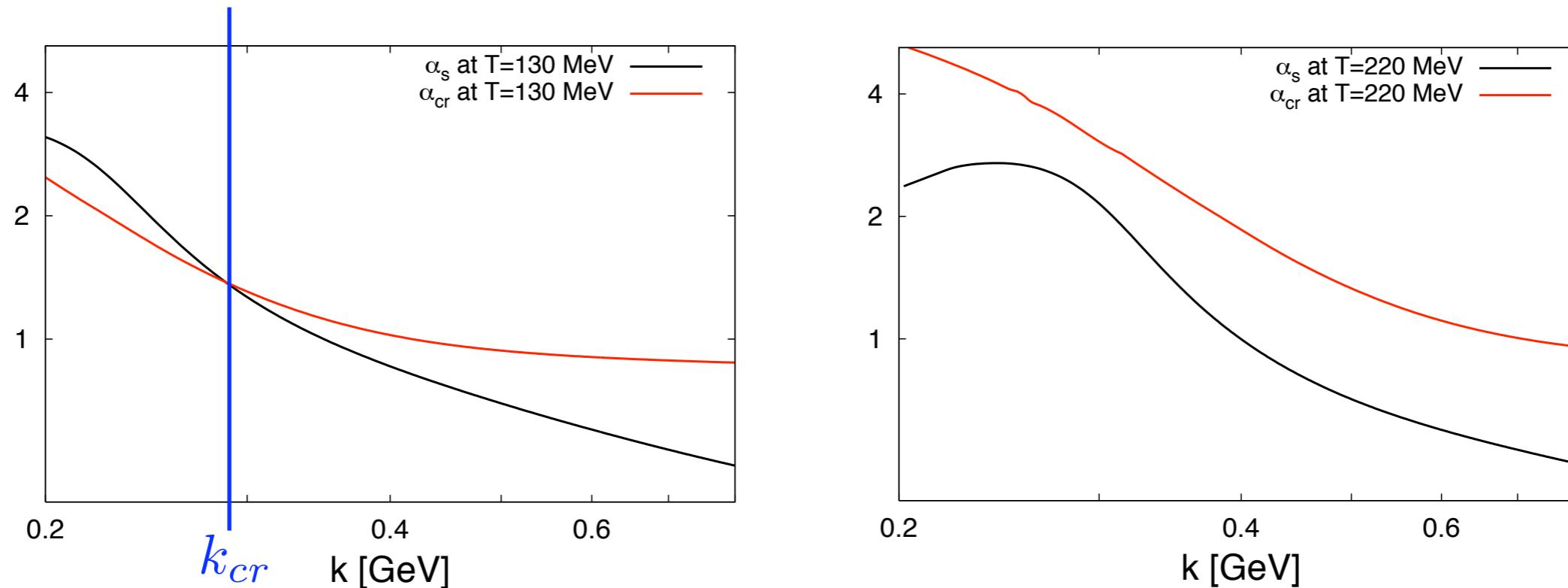


- pQCD at high scales: $k \gg T$
- $k_{max} \propto T$ decoupling of hard gluonic modes \rightarrow “finite-size” effect:
$$p_{g,0}^2 \equiv \omega_n^2 = 4n^2\pi^2T^2 \quad \rightarrow \quad \omega_0^2 = 0$$
- decrease for $T \gtrsim k$ due to existence of a non-trivial IR fixed point
in 3d Yang-Mills theory: strong interactions at high temperatures (JB, H. Gies '06)

$$\alpha_{4D} \approx \alpha_{3D}^* \frac{k}{T} + \mathcal{O}((k/T)^2) \quad \text{with} \quad \alpha_{3D}^* \approx 2.7; \eta_{3d} \rightarrow 1$$

Chiral Phase Transition in QCD

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of α_{cr} and α_s indicates onset of χSB



- single input parameter: $\alpha_s(m_\tau) = 0.322$

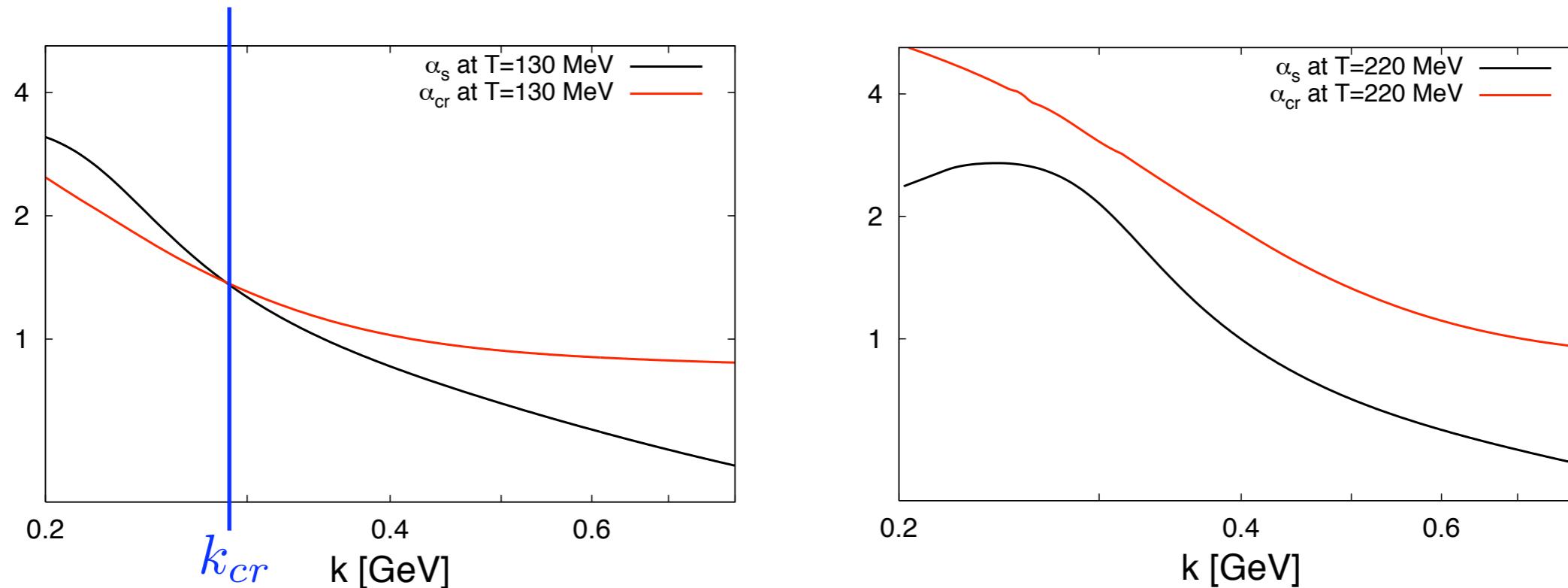
N_f	T_{cr}
2	172 MeV
3	148 MeV

(JB, H. Gies '06)

$N_f = 2 + 1$	T_{cr}
Lattice (Chen et al. '06)	192 MeV
Lattice (Aoki et al. '06)	151 MeV

