Onset of first order behaviour for the 3-state Potts Model in a finite thickness slab

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Outline

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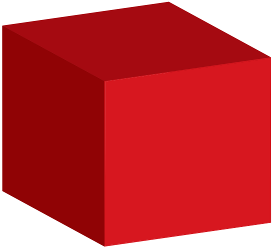
Dimensional reduction



Setup

- Lattice theory in $\mathrm{d}+1$ dimension, finite box $\underbrace{L \times \ldots}_{\mathrm{d}} \times L_t$ with lattice spacing a, external parameter β_L
- Phase transition (PT) in β_L can occur
  - Different *order* in $d+1$ ($L_t = L$) and d dim. ($L_t = 1$)
  - c.f. U(1), Z(2) LGT or classical spin systems

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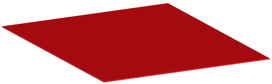

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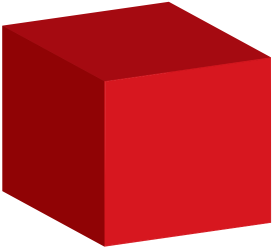

Modified setup

- Take system with $L_t << L$ and investigate criticality in the limit $L \to \infty$
- For lattice *field* theory, leads to definition of temperature T
  - System compactified in L_t - direction
  - Temperature $T \sim \frac{1}{L_t \cdot a}$
- Investigate order of PT with varying L_t and $L \to \infty$, *crossover* between d and d+1 dim. behaviour observed?

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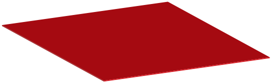

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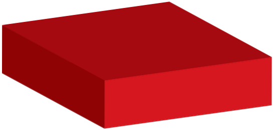

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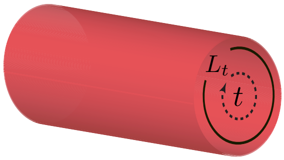

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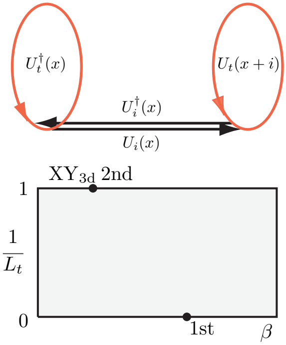
Specific Example: U(1) LGT in $(3+1)$ dimensions

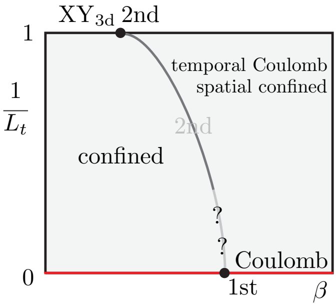
$$S_{\mathrm{Wilson}} = -\beta \sum_{\substack{x,\mu,\nu \\ \mu>\nu}} \cos\theta_{\mu\nu}(x),\ \beta = \frac{1}{g^2}$$



Phase diagram in limiting cases

- $L_t = 1 : Z_{U(1)_{L_t=1}} = Z_{\mathrm{XY_{3d}}} \cdot Z_{U(1)_{\mathrm{3d}}}$
  - $\mathrm{XY_{3d}}$ 2nd order, $\beta_c = .4542\ldots$[1]
  - $U(1)_{\mathrm{3d}}$ confined $\forall\,\beta$
- $L = L_t$: PT at $T = 0$, $\beta_c = 1.011\ldots$ is 1st order [2, 3] (no continuum limit)

Phase diagram (2004) [2]



- Phase boundary extends, separating high-T confined - temporal Coulomb phase. Order ?

1) 2nd order down to some $1/L_t^*$
   - Separation of 1st and 2nd order line by tricritical point (TP)

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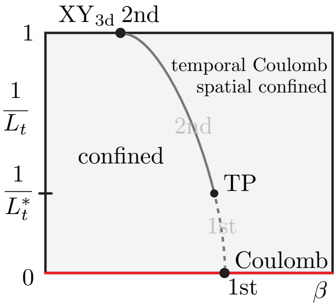

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1) 2nd order down to some $1/L_t^*$
   - Separation of 1st and 2nd order line by tricritical point (TP)
2) PT of 2nd order all the way down to - but excluding - $1/L_t = 0$
   - Counterintuitive ? (Allows for continuum limit of a *confined* Abelian Gauge Theory.)
   - Implies (new) non-trivial fixed point (FP) as Gaussian FP is not stable in d $= 3$.[4, 5]

