

Phase diagram of QCD from quark-meson models

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Outline

- Conjectured QCD Phase Diagram
- Quark-Meson Model Phase Diagram
 - ▷ Mean field approximation
 - ▷ Renormalization Group study
- Polyakov–Quark-Meson Model
- Three Flavor Quark-Meson Model

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The conjectured QCD Phase Diagram

QCD: two phase transitions:

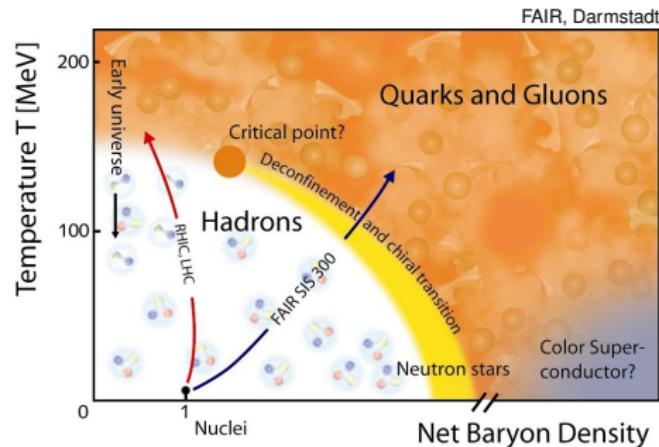
① restoration of chiral symmetry

$$SU_{L+R}(N_f) \rightarrow SU_L(N_f) \times SU_R(N_f)$$

order parameter:

$$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken}, T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase}, T > T_c \end{cases}$$

associate limit: $m_q \rightarrow 0$



chiral transition: spontaneous restoration of global $SU_L(N_f) \times SU_R(N_f)$ at high T

The conjectured QCD Phase Diagram

QCD: two phase transitions:

- ① restoration of chiral symmetry

- ② de/confinement

associate limit: $m_q \rightarrow \infty$

⇒ pure gauge theory w/ static sources

⇒ gauge action invariant

under global $Z(N_c)$ -symmetry

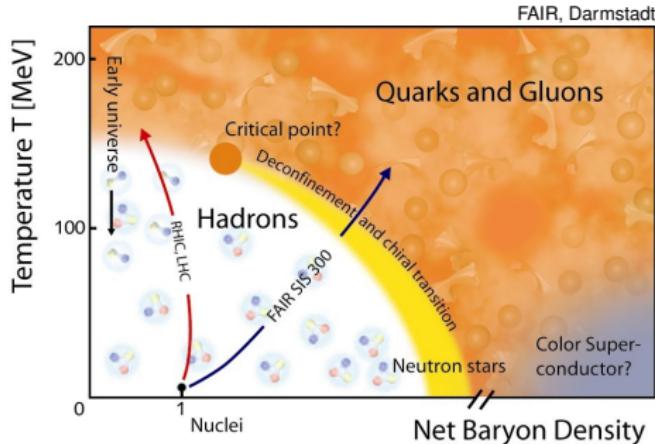
order parameter:

Polyakov loop variable $\phi = \langle \text{tr}_c \mathcal{P}(\vec{x}) \rangle / N_c$

$$\phi \begin{cases} = 0 \Leftrightarrow \text{confined phase}, & T < T_c \\ > 0 \Leftrightarrow \text{deconfining phase}, & T > T_c \end{cases}$$

free energy of static quark antiquark pair:

$$\exp\left(-\frac{F_{q\bar{q}}(r,T)}{T}\right) = \langle \text{tr}_c \mathcal{P}(x) \text{tr}_c \mathcal{P}^\dagger(y) \rangle / N_c^2$$

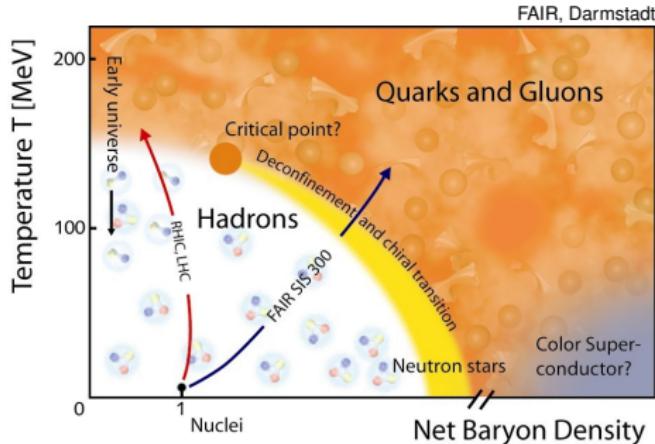


$$\mathcal{P}(\vec{x}) = \mathcal{P} e^{\int_0^\beta d\tau A_0(\tau, \vec{x})}$$

The conjectured QCD Phase Diagram

QCD: two phase transitions:

- ① restoration of chiral symmetry
- ② de/confinement



effective models:

- ① Quark-meson model or other models e.g. NJL
- ② Polyakov–quark-meson model or PNJL models

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The chiral $N_f = 2$ quark-meson model

- Lagrangian:

$$\mathcal{L}_{qm} = \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\pi}\cdot\vec{\gamma}_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

1. Mean field analysis (ignoring fluctuations)
2. Renormalization Group analysis (taking fluctuations into account)

The chiral $N_f = 2$ quark-meson model

- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\pi}\cdot\vec{\gamma}_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

1. Mean field analysis (ignoring fluctuations)

partition function:

$$\mathcal{Z}(T, \mu) = \int \mathcal{D}\bar{q} \mathcal{D}q \mathcal{D}\sigma \mathcal{D}\vec{\pi} \exp \left\{ i \int_0^{1/T} dt d^3x (\mathcal{L}_{\text{qm}} + \mu \bar{q} \gamma_0 q) \right\}.$$

$\sigma \rightarrow \langle \sigma \rangle \equiv \phi, \pi \rightarrow \langle \pi \rangle = 0$, integrate quark/antiquarks

Grand canonical potential

$$\Omega(T, \mu) = -\frac{T \ln \mathcal{Z}}{V} = \frac{\lambda}{4}(\langle \sigma \rangle^2 - v^2)^2 - c\langle \sigma \rangle + \Omega_{\bar{q}q}(T, \mu)$$

with

$$\Omega_{\bar{q}q}(T, \mu) = -2N_c N_f T \int \frac{d^3 k}{(2\pi)^3} \left\{ \ln(1 + e^{-(E_q - \mu)/T}) + \ln(1 + e^{-(E_q + \mu)/T}) \right\}$$

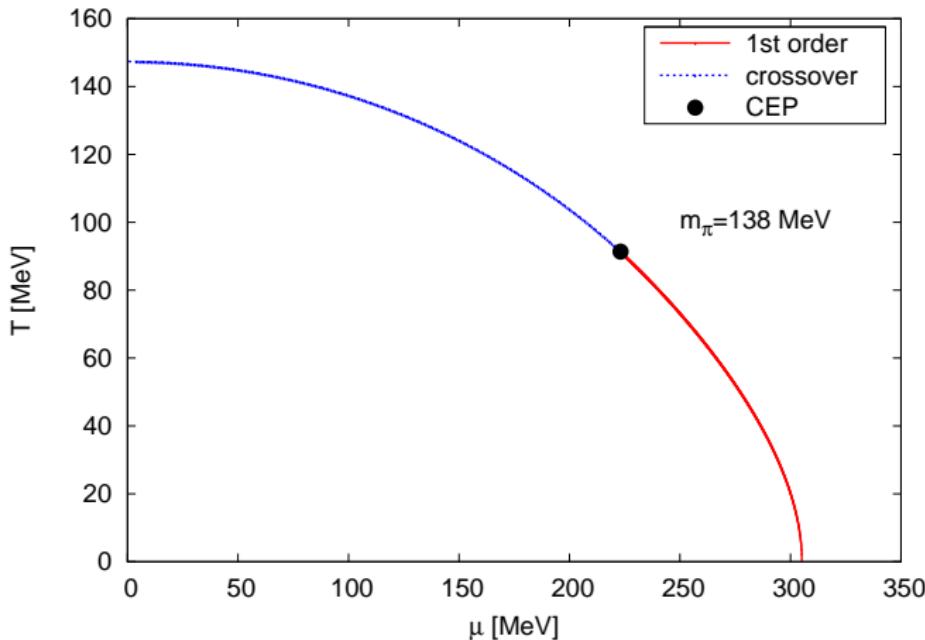
[Scavenius et al. '01]