Chiral Phase Transition in QCD: Error estimate

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of α_{cr} and α_s indicates onset of χSB



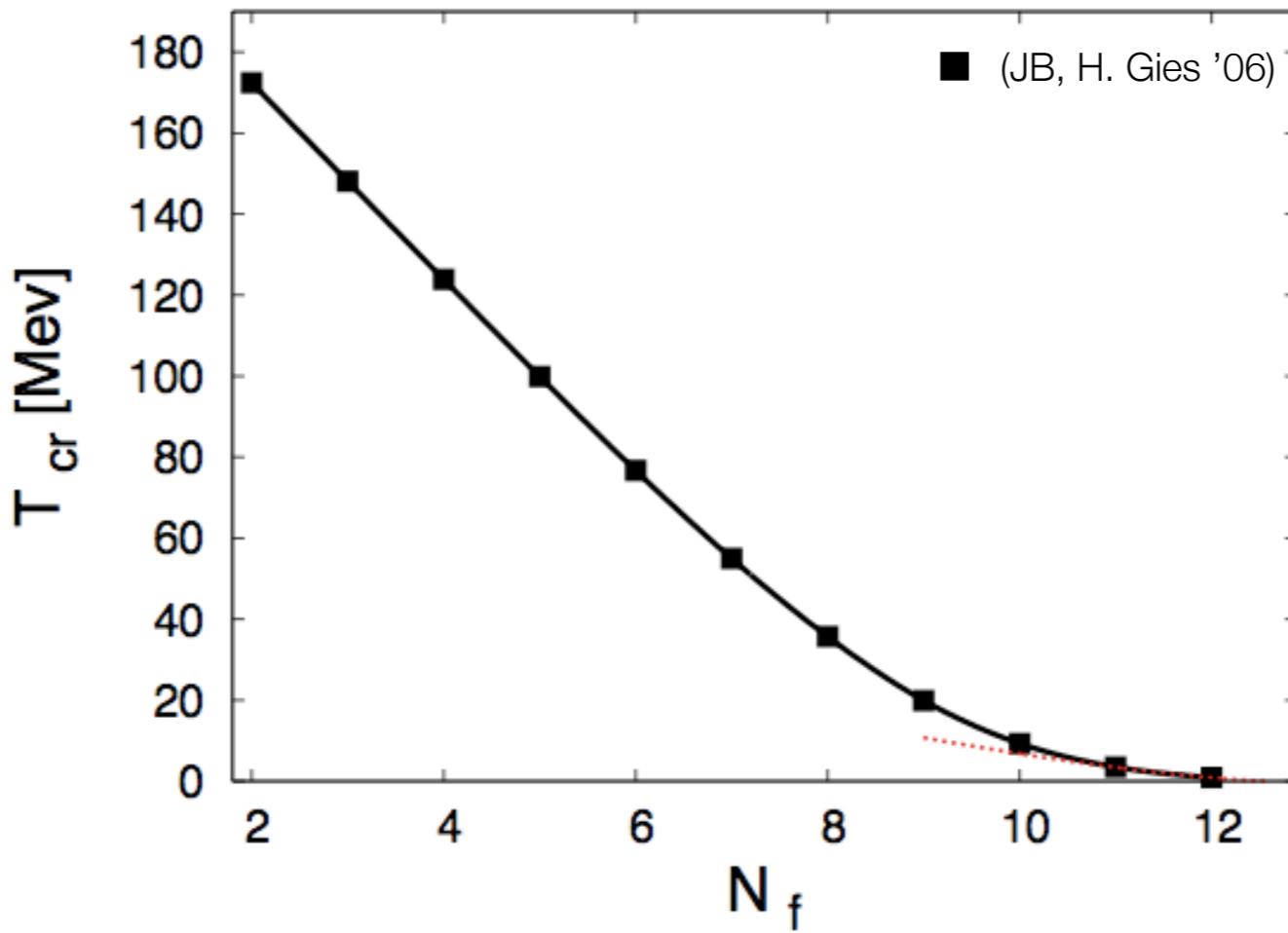
- single input parameter: $\alpha_s(m_\tau) = 0.322 \pm 0.03$

N_f	T_{cr}
2	172^{+40}_{-34} MeV
3	148^{+32}_{-31} MeV

(JB, H. Gies '06)

