Constraints on the infrared behavior of Landau-gauge gluon propagators in Yang-Mills theories

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Abstract

Several analytic approaches predict for $SU(N)$ Yang-Mills theories (in 2, 3 and 4 space-time dimensions) a suppressed gluon propagator at small momenta in Landau gauge, with a null value at $p = 0$. Numerical studies indeed support an IR finite gluon propagator. However, the agreement between analytic and numerical studies is only at the qualitative level in 3d and 4d, since the gluon propagator seems to display a (finite) nonzero value at $p = 0$. This might be due to finite-size effects. Here we present data for lattices sizes of up to 320^3 at $\beta = 3.0$ and up to 128^4 at $\beta = 2.2$, corresponding to $V \approx (85fm)^3$ and $V \approx (27fm)^4$. We see no sign of a null gluon propagator in the IR limit and discuss possible explanations for this.

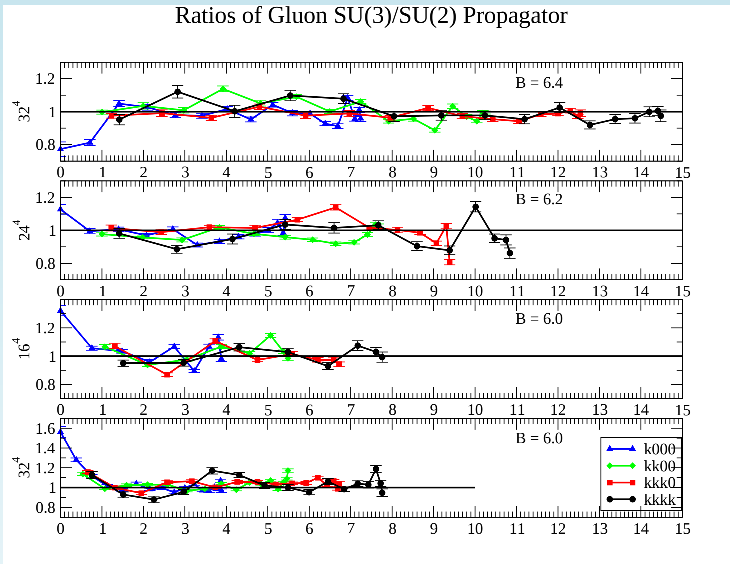
IR gluon propagator and confinement

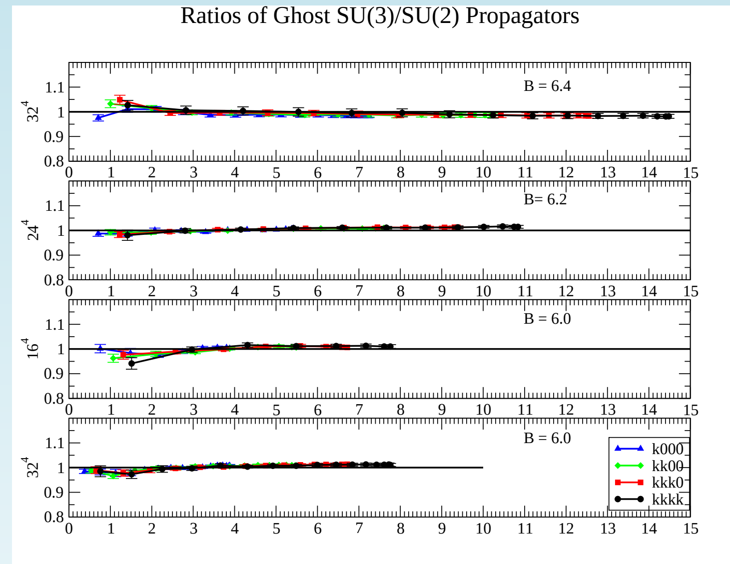
- Gribov-Zwanziger confinement scenario in Landau gauge predicts a gluon propagator $D(p^2)$ suppressed in the IR limit.
- In particular, $D(0) = 0$ implying that reflection positivity is maximally violated.
- This result may be viewed as an indication of gluon confinement.
- Above results are confirmed by functional methods.
- On large lattice volumes the gluon propagator decreases in the limit $p \to 0$, but $D(0) > 0$.

Can one find $D(0) = 0$ using lattice numerical simulations? Yes in 2d (A. Maas) using lattices up to $(42.7 fm)^2$.

$SU(2)$ vs. $SU(3)$

A. Cucchieri, T.M., O. Oliveira and P. Silva (2007)

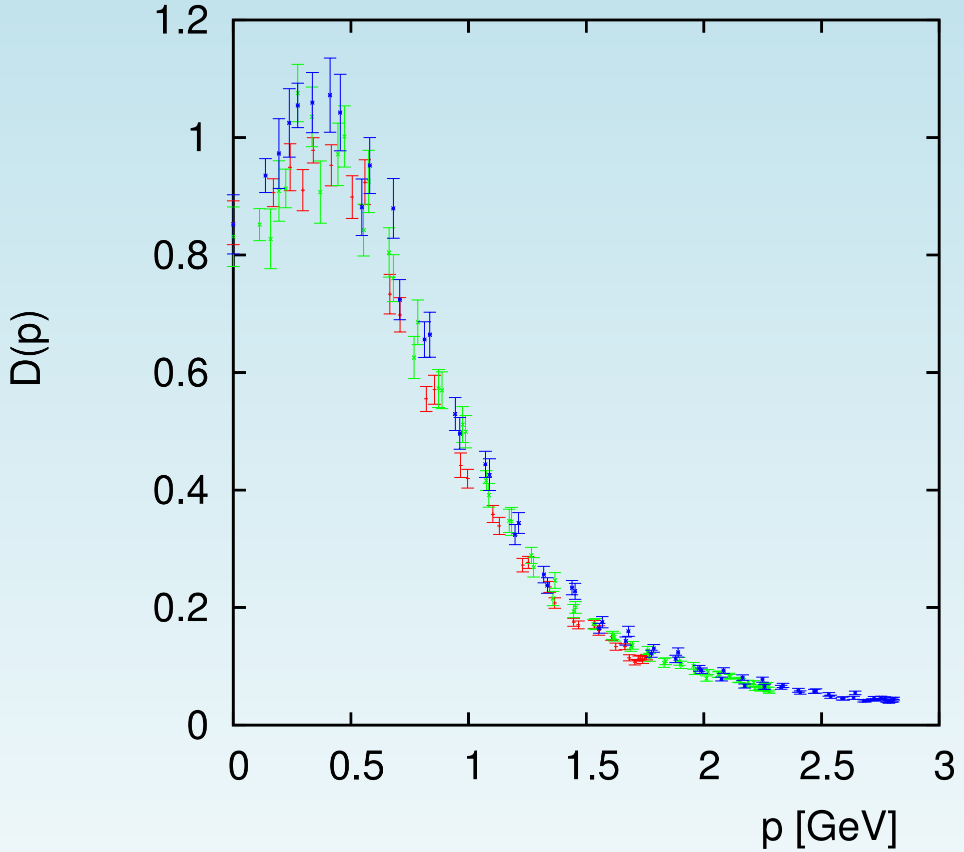




Ratio $SU(3)/SU(2)$ for the Landau-gauge gluon propagator.