Mean field analysis

- Lagrangian:

$$\mathcal{L}_{qm} = \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

1. Mean field analysis

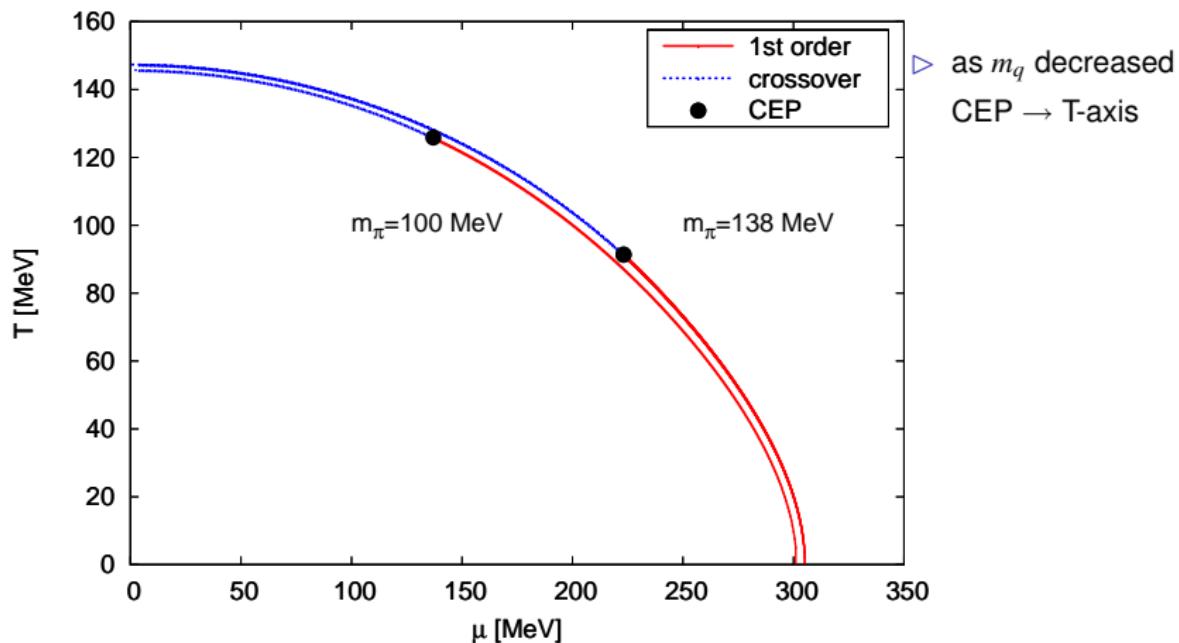


Mean field analysis

- Lagrangian:

$$\mathcal{L}_{\text{qm}} = \bar{q} [i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

1. Mean field analysis

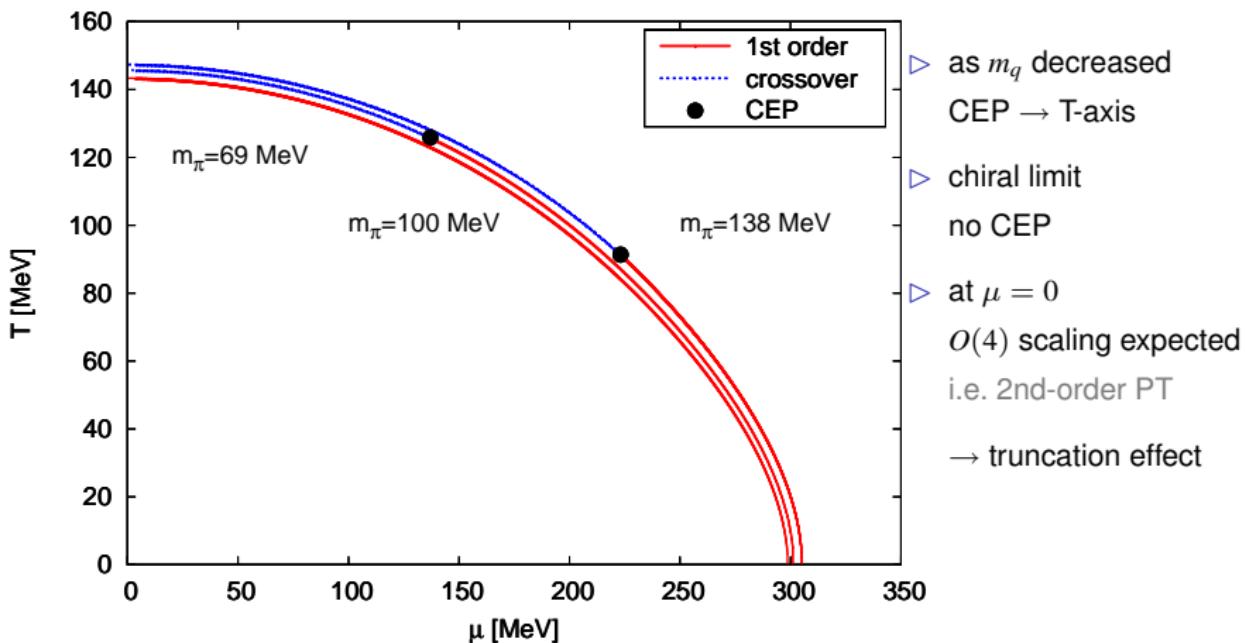


Mean field analysis

- Lagrangian:

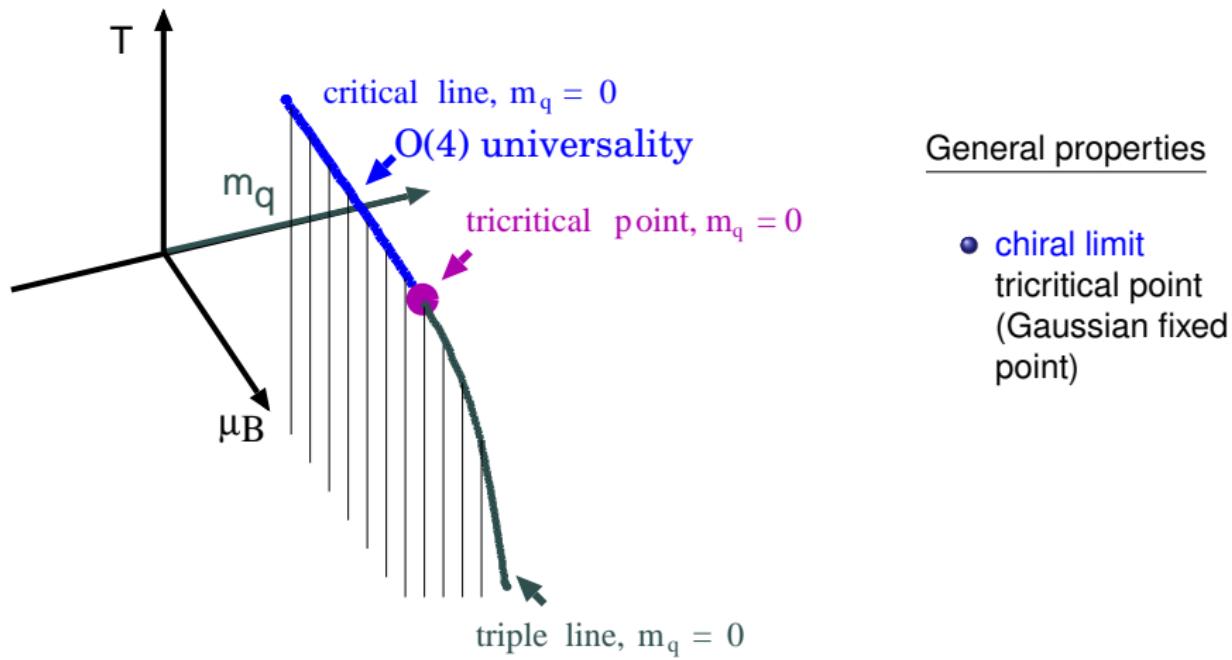
$$\mathcal{L}_{\text{qm}} = \bar{q} [i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

1. Mean field analysis



Phase diagram in (T, μ_B, m_q) -space

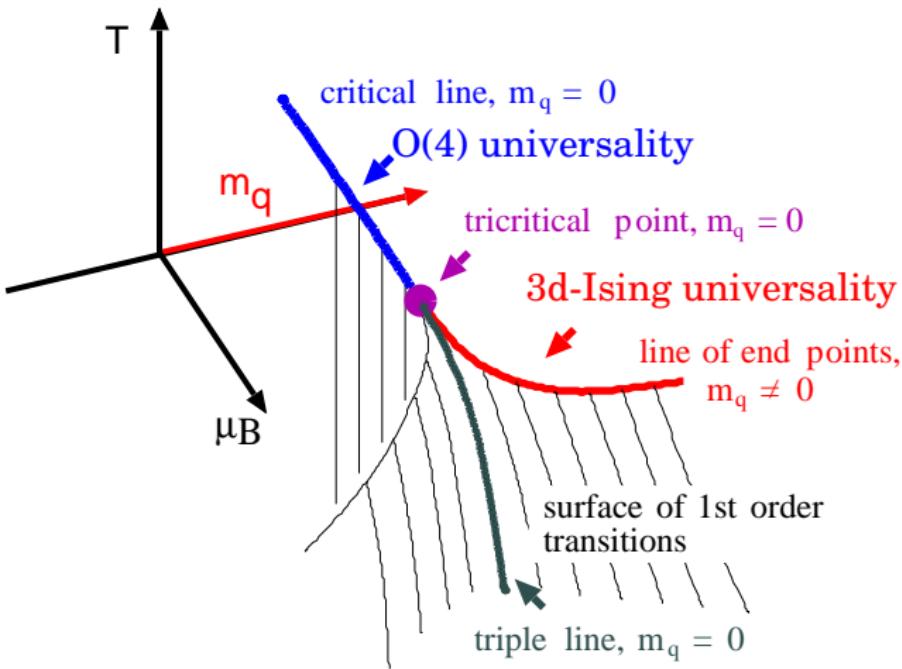
Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$ -symmetry — \rightarrow 4 modes critical $\sigma, \vec{\pi}$



Phase diagram in (T, μ_B, m_q) -space

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$ -symmetry —> 4 modes critical $\sigma, \vec{\pi}$

$m_q \neq 0$: no symmetry remains —> only one critical mode σ (**I**sing) ($\vec{\pi}$ massive)



General properties

- **chiral limit**
tricritical point
(Gaussian fixed point)
- **finite m_q**
critical endpoints
(3D-Ising class)

RG Approaches

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k or f_k regulators

1 Exact RG

ERG (average effective action)

[Wetterich]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi}$$

2 Proper-time RG

PTRG

[Liao]

$$\partial_t \Gamma_k[\phi] = -\frac{1}{2} \int_0^\infty \frac{d\tau}{\tau} \left[\partial_t f_k(\Lambda^2 \tau) \right] \text{Tr} \exp \left(-\tau \Gamma_k^{(2)} \right)$$