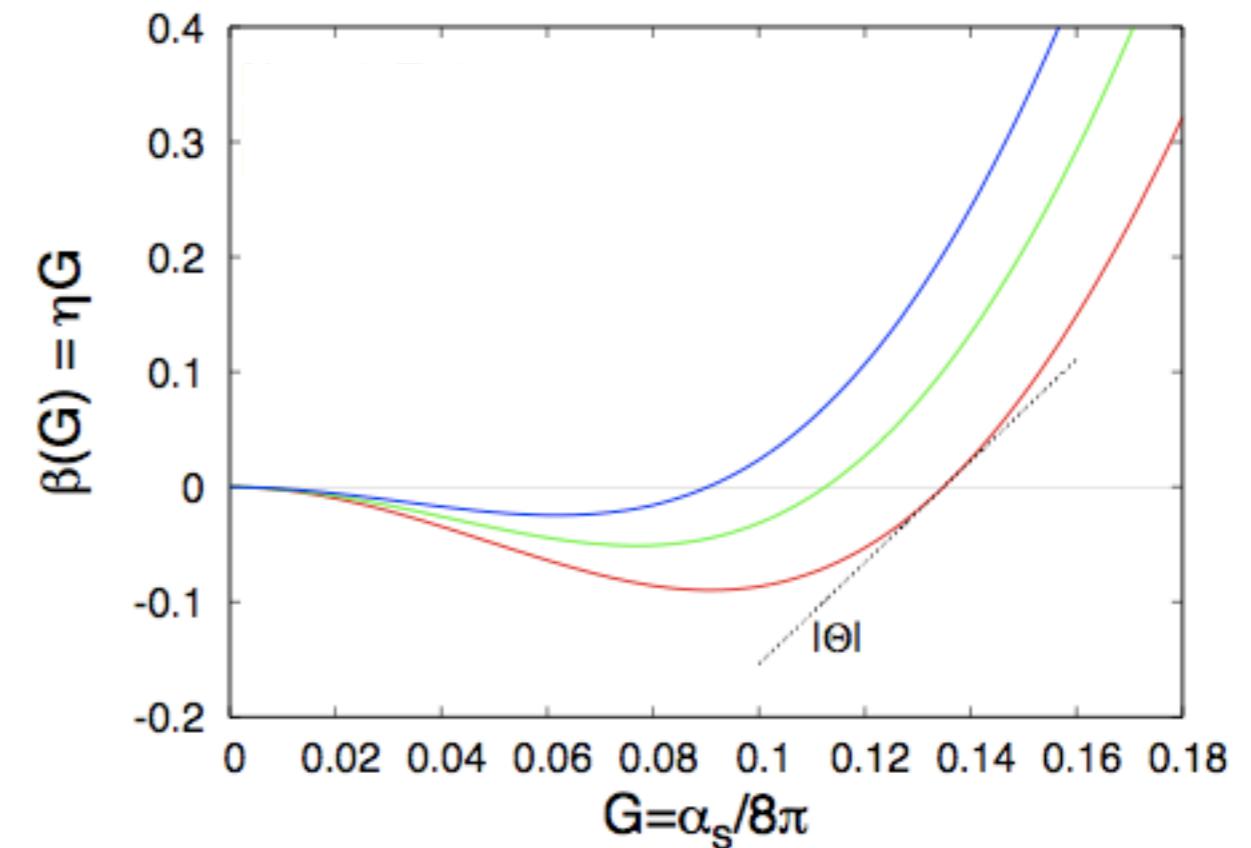
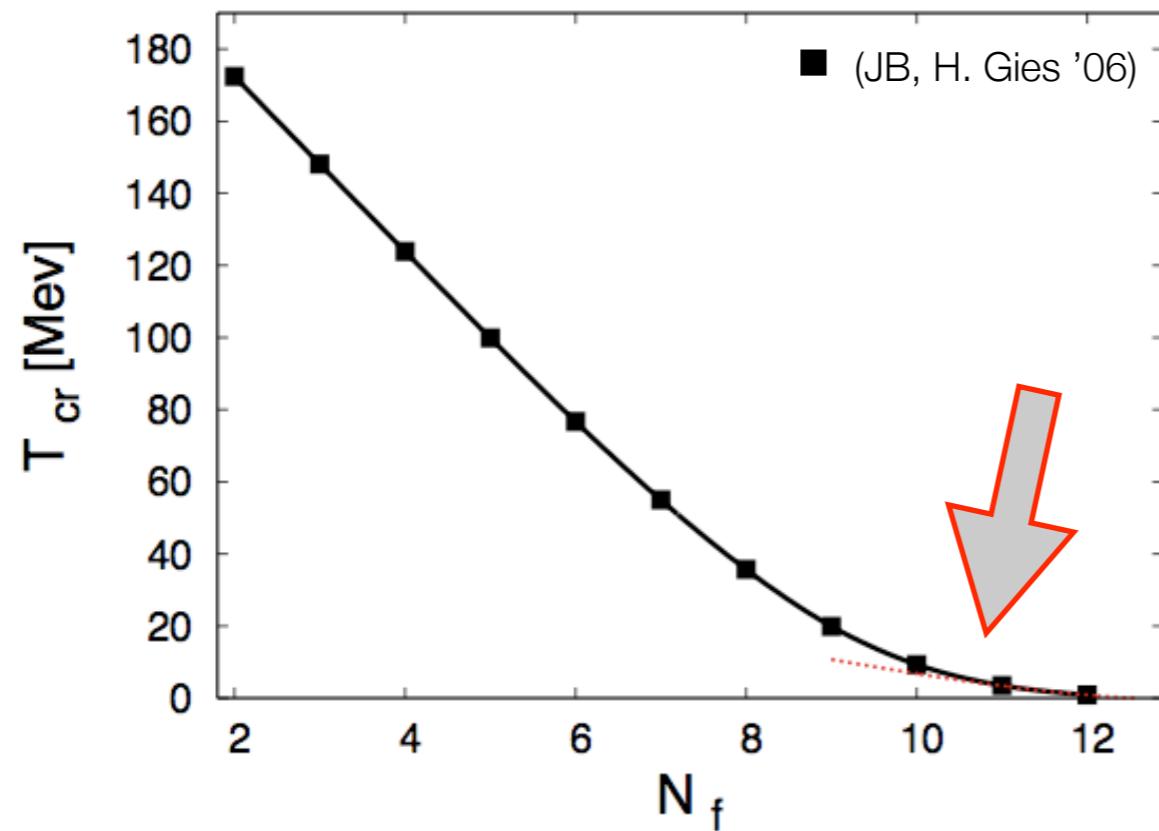
$N_f = 2 + 1$	T_{cr}
Lattice (Chen et al. '06)	192 MeV
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Many-flavor QCD



- small N_f : fermionic screening
- critical number of quark flavors: $N_{f,cr} = 12$ ($N_{f,cr} = 10$ from RG at $T=0$, Gies & Jaeckel '05)
- “conformal phase” for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no χSB

Many-flavor scaling regime



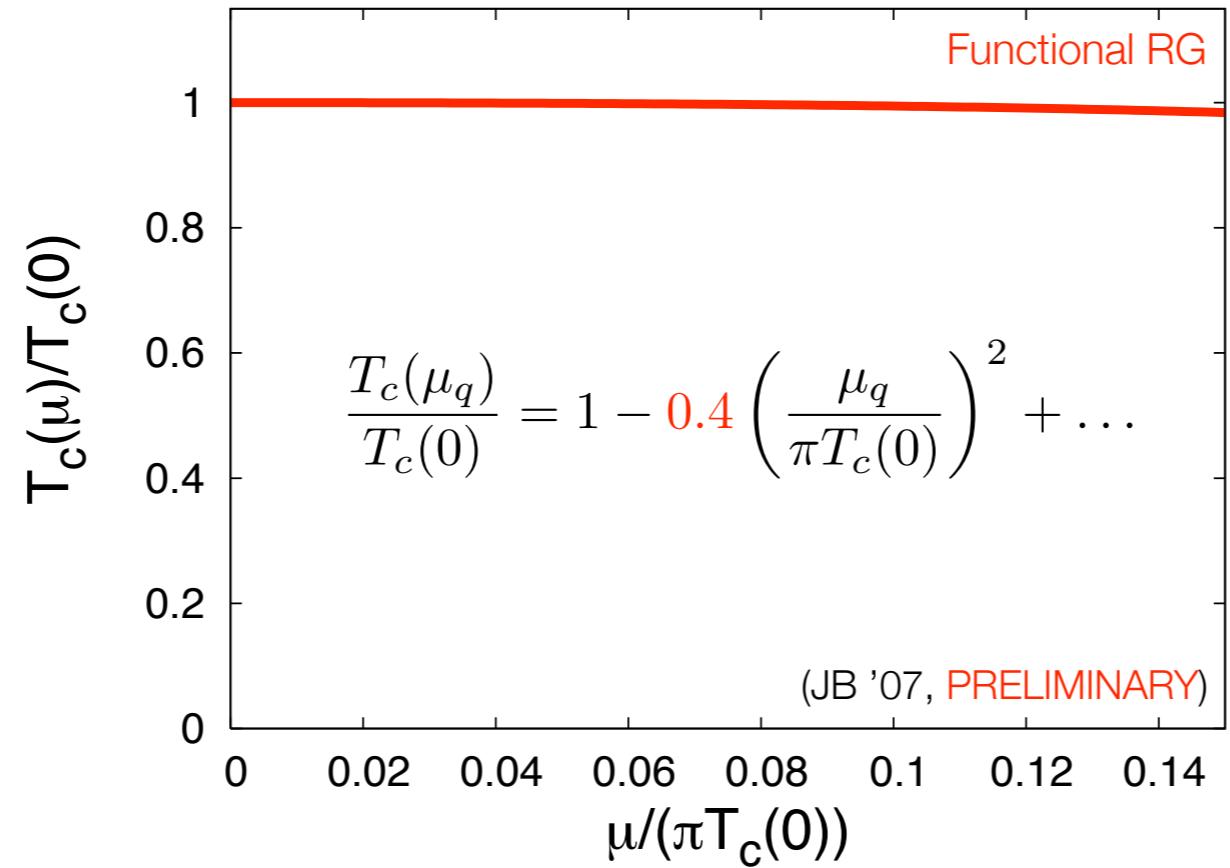
- fixed-point regime for large N_f : critical exponent $|\Theta|$

$$\partial_t g^2 \approx |\Theta|(g^2 - g_*^2)$$

- shape of the phase boundary for $N_f \approx N_{f,cr}$ (JB, H. Gies '06)

$$T_{cr} \propto |N_f - N_{f,cr}|^{\frac{1}{|\Theta|}} \quad \text{with} \quad |\Theta| \approx 0.71$$