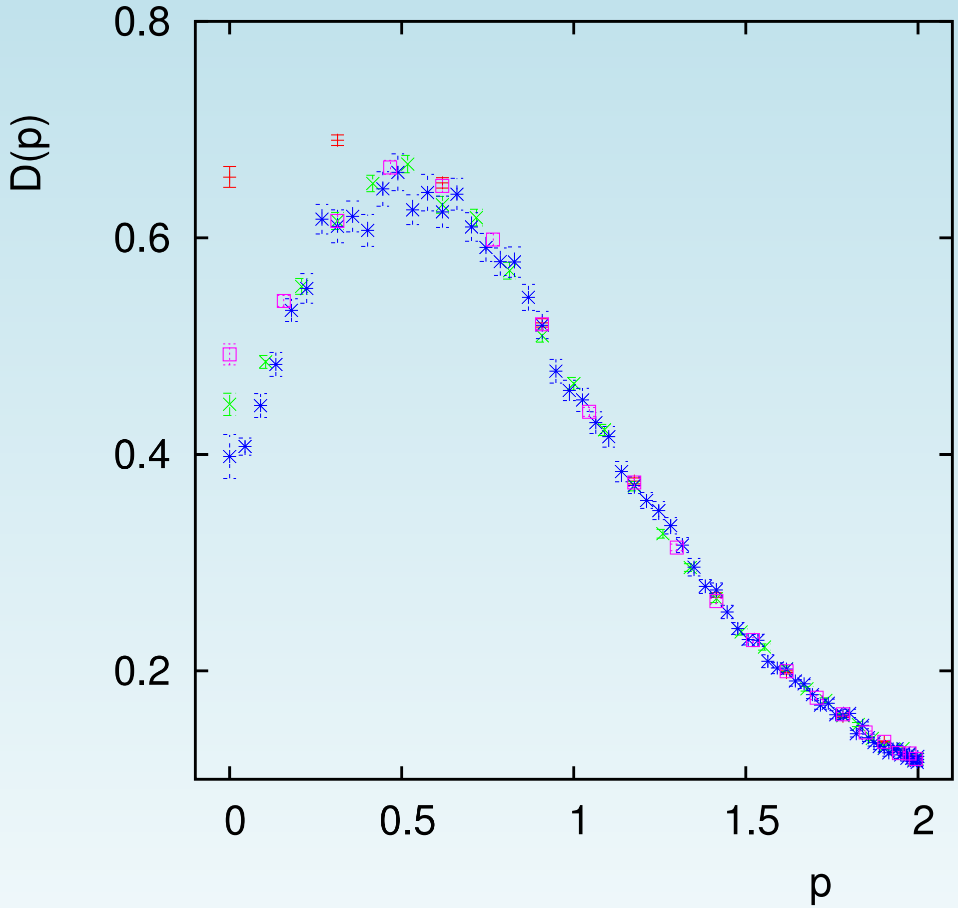
Ratio $SU(3)/SU(2)$ for the Landau-gauge ghost propagator.

Infinite-volume limit in 3d (I)



Gluon propagator as a function of the lattice momentum p for $\beta = 3.4$ and 32^3 (+), $\beta = 4.2$ and 64^3 ($\times$), $\beta = 5.0$ and 64^3 ($*$) (A. Cucchieri, Phys. Rev. D60 034508, 1999). About 100 days using a 0.5 Gflops workstation.

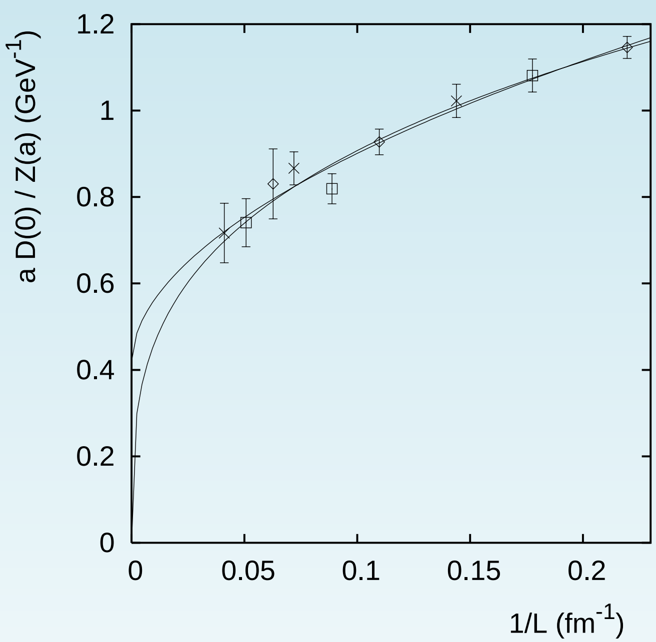
Infinite-volume limit in 3d (II)



Gluon propagator as a function of the lattice momentum p for lattice volumes $V = 20^3$, 40^3, 60^3 and 140^3 at $\beta = 3.0$ (A. Cucchieri, T. M. and A. Taurines, Phys. Rev. D67 091502, 2003). About 100 days using a 13 Gflops PC cluster.

Old results in 3d

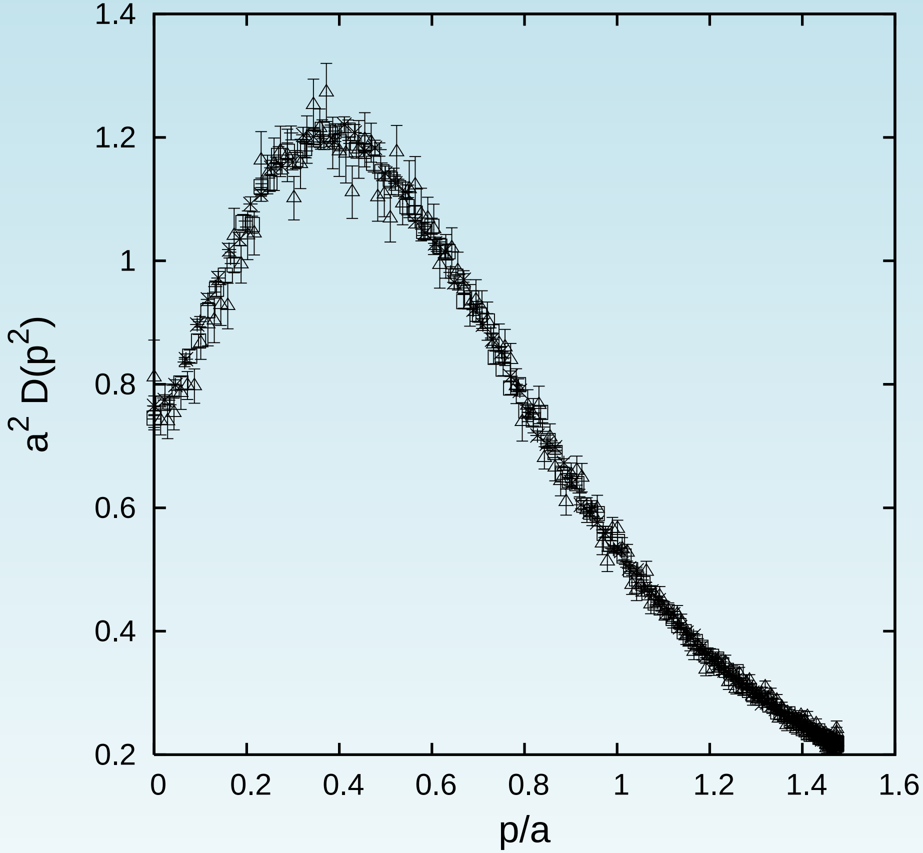
The gluon propagator using lattice volumes up to 140^3 and β values $4.2, 5.0, 6.0 \longrightarrow$ physical lattice sides almost as large as 25 fm.



Plot of the rescaled gluon propagator at zero momentum as a function of the inverse lattice side for $\beta = 4.2\,(\times),\ 5.0\,(\Box),\ 6.0\,(\Diamond)$. We also show the fit of the data using the Ansatz $d + b/L^c$ both with $d = 0$ and $d \neq 0$.