3 other approximations

RG analysis

- QM model:

$$\mathcal{L}_{\text{qm}} = \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V(\phi^2)$$

2. PTRG analysis

$$V(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

⇒ ansatz for Γ_k at UV: $\Gamma_{k=\Lambda} = \int d^4x V_{k=\Lambda}$

flow for grand canonical potential

[BJS, J.Wambach]

$$\begin{aligned} \partial_t \Omega_k(T, \mu; \phi) &= \frac{k^4}{12\pi^2} \left[\frac{3}{E_\pi} \coth \left(\frac{E_\pi}{2T} \right) + \frac{1}{E_\sigma} \coth \left(\frac{E_\sigma}{2T} \right) \right. \\ &\quad \left. - \frac{2N_c N_f}{E_q} \left\{ \tanh \left(\frac{E_q - \mu}{2T} \right) + \tanh \left(\frac{E_q + \mu}{2T} \right) \right\} \right] \end{aligned}$$

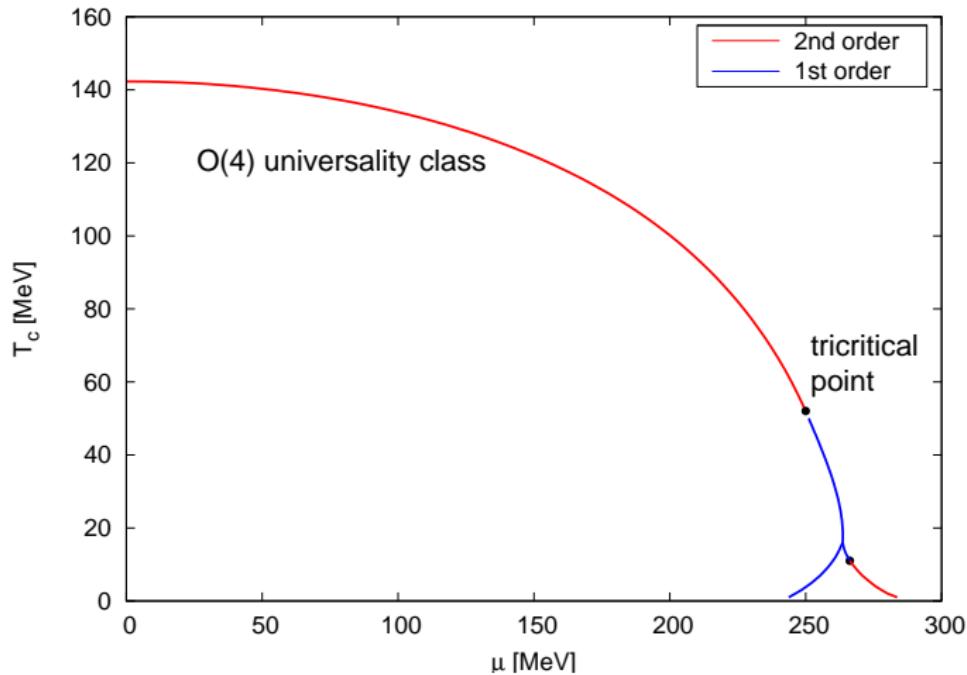
$$\begin{aligned} E_\pi^2 &= 1 + 2\Omega'_k/k^2, & E_\sigma^2 &= 1 + 2\Omega'_k/k^2 + 4\phi^2\Omega''_k/k^2, & E_q^2 &= 1 + g^2\phi^2/k^2 \\ \phi &\sim \langle \bar{q}q \rangle, & \Omega'_k &= \partial\Omega_k/\partial\phi & \text{etc} \end{aligned}$$

- quark fluctuations: chiral symmetry breaking
- meson fluctuations: chiral symmetry restoration

Chiral Phase Diagram $N_f = 2$ & $m_q \sim 280$ MeV

$O(4) \sim SU(2) \times SU(2)$

chiral limit

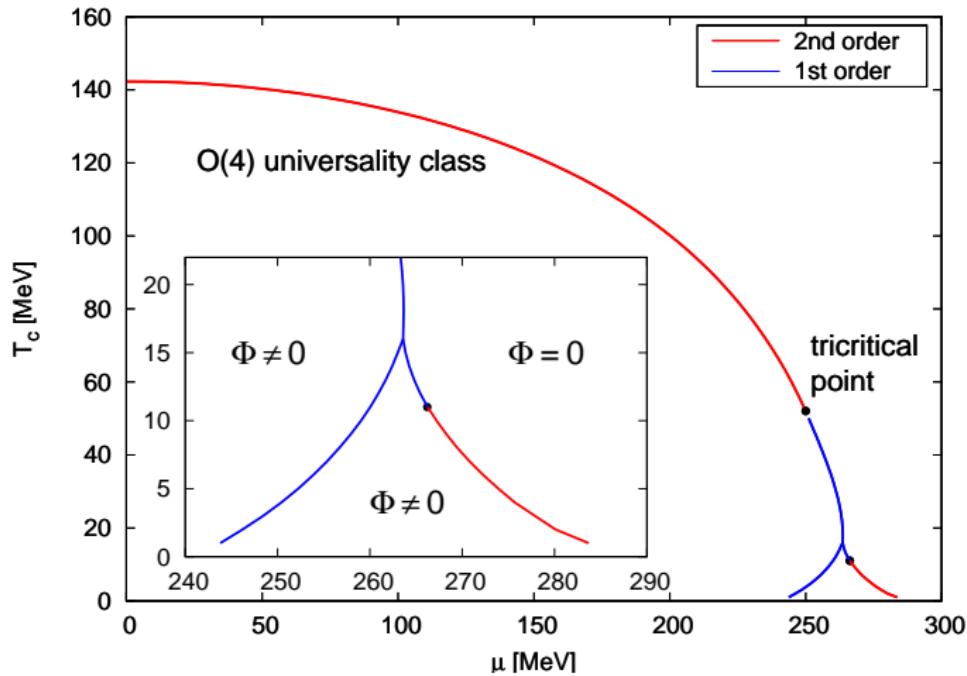


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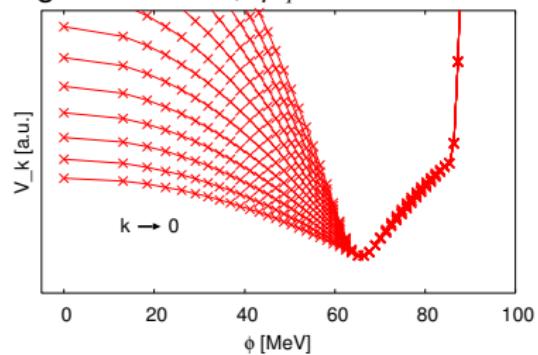
chiral limit

no spinodal lines!

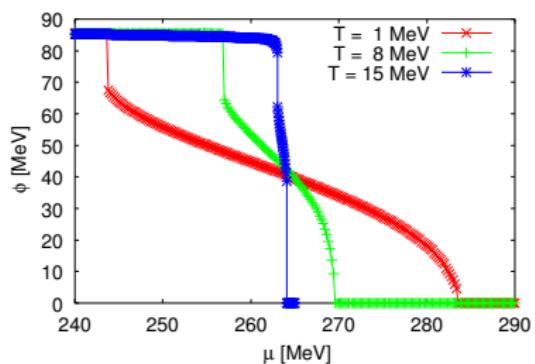
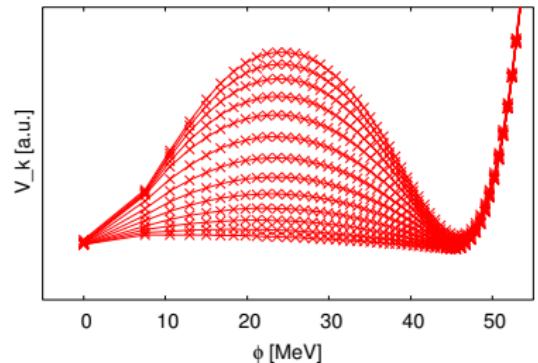


A Second (new) Phase Transition

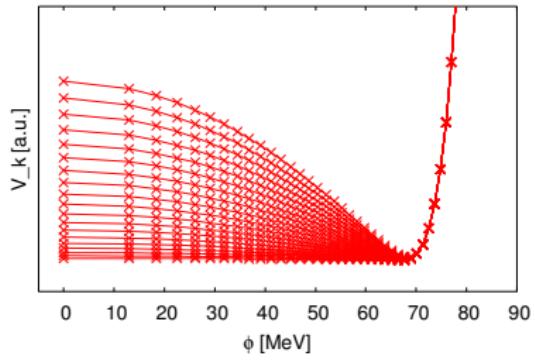
e.g. $T = 6 \text{ MeV}$, $\mu_q = 254 \text{ MeV}$