QCD with one quark flavor



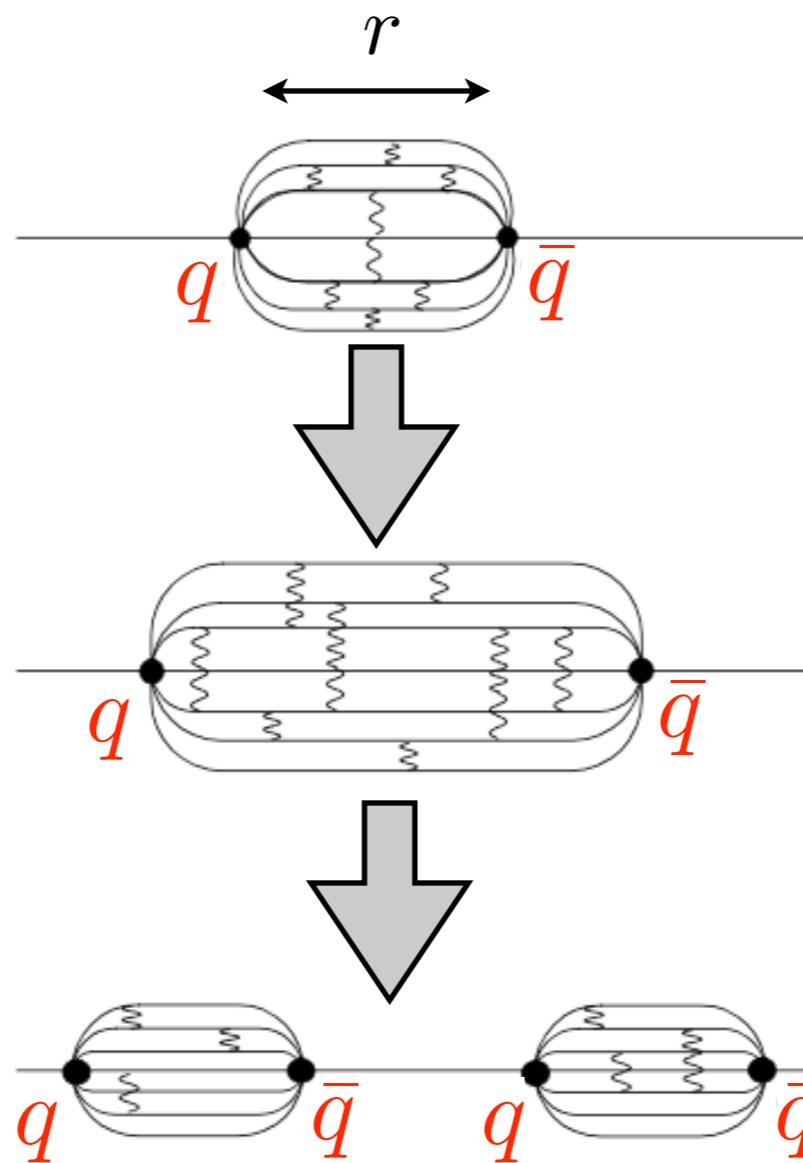
- first Taylor coefficient from Lattice QCD

Lattice QCD (de Forcrand & Philipsen '02,'06)	t_2
$N_f = 2$	0.500
$N_f = 3$	0.667

(De-)Confinement Phase Transition in Yang-Mills theory

Confinement at zero temperature

potential of a quark-antiquark pair: $\mathcal{F}_{q\bar{q}}(r) \propto \sigma r$

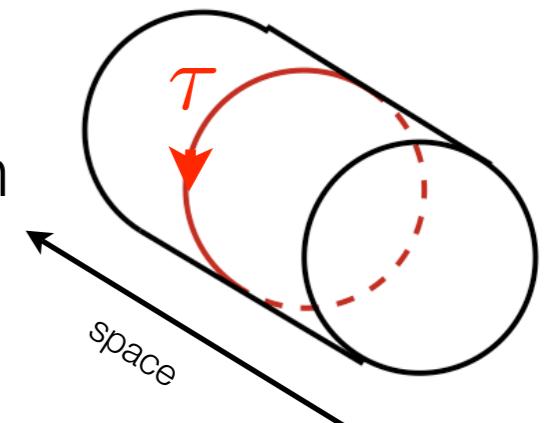


confinement at finite temperature

- infinitely heavy quark moving in Euclidean time direction:

$$\frac{\partial \Psi_q}{\partial \tau} = i\bar{g} A_0 \Psi_q \quad \longrightarrow \quad \Psi_q(\vec{x}, \tau) = \left[\text{P exp} \left(i\bar{g} \int_0^\tau dt A_0 \right) \right] \Psi_q(\vec{x}, 0)$$

infinitely heavy quark propagating in (Euclidean) time direction



- Polyakov-Loop: $\tau = \beta = 1/T$ (Polyakov '78, Susskind '79)

$$\mathcal{P}(\vec{x}) = \frac{1}{N_c} \text{P exp} \left(i\bar{g} \int_0^\beta dt A_0 \right)$$

confinement at finite temperature

- expectation value of Polyakov-loop is related to the quark free energy \mathcal{F}_q :

$$\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \sim \int \mathcal{D}A \text{Tr}_F \mathcal{P}(\vec{x}) e^{-S} \sim e^{-\beta \mathcal{F}_q}$$

→ deconfinement:

$$\mathcal{F}_q \text{ finite} \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0$$