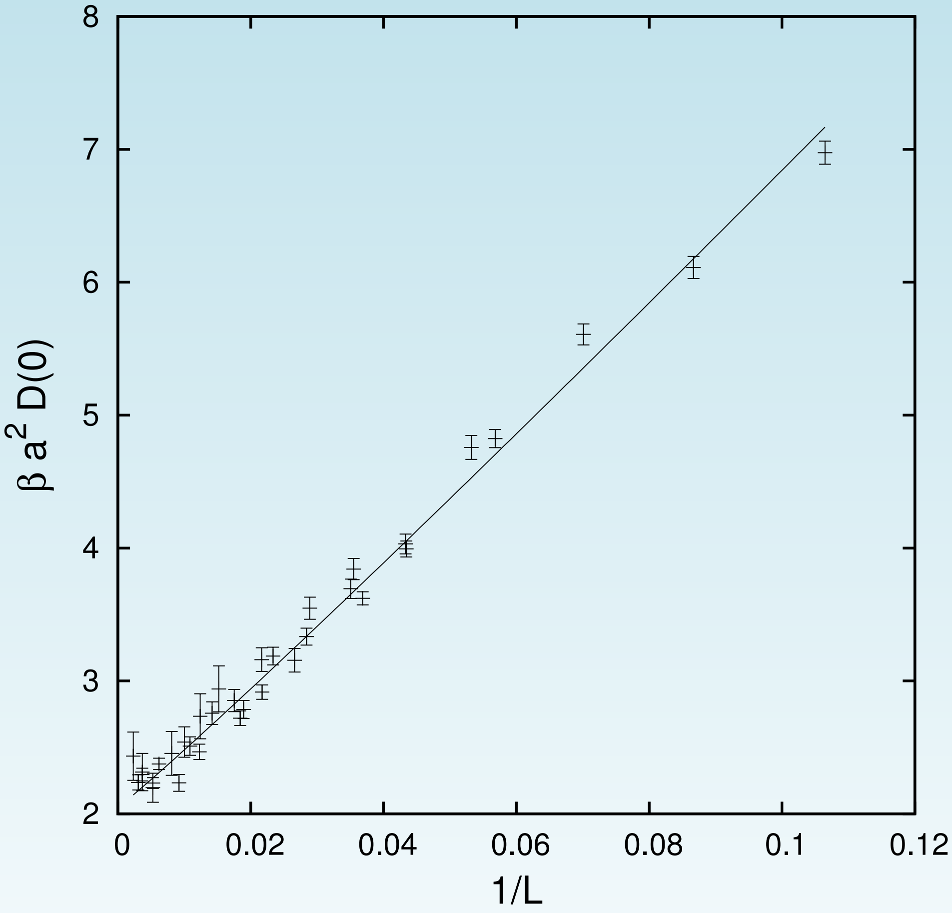
Can we go to even larger lattice volumes?

Infinite-volume limit in 3d (III)



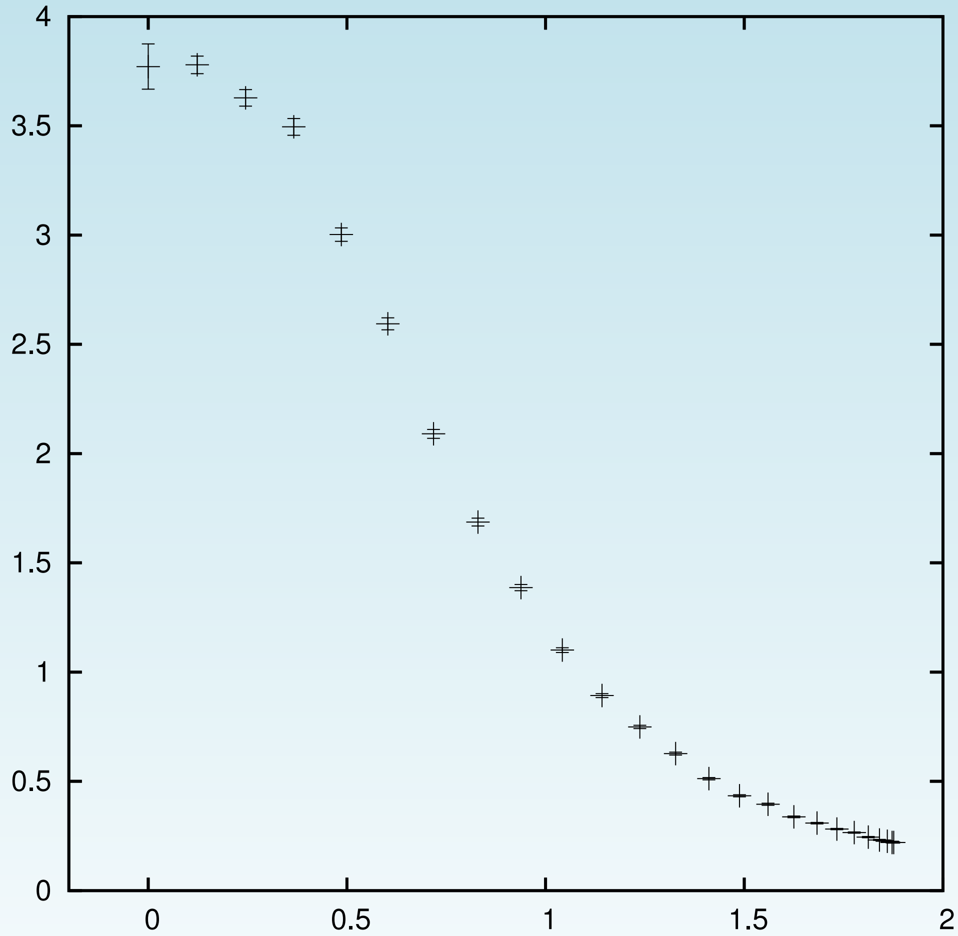
Gluon propagator as a function of the lattice momentum p including lattices of up to 320^3 in the scaling region. (A. Cucchieri, T. M., 2007) About 5 days on a 4.5Tflops IBM supercomputer.

New data: infinite-volume limit in 3d



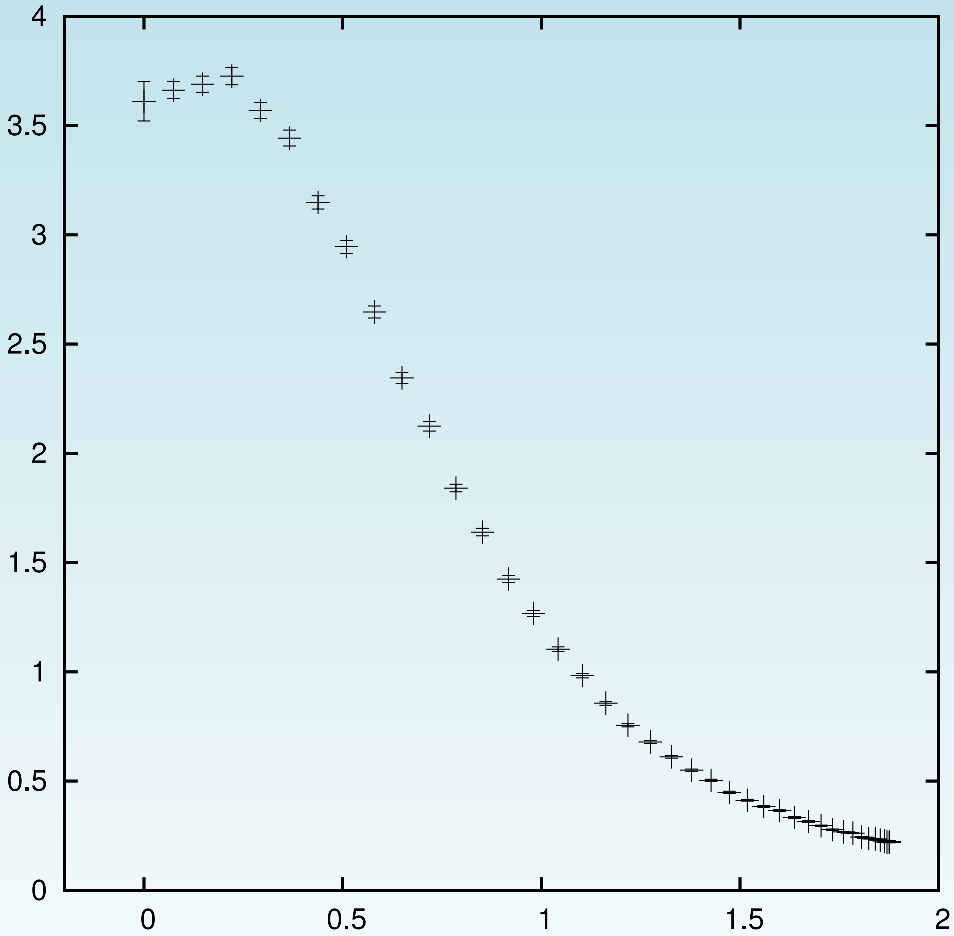
Gluon propagator at zero momentum as a function of the inverse lattice side $1/L$ (in fm^{-1}) and extrapolation to infinite volume. New data, up to 320^3 for $\beta = 3.0$.