first-order phase transition

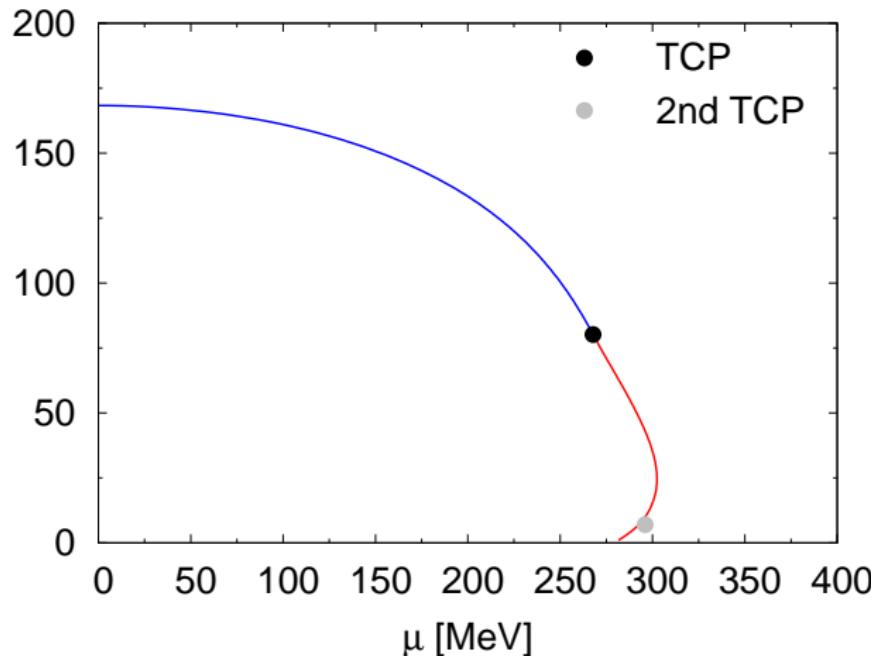


second-order phase transition



RG Phase Diagram

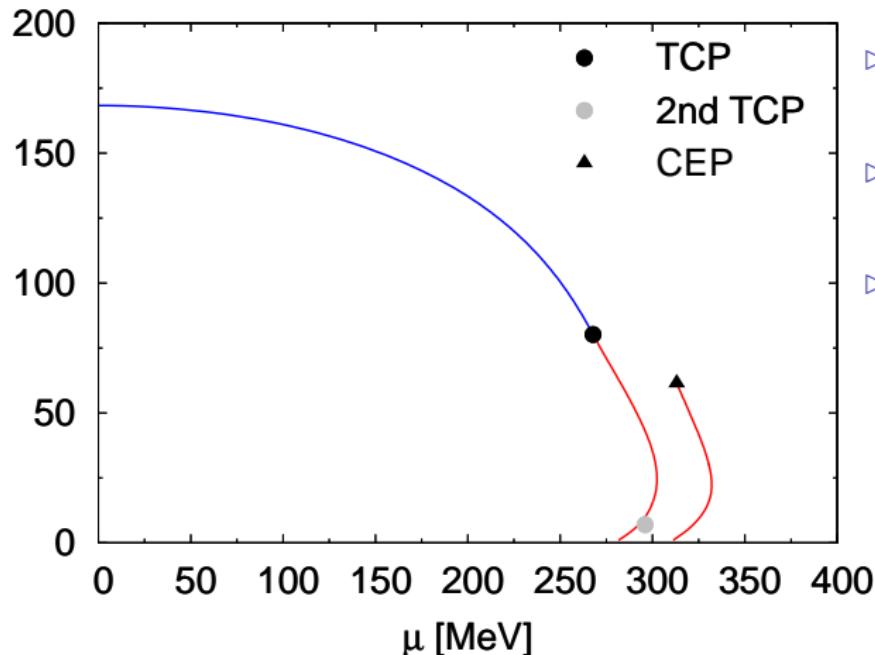
chiral limit: $m_\pi = 0$



[BJS,Wambach '05 & '06]

RG Phase Diagram

$m_\pi \sim 138$ MeV



- ▷ bending usual for RG
- ▷ 2nd tricritical point
- ▷ features
parameter independent
- but locations TCP/CEP
parameter dependent

[BJS,Wambach '05 & '06]

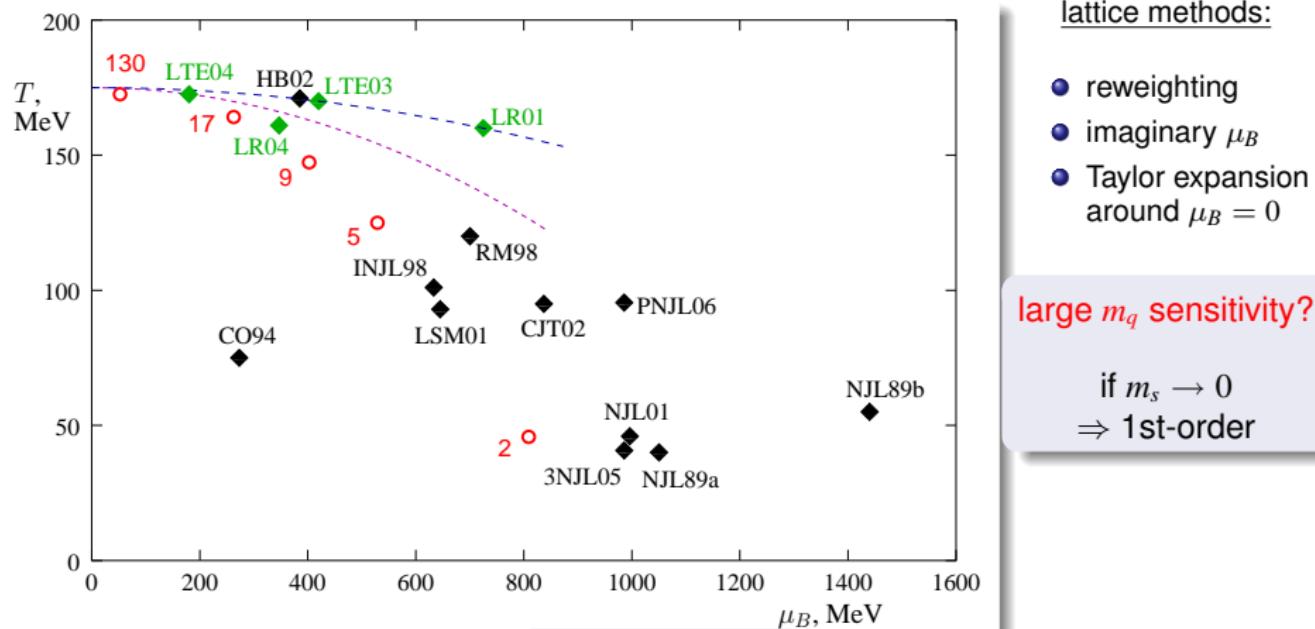
Charts of QCD Critical End Points

model studies vs. lattice simulations

Black points: models

Lines & green points: lattice

Red points: Freezeout points for HIC



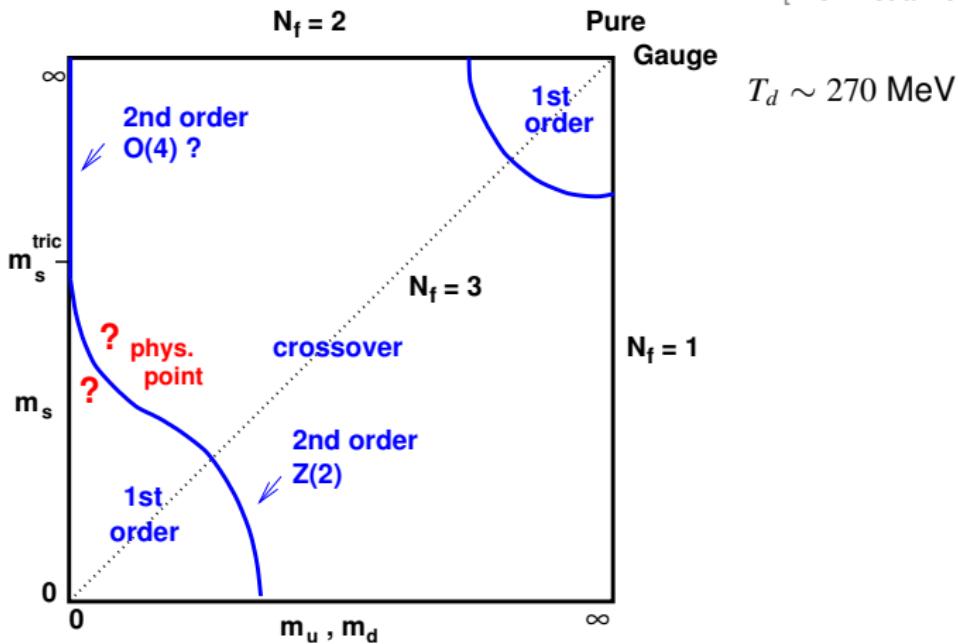
Mass Sensitivity (lattice, $N_f = 3, \mu_B = 0$)

Columbia plot:

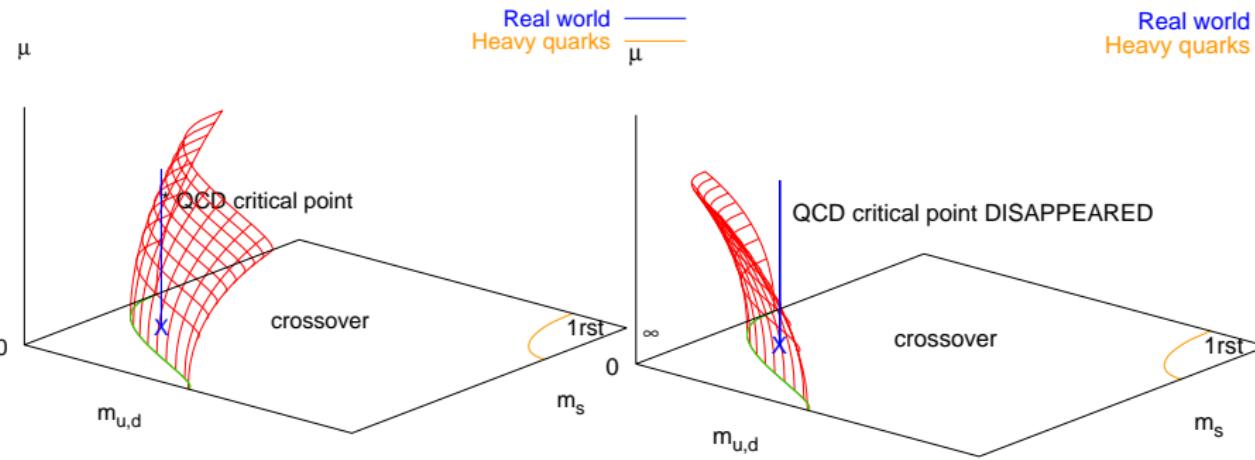
$$T_{\chi}^{N_f=2} \sim 175 \text{ MeV}$$