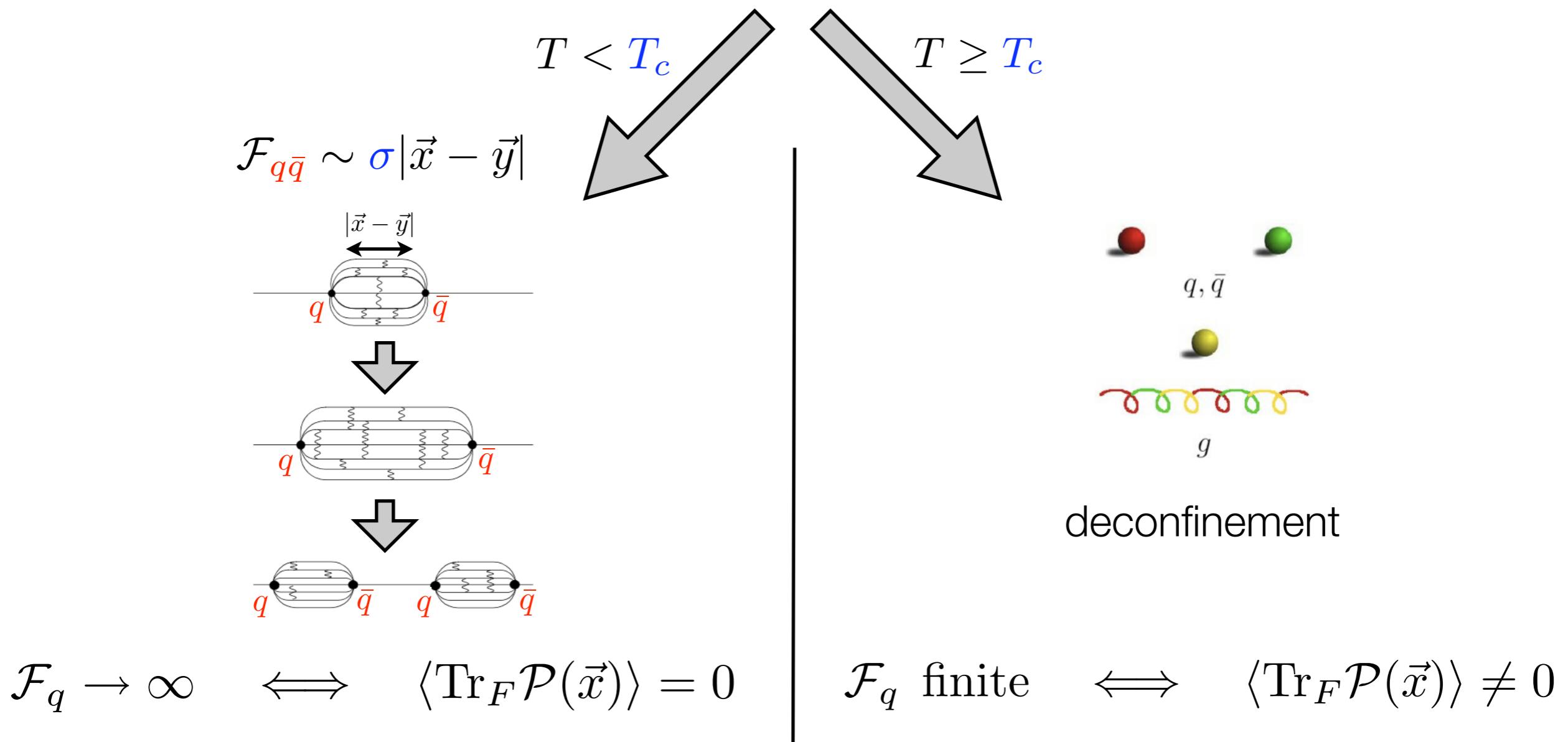
→ confinement:

$$\mathcal{F}_q \rightarrow \infty \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0$$

confinement at finite temperature

- quark-antiquark correlator:

$$\lim_{|\vec{x} - \vec{y}| \rightarrow \infty} e^{-\beta \mathcal{F}_{q\bar{q}}} \sim \lim_{|\vec{x} - \vec{y}| \rightarrow \infty} \langle \text{Tr } \mathcal{P}(\vec{x}) \cdot \text{Tr } \mathcal{P}^\dagger(\vec{y}) \rangle \leq |e^{-\beta \mathcal{F}_q}|^2$$



perturbative Polyakov-Loop potential

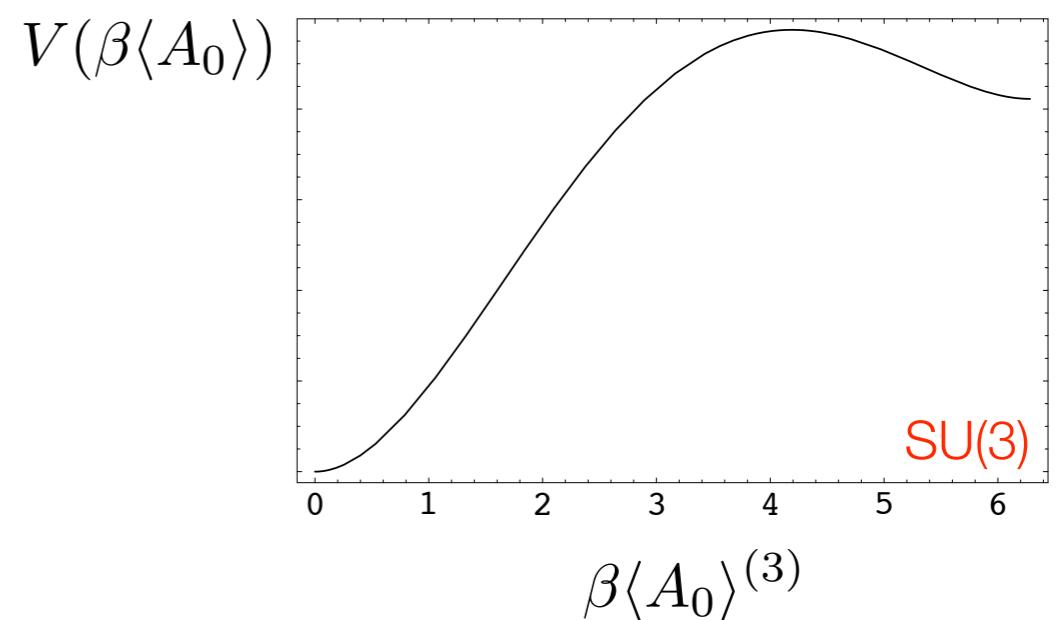
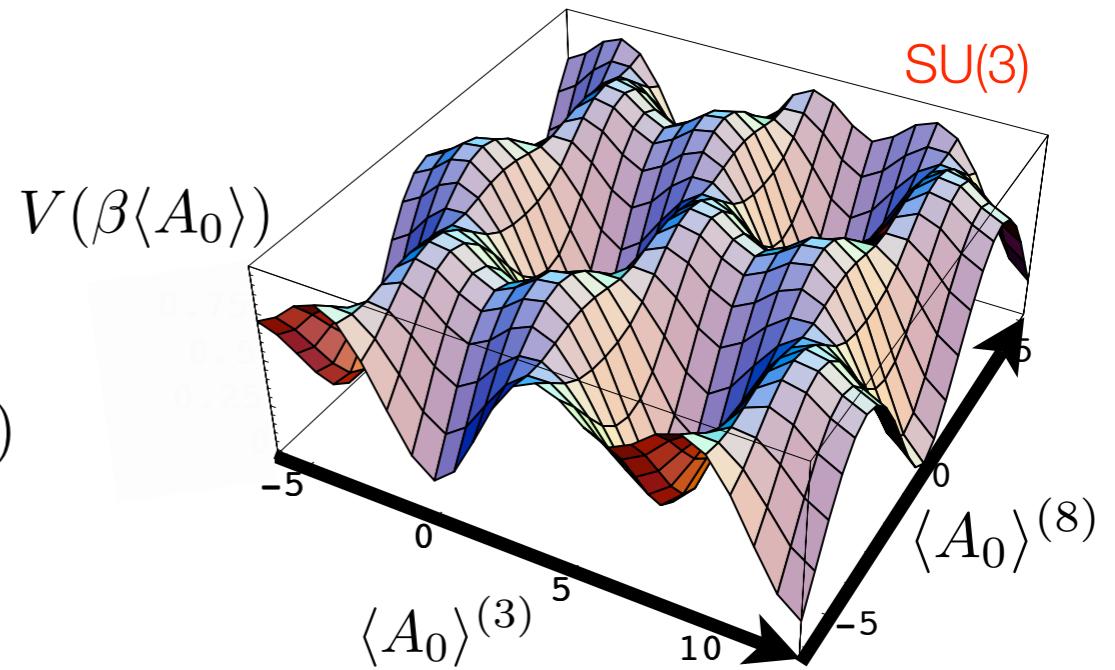
- perturbative Polyakov-loop potential in background-field gauge $A_\mu = \delta_{\mu 0} \langle A_0 \rangle$

