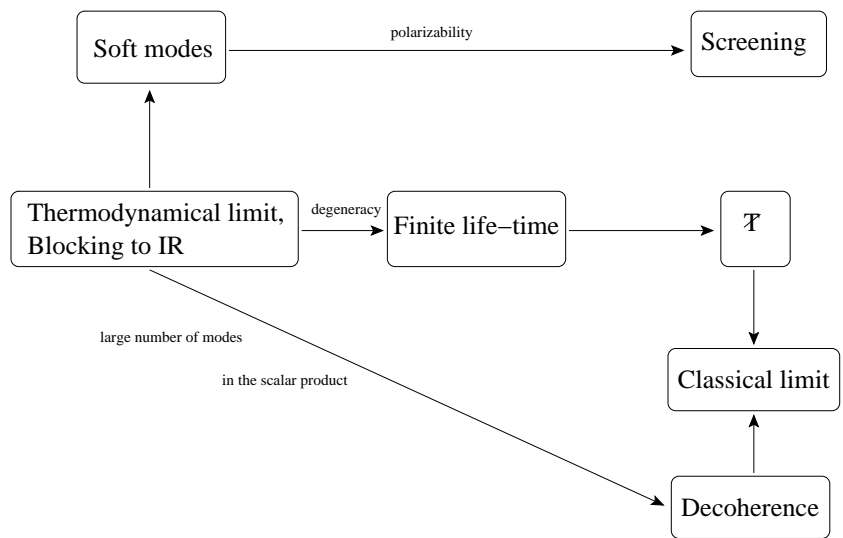


Decoherence of the Coulomb field

Q: How is the classical behaviour established in the Coulomb gas?

A: Decoherence, \mathcal{T} and screening generated at the same scale.



Plan

- | | |
|--|---|
| 1. Schwinger's CTP formalism | A: Expectation values rather than transition amplitudes
B: Radiation field
C: Decoherence
D: \mathcal{T} |
| 2. Effective action for the Coulomb field | Decoherence, \mathcal{T} and screening for low spatial resolution |
| 3. Pointer states: | Strong \mathcal{T} and overlap with a large number of states
Coulomb field: Zero-sound collective modes |

1. Schwinger's closed time path (CTP) method

A: Dynamics of expectation values of local observables

Schwinger: Expectation values in the Heisenberg representation

$$\langle \psi(t) | A | \psi(t) \rangle = \langle \psi_i | e^{i(t-t_i)H} A e^{-i(t-t_i)H} | \psi_i \rangle = \text{Tr} \left[A \underbrace{e^{-i(t-t_i)H} \overbrace{|\psi_i\rangle\langle\psi_i|}^{\rho_i} e^{i(t-t_i)H}}_{\rho(t)} \right]$$

Feynman: Transition amplitudes with operator insertion (expectation value for $|\psi_i\rangle = |0\rangle$ only)

$$\mathcal{A} = \langle \psi_i | e^{-i(t_f-t)H} A e^{-i(t-t_i)H} | \psi_i \rangle = \langle \psi_i | e^{-i(t_f-t_i)H} e^{i(t-t_i)H} A e^{-i(t-t_i)H} | \psi_i \rangle \neq \langle \psi_i | e^{i(t-t_i)H} A e^{-i(t-t_i)H} | \psi_i \rangle$$

Two time axes in the natural formalism of Quantum Mechanics

Q: What is the role of the time axes?

How to find "the" time of Classical Physics?

A: Handle radiations and dissipative dynamics.
Both drive towards classical physics.

Decoherence renders them equivalent.

