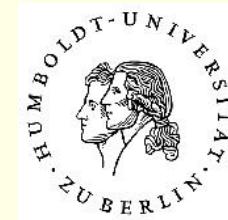


Infrared QCD on the lattice: Landau gauge gluon and ghost propagators

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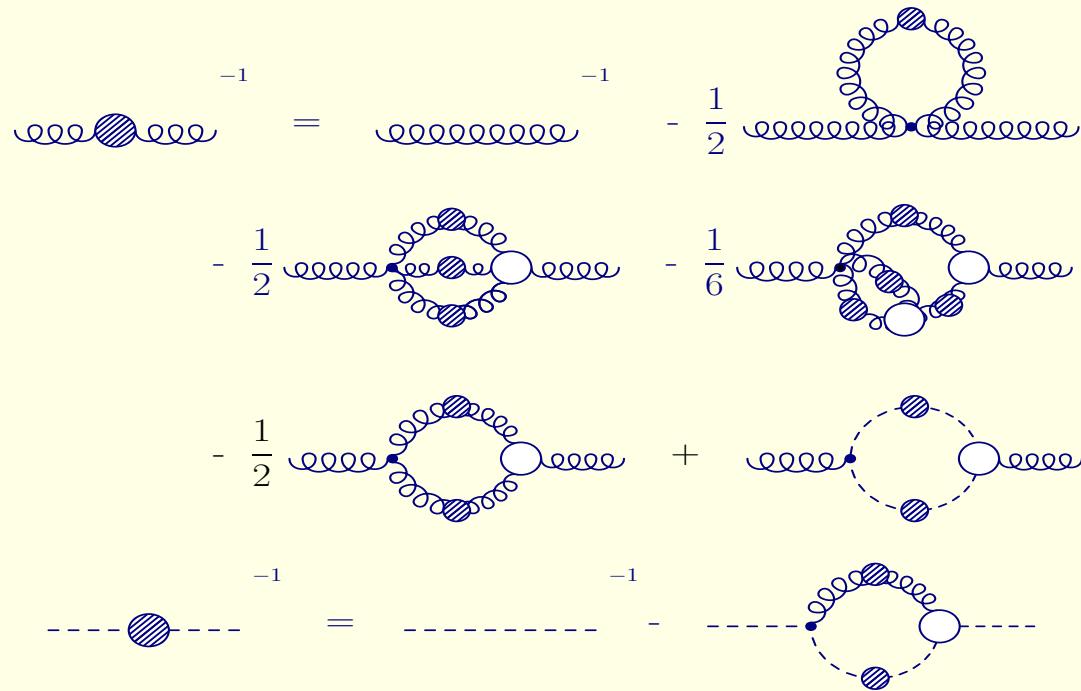
Outline of the talk

1. Introduction, motivation
2. Gluon and ghost propagators:
lattice and DSE results at finite volume
3. The running coupling
4. Gluon and ghost propagators:
recent lattice results
5. Improved gauge fixing: new hope?
6. Conclusion and outlook

1. Introduction, Motivation

Landau gauge gluon and ghost propagators computed from
non-perturbative (truncated) Dyson-Schwinger Equations (DSE)

[Alkofer, Fischer, Maas, Pawłowski, von Smekal, ..., Zwanziger ('97 - '07)]

$$\begin{aligned} \text{Gluon Propagator} &= \text{bare propagator} - \frac{1}{2} \text{loop correction} \\ &\quad - \frac{1}{2} \text{loop correction} - \frac{1}{6} \text{loop correction} \\ &\quad - \frac{1}{2} \text{loop correction} + \text{loop correction} \\ \text{Ghost Propagator} &= \text{bare propagator} - \text{loop correction} \end{aligned}$$


Propagators and Vertex functions = input for hadron phenomenology:
Bethe-Salpeter eqs. for mesons, Faddeev eqs. for baryons.

In the infrared limit $q^2 \rightarrow 0$

DSE provide asymptotic power-like solutions:

$$D_{\mu\nu}^{ab} = \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{Z(q^2)}{q^2}, \quad Z(q^2) \propto (q^2)^{-\kappa_D}$$
$$G^{ab} = \delta^{ab} \frac{J(q^2)}{q^2}, \quad J(q^2) \propto (q^2)^{-\kappa_G}$$

are claimed

- to be unique, when DSE combined with functional renormalization group,
- to hold without any DSE truncation,
- to be independent of the number of colors N_c ,
- to look qualitatively the same in any dimension $d = 2, 3, 4$.

$$\kappa_D = 2 \kappa_G + (4 - d)/2,$$

$$d = 4 : \quad \kappa_G \simeq 0.59 \quad \text{and} \quad \kappa_D = 2 \kappa_G.$$

(Conflicting claims: Boucaud et al. ('05 -'07), Aguilar, Natale ('05-'07))

Running coupling from ghost-ghost-gluon vertex in MOM scheme
assuming $Z_1 \equiv 1$:

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi} Z(q^2) \cdot [J(q^2)]^2$$

$d = 4$: $0 < \alpha_s(q^2) < \infty$, i.e. finite in the infrared $q^2 \rightarrow 0$.

Compare also with analytic perturb. theory
[D.V. Shirkov, I.L. Solovtsov ('97 - '02)].

Infrared power behavior of Z , J in agreement with confinement scenarios:

- Kugo-Ojima confinement criterion [Ojima, Kugo ('78 - '79)]:
absence of colored physical states \iff ghost (gluon)
propagator more (less) singular than simple pole for $q^2 \rightarrow 0$.
- Gribov-Zwanziger confinement scenario
[Gribov ('78), Zwanziger ('91 - ...)]:
gauge fields within the Gribov region

$$\Omega = \left\{ A_\mu(x) : \partial_\mu A_\mu = 0, M_{FP} \equiv -\partial D(A) \geq 0 \right\}$$

are accumulated at the Gribov horizon $\partial\Omega$:

non-trivial eigenvalues of M_{FP} : $\lambda_0 \rightarrow 0$.

\implies Ghost: $G(q^2) \rightarrow \infty$ for $q^2 \rightarrow 0$.
Gluon: $D(q^2) \rightarrow 0$??

Gribov problem:

- Existence of several gauge copies inside Ω .

- What are the right copies?

Restriction inside Ω to fundamental modular region (FMR) required?

$$\Lambda = \left\{ A_\mu(x) : F(A^g) < F(A) \text{ for all } g \neq 1 \right\}.$$

Answer in the limit of infinite volume [Zwanziger ('04)]:

Non-perturbative quantization requires only restriction to Ω ,

$$\text{i.e. } \delta_\Omega(\partial_\mu A_\mu) \det(-\partial_\mu D_\mu^{ab}) e^{-S_{YM}[A]}.$$

Expectation values taken on Ω or Λ should be equal in the thermodynamic limit.

- What happens on a (finite) torus?
- How Gribov copies influence finite-size effects?

Questions to lattice QCD:

- Do propagators show the infrared behavior proposed by DSE ?
- What is the influence of Gribov copies on the propagators?
Large-volume limit ?
- Full QCD versus quenched QCD ?
- Infrared limit of the MOM-scheme running coupling $\alpha_s(q^2)$?
- What lattice QCD can tell about various confinement criteria?
- What about the eigenvalues and eigenmodes of the Faddeev-Popov operator?

$$G(q) = \langle \sum_{i=1}^n \frac{1}{\lambda_i} \vec{\Phi}_i(k) \cdot \vec{\Phi}_i(-k) \rangle$$

2. Gluon and ghost propagators: lattice and DSE results at finite volume

A few technicalities:

- i) Generate lattice discretized gauge fields $U = \{U_{x,\mu} \in SU(N_c)\}$ by MC simulation from path integral

$$Z_{\text{Latt}} = \int \prod_{x,\mu} [dU_{x,\mu}] (\det Q(\kappa, U))^{N_f} \exp(-S_G(U))$$

- standard Wilson plaquette action

$$S_G(U) = \beta \sum_x \sum_{\mu < \nu} \left(1 - \frac{1}{N_c} \operatorname{Re} \operatorname{Tr} U_{x,\mu\nu} \right),$$

$$U_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger, \quad \beta = 2N_c/g_0^2$$

- (clover-improved) Dirac-Wilson fermion operator $Q(\kappa, U)$:
 - $N_f = 0$ – pure gauge case,
 - $N_f = 2$ – full QCD with equal bare quark masses
 - $ma = 1/2\kappa - 1/2\kappa_c$, $a(\beta)$ – lattice spacing.

- ii) Z_{Latt} is simulated with (Hybrid) Monte Carlo method without any gauge fixing.
- iii) Gauge fix each lattice field U :

$$U_{x\mu}^g = g_x \cdot U_{x\mu} \cdot g_{x+\hat{\mu}}^\dagger$$

standard gauge orbits: $\{g_x\}$ periodic on the lattice

Landau gauge: $\mathcal{A}_{x+\hat{\mu}/2,\mu} = \frac{1}{2iag_0} (U_{x\mu} - U_{x\mu}^\dagger)|_{\text{traceless}}$

$$(\partial \mathcal{A})_x = \sum_{\mu=1}^4 (\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}) = 0$$

equivalent to maximizing the gauge functional

$$F_U(g) = \sum_{x,\mu} \frac{1}{N_c} \Re \operatorname{Tr} U_{x\mu}^g = \text{Max.}$$

Maximization: by various iterative techniques possible:
overrelaxation, **simulated annealing**, Fourier acceleration,...