$$V(\beta \langle A_0 \rangle, \vartheta) = - \sum_{i=1}^{N_c^2-1} \sum_{n=1}^{\infty} \frac{4}{\pi^2 n^4} \cos(n\nu_i(\vartheta)\beta \langle A_0 \rangle)$$

minimum at $\beta \langle A_0 \rangle = 0$:
deconfinement (broken Z_3 -symmetry)

$$\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \sim e^{-\beta \mathcal{F}_q} \neq 0$$

(see N. Weiss '81, '82)



non-perturbative Polyakov-loop potential

- **RG Flow** of the Polyakov-loop potential in Landau-background-field-gauge
(J.B., H. Gies, J.M.~Pawlowski '07)

$$k\partial_k V_k(\beta\langle A_0 \rangle) = \frac{1}{\Omega T^4} \left(\frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma_{k,A}^{(2)}[\beta\langle A_0 \rangle] + R_k} - \text{Tr} \frac{\partial_t R_k}{\Gamma_{k,\text{gh}}^{(2)}[\beta\langle A_0 \rangle] + R_k} \right)$$

- we need to know the full momentum-dependent two-point functions!

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use: **e. g.**, results from FRG studies for the ghost- and gluon-dressing functions in Landau-gauge QCD (see J. M. Pawlowski '07)

or from Lattice QCD studies (see e. g. Bonnet et al. '00, Sternbeck et al. '06)

- **then**: we need to know the relation between Landau gauge and Landau background-gauge propagators: (J.B., H. Gies, J. M. Pawlowski '07)

$$\Gamma_{\text{BF}}^{(2)}[\beta\langle A_0 \rangle] = \Gamma_{\text{Landau}}^{(2)}(\textcolor{red}{p^2} \rightarrow D^2[\beta\langle A_0 \rangle]) + \mathcal{O}(F)$$

non-perturbative Polyakov-loop potential

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$$V(\beta\langle A_0 \rangle) = \frac{1}{\Omega T^4} \left(\frac{1}{2} \text{Tr} \ln \Gamma_A^{(2)}[\beta\langle A_0 \rangle] - \text{Tr} \ln \Gamma_{gh}^{(2)}[\beta\langle A_0 \rangle] \right) + \mathcal{O}(\partial_t \Gamma_k)$$

- we need to know the full momentum-dependent two-point functions:

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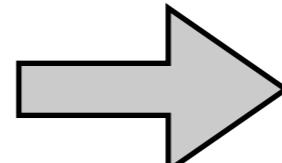
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- analytical study of RG equations yields a criterion for quark confinement:

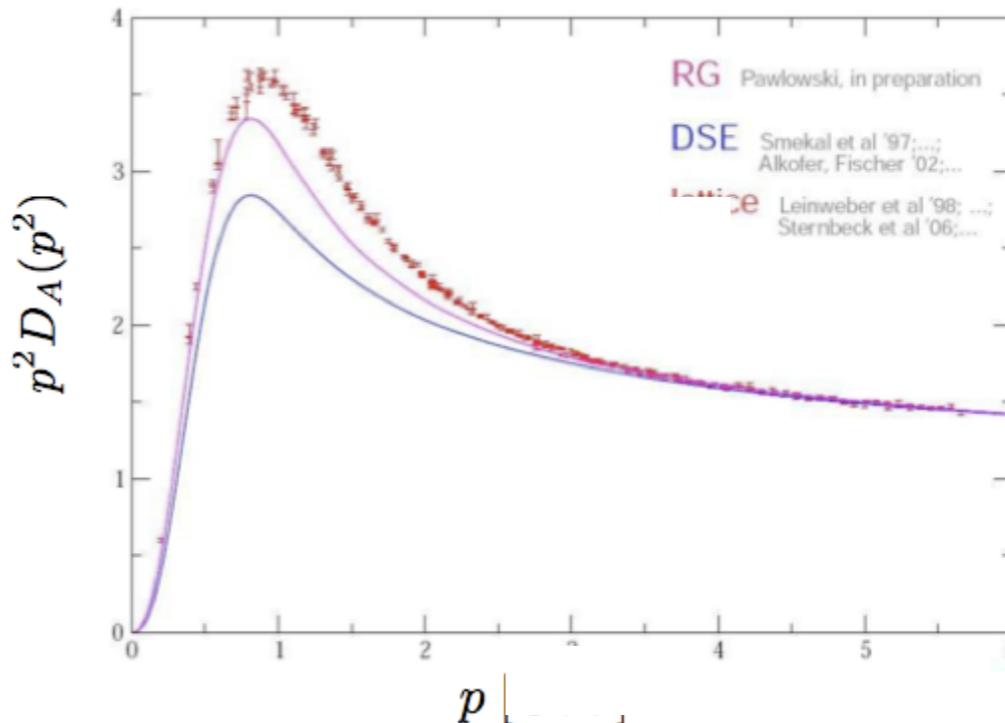
$$\begin{aligned} (\Gamma_A^{(2)})^{-1} &\xrightarrow{\text{IR}} \frac{1}{(p^2)^{1-2\kappa}} \\ (\Gamma_{gh}^{(2)})^{-1} &\xrightarrow{\text{IR}} \frac{1}{(p^2)^{1+\kappa}} \end{aligned}$$



$$\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0 : \quad \kappa > \frac{d-3}{4}$$

Landau-gauge propagators & color confinement

$$(\Gamma_A^{(2)})^{-1} = D_A(p^2) \xrightarrow{\text{IR}} \frac{1}{(p^2)^{1-2\kappa}}$$



- for $\kappa > 1/2$, we have $\lim_{p \rightarrow 0} p^2 D(p^2) = 0$ (D. Zwanziger '91)

→ signature for color confinement (Kugo, Ojima '79)