Infinite-volume limit in 4d (I)



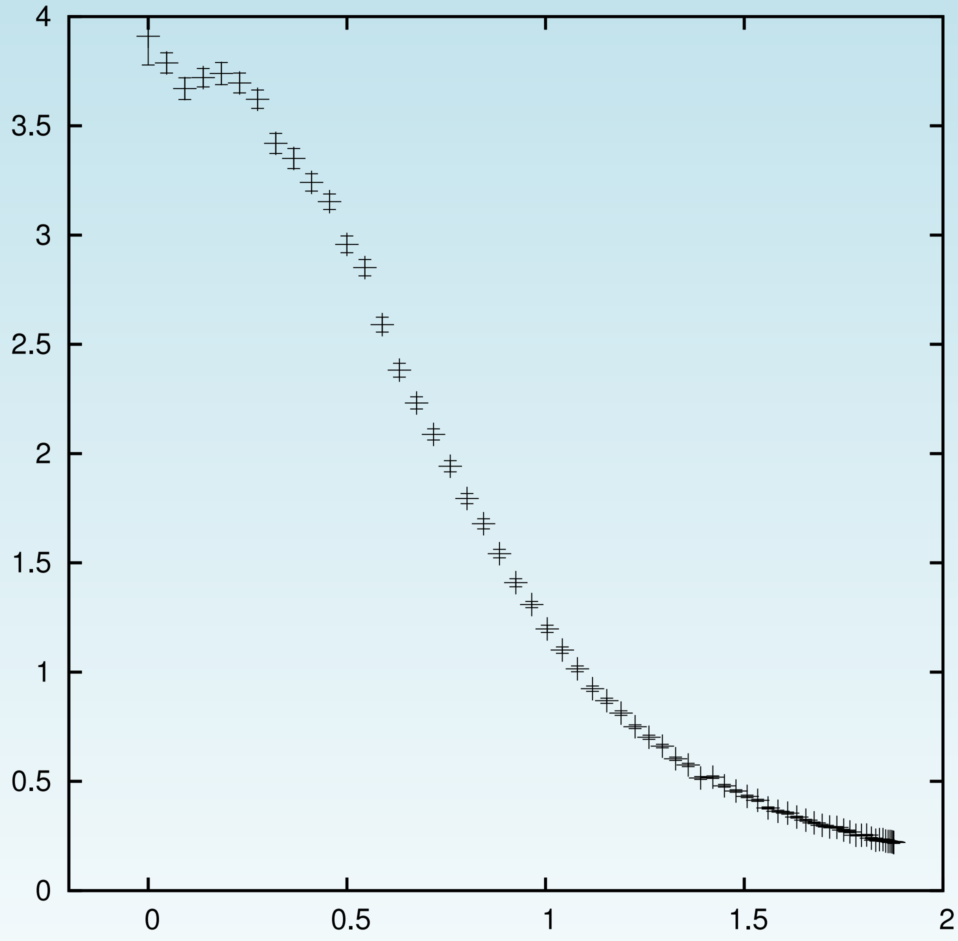
Gluon propagator as a function of the lattice momentum p for lattice volume $V = 48^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d (II)



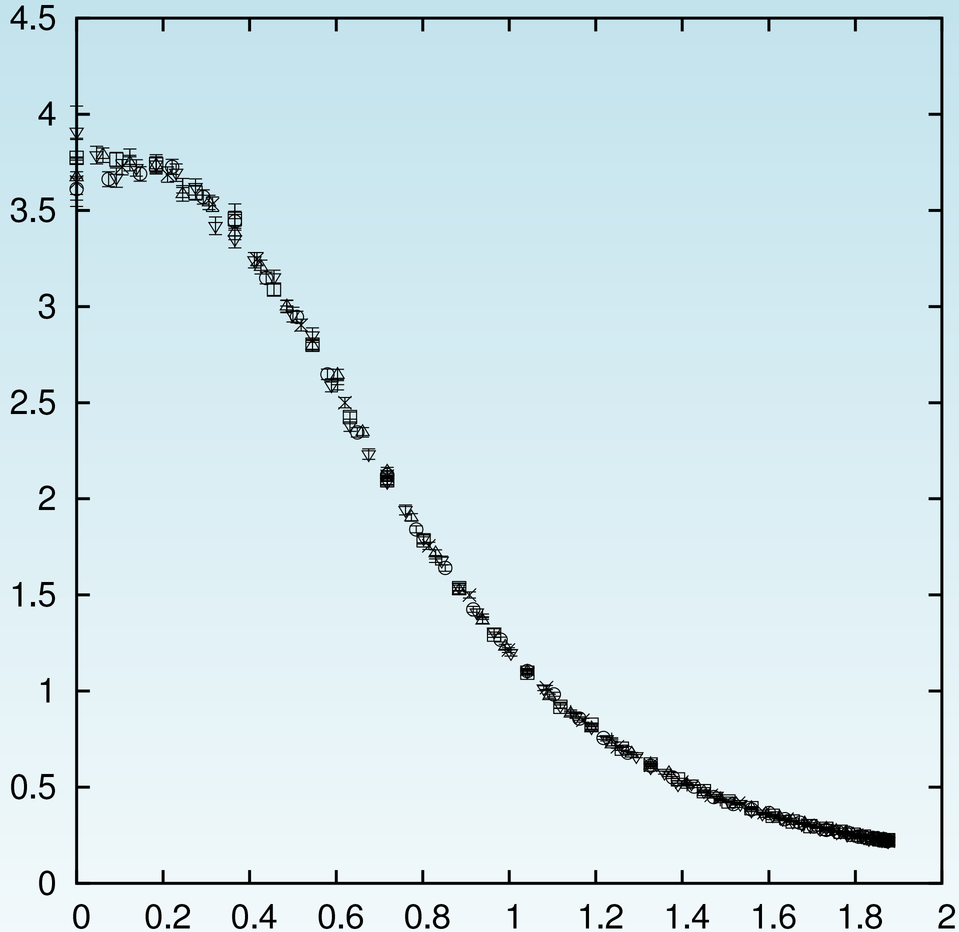
Gluon propagator as a function of the lattice momentum p for lattice volume $V = 80^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d (III)



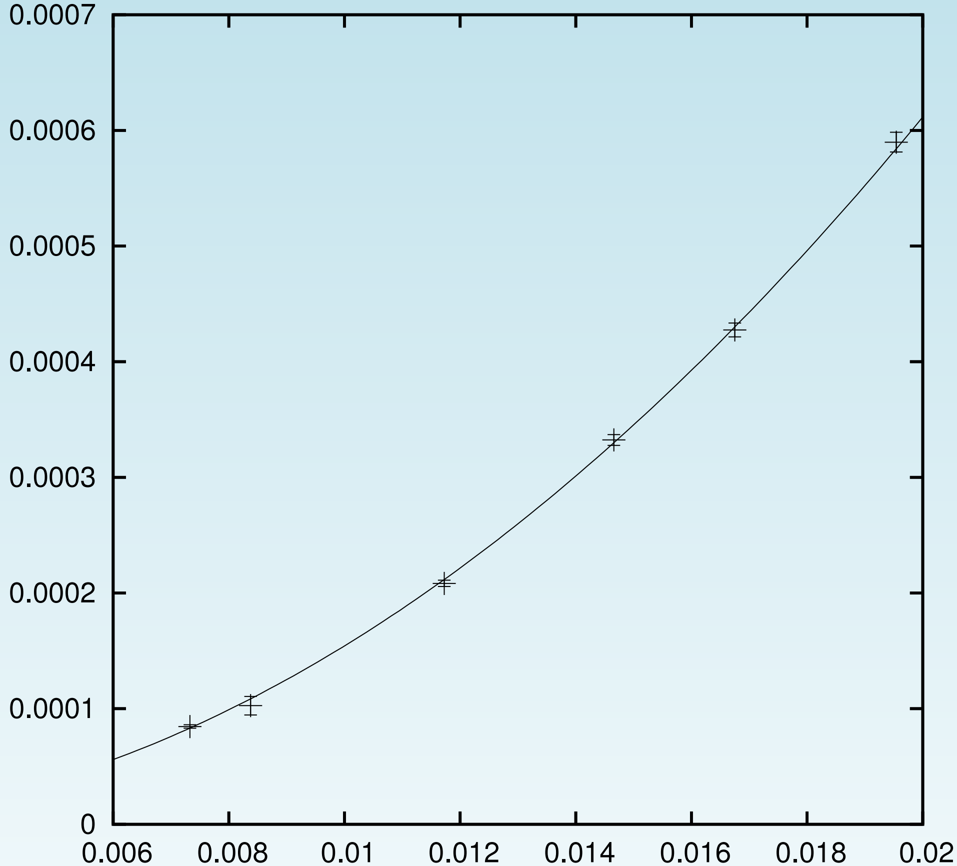
Gluon propagator as a function of the lattice momentum p for lattice volume $V = 128^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d (IV)