[Brown et al. '90]

$$T_{\chi}^{N_f=3} \sim 155 \text{ MeV}$$



Mass Sensitivity (lattice, $N_f = 3$, $\mu_B \neq 0$)



standard scenario: $m_c(\mu)$ increasing

non-standard scenario: $m_c(\mu)$ decreasing

[de Forcrand, Philipsen '05]

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... is a combination of

1. the chiral quark-meson model (here $N_f = 2$)
2. with Polyakov loop dynamics: including now color dof's. → confinement

It's a new model (cf. PNJL)

Polyakov–quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$

- Polyakov loop potential:

Polyakov 1978

Pisarski 2000

$$\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$$

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(\textcolor{red}{T}, \textcolor{red}{T}_0)}{2} \phi \bar{\phi} - \frac{b_3}{6} (\phi^3 + \bar{\phi}^3) + \frac{b_4}{16} (\phi \bar{\phi})^2$$

Ratti, Weise et al. 2004

Dumitru, Pisarski 2004

Friman, Redlich, Sasaki 2006

⇒ first-order transition at $T_0 = 270$ MeV

in presence of dynamical quarks: $T_0 = T_0(N_f)$

BJS, Pawlowski, Wambach, 2007

| N_f | | 0 | 1 | 2 | 2 + 1 | 3 |
|-------------|--|-----|-----|-----|-------|-----|
| T_0 [MeV] | | 270 | 240 | 208 | 187 | 178 |

Polyakov–quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$

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Ratti, Weise et al. 2004

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⇒ first-order transition at $T_0 = 270$ MeV

$$\mu \neq 0 : \quad T_0 = T_0(N_f, \mu)$$

BJS, Pawłowski, Wambach, 2007

$$\bar{\phi} \neq \phi^*$$

Mean-field approximation

- grand canonical potential:

$$\Omega(T, \mu) = \mathcal{U}(\phi, \bar{\phi}) + V_{\text{renorm}}(\langle \sigma \rangle, \vec{0}) + \Omega_{\bar{q}q}(T, \mu)$$

with fermi contribution:

$$\begin{aligned}\Omega_{\bar{q}q} = -2N_f T \int \frac{d^3 p}{(2\pi)^3} & \left\{ \ln \left[1 + 3(\phi + \bar{\phi} e^{-(E_p - \mu)/T}) e^{-(E_p - \mu)/T} + e^{-3(E_p - \mu)/T} \right] \right. \\ & \left. + \ln \left[1 + 3(\bar{\phi} + \phi e^{-(E_p + \mu)/T}) e^{-(E_p + \mu)/T} + e^{-3(E_p + \mu)/T} \right] \right\}\end{aligned}$$

$$E_p = \sqrt{p^2 + m_q^2}$$

confined phase $\phi = 0$: 1q- & 2q-states suppressed, only 3 quark states

deconfined phase $\phi \neq 0$: 1q-, 2q- & 3q-states

- three EoM:

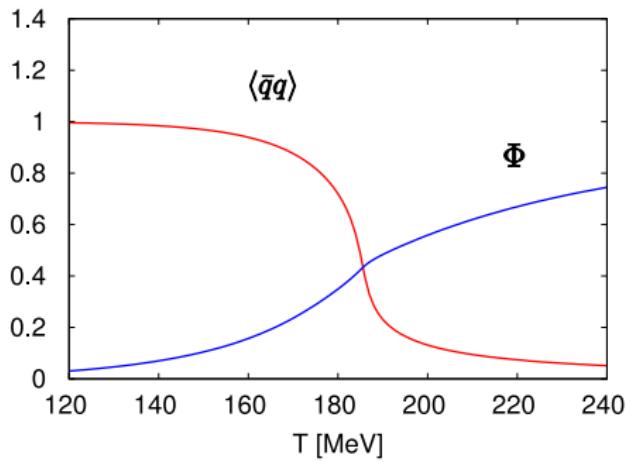
$$\frac{\partial \Omega}{\partial \sigma} = 0 , \quad \frac{\partial \Omega}{\partial \phi} = 0 , \quad \frac{\partial \Omega}{\partial \bar{\phi}} = 0 .$$

Finite temperature and $\mu = 0$

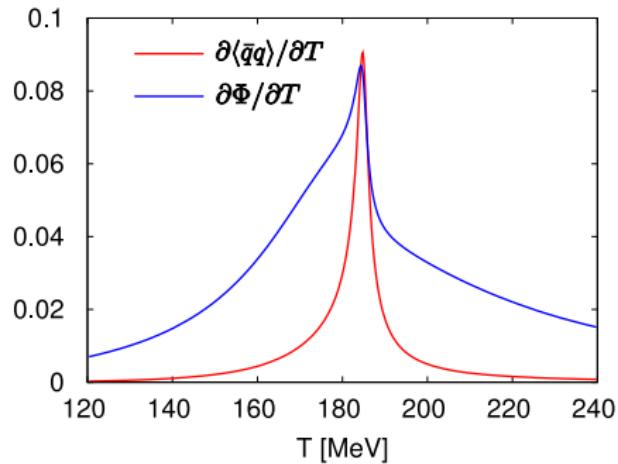
[BJS, Pawlowski, Wambach '07]

Numerical results:

order parameters



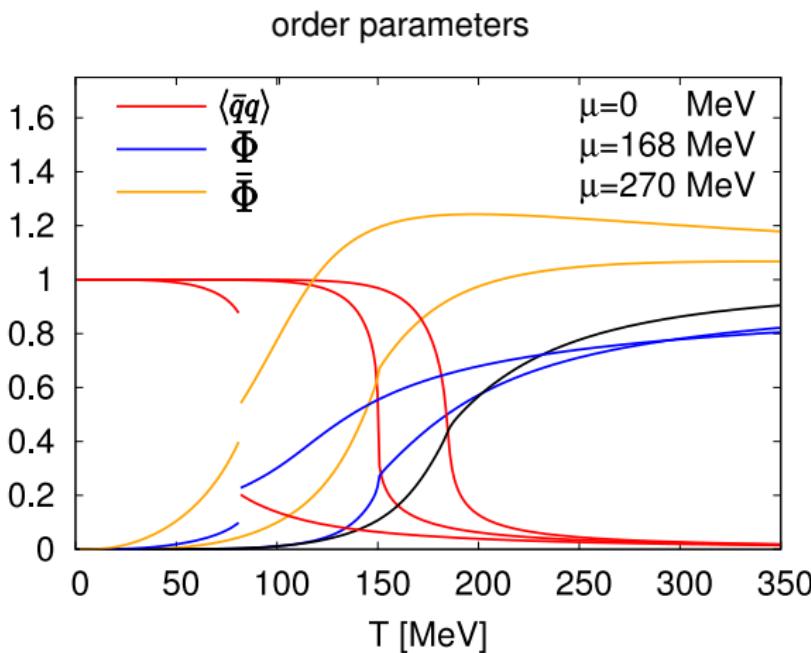
T -derivatives of order parameters



Finite temperature and finite μ

[BJS, Pawlowski, Wambach '07]

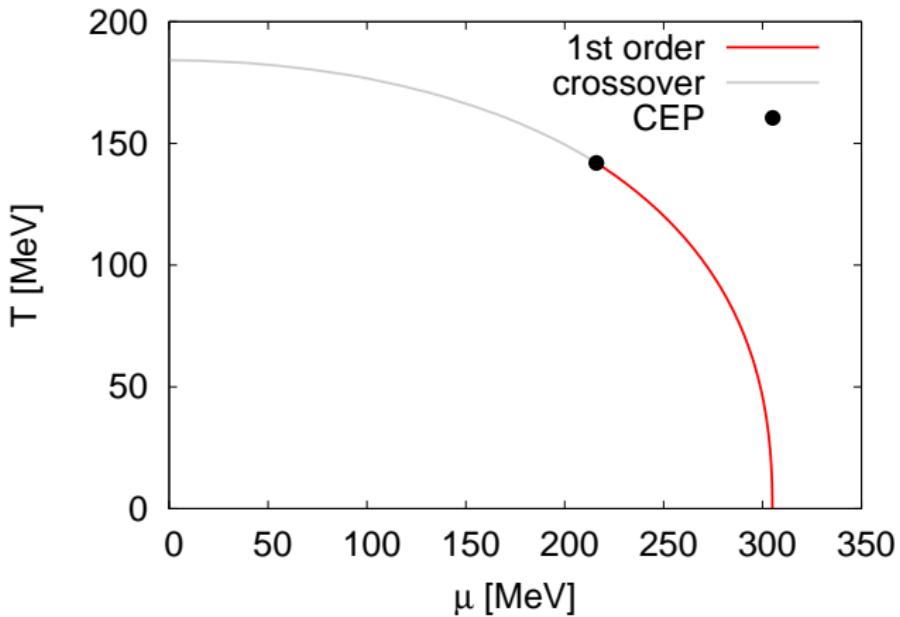
without μ -modifications in Polyakov potential:



in mean field approximation

chiral transition and 'deconfinement' coincide

● for PQM model

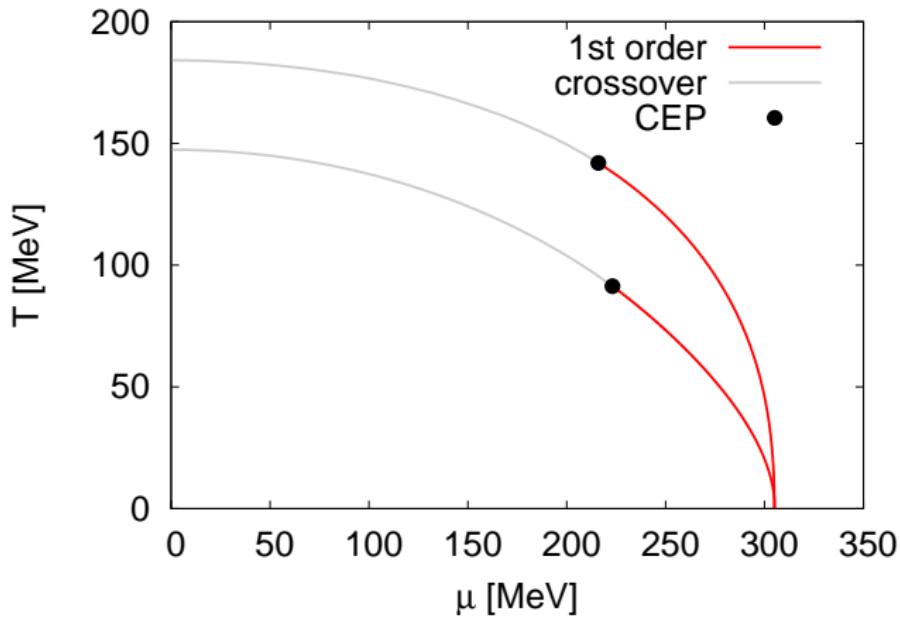


[BJS, Pawlowski, Wambach '07]

in mean field approximation

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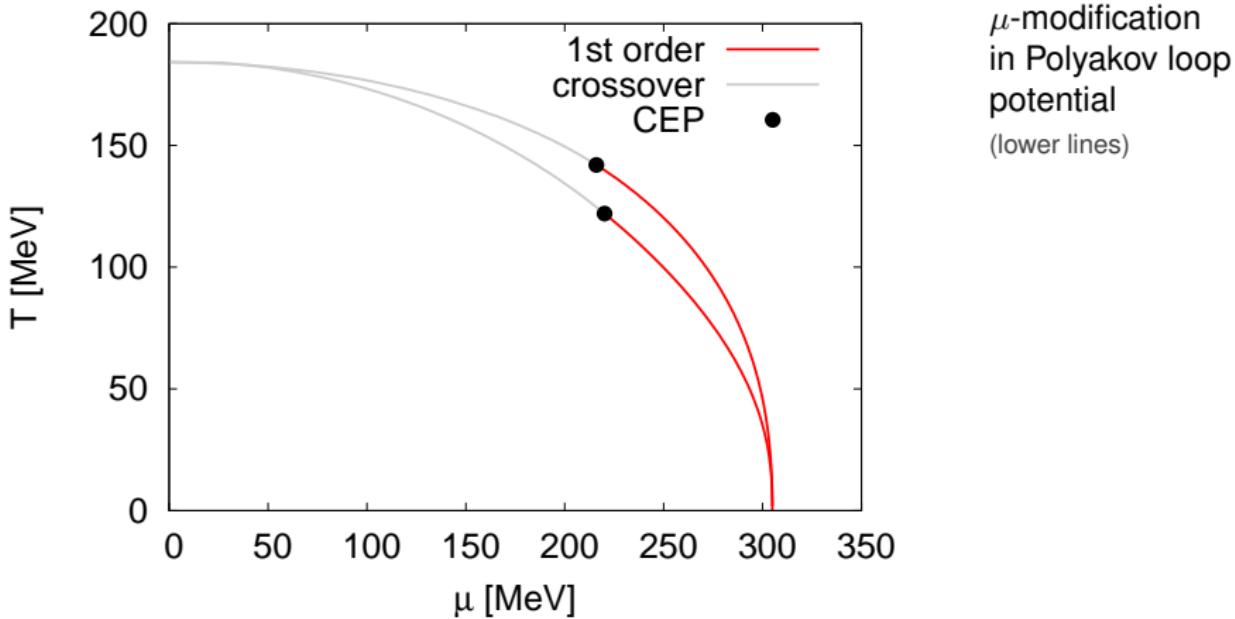
- for PQM model
- for QM model
(lower lines)



[BJS, Pawlowski, Wambach '07]

in mean field approximation

chiral transition and 'deconfinement' coincide



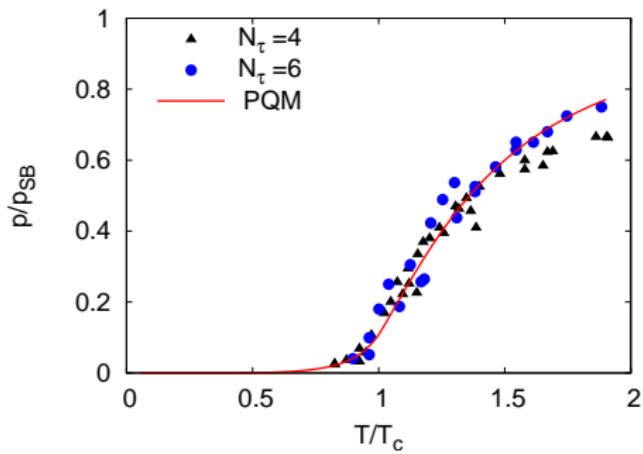
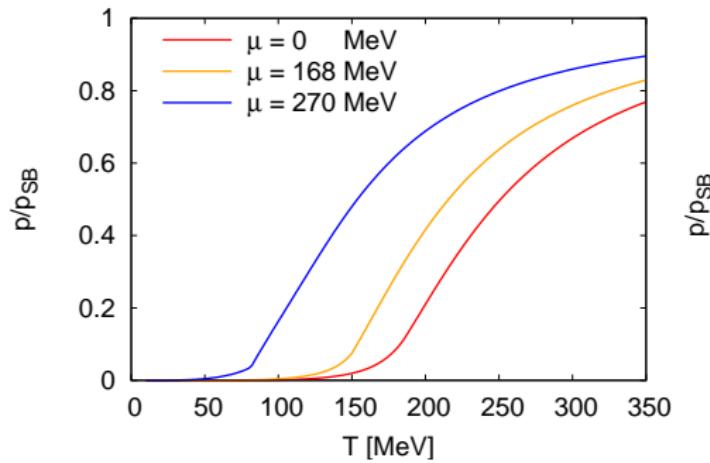
Pressure

- perturbative pressure of QCD with N_f massless quarks

$$\frac{p}{T^4} = (N_c^2 - 1) \frac{\pi^2}{45} + N_f \left[\frac{7\pi^2}{60} + \frac{1}{2} \left(\frac{\mu}{T} \right)^2 + \frac{1}{4\pi^2} \left(\frac{\mu}{T} \right)^4 \right].$$

- $N_f = 2$:

lattice: $N_\tau = 4, 6$; $\mu = 0$



[Ali Khan et al. '01]

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$N_f = 3$ quark-meson model

[BJS, M. Wagner, work in progress]

- Lagrangian $\mathcal{L} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\cancel{\partial} - g \frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q \quad \text{and } \sigma_a \text{ scalar and } \pi_a \text{ pseudoscalar nonet}$$

$$\begin{aligned}\mathcal{L}_{\text{meson}} = & \text{tr}(\partial_\mu \phi^\dagger \partial^\mu \phi) - m^2 \text{tr}(\phi^\dagger \phi) - \lambda_1 (\text{tr}(\phi^\dagger \phi))^2 \\ & - \lambda_2 (\text{tr}((\phi^\dagger \phi)^2) + c (\det(\phi) + \det(\phi^\dagger))) \\ & + \text{tr}H(\phi + \phi^\dagger)\end{aligned}$$

fields: $\phi = \sum_a \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$ and $H = \sum_a \frac{\lambda_a}{2}h_a$

symmetries:

- $H = 0, c = 0, (\lambda_2 > 0, m^2 < 0) : SU(N_f)_V \times U(N_f)_A \longrightarrow SU(N_f)_V$
- $H = 0, c \neq 0 : SU(N_f)_V \times SU(N_f)_A \longrightarrow SU(N_f)_V$
- $H \neq 0 (h_0 \neq 0, h_8 \neq 0)$: explicit sym. breaking:

$N_f = 3$ quark-meson model

[BJS, M. Wagner, work in progress]

- Lagrangian $\mathcal{L} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial - g \frac{\lambda_a}{2}(\sigma_a + i\gamma_5 \pi_a))q \quad \sigma_a \text{ scalar and } \pi_a \text{ pseudoscalar nonet}$$

$$\begin{aligned}\mathcal{L}_{\text{meson}} = & \text{tr}(\partial_\mu \phi^\dagger \partial^\mu \phi) - m^2 \text{tr}(\phi^\dagger \phi) - \lambda_1 (\text{tr}(\phi^\dagger \phi))^2 \\ & - \lambda_2 (\text{tr}((\phi^\dagger \phi)^2) + c (\det(\phi) + \det(\phi^\dagger))) \\ & + \text{tr}H(\phi + \phi^\dagger)\end{aligned}$$

fields: $\phi = \sum_a \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$ and $H = \sum_a \frac{\lambda_a}{2}h_a$

- Seven parameters: $m^2, \lambda_1, \lambda_2, c, g, h_0, h_8$
and two condensates: $\langle \phi \rangle \longrightarrow \sigma_0, \sigma_8$
- Fixed to $m_\pi, m_K, f_\pi, f_K, m_\sigma$ and $m_\eta^2 + m_{\eta'}^2$ (mesonic level)
and m_q (light quark mass)