- results for κ

Method	κ
DSE/SQ	0.595
FRG	$0.539 \leq \kappa \leq 0.595$
Lattice	> 0.5

(Lerche, v. Smekal '02; Zwanziger '02)

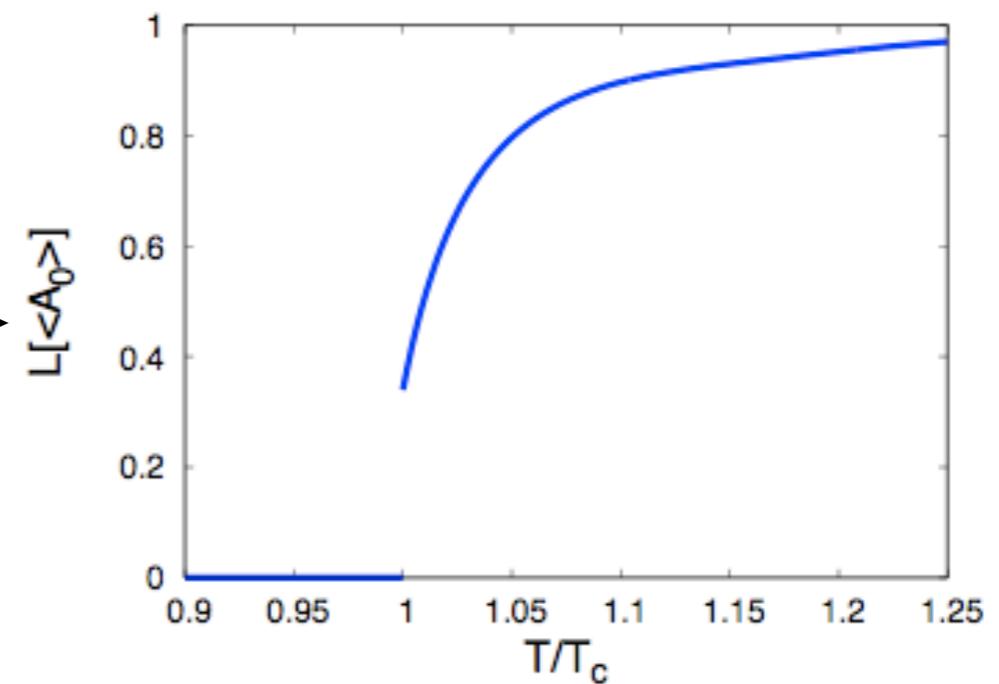
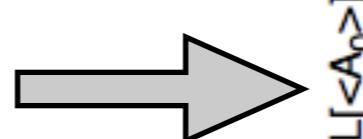
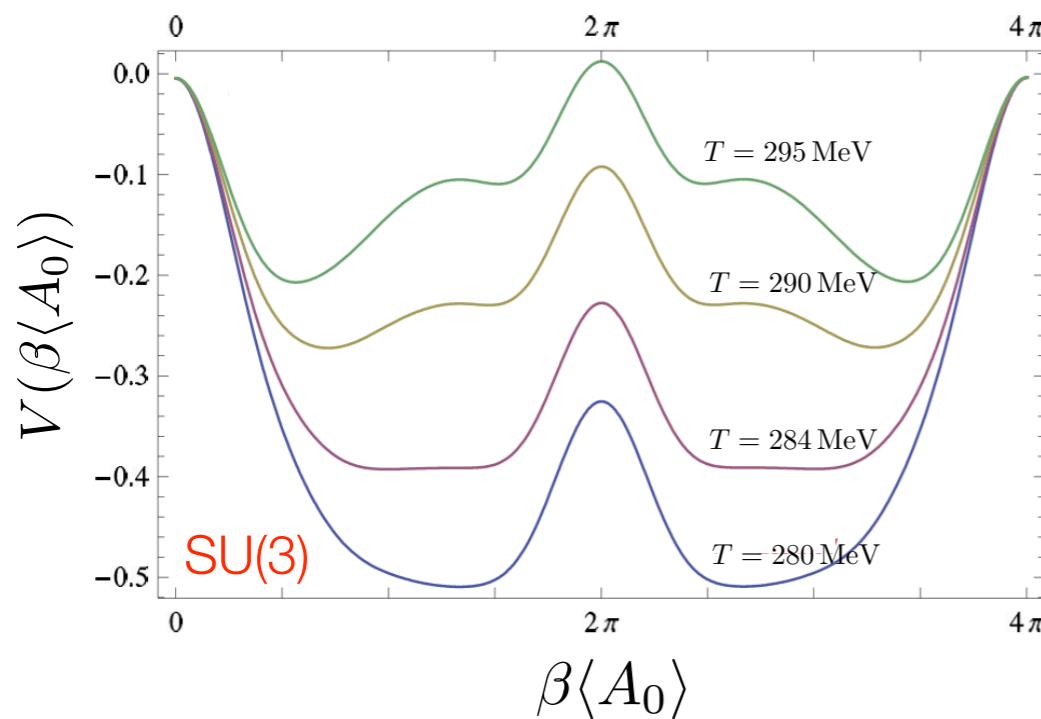
(Pawlowski, Litim, Nedelko, v. Smekal '03; Fischer, Gies '04)

(Sternbeck et al.'05; Olivera, Silva '06; Cucchieri, Mendes '06; Cucchieri, Mendes '07; Sternbeck et al.'07)

Polyakov-Loop Potential in Landau-gauge

(JB, H. Gies, J. M. Pawłowski '07)

- order parameter $L[\langle A_0 \rangle] = \frac{1}{N_c} \text{P exp} \left(i \int_0^\beta dt \langle A_0 \rangle \right)$



- first order phase transition for SU(3) (and second order for SU(2))
- SU(3): $T_c = 284 \text{ MeV} (= 0.646\sqrt{\sigma})$ Lattice QCD: $T_c = 0.646\sqrt{\sigma}$ (Kaczmarek et al.)

Conclusions

- Functional RG allows for a consistent and systematic expansion of QCD
- shape of the phase boundary near $N_{f,cr}$ is determined by the underlying IR fixed point scenario (**testable prediction!**)
- critical number of quark flavors for SU(3): $N_{f,cr} = 12$
- first results for the finite-density phase boundary are consistent with lattice QCD studies
- good agreement with Lattice QCD for the **chiral** as well as the **deconfinement phase transition**