Gluon propagator as a function of the lattice momentum p for lattice volume up to $V = 128^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d

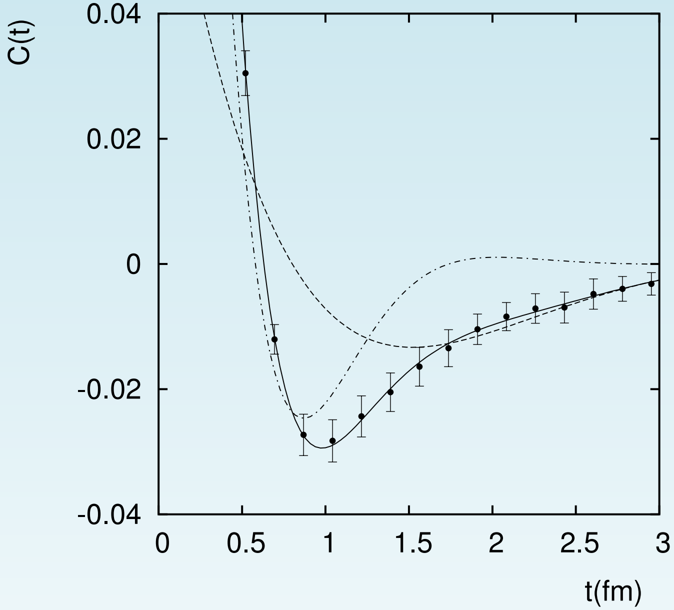


Average absolute value of the gluon field at zero momentum $|A_\mu^b(0)|$ (for $\beta = 2.2$) as a function of the inverse lattice side $1/L$ (in fm^{-1}) and extrapolation to infinite volume. Recall that $D(0) \propto V \sum_{\mu,b} |A_\mu^b(0)|^2$. We also show the fit of the data using the Ansatz b/L^c (with $c = 1.99 \pm 0.02$).

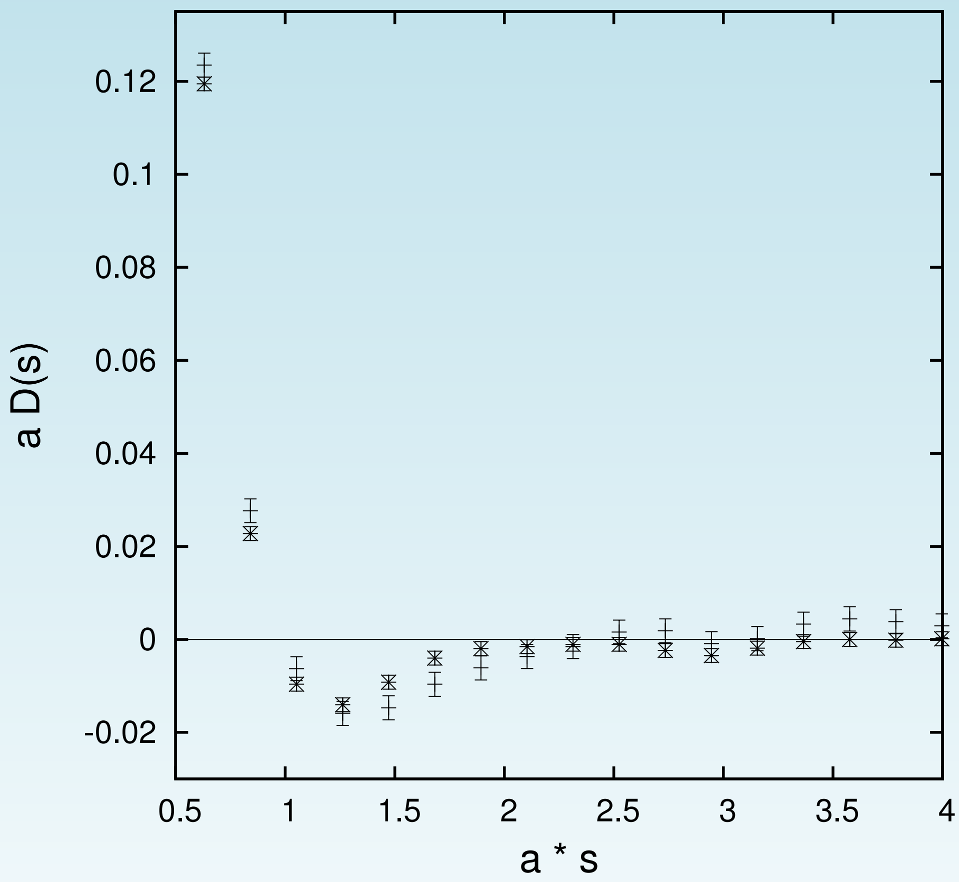
Violation of reflection positivity in 3d

- The transverse gluon propagator decreases in the IR limit for momenta smaller than p_{dec}, which corresponds to the mass scale λ in a Gribov-like propagator $p^2/(p^4 + \lambda^4)$. We can estimate $p_{dec} = 350^{+100}_{-50}$ MeV.

- Clear violation of reflection positivity: this is one of the manifestations of gluon confinement. In the scaling region, the data are well described by a sum of Gribov-like formulas, with a light-mass scale $M_1 \approx 0.74(1)\sqrt{\sigma} = 325(6)\,MeV$ and a second mass scale $M_2 \approx 1.69(1)\sqrt{\sigma} = 745(5)\,MeV$.