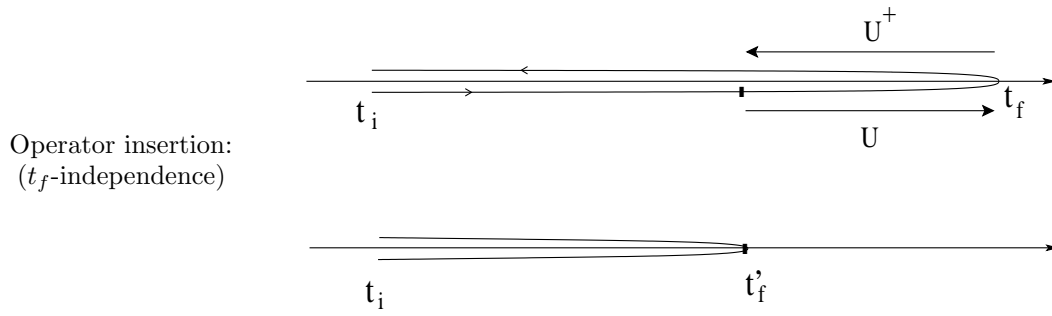
Generating functional: insertions for $\langle \psi(t) | A | \psi(t) \rangle = \text{Tr}[A e^{-i(t-t_i)H} \rho_i e^{i(t-t_i)H}]$:

$$\begin{aligned}
 H &\rightarrow H^\pm(t) = H \mp \sum_a \int d^3x j_a^\pm(t, \mathbf{x}) O_a(t, \mathbf{x}) \\
 e^{iW[j^+, j^-]} &= \text{Tr} T[e^{i \int_{t_i}^{t_f} dt' [-H(t') + \sum_a \int d^3x j_a^+(t, \mathbf{x}) O_a(t, \mathbf{x})]}] \rho_i \bar{T}[e^{i \int_{t_i}^{t_f} dt' [H(t') + \sum_a \int d^3x j_a^-(t, \mathbf{x}) O_a(t, \mathbf{x})]}] \\
 &= \text{Tr} T^*[e^{i \int_{t_i}^{t_f} dt' \sum_a j_a^+(t') O_a(t')}] \rho_i e^{i \int_{t_i}^{t_f} dt' \sum_a j_a^-(t') O_a(t')}
 \end{aligned}$$

NB. Time passes back and forth, retarded and advanced effects separated:

$$j^\pm = \frac{1}{2} j^a \pm j^r \implies \begin{aligned} j^r &: \text{physical, retarded propagators,} \\ j^a &: \text{non-physical, advanced propagators and is suppressed by decoherence} \end{aligned}$$

Expectation values:



B: Radiation field

CTP propagator for a relativistic scalar field:

$$e^{iW[\hat{j}]} = \langle 0 | \bar{T} [e^{i \int dt' [H(t') + j_-(t') \phi_-(t')]}] T [e^{-i \int dt' [H(t') - j_+(t') \phi_+(t')]}] | 0 \rangle = \int D[\hat{\phi}] e^{\frac{i}{2} \hat{\phi} \cdot \hat{D}^{-1} \cdot \hat{\phi} + i \hat{j} \cdot \hat{\phi}} = e^{-\frac{i}{2} \hat{j} \cdot \hat{D} \cdot \hat{j}}$$

$$\begin{pmatrix} D & D^{+-} \\ D^{-+} & D^{--} \end{pmatrix}_{xx'} = \begin{pmatrix} -D^n + i\Im D & \frac{1}{2}D^f + i\Im D \\ -\frac{1}{2}D^f + i\Im D & D^n + i\Im D \end{pmatrix}_{xx'} = -i \begin{pmatrix} \langle 0 | T[\phi_x \phi_{x'}] | 0 \rangle & \langle 0 | \phi_{x'} \phi_x | 0 \rangle \\ \langle 0 | \phi_x \phi_{x'} | 0 \rangle & \langle 0 | T[\phi_x \phi_{x'}] | 0 \rangle^* \end{pmatrix}$$

$$\begin{aligned} \text{CTP symmetry :} \quad T[\phi_a \phi_b] + \bar{T}[\phi_a \phi_b] &= \phi_a \phi_b + \phi_b \phi_a \\ D - D^* &= D^{+-} - D^{+-*} \end{aligned}$$

Three real functions: $D^n = D^{n\text{tr}}$ (near field), $D^f = -D^{n\text{tr}}$ (far or radiation field) and $\Im D = \Im D^{\text{tr}}$