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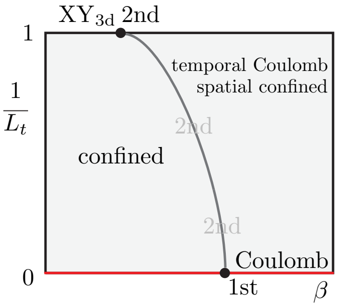

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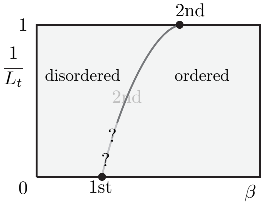
Outline

Simpler example: $q = 3$ Potts Model [6] in $(2+1)$ dimensions (this study)

$$-\beta H = \beta J \sum_{<ij>} \delta_{s_i s_j}, \quad s_k = 1, \ldots q, \; J > 0$$

- d $= 2$: 2nd order PT $\beta_c J = \ln\left(1 + \sqrt{q}\right)$, $q \leq 4$ (known exactly [7]) critical exponents known
- d $= 3$: PT (weak) 1st order, $\beta_c = 0.550565(10)$ [9]



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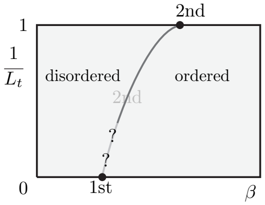

q-state Potts Model (d = 2) - extended phase diagram

Extension to Potts Lattice Gas (PLG)

Allow for vacancies on lattice, $t_k = 0(1)$ corresponds to vacant (spin occupied) site, vacancies are controlled via chem.potential Δ.

Renormalization group analysis [10, 11] for $d = 2$ yielded extended phase diagram: PT can be driven from 2nd to 1st order by increasing $q > q_c(d)$ (1st order for $q > q_c(d)$, $q_c(2) = 4$) or by increasing the vacancy concentration.

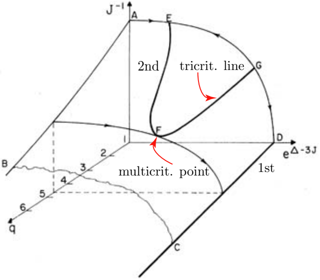

FIG. 1. Phase diagram of the Potts lattice gas in the space of temperature, J^{-1}, fugacity, $e^{\Delta-3J}$, and number of states, q.

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Besides RG relevant scaling field ϕ (thermal), Δ amounts to new scaling field ψ. RG eigenvalue y_ψ changes sign at multicritical point (logarithmic corrections to scaling).

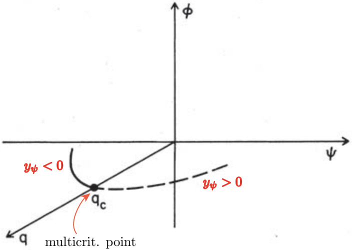


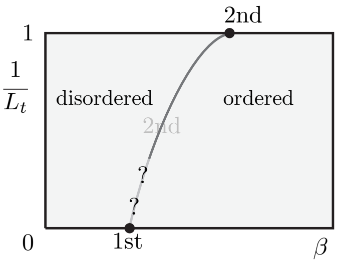
FIG. 1. Fixed point structure in the q-ψ-φ space. A continuous line of critical points (solid curve) meets a continuous line of tricritical points at a multicritical point ($q = q_c$, $\psi = \varphi = 0$).

Potts Model phase diagram - lessons to learn

- PM with $q = 4$ in $d = 2$ (2nd order) - difficult to numerically determine scaling (logarithmic corrections).
- $q = 3$, $d = 2$ universal entities (exponents) known, *no* logarithmic corrections.

Expectations

- $q = 3$ PM ($d = 2$) with $L_t > 1$ falls - if 2nd order - in universality class of $d = 2$.
- Iff scenario 1): Separating TP has exponents which are known.
- Correlation length $\xi_0 \approx 10$ [9] for weak 1st order PT, naive expectation: $L_t^* \sim \mathcal{O}(10)$



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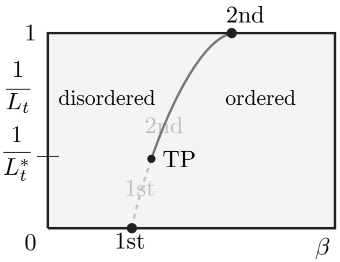


Outline

1. Motivation
   - Phase transitions and dimensional reduction
   - The phase diagram of $U(1)$ LGT in $(3+1)$ dimensions
2. The Potts Model
   - Definition
   - Phase diagram
3. Numerical experiment
   - Task
   - Results
4. Summary and outlook

Criteria at hand

Task

- Perform MC runs for PM-systems of size $L^2 \times L_t$.
- Tune systems to criticality, FSS for $L \longrightarrow \infty$, keeping L_t fixed.
- Distinguish weak 1st from 2nd order PT by :
  1) Surface tension and energy histogram inspection,
  2) Asymptotic behaviour of correlation function $G(i-j) =< s_i s_j >$,
  3) Estimation of critical exponents.