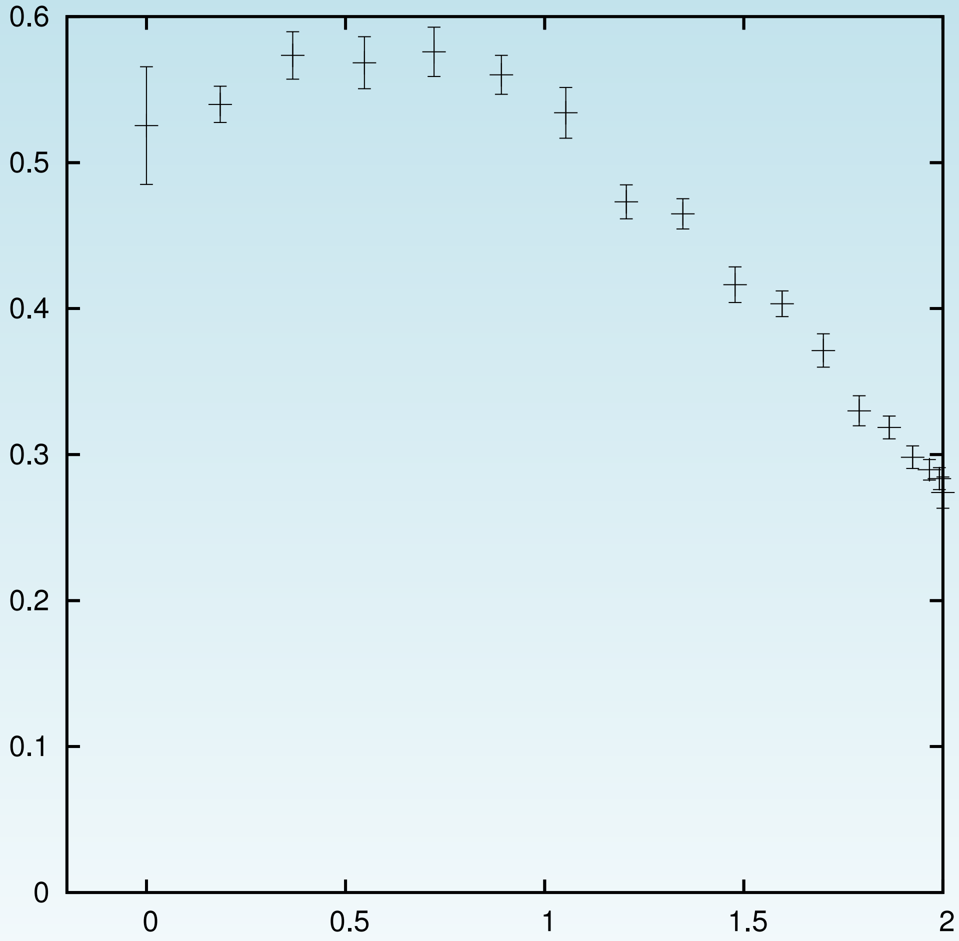


Violation of reflection positivity in 4d



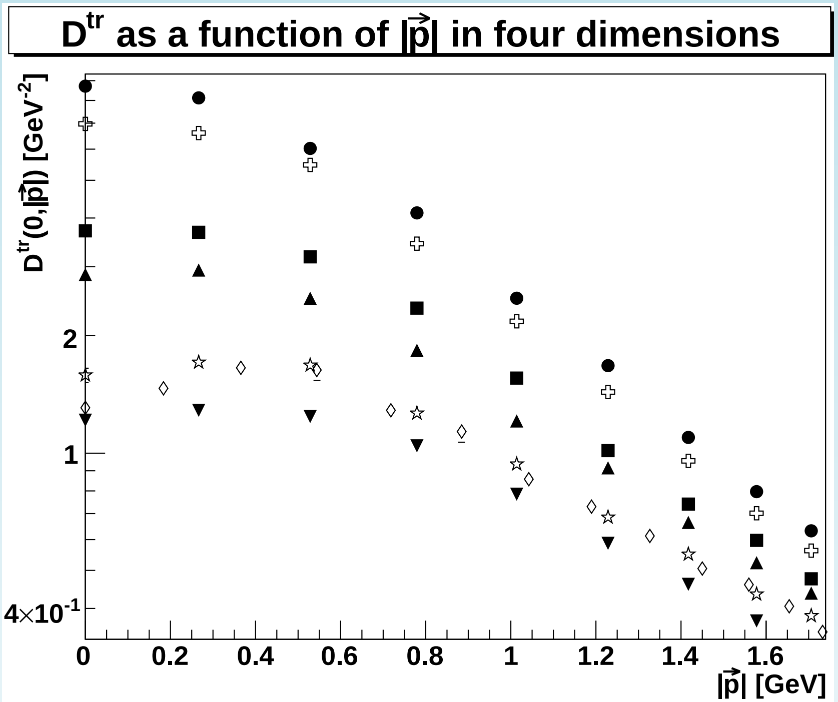
Clear violation of reflection positivity for lattice volume $V = 128^4$ at $\beta = 2.2$.

Small β in 4d



The gluon propagator decreases for small momenta in the strong-coupling regime. Here we consider $V = 34^4$ and $\beta = 1.4$.

Other gauges in 4d



The gluonic correlation function $D^{tr}(0, |\vec{p}|)$ decreases for small momenta in the so-called λ-gauge for small values of λ. Here, diamonds correspond to a $40^4 \approx$ (8.4 fm)^4 lattice at $\lambda = 1/100$.

Questions

- Just considering very large lattice volumes is not enough?
- Gribov-copy effects?
- What about the gluon propagator in strong coupling?
- Partially-wrong scenarios?
- What about Coulomb gauge and the interpolating (λ) gauge?

Lower bound for $D(0)$

We can obtain a lower bound for the gluon propagator at zero momentum $D(0)$ by considering the quantity

$$M(0) \;=\; \frac{1}{d(N_c^2-1)} \sum_{b,\mu} \langle | A_\mu^b(0) | \rangle \;.$$

Consider the Cauchy-Bunyakovski-Schwarz inequality $|\vec{x} \cdot \vec{y}|^2 \leq \|\vec{x}\|^2 \|\vec{y}\|^2$, a vector $\vec{y}$ with all components equal to 1 and a vector $\vec{x}$ with components x_i, we find

$$\left(\frac{1}{m} \sum_{i=1}^{m} x_i \right)^2 \;\leq\; \frac{1}{m} \sum_{i=1}^{m} x_i^2 \;,$$

where m is the number of components of the vectors $\vec{x}$ and $\vec{y}$.

Lower bound for $D(0)$ (II)

We can now apply this inequality first to the vector with $m = d(N_c^2 - 1)$ components $\langle |A_\mu^b(0)| \rangle$, where

$$A_\mu^b(0) \;=\; \frac{1}{V} \sum_x A_\mu^b(x)$$

is the gluon field at zero momentum. This yields

$$M(0)^2 \;\leq\; \frac{1}{d(N_c^2 - 1)} \sum_{b,\mu} \langle |A_\mu^b(0)| \rangle^2 \; .$$

Then, we can apply the same inequality to the Monte Carlo estimate of the average value

$$\langle |A_\mu^b(0)| \rangle \;=\; \frac{1}{n} \sum_c |A_{\mu,c}^b(0)| \; ,$$

where n is the number of configurations. In this case we obtain

$$\langle |A_\mu^b(0)| \rangle^2 \;\leq\; \langle |A_\mu^b(0)|^2 \rangle \; .$$

Lower bound for $D(0)$ (III)

Thus, by recalling that

$$D(0) \; = \; \frac{V}{d(N_c^2 - 1)} \sum_{b,\mu} \langle |A_\mu^b(0)|^2 \rangle \; ,$$