Free photons:

$$\begin{aligned}
e^{iW[j^+, j^-]} &= \text{Tr} T[e^{-i \int_{t_i}^{t_f} dt \int d\mathbf{x} [H(x) - A^+(x) j^+(x)]}] \rho_i \bar{T}[e^{i \int_{t_i}^{t_f} dt \int d\mathbf{x} [H(x) + A^-(x) j^-(x)]]] \\
&= \int D[A^+] D[A^-] e^{iS[A^+] + i \int dx j^{+\mu} A_\mu^+ - iS^*[A^-] + i \int dx j^{-\mu} A_\mu^- + iS_{\text{BC}}[A^+, A^-]} \\
&= \int D[\hat{A}] e^{\frac{i}{2} \hat{A} \cdot \hat{D}^{-1} \cdot \hat{A} + i \hat{j} \cdot \hat{A}}
\end{aligned}$$

$$\hat{A} = \begin{pmatrix} A^+ \\ A^- \end{pmatrix}, \quad \hat{j} = \begin{pmatrix} j^+ \\ j^- \end{pmatrix}, \quad \hat{D}^{-1} = \begin{pmatrix} D_0^{-1} & 0 \\ 0 & -D_0^{-1*} \end{pmatrix} + \underbrace{\hat{D}_{\text{BC}}^{-1}}_{t=t_i, t_f}$$

CTP propagator:

$$\begin{aligned}
\hat{D} &= \begin{pmatrix} D^n + i\Im D & -\frac{1}{2}D^f + i\Im D \\ \frac{1}{2}D^f + i\Im D & -D^n + i\Im D \end{pmatrix} \\
\text{casual} \quad D_{xy}^n + i\Im D_{xy} &= \frac{1}{4\pi} \delta((x-y)^2) - \frac{i}{4\pi^2} P \frac{1}{(x-y)^2} \\
\text{retarded} \quad D_{xy}^r &= D_{xy}^n + \frac{1}{2}D_{xy}^f = \frac{1}{2\pi} \Theta(x^0 - y^0) \delta((x-y)^2) \\
\text{advanced} \quad D_{xy}^a &= D_{xy}^n - \frac{1}{2}D_{xy}^f = \frac{1}{2\pi} \Theta(y^0 - x^0) \delta((x-y)^2)
\end{aligned}$$

\nearrow **near-field** \nwarrow **far-field**
 Green functions (Dirac)
 $D^{n \ tr} = D^n$ $D^{f \ tr} = -D^f$

Radiation and coupling of the time axes: reflection from the 'end of time'

C: Decoherence of non-relativistic charges

Decoherence: $\mathcal{H} = \mathcal{H}_{\text{system}} \otimes \mathcal{H}_{\text{environment}}$

$$\begin{aligned} \text{pre measurement : } |\Psi\rangle &= \sum_n c_n |n\rangle \otimes |\phi_0\rangle \rightarrow \sum_n c_n |n\rangle \otimes |\phi_n\rangle \\ \rho^{\text{sys}} &= |\phi_0\rangle \otimes \langle\phi_0| \rightarrow \sum_{nn'} c_n^* c_{n'} |n\rangle \otimes \langle n'| \langle\phi_{n'}|\phi_n\rangle \approx \sum_n |c_n|^2 |n\rangle \otimes \langle n| \end{aligned}$$

1. Suppression of the off-diagonal (in the pointer base $|n\rangle$) matrix elements by $\langle\phi_{n'}|\phi_n\rangle$
2. Quantum probability \implies classical probability
3. Has no direct impact on the 'main issue' of measurement theory, on the choice of the observed result
4. The many world interpretation is avoided by the smallness of $1/\sqrt{N_{\text{Avogadro}}}$

D: Time-reversal invariance

(\mathcal{T} to stabilise the records of observation)

Classical physics: Setup of an observation: given initial state, tailored coordinate system/base
 Complexity of the dynamics (ergodicity, mixing) \implies information is diffused and
 can not be regained due to the
 finite resolution/resources
 (as ink in paper)
 Irreversibility results from our accepting that there are things that we can not do (A. Peres)