Mean field approximation

- partition function:

$$\mathcal{Z}(T, \mu) = \int \mathcal{D}\bar{q} \mathcal{D}q \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \exp \left\{ i \int_0^{1/T} dt d^3x \left(\mathcal{L}_{N_f=3} + \sum_f \mu_f \bar{q}_f \gamma_0 q_f \right) \right\}.$$

- $\sigma_a \rightarrow \sigma_0, \sigma_8$ (otherwise 0) , $\pi_a \rightarrow \langle \pi \rangle = 0$, integrate quark/antiquarks yields

Grand canonical potential

$$\Omega(T, \mu) = -\frac{T \ln \mathcal{Z}}{V} = U(\sigma_0, \sigma_8) + \Omega_{\bar{q}q}(T, \mu)$$

with

$$\Omega_{\bar{q}q}(T, \mu) = -2N_c T \sum_f \int \frac{d^3k}{(2\pi)^3} \left\{ \ln(1 + e^{-(E_{q,f} - \mu_f)/T}) + \ln(1 + e^{-(E_{q,f} + \mu_f)/T}) \right\}$$

and mesonic potential $U(\sigma_0, \sigma_8)$

- EoM

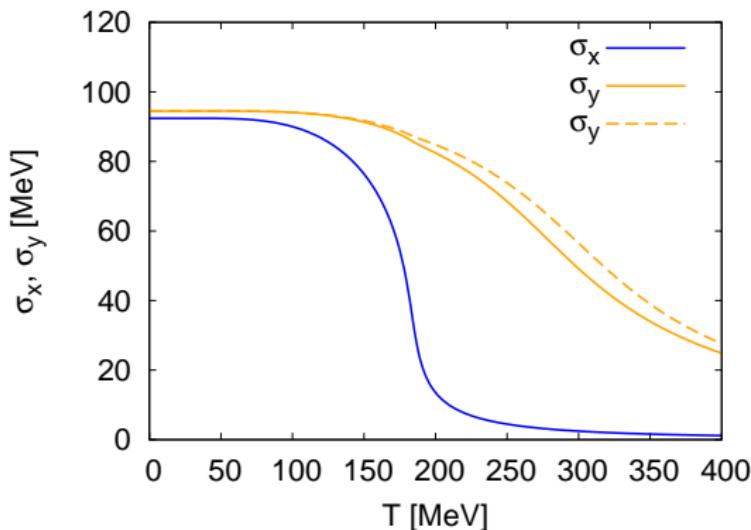
$$\frac{\partial \Omega}{\partial \sigma_0} = \frac{\partial \Omega}{\partial \sigma_8} \Big|_{\sigma_0=\sigma_x, \sigma_8=\sigma_y} = 0$$

Chiral symmetry restoration

change of base: $(\sigma_0, \sigma_8) \longrightarrow (\sigma_x, \sigma_y)$ (nonstrange, strange)

→ two condensates: nonstrange $\sigma_x(T, \mu_f)$ and strange $\sigma_y(T, \mu_f)$

with (solid) and without (dashed) $U(1)_A$ anomaly



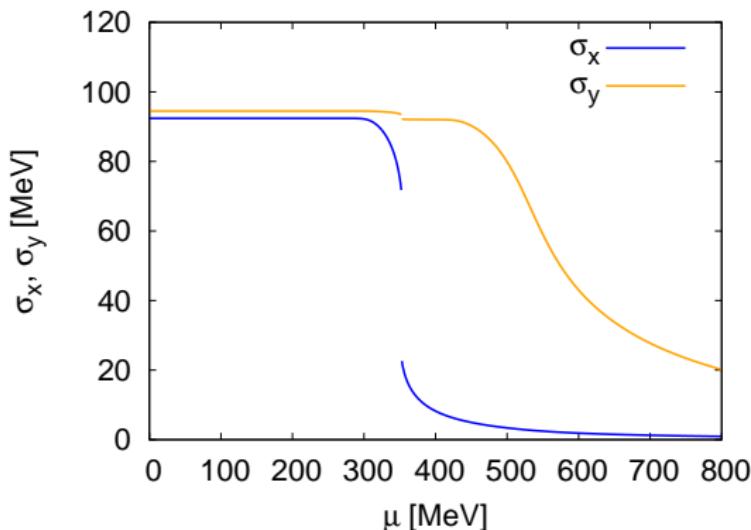
- ▷ almost no effect due $U(1)_A$ anomaly
- ▷ T_c depends on m_σ

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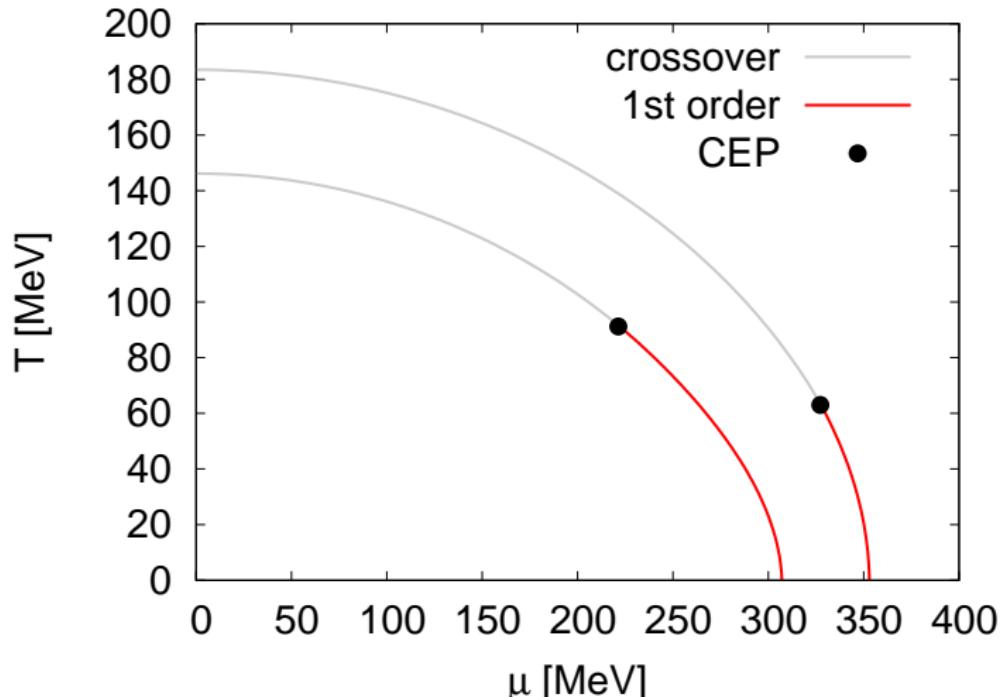


- ▷ almost no effect due $U(1)_A$ anomaly
- ▷ T_c depends on m_σ
- ▷ $\mu \equiv \mu_q = \mu_s$
- ▷ μ_c depends on m_σ

Phase diagram $N_f = 3$

for $\mu \equiv \mu_q = \mu_s$

$m_\sigma = 600$ MeV (lower lines) and $m_\sigma = 800$ MeV



In-medium meson masses

- ▷ genuine problem of linear sigma model (w/ and w/o quarks) finite T
 - negative meson masses

In-medium meson masses

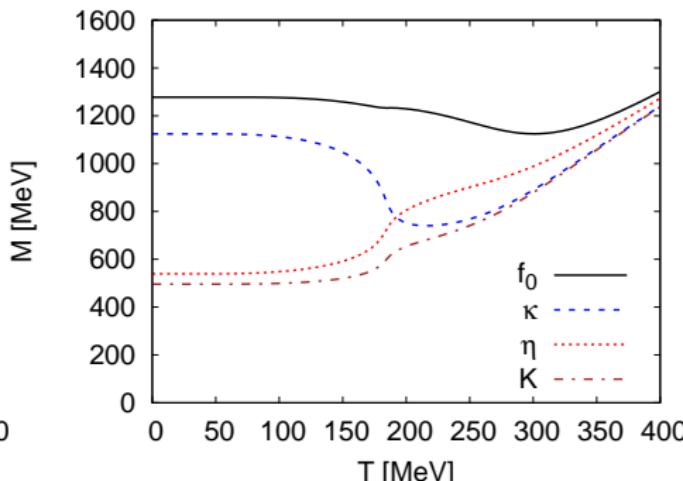
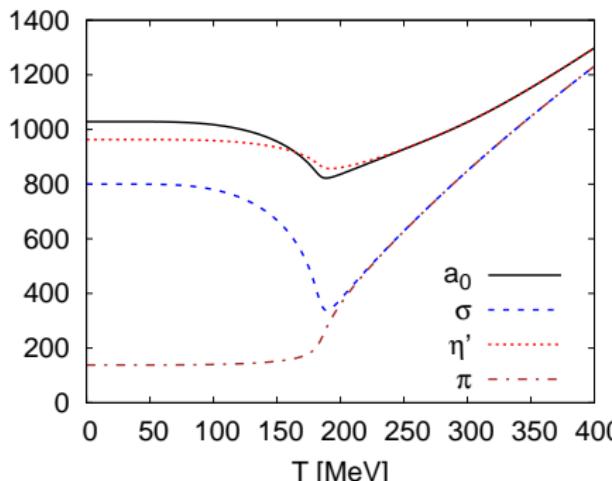
- ▷ generalize tree-level Ward identities to finite T, μ_f

$$h_x = f_\pi m_\pi^2 \quad \rightarrow \quad h_x = f_\pi(T, \mu_f) m_\pi^2(T, \mu_f)$$