Gribov problem: large number of local maxima of $F_U(g)$.

Practical solution: Initial **random gauges**

- ⇒ best copies (bc) from subsequent maximizations,
- ⇒ compared with first copies (fc)).

iv) Compute propagators

- Gluon propagator:

$$D_{\mu\nu}^{ab}(q) = \left\langle \tilde{A}_\mu^a(k) \tilde{A}_\nu^b(-k) \right\rangle \equiv \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D(q^2)$$

for lattice momenta

$$q_\mu(k_\mu) = \frac{2}{a} \sin \left(\frac{\pi k_\mu}{L_\mu} \right), \quad k_\mu \in (-L_\mu/2, L_\mu/2]$$

- Ghost propagator:

$$G^{ab}(q) = \frac{1}{V^{(4)}} \sum_{x,y} \left\langle e^{-2\pi i k \cdot (x-y)} [M^{-1}]_{xy}^{ab} \right\rangle \equiv \delta^{ab} G(q).$$

$M \sim \partial_\mu D_\mu$ - Landau gauge Faddeev-Popov operator

$$M_{xy}^{ab}(U) = \sum_{\mu} A_{x,\mu}^{ab}(U) \delta_{x,y} - B_{x,\mu}^{ab}(U) \delta_{x+\hat{\mu},y} - C_{x,\mu}^{ab}(U) \delta_{x-\hat{\mu},y}$$

$$\begin{aligned} A_{x,\mu}^{ab} &= \Re \operatorname{Tr} \left[\{T^a, T^b\} (U_{x,\mu} + U_{x-\hat{\mu},\mu}) \right], \\ B_{x,\mu}^{ab} &= 2 \cdot \Re \operatorname{Tr} \left[T^b T^a U_{x,\mu} \right], \\ C_{x,\mu}^{ab} &= 2 \cdot \Re \operatorname{Tr} \left[T^a T^b U_{x-\hat{\mu},\mu} \right], \quad \operatorname{Tr} [T^a T^b] = \delta^{ab}/2. \end{aligned}$$

M^{-1} from solving

$$M_{xy}^{ab} \phi^b(y) = \psi_c^a(x) \equiv \delta^{ac} \exp(2\pi i k \cdot x)$$

with (preconditioned) conjugate gradient algorithm.

Lattice studies of ghost and gluon propagators:

SU(2):

Cucchieri, Maas, Mendes ('96-'07); Gattnar, Langfeld, Reinhardt,... ('02 - '03);
Bloch, Cucchieri, Mendes, Langfeld ('04).

Finite-size and Gribov copy effects:

Bakeev, Ilgenfritz, Mitrjushkin, M.-P., PRD 69, 074507 (2004), hep-lat/0311041;
Bogolubsky, Burgio, Mitrjushkin, M.-P., PRD 74, 034503 (2006), hep-lat/0511056;
Bogolubsky et al., arXiv:0707.3611 [hep-lat], poster contr. LATTICE '07.

SU(3):

Suman, Schilling ('96); Bonnet, Leinweber, Williams,... ('99 - '06);
Furui, Nakajima ('03 - '06); Boucaud et al. ('98-'05); Oliveira, Silva ('05 - '07).

Finite-size and Gribov copy effects:

Sternbeck, Ilgenfritz, M.-P., Schiller, PRD 72, 014507 (2005), hep-lat/0506007;
Sternbeck, Ilgenfritz, M.-P., PRD 73, 014502 (2006), hep-lat/0510109;
Bogolubsky, Ilgenfritz, M.-P., Sternbeck, poster contr. LATTICE '07.

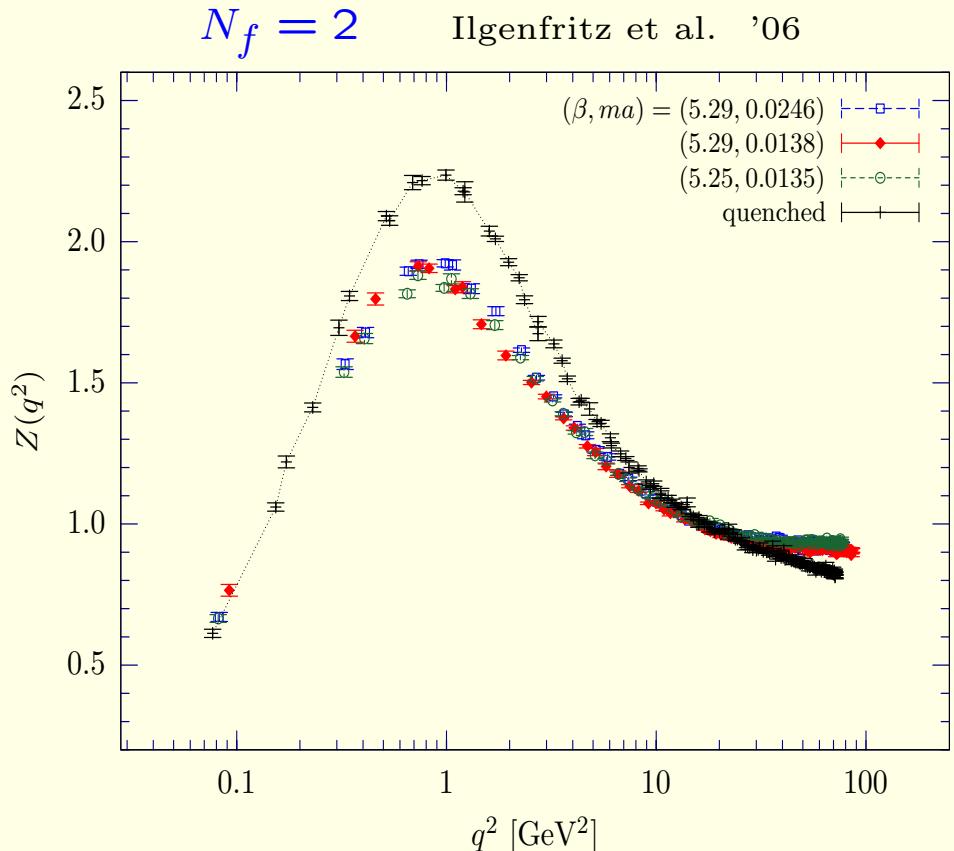
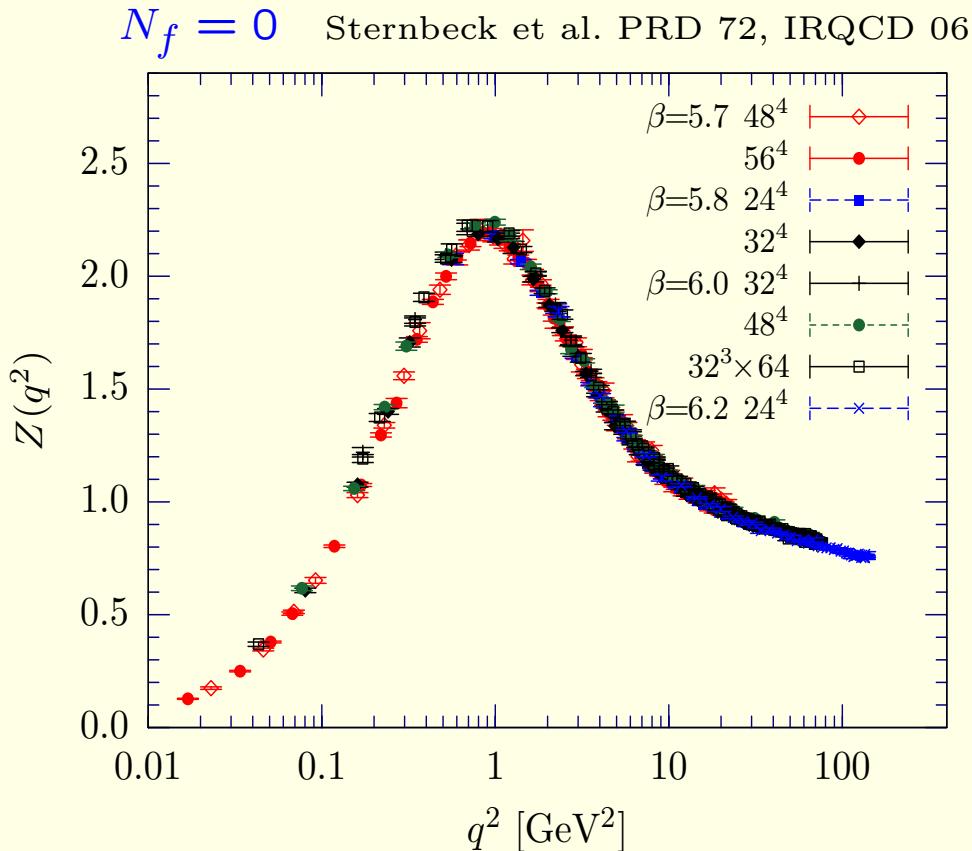
SU(3) study: pure gauge theory versus full QCD

- Pure gauge case $N_f = 0$:
bare coupling $\beta = 5.7, 5.8, 6.0, 6.2$;
lattice sizes $12^4, \dots, 56^4$, and recently $64^4, \dots, 80^4$.
- Full QCD case $N_f = 2$:
thanks: configurations provided by QCDSF - collaboration,
bare coupling $\beta = 5.29, 5.25$; mass parameter
 $\kappa = 0.135, \dots, 0.13575$; lattice size $16^3 \times 32, 24^3 \times 48$.
- Gauge fixing:
start with random gauge copies and apply
standard over-relaxation (OR)
compare first (fc) and best gauge copies (bc)
- Dressing functions:

$$\text{Gluon} \quad Z(q^2) \equiv q^2 D(q^2), \quad \text{Ghost} \quad J(q^2) \equiv q^2 G(q^2)$$