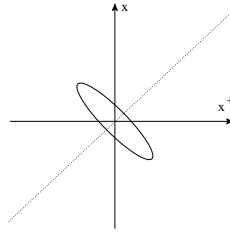
Quantum physics: Finite life-time, $|\gamma\rangle \rightarrow |\gamma\rangle|\gamma, e^+e^-\rangle + |\gamma, e^+e^-, e^+e^-\rangle + \dots$ and mixed states
 "Mixing" in the Fock-space (increased sensitivity to perturbations)

CTP: $\Im S_{\text{eff}}$ has double role: $\Im S_{\text{eff}} > 0 \implies$ Finite life-time (\mathcal{T}) and decoherence

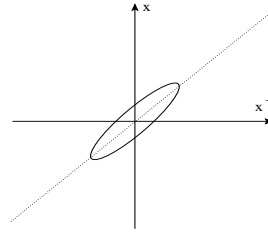
$$\rho\left(x + \frac{\Delta x}{2}, x - \frac{\Delta x}{2}\right) = N e^{-\Delta x^2 b + i x \Delta x a} \implies S_{\text{eff}} = i \Delta x^2 b + x \Delta x a$$

Classicality:

Ellipses: equal density-matrix curves of a peak at the origin



Quantum



Classical

$$\rho\left(x + \frac{\Delta x}{2}, x - \frac{\Delta x}{2}\right) = N e^{-\Delta x^2 b + i x \Delta x a}, \quad \overline{\Delta x} = \frac{1}{\sqrt{b}}, \quad \bar{x} = \frac{1}{\Delta x a}$$

Ratio of the off diagonal and quantum fluctuations:

$$\frac{1}{C} = \frac{\overline{\Delta x}}{\bar{x}} = \frac{a}{b}$$

Strongly coupled quantum-classical crossover:

$$\begin{aligned}
Z &= \int D[\hat{A}] D[\hat{\mathbf{x}}] e^{-\frac{i}{2} \hat{A} \cdot \hat{D}_0^{-1} \cdot \hat{A} + i S_c[\mathbf{x}^+] - i S_c^*[\mathbf{x}^-] - i e j[\mathbf{x}^+] \cdot A^+ + i e j[\mathbf{x}^-] \cdot A^-} \\
&= \int D[\hat{\mathbf{x}}] e^{i S_c[\mathbf{x}^+] - i S_c^*[\mathbf{x}^-] + i W[-e j[\mathbf{x}^+], e j[\mathbf{x}^-]]} \\
\text{decoherence : } \mathbf{x}^+ \approx \mathbf{x}^- &\implies \approx \int D[\mathbf{x}] e^{i S_c[\mathbf{x}] - i S_c^*[\mathbf{x}] + i S_1[e j[\mathbf{x}]]} \\
&= \int D[\mathbf{x}] e^{i S_1[e j[\mathbf{x}]]} \\
&\approx \int D[\mathbf{x}]
\end{aligned}$$

Transition amplitude between pure states (Feynman): \implies rigid trajectories

Expectation values (Schwinger): \implies soft trajectories (degeneracy), strong coupling

2. Effective theory for the Coulomb field

$$\begin{aligned}
Z &= \int D[\hat{u}] D[\hat{\psi}] D[\hat{\psi}] e^{i\hat{\psi} \cdot [\hat{G}^{-1} - e\hat{\sigma}\hat{u}] \cdot \hat{\psi} + \frac{i}{2}\hat{u} \cdot \hat{D}_0^{-1} \cdot \hat{u} - ie\hat{u} \cdot \hat{n}} \\
&= \int D[\hat{u}] e^{\text{Tr} \ln[\hat{G}^{-1} - e\hat{\sigma}\hat{u}] + \frac{i}{2}\hat{u} \cdot \hat{D}_0^{-1} \cdot \hat{u}} \\
&= \int D[\hat{u}] e^{\text{Tr} \ln \hat{G}^{-1} - \sum_{n=1}^{\infty} \frac{1}{n} \text{Tr}(e\hat{G} \cdot \hat{\sigma}\hat{u})^n + \frac{i}{2}\hat{u} \cdot \hat{D}_0^{-1} \cdot \hat{u}} \\
&= \int D[\hat{u}] e^{\text{Tr} \ln \hat{G}^{-1} - ie\hat{j} \cdot \hat{u} + \frac{i}{2}\hat{u} \cdot \hat{D}^{-1} \cdot \hat{u} + \mathcal{O}(\hat{u}^3)}
\end{aligned}$$