Outlook

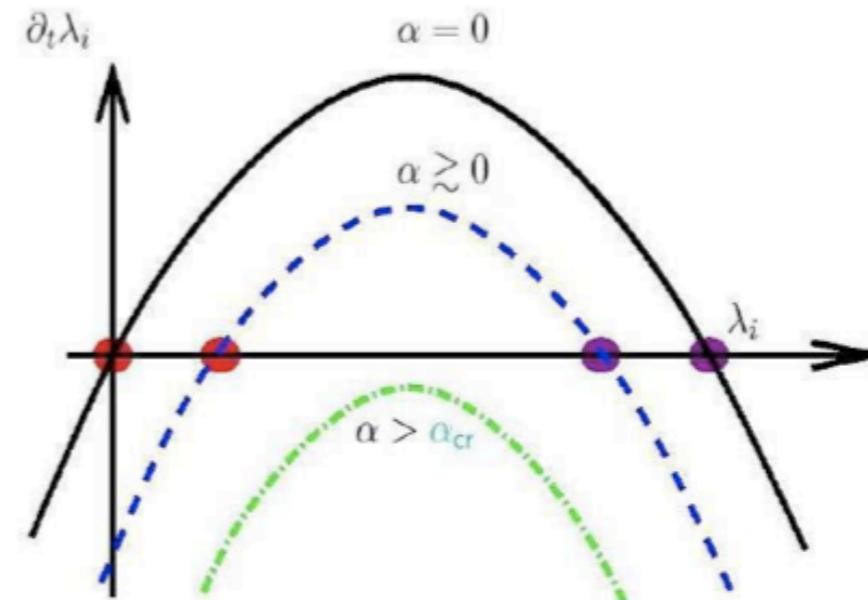
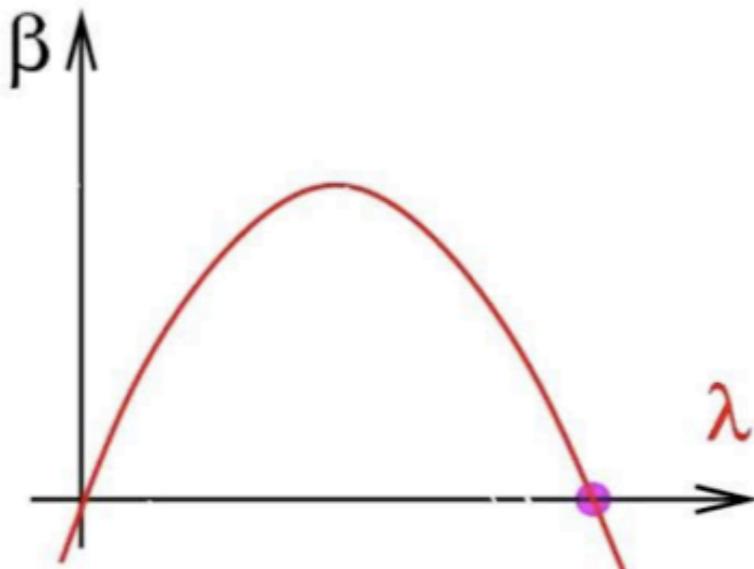
- study of QCD phase boundary at finite chemical potential with **more than one flavor**
- confinement: full thermal propagators, spatial Wilson loops ... (see J. Pawłowski's talk)

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- **condensed matter:** S. Diehl (Innsbruck U.), J. Drut (INT, Seattle), M. Ku (U. of
British Columbia), A. Schwenk (TRIUMF)
- **nuclear physics:** J. Polonyi (Louis Pasteur), A. Schwenk (TRIUMF)

Appendix

Quark-mass dependence?



NJL, Quark-Meson model

$$\frac{T_{cr}(m_c) - T_{cr}(0)}{T_{cr}(0)} \approx 0.3$$

$m_c \lesssim 10 \text{ MeV} \Leftrightarrow m_\pi \lesssim 200 \text{ MeV}$

(JB et al. '05)

QCD RG flow

$$\frac{T_{cr}(m_c) - T_{cr}(0)}{T_{cr}(0)} \approx 0.01$$

$m_c \lesssim 10 \text{ MeV} \Leftrightarrow m_\pi \lesssim 200 \text{ MeV}$

(JB '06)

$$\frac{T_{cr}(m_c) - T_{cr}(0)}{T_{cr}(0)} \approx 0.04$$

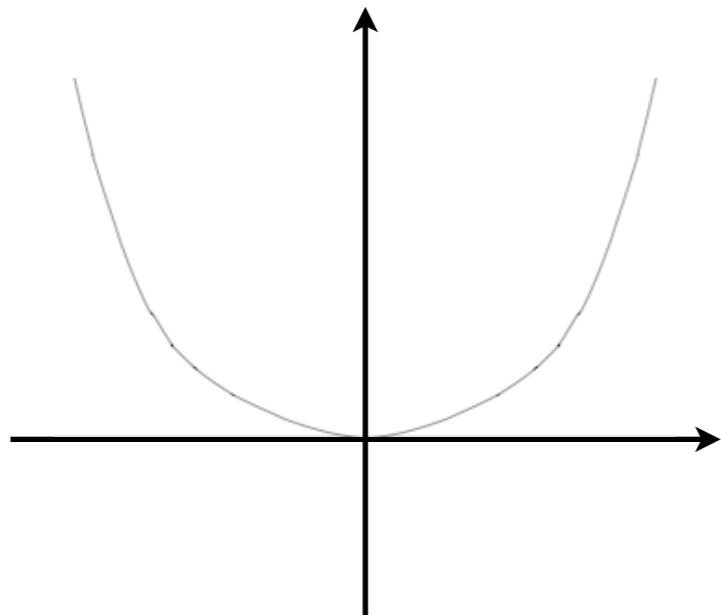
(Karsch, Laermann, Peikert '01)

confinement at finite temperature: symmetries

- symmetric under gauge transformations?

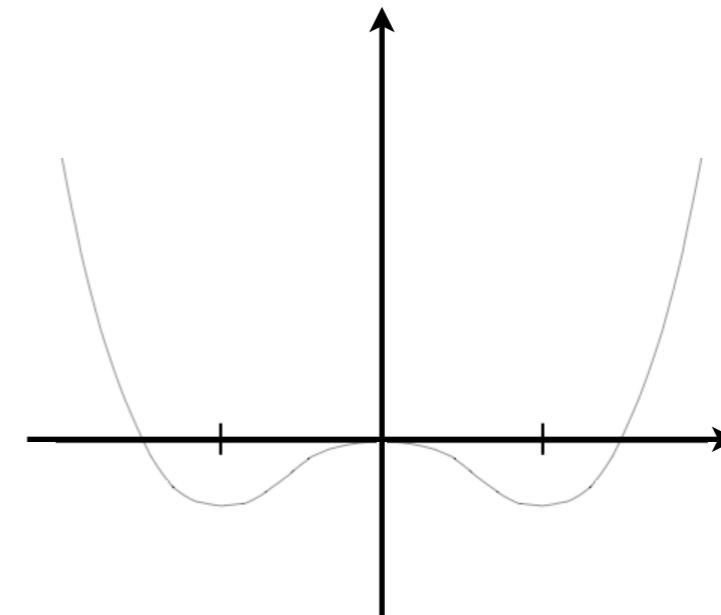
$$\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \rightarrow e^{\frac{2i\pi N}{N_c}} \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle$$

- e. g., SU(2)



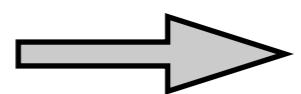
confining phase: symmetric

$$\mathcal{F}_q \rightarrow \infty \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0$$



non-confining phase: broken symmetry

$$\mathcal{F}_q \text{ finite} \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0$$



$\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle$ is a proper order parameter