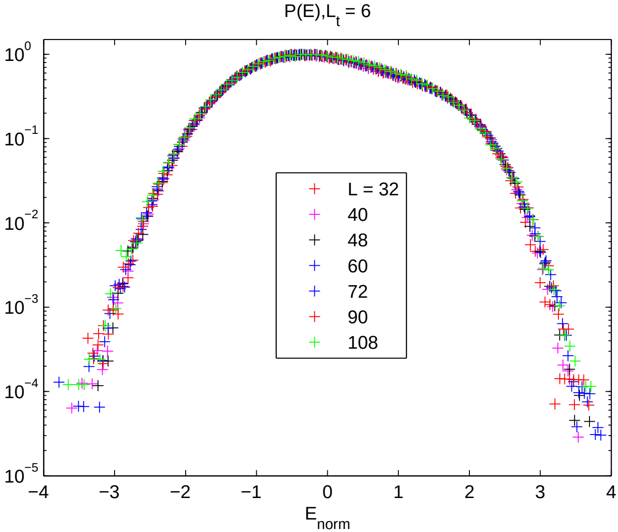
Surface tension and energy histogram

- Coexistence of ordered and disordered phase (mixing) is characteristic for 1st PT: For observables M, E

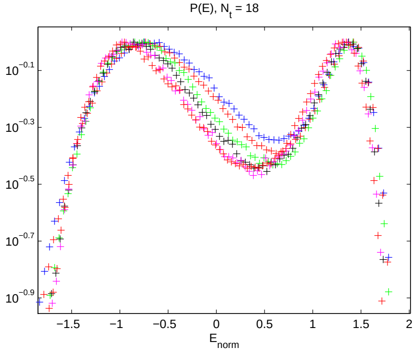
  $$P(M) = c_o \mathrm{e}^{-(M-M_o)^2/d_o} + c_d \mathrm{e}^{-(M-M_d)^2/d_d} + c_m \mathrm{e}^{-\sigma 2 A \beta}, \ (\mathrm{at} \beta_c)$$

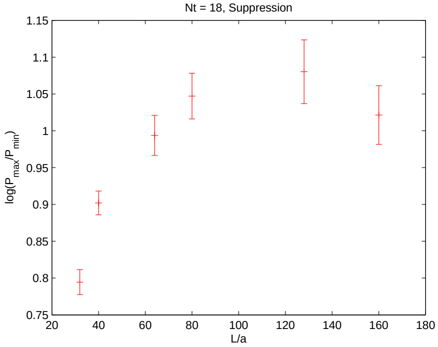
  A - area of interface, σA - surface energy $(A = L_t L)$
- From distribution obtain $2\sigma L L_t = \log(P_{\max}/P_{\min})$ (via FSS).
- Note also that $\log(P_{\max}) - \log(P_{\min})$ has to grow with L (for 1st order PT).

Energy histogram, $L_t = 6$

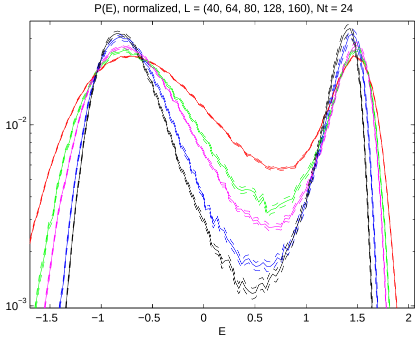


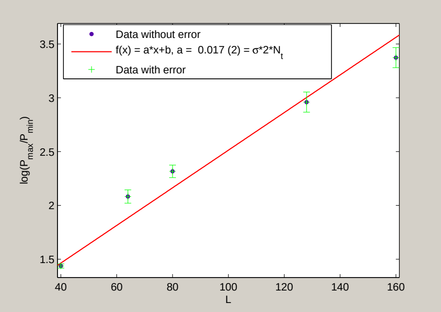
Surface tension and energy histogram, $L_t = 18$