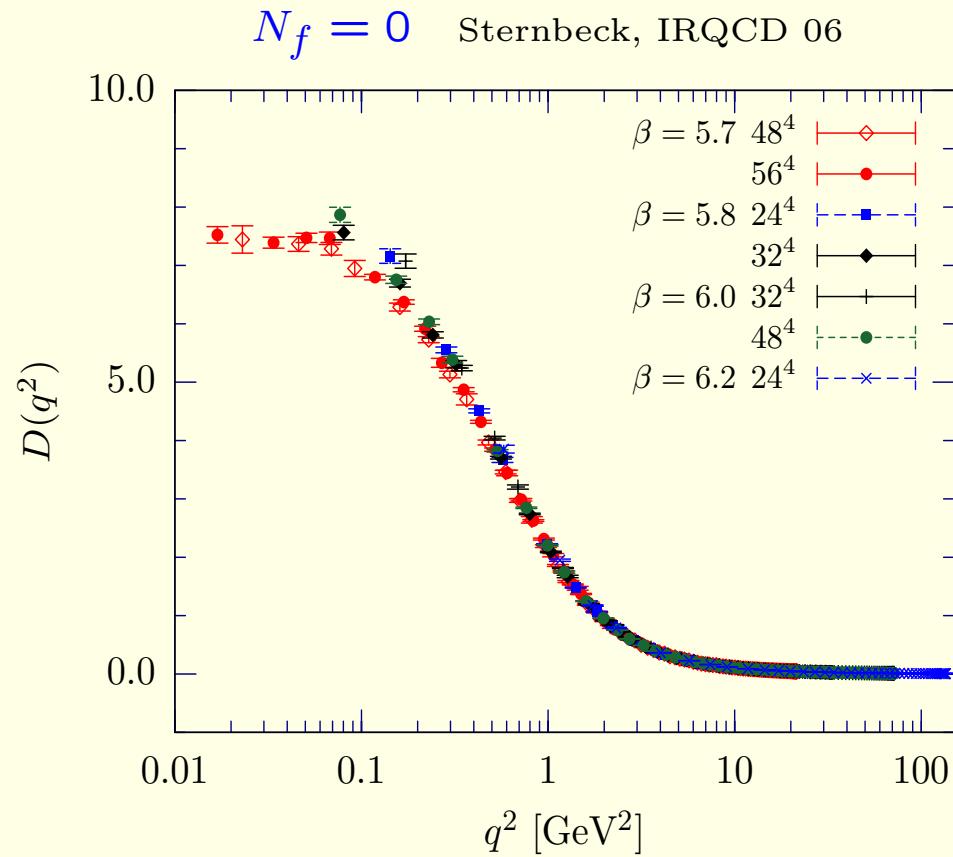
Gluon dressing functions from first copies:

renormalization point: $q = \mu = 4\text{GeV}$



- ⇒ Influence of virtual quark loops clearly visible.
- ⇒ $D(q^2) = Z(q^2)/q^2$ vanishing in the infrared ?

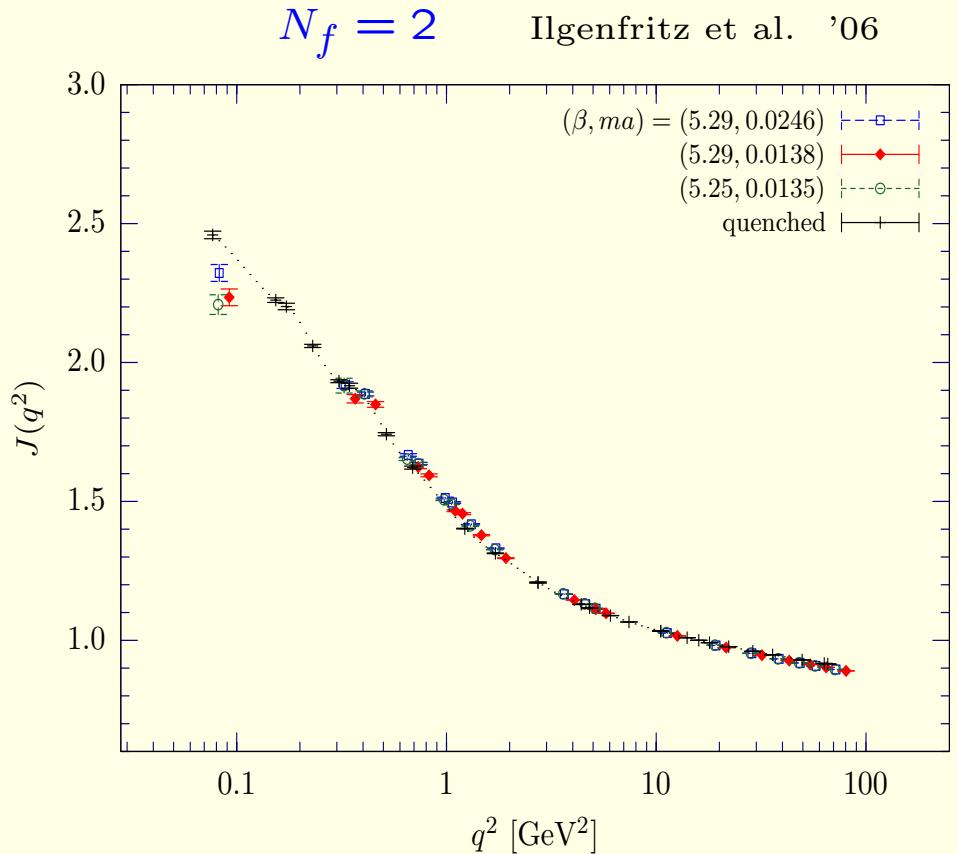
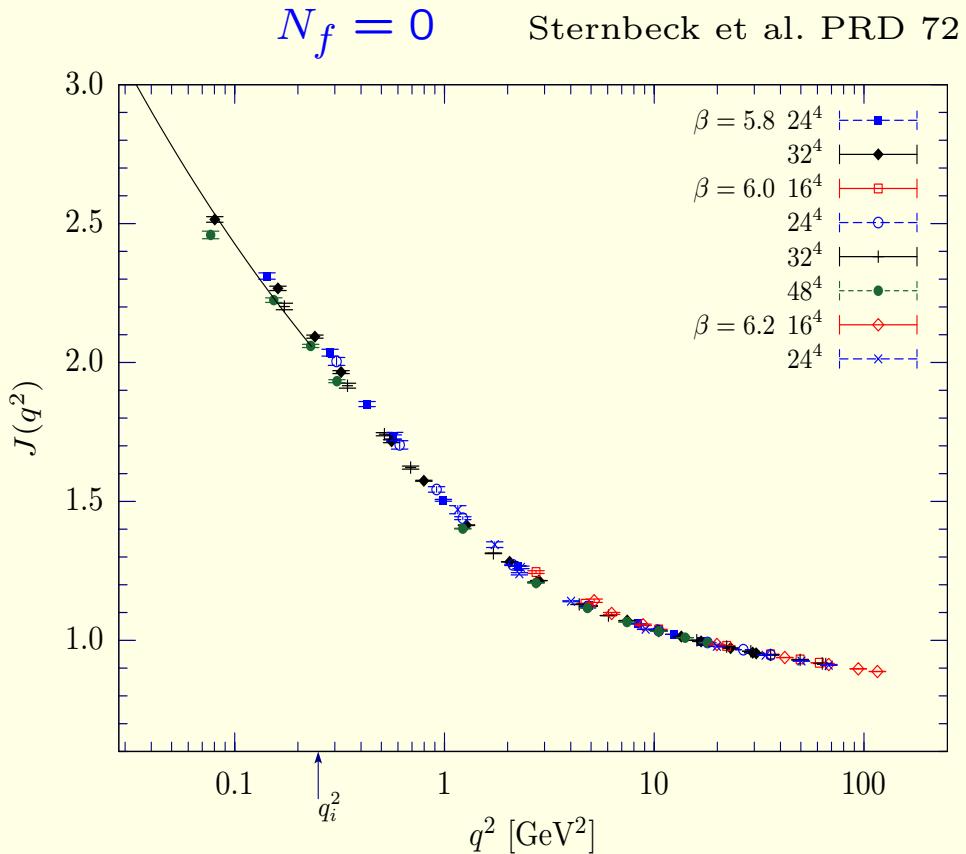
Gluon propagator from first copies:



$\implies D(q^2) = Z(q^2)/q^2$ shows a plateau at small q^2 ??.