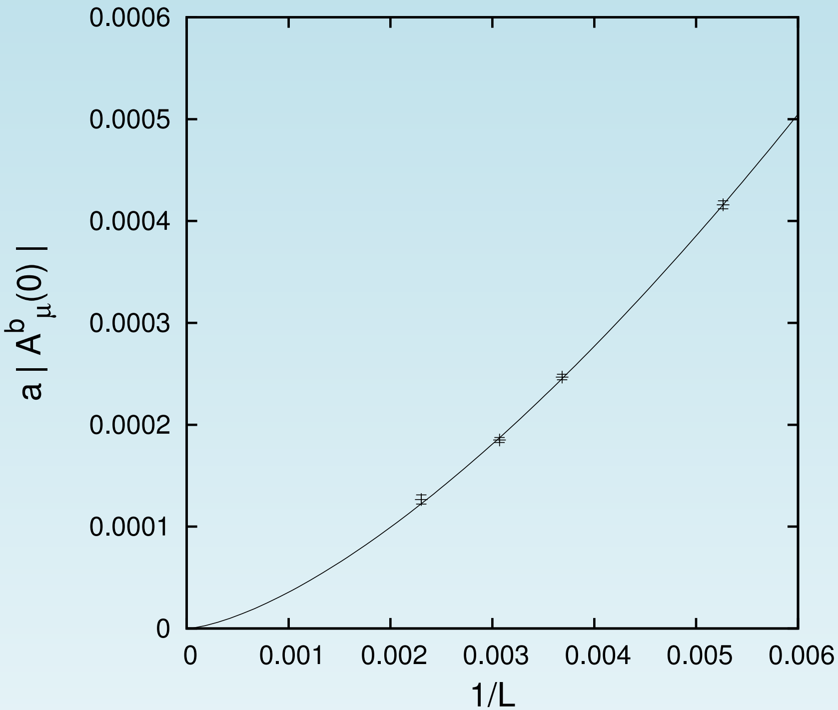
we find

$$\left[V^{1/2} M(0) \right]^2 \; \leq \; D(0) \; .$$

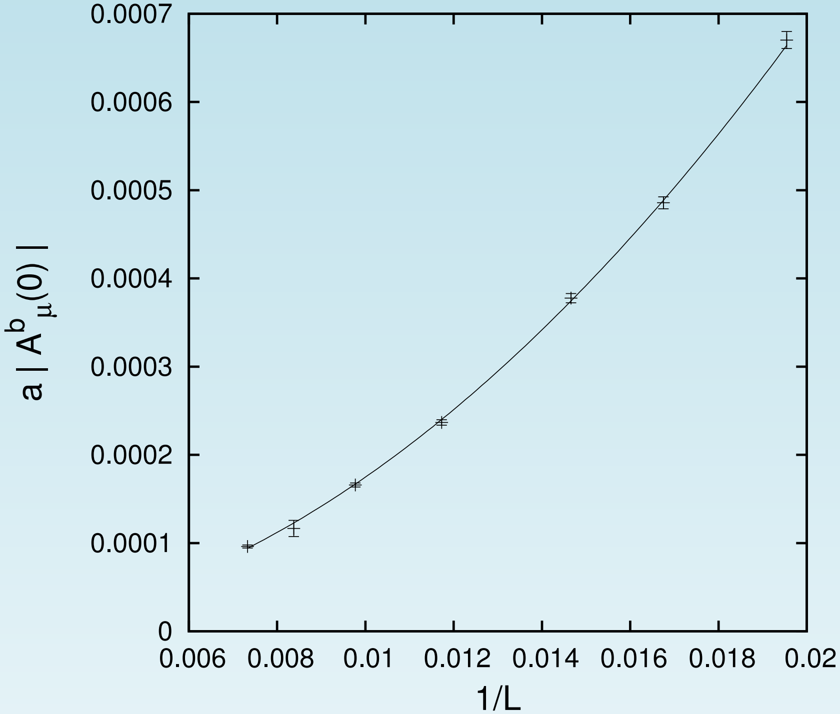
From our fits we obtain that $M(0)$ goes to zero exactly as $1/V^{1/2}$!

This gives $D(0) \geq 0.5(1)$ (GeV^{-2}) in 3d and $D(0) \geq 2.5(3)$ (GeV^{-2}) in 4d.

Lower bound for $D(0)$ (IV)



Fit of $M(0)$ using the Ansatz B/L^c, with $B = 1.0(1)$ (GeV^{-2}), $c = 1.48(3)$ and $\chi/d.o.f. = 1.00$ in 3d.



Fit of $M(0)$ using the Ansatz B/L^c, with $B = 1.7(1)$ (GeV^{-2}), $c = 1.99(2)$ and $\chi/d.o.f. = 0.91$ in 4d.

Upper bound for $D(0)$

We can now consider the inequality

$$\langle \sum_{\mu,b} |A_\mu^b(0)|^2 \rangle \leq \langle \left\{ \sum_{\mu,b} |A_\mu^b(0)| \right\}^2 \rangle .$$

This implies

$$D(0) \leq V d(N_c^2 - 1) \langle M(0)^2 \rangle .$$

Thus

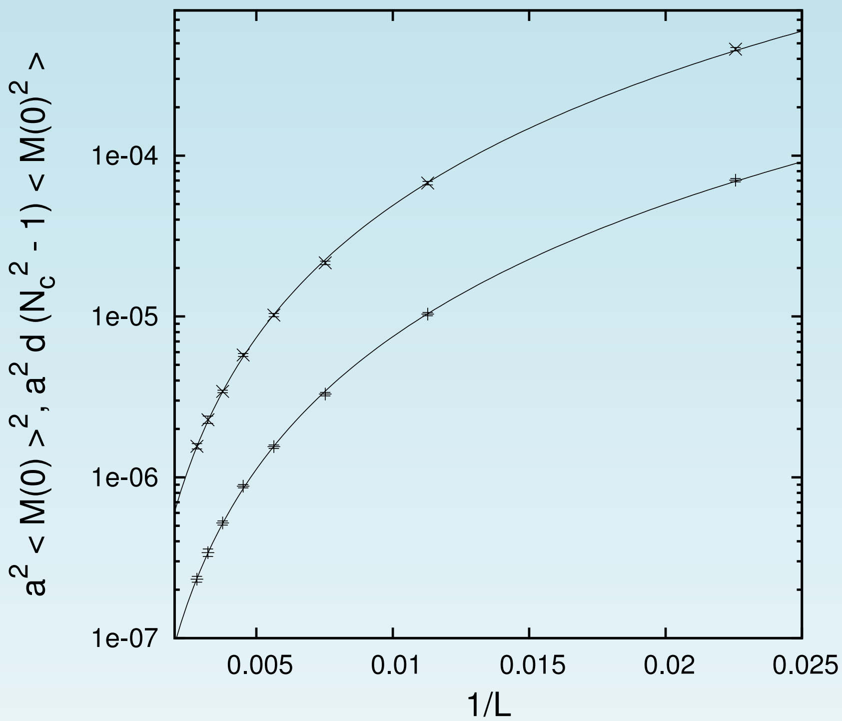
$$V \langle M(0) \rangle^2 \leq D(0) \leq V d(N_c^2 - 1) \langle M(0)^2 \rangle .$$

In summary, if $M(0)$ goes to zero as $V^{-\alpha}$ we find that

$$D(0) \to 0, \quad 0 < D(0) < +\infty \quad \text{or} \quad D(0) \to +\infty$$

respectively if α is larger than, equal to or smaller than 1/2.