Surface tension and energy histogram, $L_t = 24$





Asymptotic behaviour of $\bar{G}(x, y)$

- $\bar{G}(\mathbf{r_1} - \mathbf{r_2}) = \langle \bar{s}(\mathbf{r_1}) \bar{s}(\mathbf{r_2}) \rangle$, where $\bar{s}(x, y) = \frac{1}{L_t} \sum_z s(x, y, z)$
- By dimensional reduction ($\xi \to \infty$) $\bar{G}(\mathbf{r}) \to G_{2d}(x, y)$ at 2nd order criticality, where

$$G_{2d}(r) \sim r^{-(\eta_{\text{crit/tricrit}} + d - 2)},$$

  at the critical/tricritical point.
- If $\xi \to \infty$ and $d = 2$, $G(r) \sim f(r/L), r < L$ (by dim. reasons), s.t. $G(r) \approx 1/L^\eta f(r/L)$ which is used for a data collapse rescaling: $G_L(r) \cdot L^\eta \approx f(r/L)$

Critical Exponents

Recall

Thermodynamic quantities $(\chi, c_v, \ldots)$ exhibit divergent behaviour at PT $(V \to \infty)$, $t = \frac{T - T_c}{T_c}$:

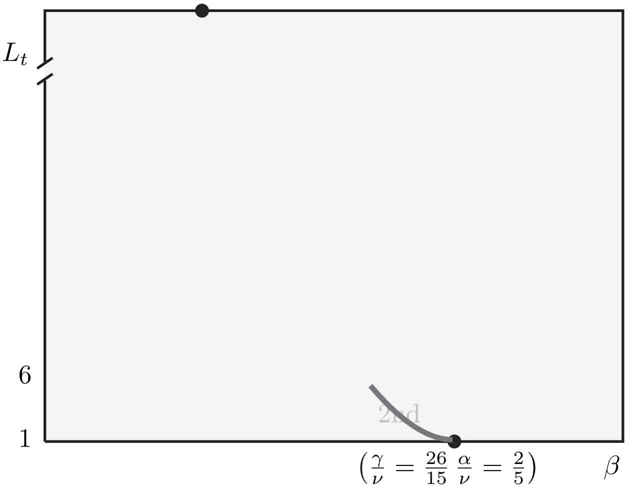
- Continuous PT: $\xi \sim |t|^{-\nu_{\mathrm{crit}}}$, $\chi \sim |t|^{-\gamma_{\mathrm{crit}}}$, $c_v \sim |t|^{-\alpha_{\mathrm{crit}}}$, $\alpha_{\mathrm{crit}}, \gamma_{\mathrm{crit}}$ etc. universal
- PT of 1st order: $1/\nu = d$ and $\alpha, \gamma = 1$
- At TP obtain critical exponents (universal) where

$$\varsigma_{\mathrm{crit}} < \varsigma_{\mathrm{tricrit}} < \varsigma_{\mathrm{1st}}$$

- FSS in L allows determination:

$$c(L)_{v,\mathrm{max}} = a_1 L^{\frac{\alpha}{\nu}} + a_2 + \mathrm{corr.}$$

Critical Exponents



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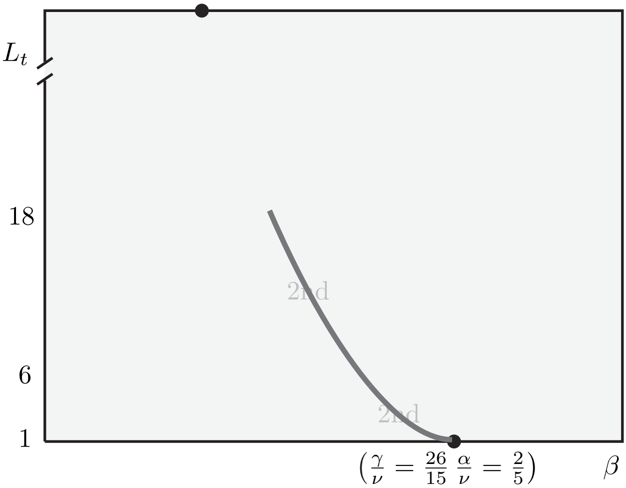