$$\text{Bare action : } S[\hat{u}] = \frac{1}{2}\hat{u} \cdot \hat{D}_0^{-1} \cdot \hat{u}$$

$$\hat{D}_0^{-1} = \begin{pmatrix} D_0^{-1} & 0 \\ 0 & -D_0^{-1*} \end{pmatrix} + \underbrace{\hat{D}_{\text{BC}}^{-1}}_{t=t_f}, \quad D_0^{-1} = -\Delta + i\epsilon$$

$$\text{Effective action : } S_{\text{eff}}[\hat{u}] = \frac{1}{2}\hat{u} \cdot \hat{D}^{-1} \cdot \hat{u} + \mathcal{O}(\hat{u}^3), \quad \hat{D}^{-1} = \hat{D}_0^{-1} - \hat{\Sigma}$$

$$\text{Self energy : } \hat{\Sigma}_{xx'}^{\sigma\sigma'} = -i\sigma\sigma' G_{xx'}^{\sigma\sigma'} G_{x'x}^{\sigma'\sigma}$$

$$\text{Electron propagator : } \hat{G}_{\omega, \mathbf{k}} = \begin{pmatrix} \frac{1}{\omega - \epsilon_{\mathbf{k}} + i\epsilon} + i2\pi\delta(\omega - \epsilon_{\mathbf{k}})n_{\mathbf{k}} & -i2\pi\delta(\omega - \epsilon_{\mathbf{k}})n_{\mathbf{k}} \\ i2\pi\delta(\omega - \epsilon_{\mathbf{k}})(1 - n_{\mathbf{k}}) & \frac{1}{\epsilon_{\mathbf{k}} - \omega + i\epsilon} + i2\pi\delta(\omega - \epsilon_{\mathbf{k}})n_{\mathbf{k}} \end{pmatrix}$$

One-loop integral: quantum fluctuations finite lifetime (particle-hole pairs)

\downarrow \swarrow

$$\hat{D}_{\omega, \mathbf{k}}^{-1} = \begin{pmatrix} \mathbf{q}^2 - L + i\epsilon & 0 \\ 0 & -\mathbf{q}^2 + L + i\epsilon \end{pmatrix} + i \begin{pmatrix} r_{|\omega|, \mathbf{q}} & -2\Theta(-\omega)r_{|\omega|, \mathbf{q}} \\ -2\Theta(\omega)r_{|\omega|, \mathbf{q}} & r_{|\omega|, \mathbf{q}} \end{pmatrix} + \hat{D}_{\text{BC}}^{-1}$$

$$S_e[\hat{u}] = \frac{1}{2} \hat{u} \cdot \hat{D}^{-1} \cdot \hat{u} = \frac{i}{4} v \cdot r \cdot v - u \cdot (\Delta + L) \cdot v, \quad \hat{u} = \begin{pmatrix} u + \frac{v}{2} \\ u - \frac{v}{2} \end{pmatrix}$$

\uparrow
 decoherence

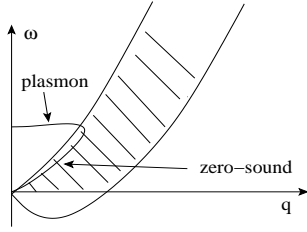
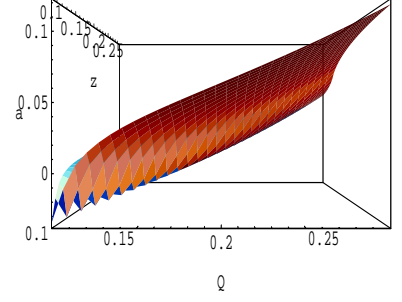
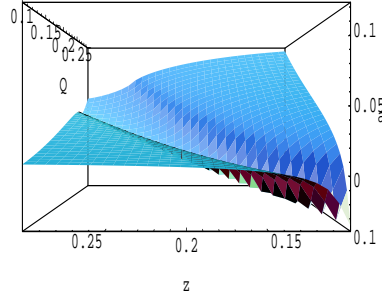
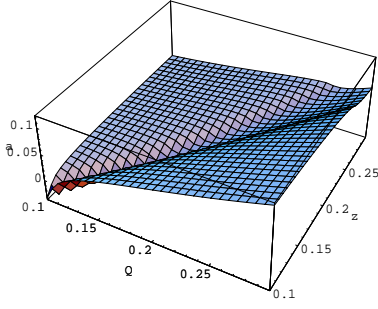
$$\rho(u_1, u_2) = e^{iS_C[u_1, u_2]} \langle \Psi_e[u_1] | \Psi_e[u_2] \rangle$$