Ghost dressing functions from first copies:

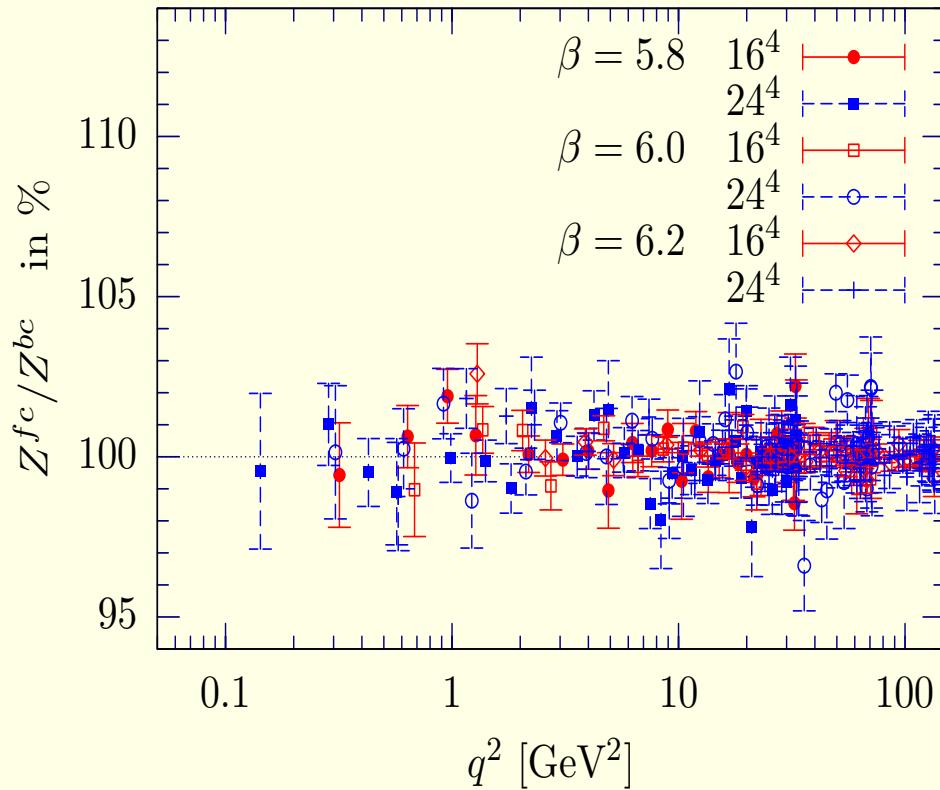
renormalization point: $q = \mu = 4\text{GeV}$



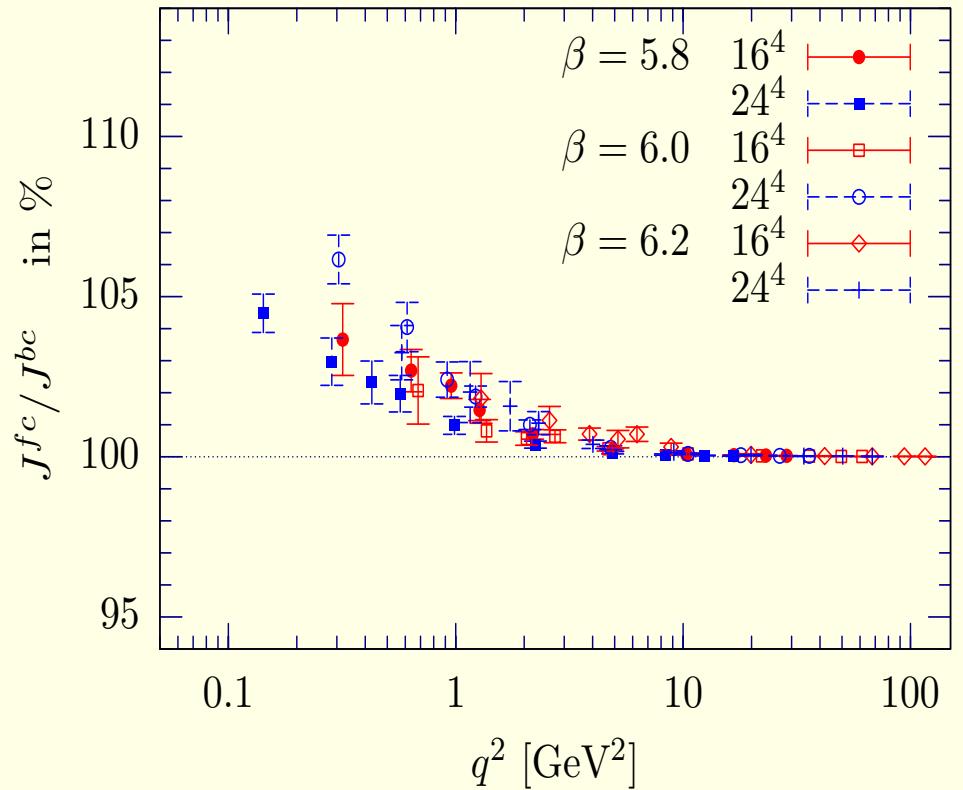
- ⇒ no finite-volume effects?
- ⇒ no quenching effect!
(ghosts do not directly couple to quarks).

Systematic effects: Gribov copies fc / bc - ratios ($N_f = 0$)
 overrelaxation algorithm (OR), gauge transformations strictly periodic on the torus

gluon dressing fct.



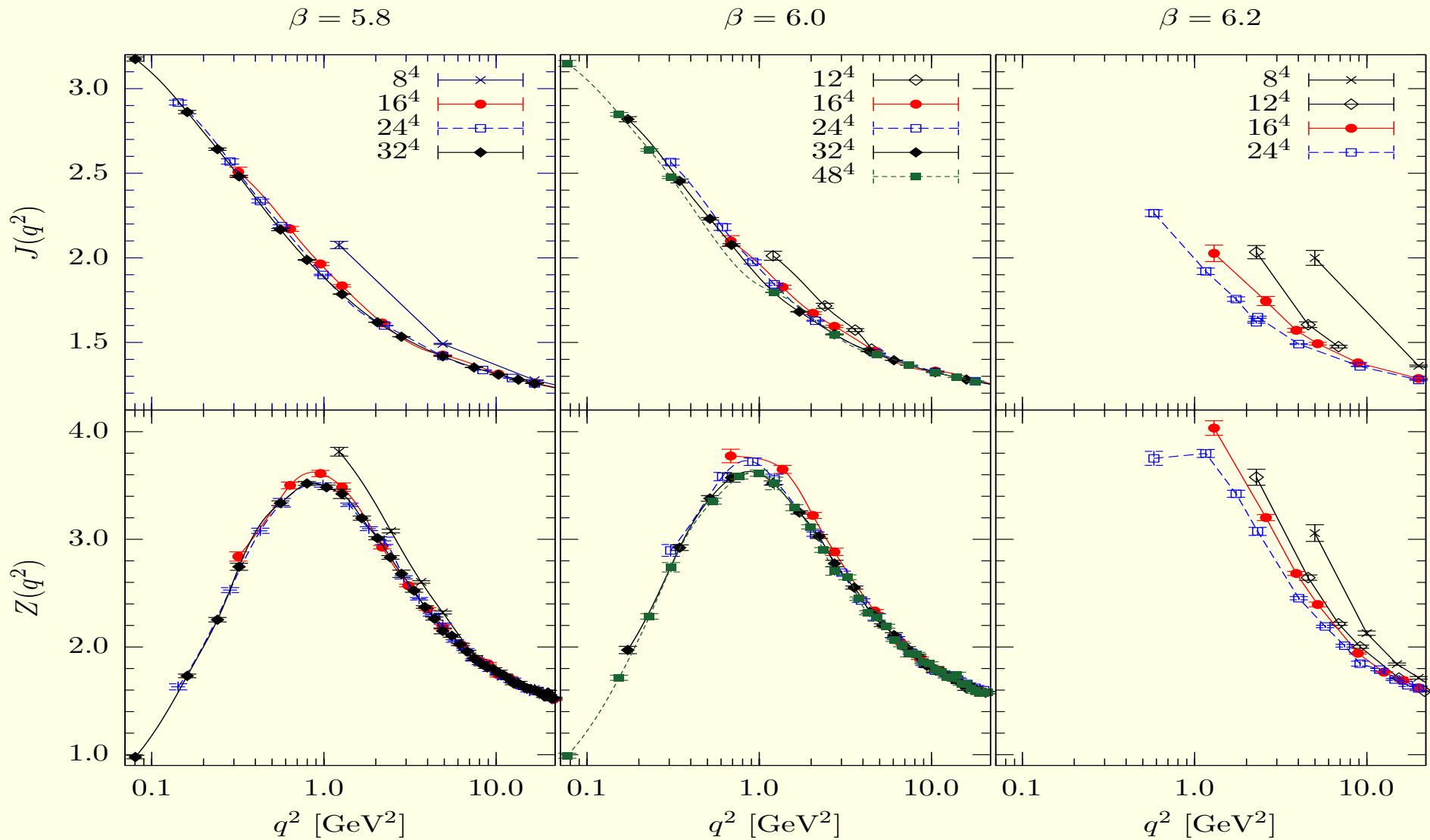
ghost dressing fct.



- ⇒ Gribov problem visible in the infrared for the ghost ($O(5\%)$ effect), still not visible for the gluon propagator,
- ⇒ seems slightly to weaken as the volume increases (in acc. with Zwanziger),
- ⇒ more thorough studies under way for $SU(2)$.