similar for strange sector

$$h_y = \sqrt{2} f_K m_K^2 - \frac{1}{\sqrt{2}} f_\pi m_\pi^2$$

masses with $U(1)_A$ anomaly



In-medium meson masses

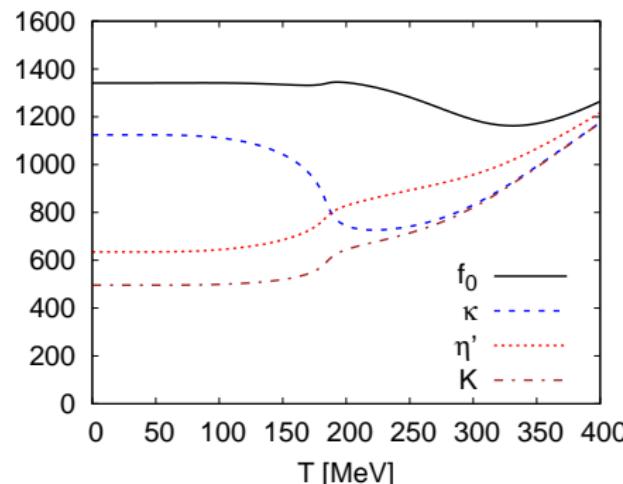
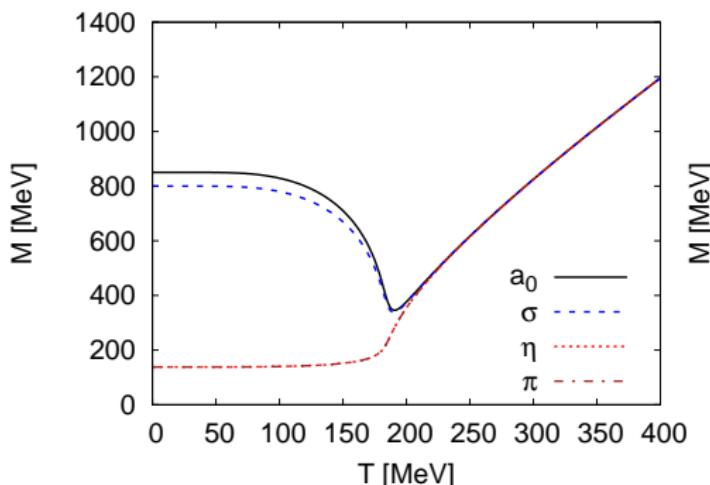
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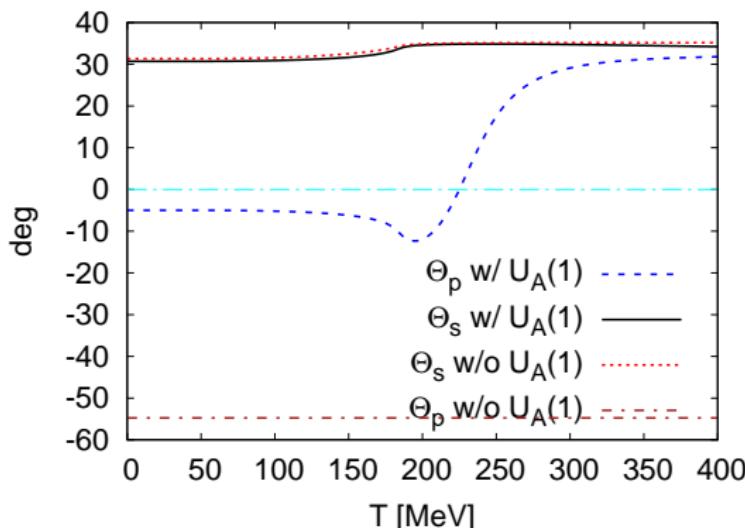
masses without $U(1)_A$ anomaly



η - η' mixing

In vacuum: physical (η, η') close to (η_8, η_0)

- mixing angle θ_p
- pseudoscalar and scalar mixing angles
as a function of T (for $\mu = 0$)
with and without $U(1)_A$ anomaly



▷ without $U(1)_A$ anomaly:

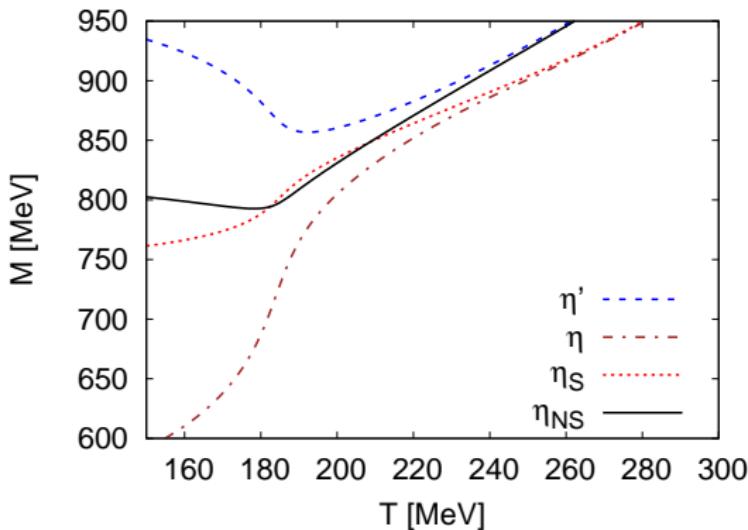
$$\begin{aligned}\eta' &\rightarrow \eta_S \\ \eta &\rightarrow \eta_{NS}\end{aligned}$$

η - η' mixing

In vacuum: physical (η, η') close to (η_8, η_0)

→ mixing angle θ_p

→ identity switching above T_c

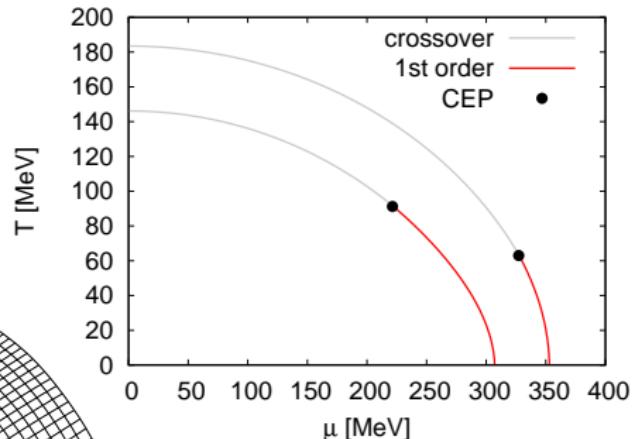
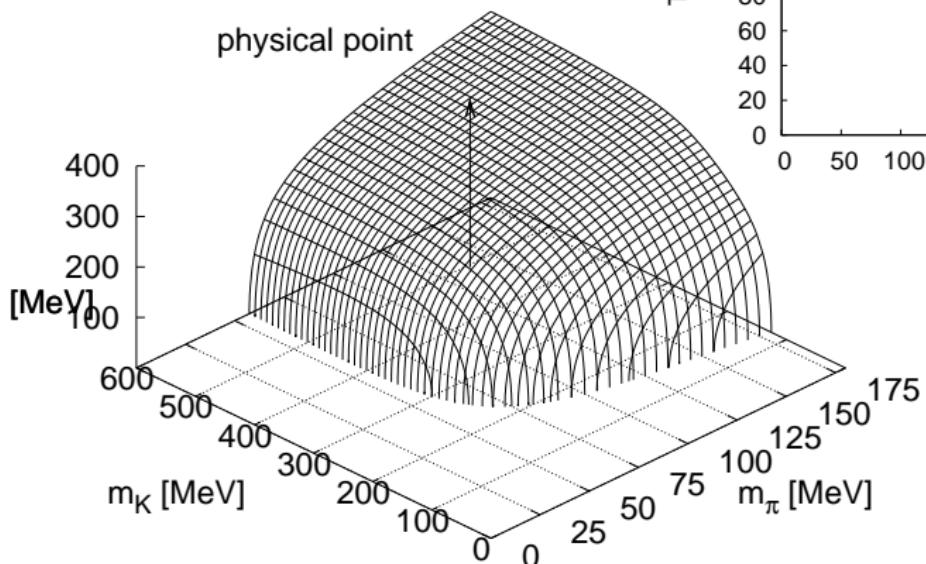


- ▷ without $U(1)_A$ anomaly:
 - $\eta' \rightarrow \eta_S$
 - $\eta \rightarrow \eta_{NS}$
- ▷ with $U(1)_A$ anomaly:
 - $\eta' \rightarrow \eta_{NS}$
 - $\eta \rightarrow \eta_S$ for $T > 250$ MeV
- ▷ no Witten-Veneziano relation has been used

Chiral critical surface

$m_\sigma = 600 \text{ MeV}$ (lower lines) and $m_\sigma = 800 \text{ MeV}$

chiral critical surface:



Summary & Outlook

Summary

Quark-meson model study for $N_F = 2$

→ Mean field versus RG

Influence of fluctuations on phase diagram

Findings:

- ▷ MF phase diagram: no TCP (in chiral limit) found
- ▷ RG phase diagram: two TCP's (in chiral limit) & CEP found
- ▷ Size of critical region via susceptibilities: “**compressed**” with fluctuations

Quark-meson model study for $N_F = 3$

→ preliminary Mean field approximation

no need for Optimized Perturbation Theory

with and without axial anomaly

Polyakov–quark-meson model study for $N_F = 2$

→ only mean-field approximation

Findings:

- Parameter in Polyakov loop potential: $T_0 \Rightarrow T_0(N_f, \mu)$

pure gauge: $T_0 \sim 270$ MeV

$N_f = 2$: $T_0 \sim 210$ MeV

- Chiral & deconfinement transition coincide
- Mean-field approximation encouraging

Quark-meson model is renormalizable

→ no UV cutoff parameter (cf. PNJL model)

Outlook

- include quark-meson dynamics in PQM model with RG
- include glue dynamics with RG → full QCD
(step by step)