Zero-temperature:

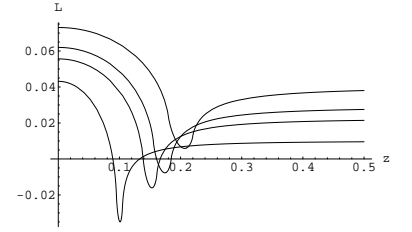
$$L_{\omega, \mathbf{q}} = 2e^2 P \int_{\mathbf{k}} \frac{n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{q}}}{\omega - \epsilon_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}}} \quad (\text{Lindhart}), \quad z = \frac{\omega m}{\hbar k_F^2}, \quad q = \frac{|\mathbf{q}|}{k_F}, \quad r_s = \frac{r}{a_0}, \quad s = \left(\frac{4}{9\pi} \right)^{\frac{1}{3}}$$

$$\mathbf{q}^2 - L = k_F^2 \left[q^2 - \frac{2sr_s}{\pi} \left\{ \frac{1}{2q} \left[1 - \left(\frac{z}{q} - \frac{q}{2} \right)^2 \right] \ln \left| \frac{1 + \frac{z}{q} - \frac{q}{2}}{1 - \frac{z}{q} + \frac{q}{2}} \right| - \frac{1}{2q} \left[1 - \left(\frac{z}{q} + \frac{q}{2} \right)^2 \right] \ln \left| \frac{1 + \frac{z}{q} + \frac{q}{2}}{1 - \frac{z}{q} - \frac{q}{2}} \right| - 1 \right\} \right]$$

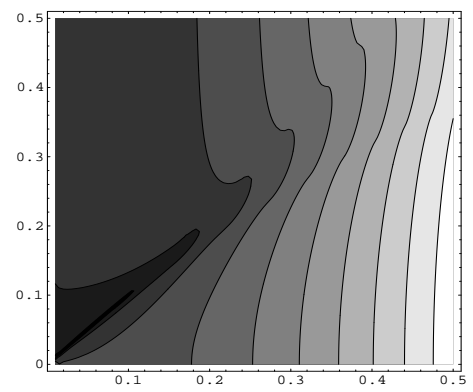
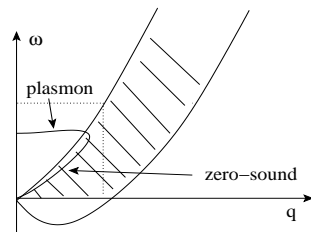
$$q^2 - L = k_F^2 \left[q^2 - \frac{2sr_s}{\pi} \left\{ \frac{1}{2q} \left[1 - \left(\frac{z}{q} - \frac{q}{2} \right)^2 \right] \ln \left| \frac{1 + \frac{z}{q} - \frac{q}{2}}{1 - \frac{z}{q} + \frac{q}{2}} \right| - \frac{1}{2q} \left[1 - \left(\frac{z}{q} + \frac{q}{2} \right)^2 \right] \ln \left| \frac{1 + \frac{z}{q} + \frac{q}{2}}{1 - \frac{z}{q} - \frac{q}{2}} \right| - 1 \right\} \right]$$