Systematic effects: finite-size dependence ($N_f = 0$)

Sternbeck thesis '06

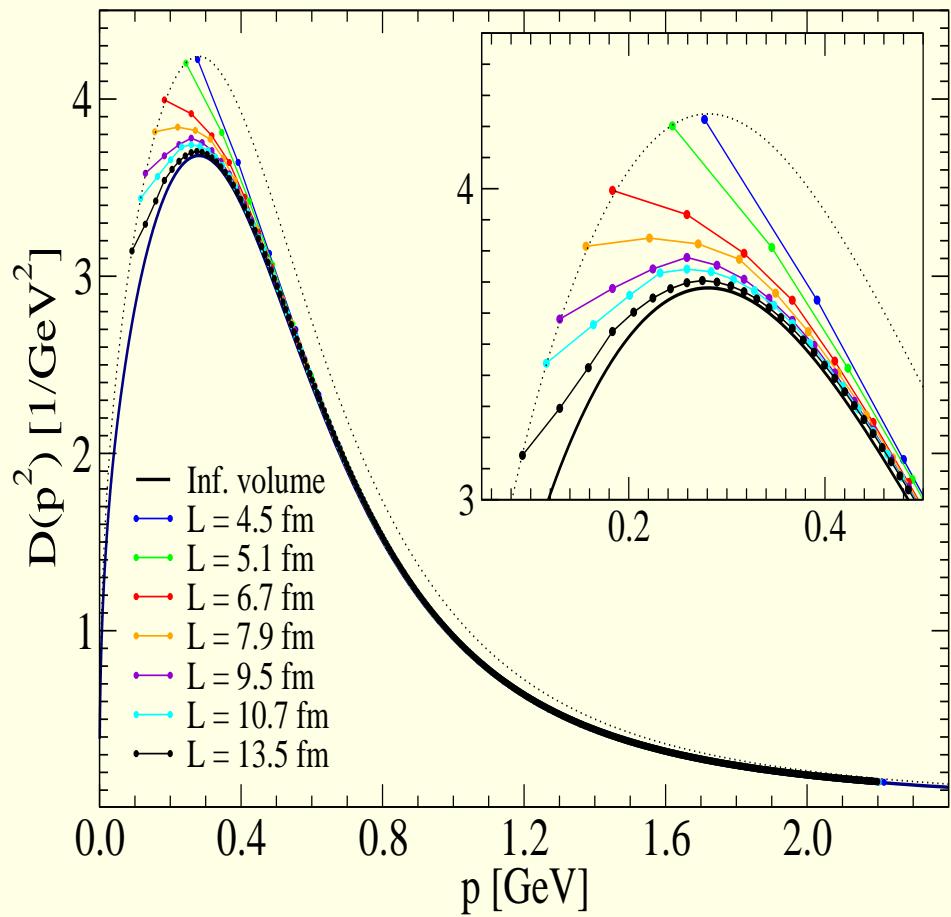


⇒ For $L \geq 16$ $a(\beta = 5.8) \simeq 24$ $a(\beta = 6.0) \simeq 2.2$ fm
finite-size effect hard to resolve ?

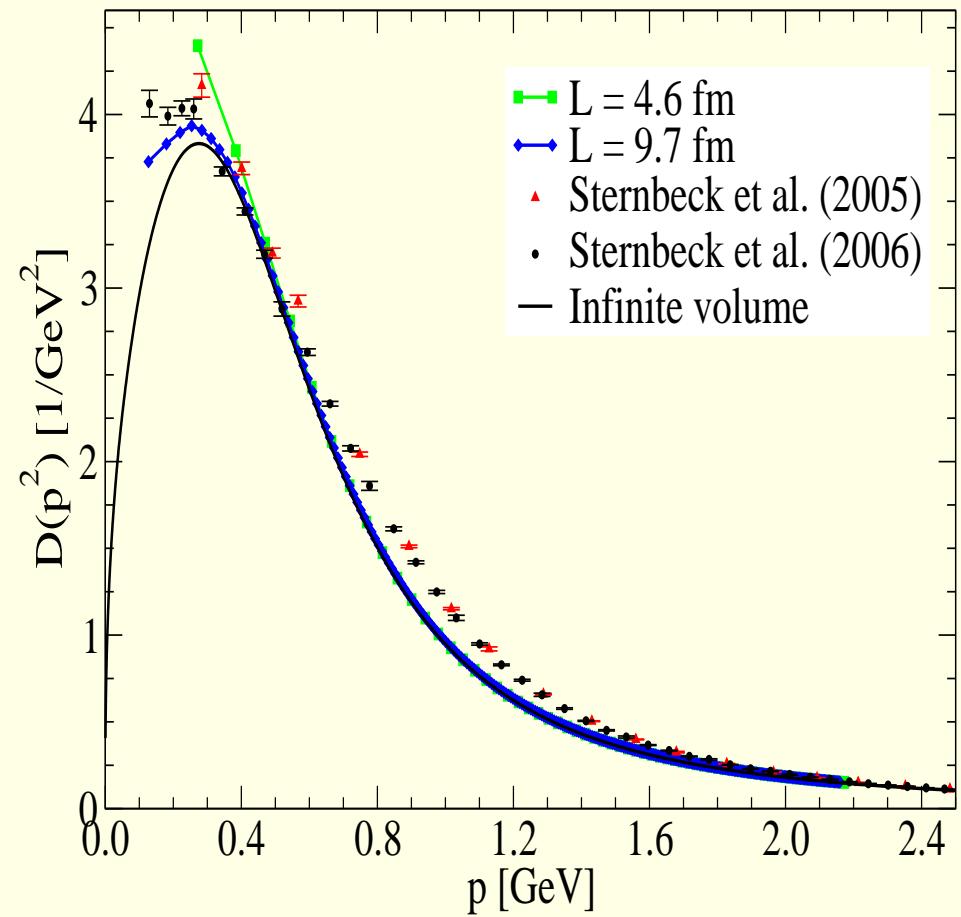
DSE results for gluon propagator: infinite volume vs. torus