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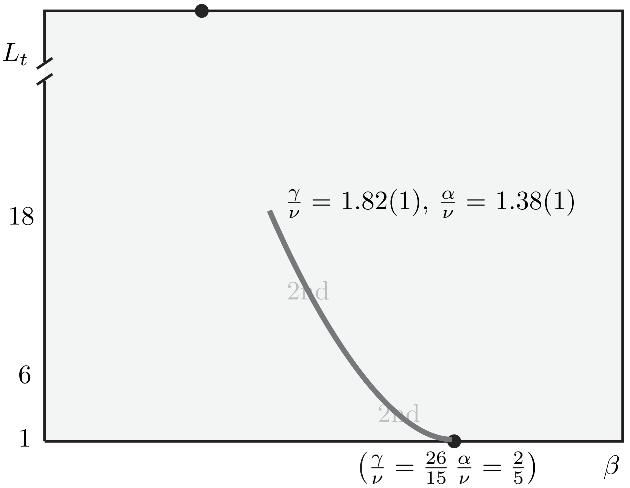

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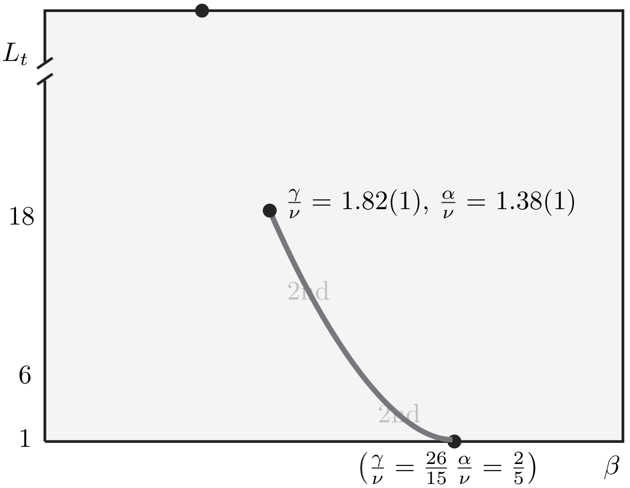

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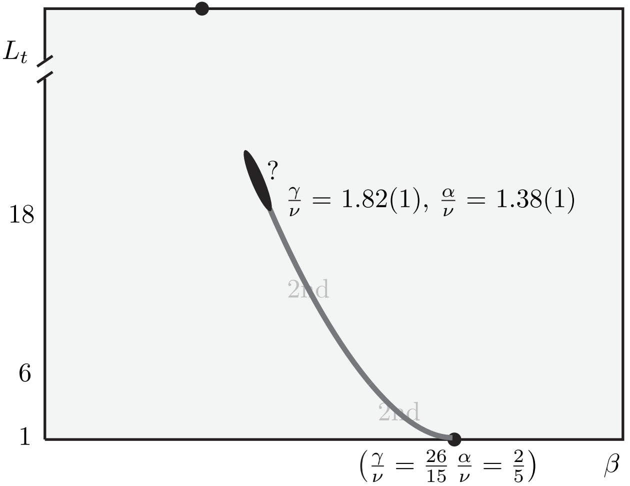

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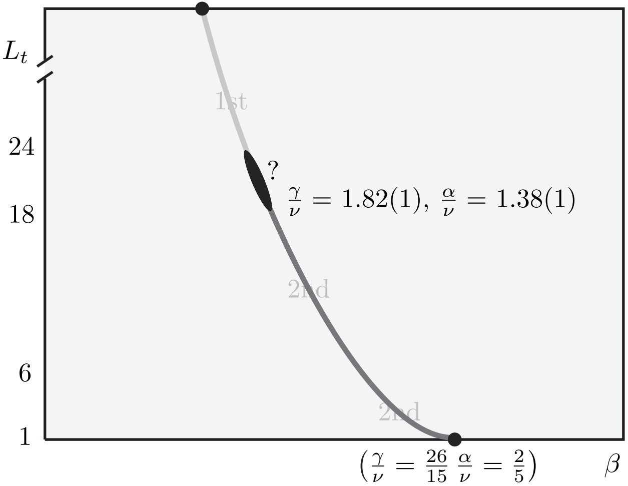

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- There is a TP at finite L_t (and for U(1) LGT it is of relevance).
- $L_t^* \approx 18$ agrees with initial argument $L_t^* \sim \mathcal{O}(10)$
- Question: PT changes from 2nd to 1st order by going from d to $d+1$ but nerver v.v.?

For further reading

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