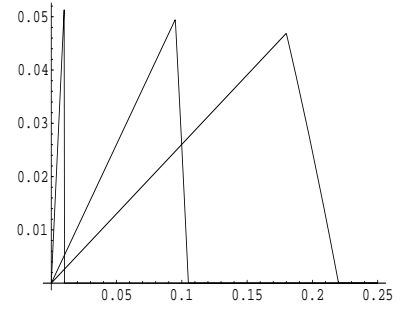
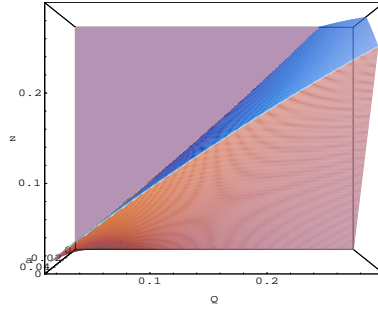
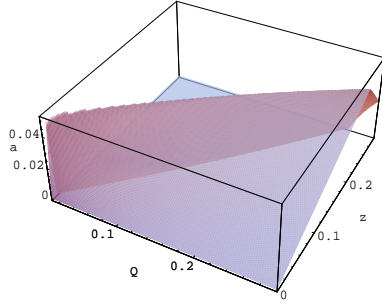
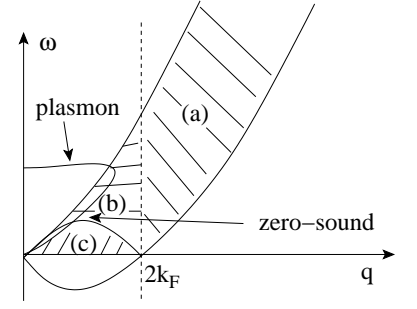
Rightmost end of the zero-sound wal-
ley:
 $q = 0.1, 0.15, 0.17, 0.2$
 $q_{cr} \approx 0.1875$
 $r_s = 0.05$



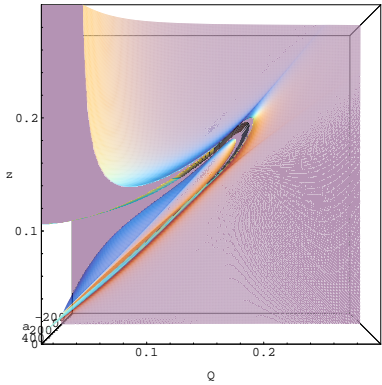
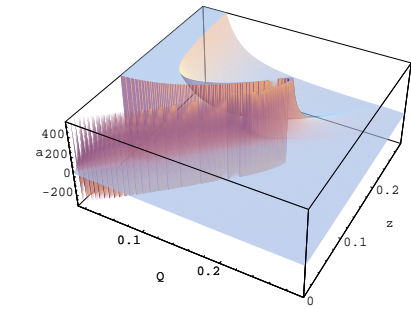
Contour of $q^2 - L_{\omega,q} = 0$:



$$\begin{aligned}
r_{\omega, q} &= 2\pi e^2 \int_{\mathbf{k}} \delta(|\omega| - \epsilon_{\mathbf{k}-\mathbf{q}} + \epsilon_{\mathbf{k}}) n_{\mathbf{k}} (1 - n_{\mathbf{k}-\mathbf{q}}) \\
&= k_F^2 \frac{s r_s}{q} \begin{cases} 1 - \left(\frac{z}{q} - \frac{q}{2}\right)^2 & q > 2, \quad \frac{Q^2}{2} - q < z < \frac{q^2}{2} + q \quad (a) \\ 1 - \left(\frac{z}{q} - \frac{q}{2}\right)^2 & q < 2, \quad q - \frac{q^2}{2} < z < \frac{q^2}{2} + q \quad (b) \\ 2z & q < 2, \quad \frac{q^2}{2} - q < z < q - \frac{q^2}{2} \quad (c) \end{cases}
\end{aligned}$$



$$\frac{1}{q^2-L_{\omega,q}}:$$



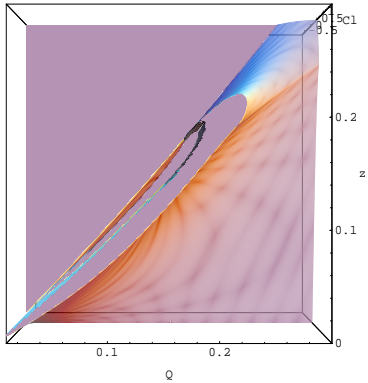
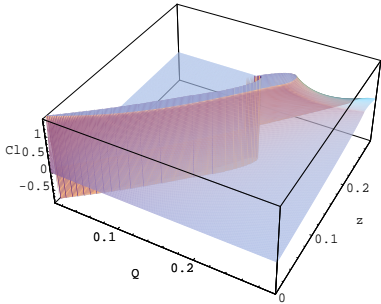
Classicality:

Inverse ratio of the off diagonal and quantum fluctuations:

$$C = \frac{\Im D^{-1}}{\Re D^{-1}} = \frac{r_{\omega,q}}{q^2-L_{\omega,q}}$$

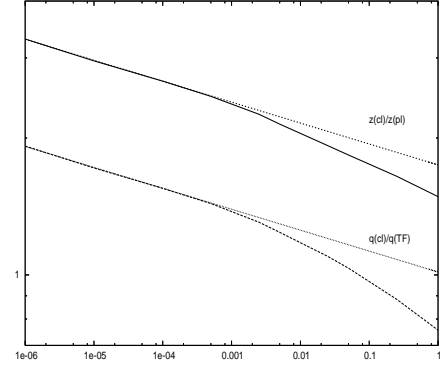
Inverse ratio of time scales for \mathcal{T} :

$$\frac{\text{life-time}}{\text{q. osc. time}} = \frac{\Re D^{-1}}{\Im D^{-1}} = \frac{1}{C}$$



Density dependence

$$\begin{aligned}
 \frac{V}{N} &= \frac{4\pi}{3} r_0^3, \quad r_s = \frac{r_0}{a_0}, \quad k_F = \frac{1.92}{a_0 r_s} \\
 \text{plasmon : } z_{\text{pl}} &= 0.471 \sqrt{r_s}, \quad z = \frac{\omega m}{\hbar k_F^2} \\
 z_{\text{cl}} &= 1.745 \cdot r_s^{-0.046} \cdot z_{\text{pl}} = 0.822 \cdot r_s^{0.495} \\
 \frac{t_{\text{cl}}}{t_{\text{Ry}}} &= \frac{1 \text{ Ry}}{\hbar \omega_{\text{cl}}} = 0.151 \frac{r_s^2}{z_{\text{cl}}} = .184 \cdot r_s^{1.505} \\
 \text{screening : } q_{TF} &= 0.81 \sqrt{r_s} \quad (\text{in units of } k_F) \\
 q_{\text{cl}} &= 1.015 \cdot r_s^{-0.046} \cdot q_{TF} = 0.822 \cdot r_s^{0.495}
 \end{aligned}$$



Generation of pointer states

Pointer: Robust degree of freedom with strong decoherence and \mathcal{T} (to express the result of observations)

Coulomb field with $|\mathbf{q}| < q_{\text{cr}}$ has strong dissipative and decohering strength at some frequency $\omega(\mathbf{q})$.

Classical phenomena: Realized by the pointer states with dynamics given by the poles of the propagators

$$\text{Classicality: } \frac{\Im D^{-1}}{\Re D^{-1}} = \Im D^{-1} / \Re D^{-1}$$

↑
strength of \mathcal{T}

↖
distance from mass shell

Quantum phenomena diffused in an irreversible manner into a large number of channels

Pointer: State with short life-time and strong, wide spread coupling

Summary

1. CTP for measurement theory:

- (a) (reduced) density matrix rather than transition amplitude,
- (b) large number of degrees of freedom,
- (c) dynamical symmetry breaking,
- (d) classical physics:
decoherence $\Longleftrightarrow \mathcal{T} \Longleftrightarrow$ radiation field
- (e) QFT: no state vector is needed

2. Semiclassical Coulomb field:

- (a) decoherence by the small overlap of particle-hole excitations
- (b) zero-sound collective modes with finite life-time
- (c) classical behaviour for low space resolution, approximately at screening length and plasmon frequency

3. Pointer states:

- (a) macroscopic, stable records by \mathcal{T}
- (b) decoherence: classical probabilities
- (c) zero-sound modes collective modes