Fischer, Maas, Pawłowski, von Smekal, hep-ph/0701051

DSE gluon propagator

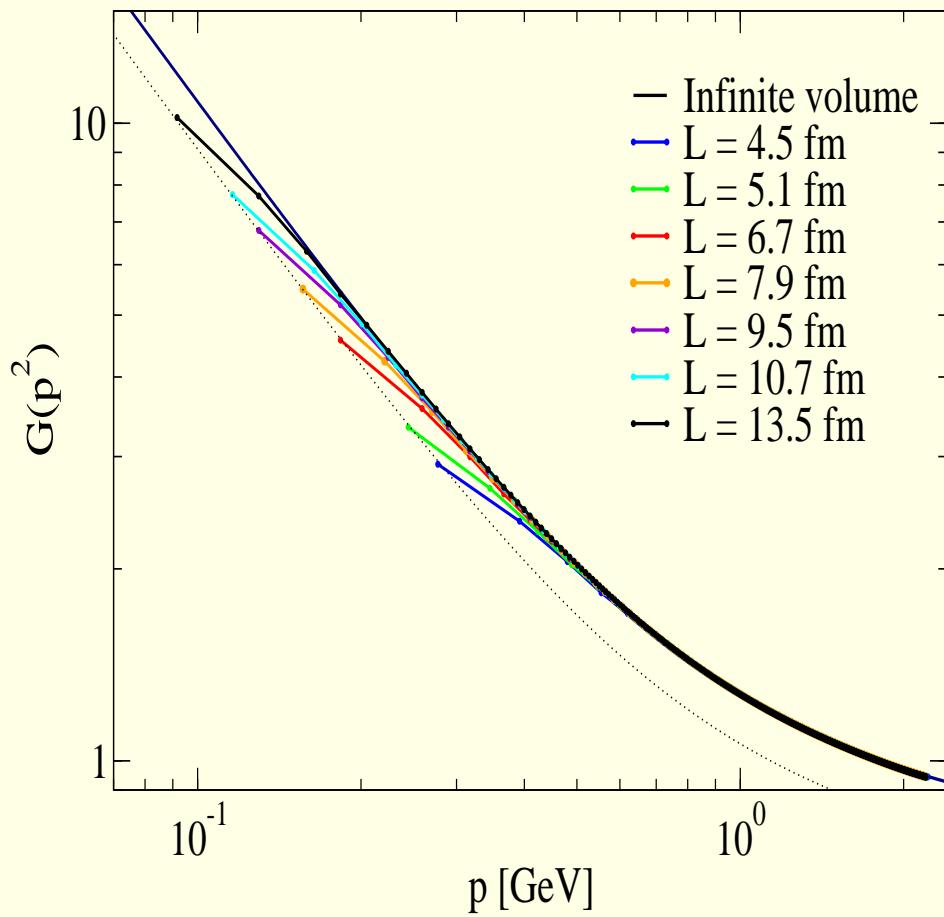


comparison with lattice

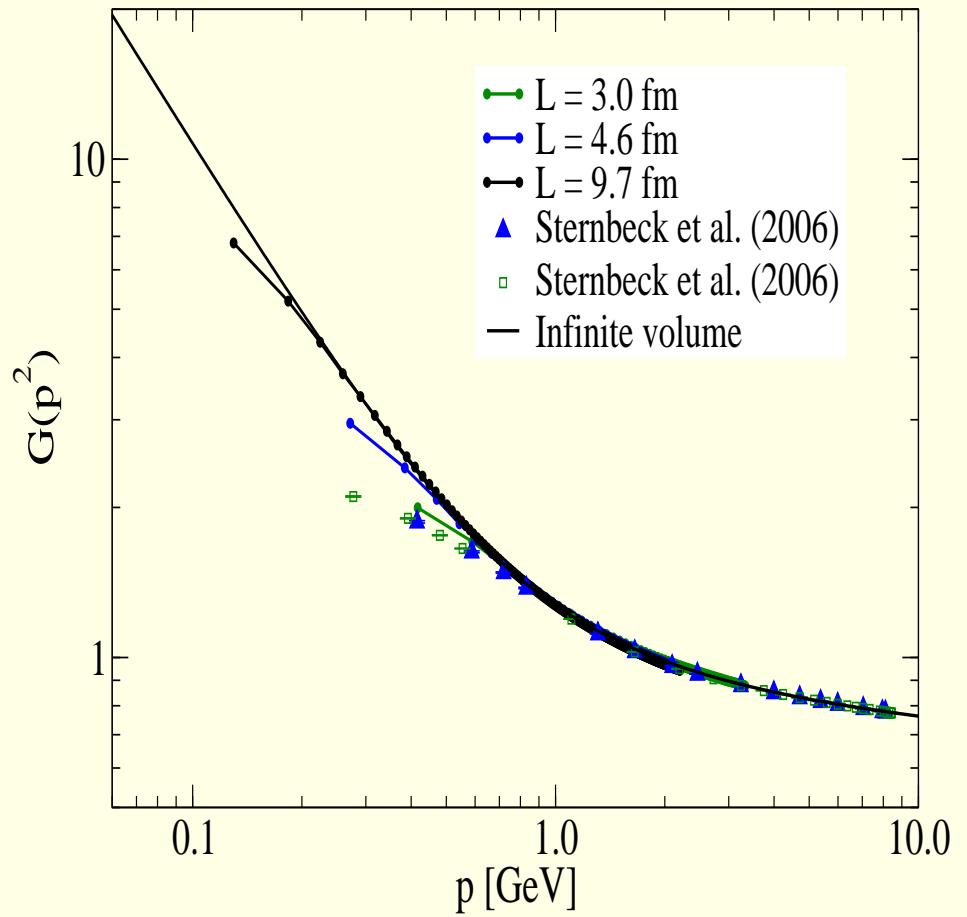


DSE results for ghost dressing function: infinite volume vs. torus

DSE ghost dressing fct.



comparison with lattice

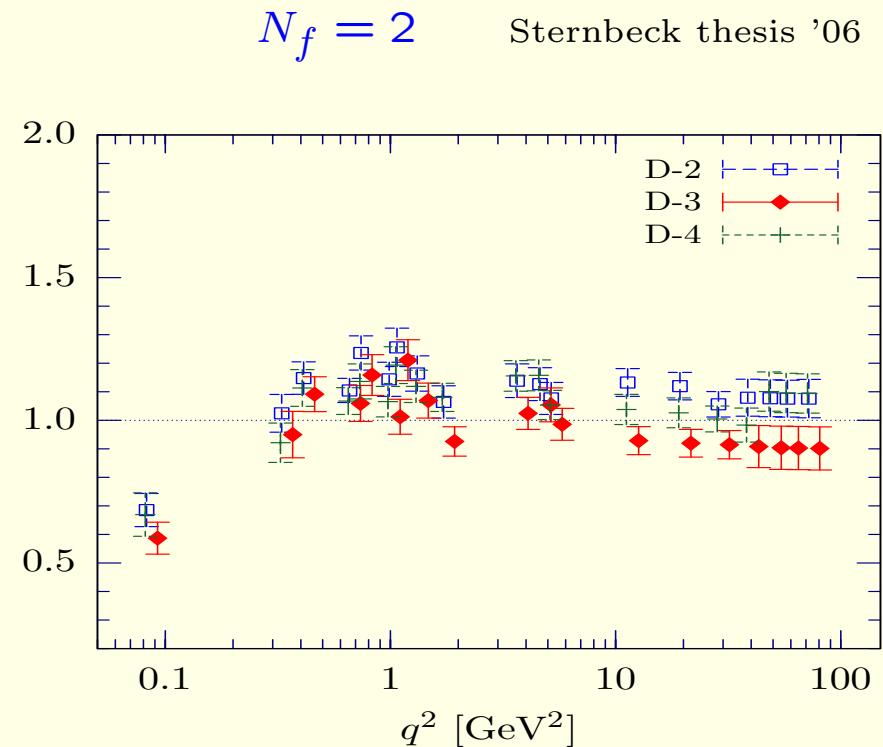
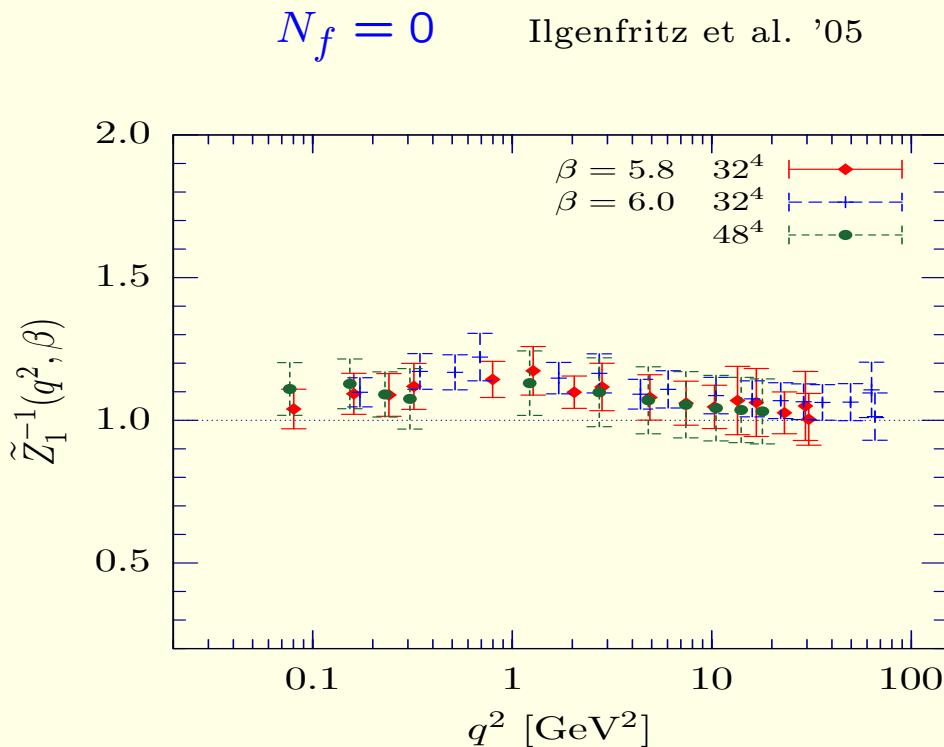


3. The running coupling

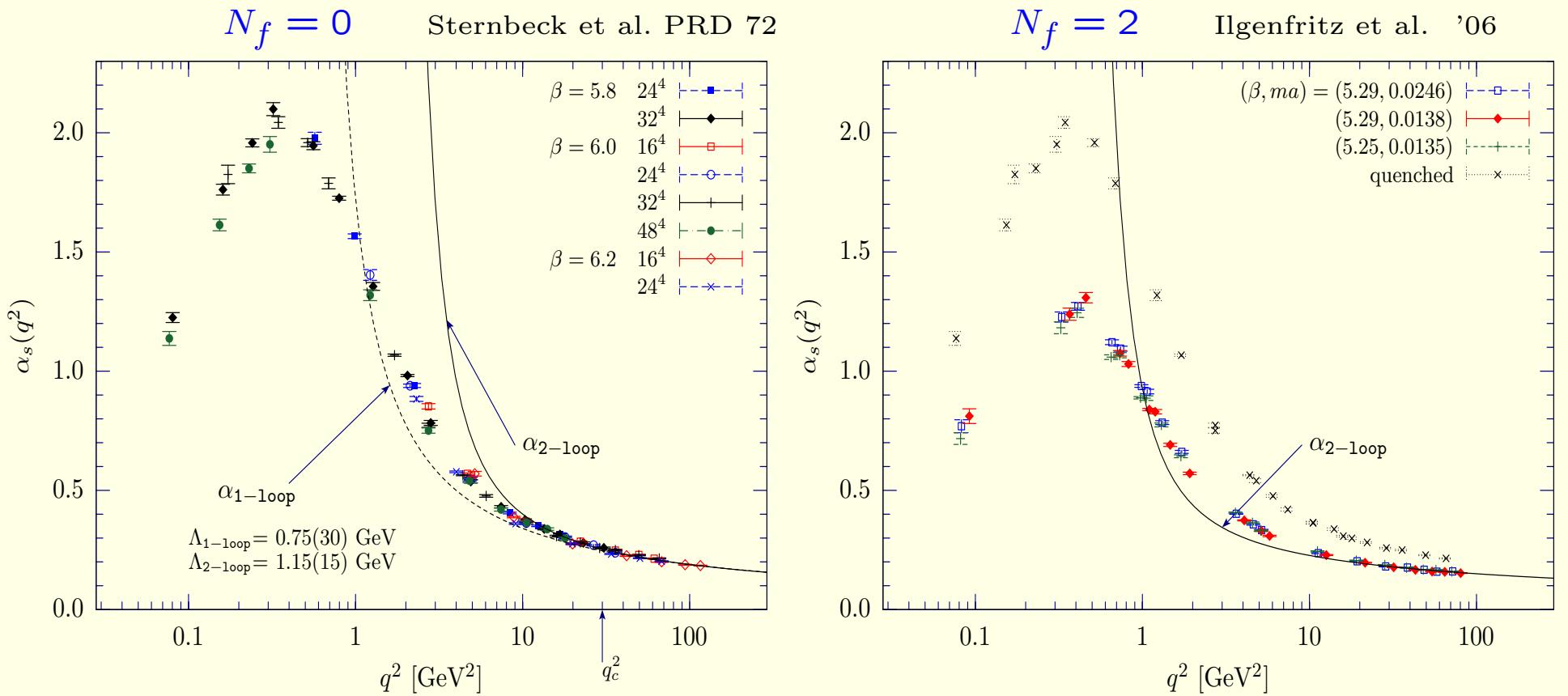
from ghost-ghost-gluon vertex: $\alpha_s(q^2) = \frac{g_0^2}{4\pi} Z(q^2) (J(q^2))^2$
assuming $Z_1(q^2) = 1$, $q > 1\text{GeV}$

[perturbation theory: Taylor ('71) / lattice: Cucchieri et al. ('04)]

The vertex renormalization function Z_1 , gluon momentum $k = 0$.