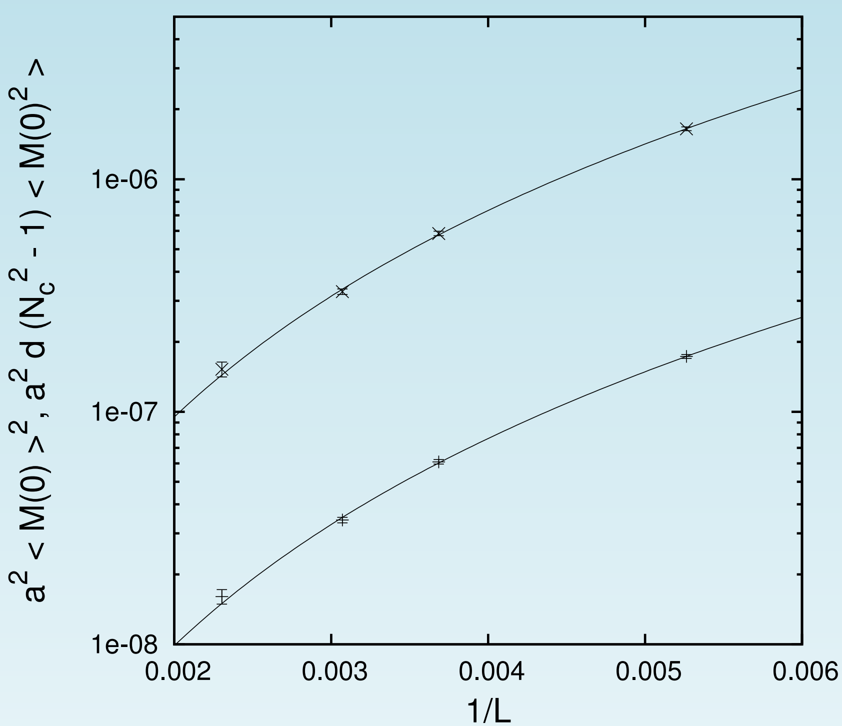
Upper and lower bounds for $D(0)$

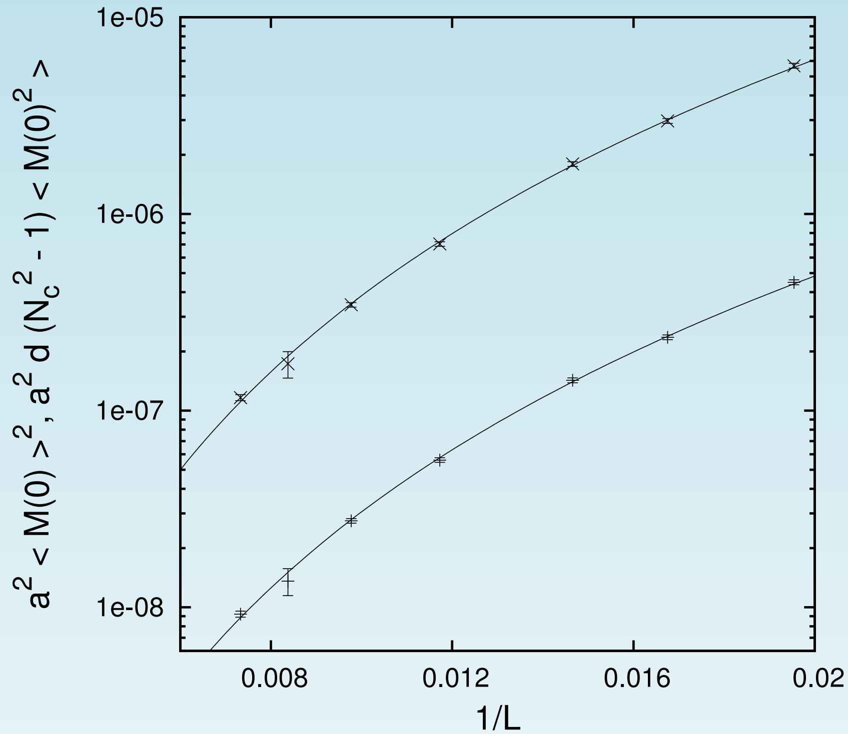


Two-dimensional case: B_l/L^c (for $a\langle M(0)\rangle$) and the Ansatz B_u/L^e (for $a^2\langle\, M(0)^2\,\rangle$), with $B_l = 1.48(6)$, $c = 1.367(8)$ and $\chi/d.o.f. = 1.00$ and $B_u = 2.3(2)$, $e = 2.72(1)$ and $\chi/d.o.f. = 1.02$.

Upper and lower bounds extrapolate to zero, implying $D(0) = 0$.

Upper and lower bounds for $D(0)$ (II)

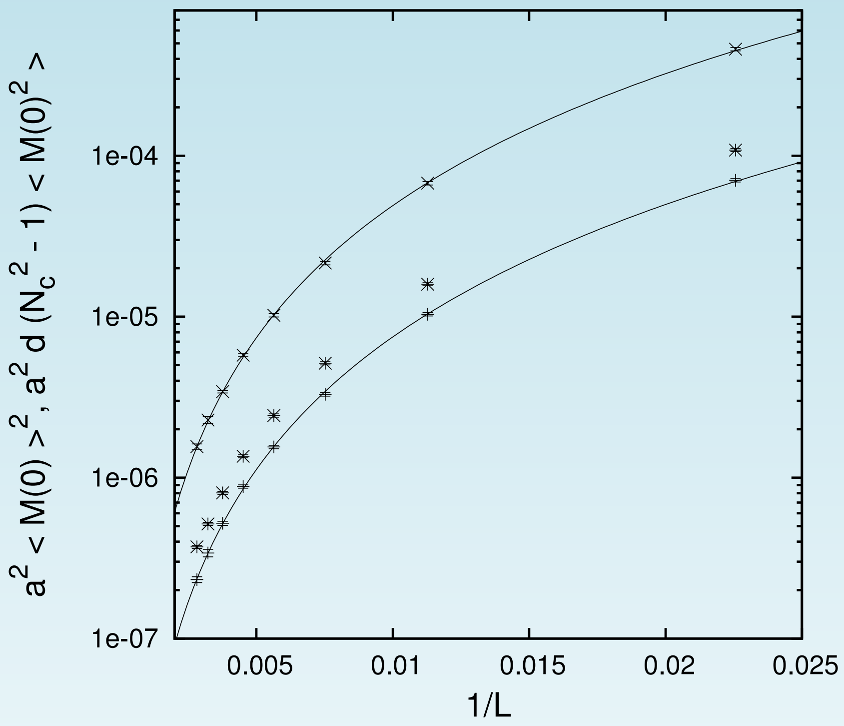




Similarly for 3d: $B_u = 1.0(3)$, $e = 2.95(5)$ and $\chi/d.o.f. = 0.95$.

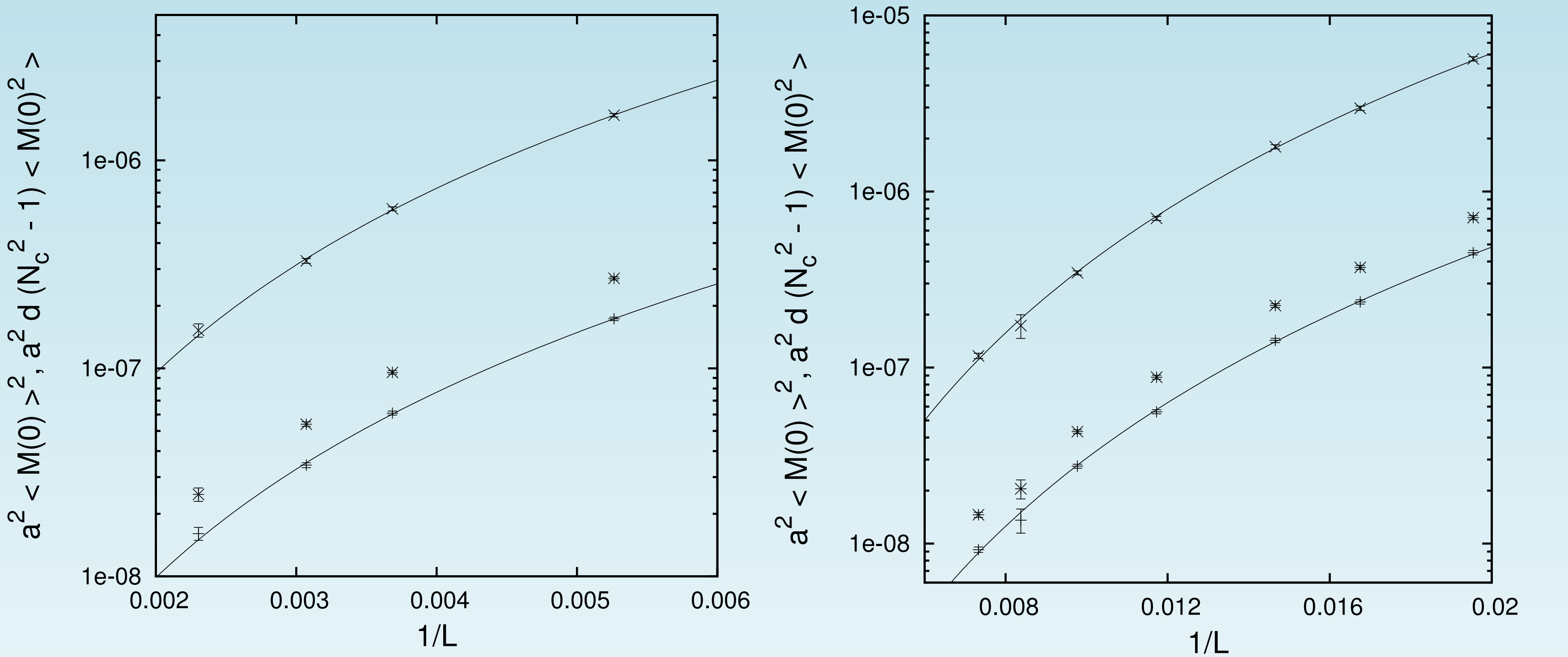
Similarly for 4d: $B_u = 3.1(5)$, $e = 3.99(4)$ and $\chi/d.o.f. = 0.96$.

Upper and lower bounds plus $D(0)/V$



2d case

Upper and lower bounds plus $D(0)/V$ (II)



3d case 4d case

Conclusion

- Gluon propagator in Landau gauge is IR finite in 3d and 4d, as a consequence of “self-averaging” of $M(0)$.
- May think of $D(0)$ as a response function (susceptibility) of the observable $M(0)$ (“magnetization”). In this case it is natural to expect $D(0) \sim const$ in the infinite-volume limit.
- 2d case is different, $M(0)$ is “over self-averaging”, the susceptibility is zero.