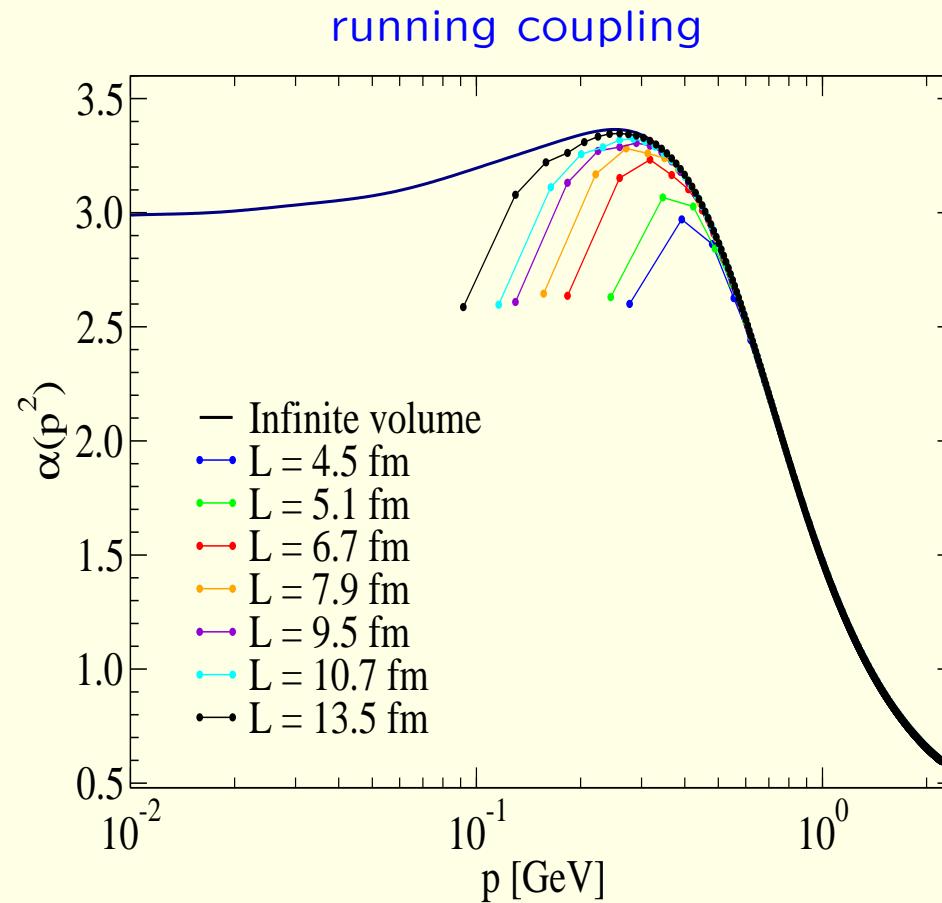


Our result for the running coupling:



- Running coupling not monotonous, passes a maximum.
- Behavior agrees with other lattice studies, in particular for the three-gluon vertex.
- $\alpha_s \rightarrow 0$ for $q \rightarrow 0$? Strong volume dependence ?

DSE results for the running coupling: infinite volume vs. torus



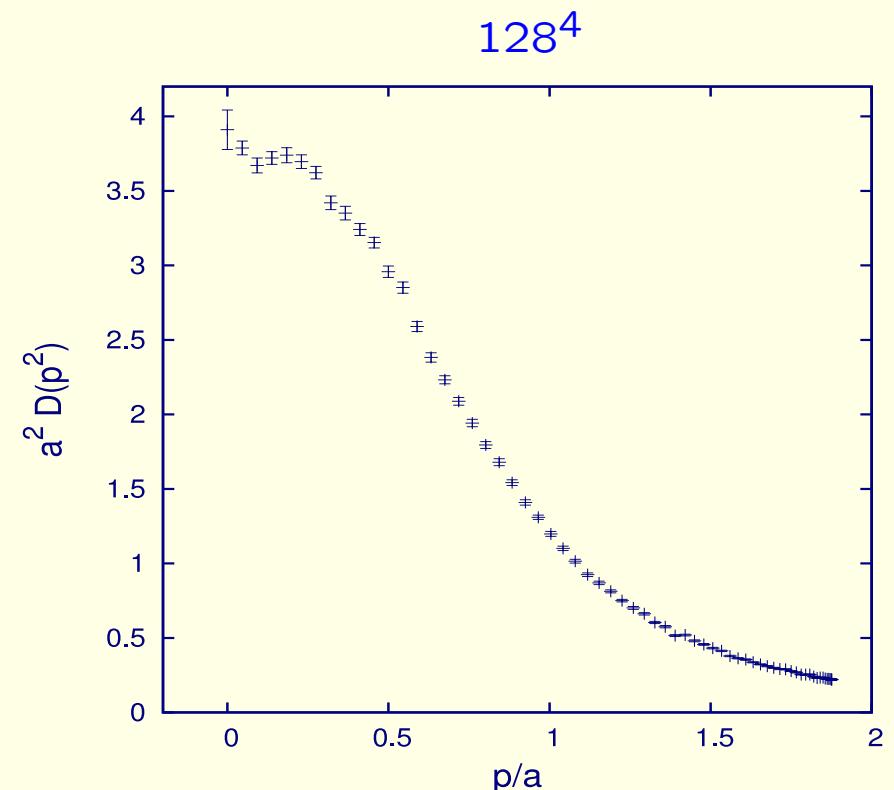
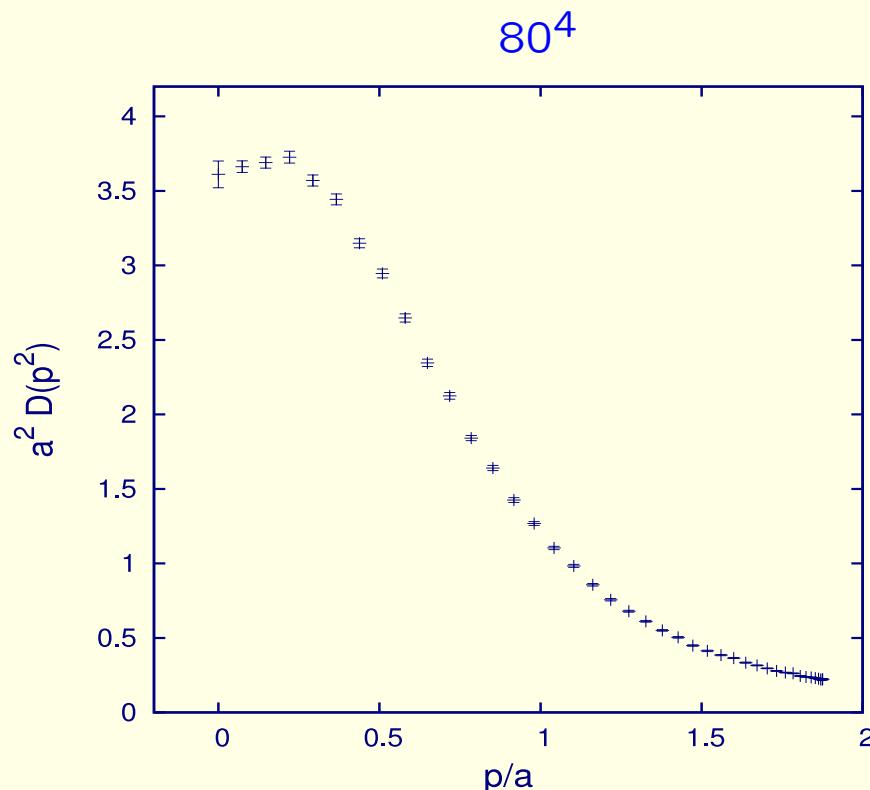
- ⇒ Extremely slow convergence to the infrared limit expected.
- ⇒ DSE on torus may provide extrapolation of lattice results.

4. Gluon and ghost propagators: recent lattice results

Unrenormalized gluon propagator for $SU(2)$ on very large lattices:

Cucchieri, Mendes, contr. LATTICE '07, arXiv:0710.0412 [hep-lat]

$$\beta = 2.20, \implies (128a)^4 \simeq (27 \text{ fm})^4$$



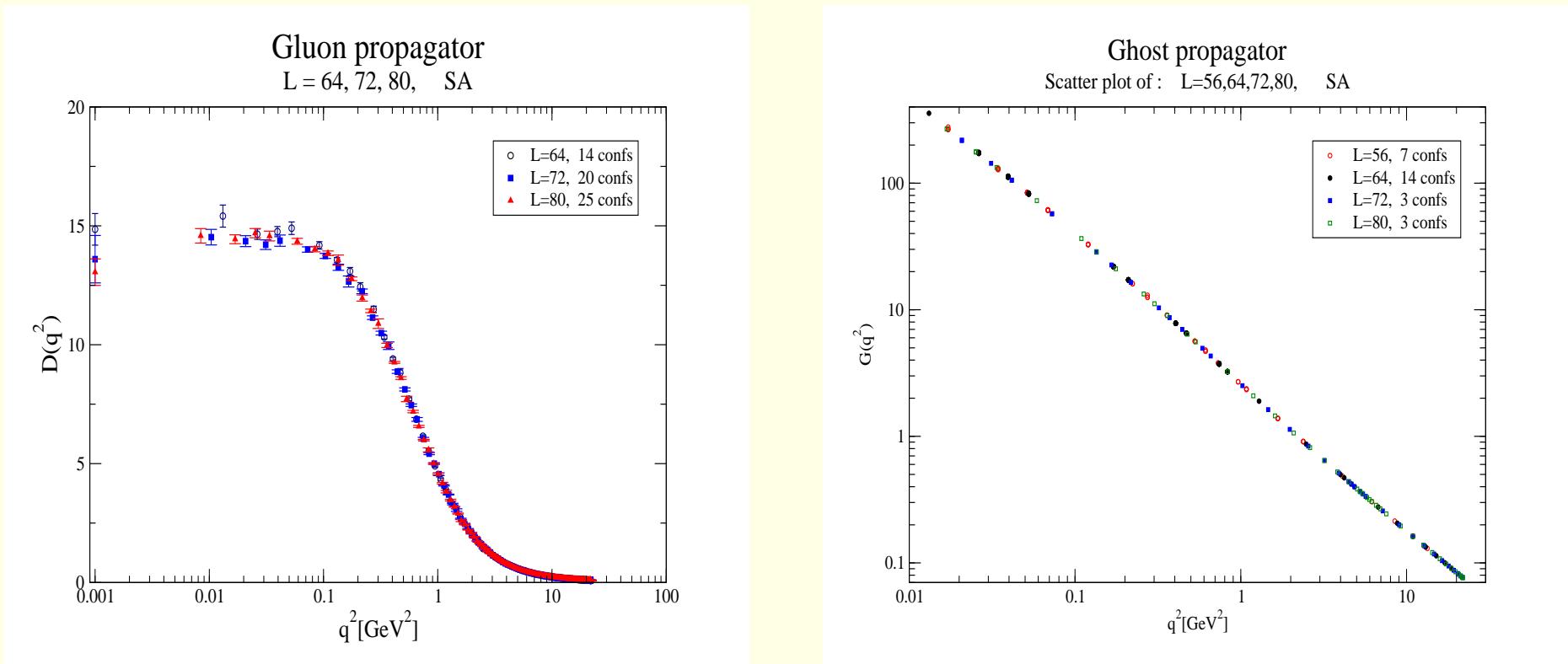
$\implies D(q^2) = Z(q^2)/q^2$ still no signal for vanishing $D \rightarrow 0$ for $q^2 \rightarrow 0$??.

Gluon and ghost propagators for $SU(3)$ on large lattices

Bogolubsky, Ilgenfritz, M.-P., Sternbeck, contr. LATTICE '07

$$\beta = 5.70, \implies (80a)^4 \simeq (13.2 \text{ fm})^4.$$

Simulated annealing used for gauge fixing.



- ⇒ Gluon propagator does not seem to tend to zero for $q^2 \rightarrow 0$.
- ⇒ Ghost propagator much less divergent, than expected from DSE.
- ⇒ Finite-size effects still weak.

5. Improved gauge fixing: new hope ?

Improved gauge fixing \implies getting closer to the FMR:
simulated annealing plus global $\mathbb{Z}(N)$ flips

- Simulated annealing (SA):
Find g 's randomly with statistical weight:

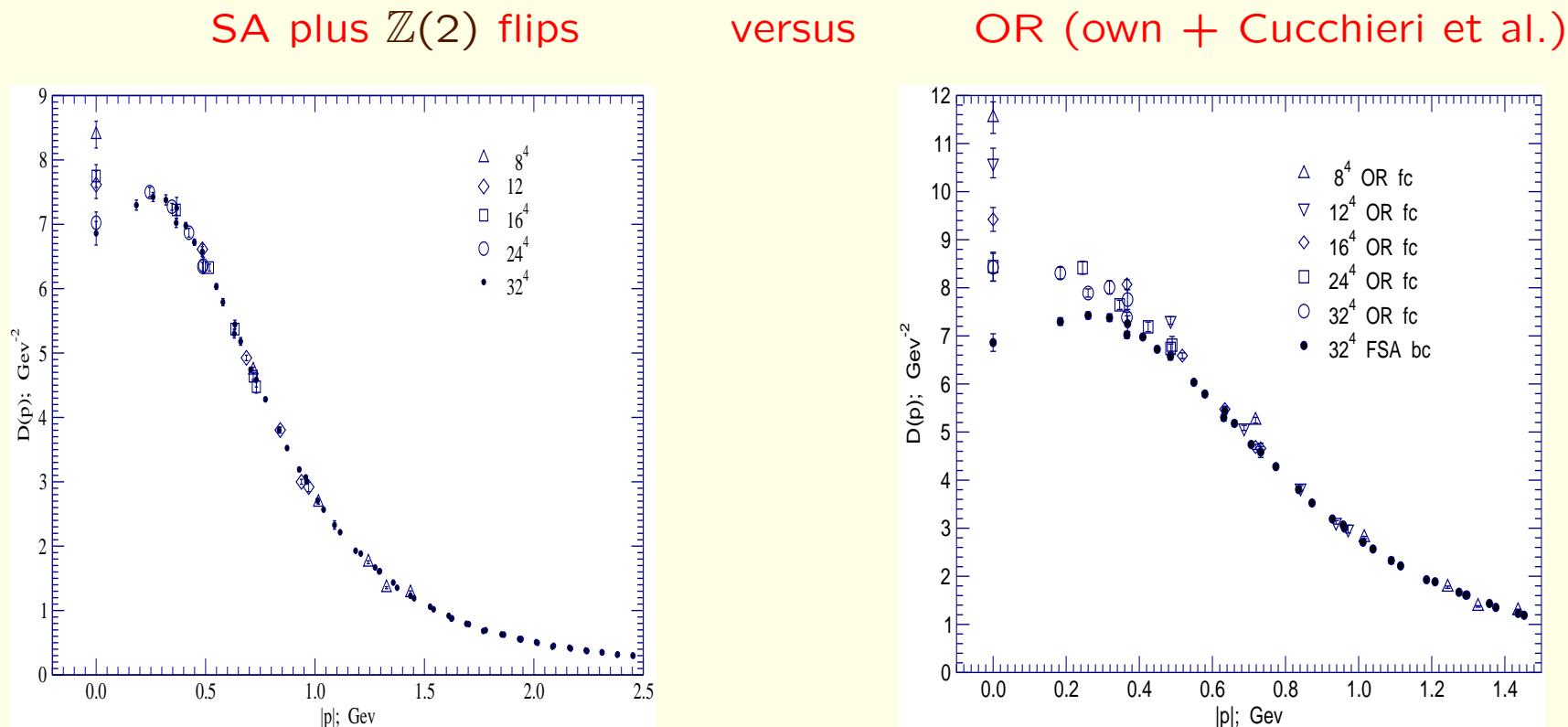
$$W \propto \exp\left(\frac{F_U(g)}{T}\right).$$

Let “temperature” T slowly decrease. Infinitely slow cooling ends at the global extremum. In practice SA clearly wins for large lattice sizes.

- $\mathbb{Z}(N)$ flips:
Gauge functional $F_U(g)$ maximized by enlarging the gauge orbit with respect to $\mathbb{Z}(N)$ non-periodic gauge transformations:

$$g(x + L\hat{\nu}) = z_\nu g(x), \quad z_\nu \in \mathbb{Z}(N).$$

SU(2) results for the gluon propagator ($8^4, \dots, 32^4$, $\beta = 2.2$):
 Bogolubsky, Bornyakov, Burgio, Ilgenfritz, Mitrjushkin, M.-P.,
 arXiv:0707.3611 [hep-lat], LATTICE '07



- ⇒ Gribov copies also important for the gluon propagator.
- ⇒ Finite-size effects weaker when approaching the FMR Λ .
- ⇒ Hope to see the turn to $D(q^2 \rightarrow 0) = 0$.

6. Conclusion and outlook

- Small-momentum behavior on the lattice (still) in conflict with asymptotic DSE power solutions.
⇒ When do we reach “asymptotia”?
- $SU(2)$: For $d = 2$ on large lattices up to $((43 \text{ fm})^4)$ a correct DSE behavior has been reported [Maas, '07].
↔ For $d = 3$ clear disagreement [Cucchieri, Mendes, '07].
- Finite-size effects on the lattice - in particular for the ghost - look different than for DSE on a torus.
⇒ DSE truncation effect ?
- Comparison with confinement scenarios:
 $D(q^2) \rightarrow 0$ for $q^2 \rightarrow 0$ could be still possible. Ghost dressing function seems to diverge, but probably weaker than expected.

- Gribov copies produce finite-size effects, $\mathbb{Z}(N_c)$ -flips important. Does this solve the puzzle ? We hope to give an answer soon.
- Recent debate on analytic results:
Dudal, Sorella, Vandersickel, Verschelde, arXiv:0711.4496 [hep-th]
 $\implies D(q^2 \rightarrow 0) \neq 0$ by modifying Zwanziger's approach.

\implies Food to think ?!

Thank you for your attention.