Plasma Instabilities & The Thermalization Puzzle

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(In collaboration with J. Berges and D. Sexty)

Outline of the talk

The thermalization puzzle

Plasma instabilities 101

Classical statistical gauge theory on the lattice

Extracting time scales

Conclusions & Outlook

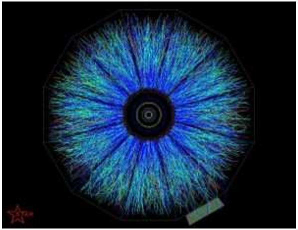
Outline of the talk

The thermalization puzzle

Applicability of hydrodynamic models at RHIC suggests thermalization within $\tau_{\text{eq}} \lesssim 0.6\,\text{fm}/c$.

Perturbatively, one finds $\tau_{\text{eq}} \gtrsim 2.6\,\text{fm}/c$, however.

(Heinz, nucl-th/0407067; Mrówczyński, NPA 774)



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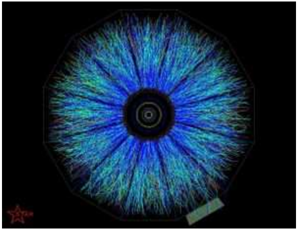

Possible ways out:

- No complete thermalization needed for hydrodynamics, *isotropization* & *prethermalization* should be sufficient.
  (Arnold et al., PRL 94; Berges et al., PRL 93)
- Local thermal equilibrium found in Boltzmann approaches for $t \lesssim 1$ fm/c (Xu, Greiner, PRC 70) but there are conceptual problems.
- Mechanisms other than perturbative scattering are relevant: $\rightsquigarrow$ *Plasma instabilities* (Arnold et al., JHEP 0308)

Instabilities & QGP- physics

An instability occurs if there is a dispersion relation $\Omega = \omega + i\gamma$ giving rise to an exponentially growing solution.

It has been known for a long time that instabilities exist in el.-magn. plasmas if the momentum distribution of the charge carriers is anisotropic (Weibel, PRL 2, 1959) .

But what about the quark-gluon plasma?

Instabilities & QGP- physics

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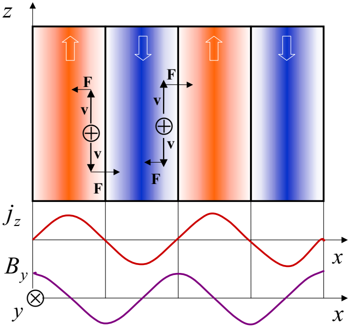
But what about the quark-gluon plasma?

- Indeed, instabilities have been predicted to exist in the QGP as well. (Mrówczyńsky, Phys. Lett. B214; Romatschke & Strickland, PRD 68; Arnold et al., JHEP 0308)
- Intense study of plasma instabilities in the framework of Vlasov- equations and HTL- approximations in the literature (e. g. Arnold et al., PRD 72; Dumitru et al., PRD 75; ...)
- So far, it seems as if instabilities are far too slow to explain the experimentally observed phenomena.

Outline of the talk

Cartoon of an instability



(Figure taken from S. Mrowczynski, hep-ph0511052)

- Consider particles with momenta $\vec{p} \perp \vec{q}$.
- Charges get 'trapped'
- 'Filamentation'
- Resulting current amplifies the existing B- field.
- $\Rightarrow$ Magnetic Instability

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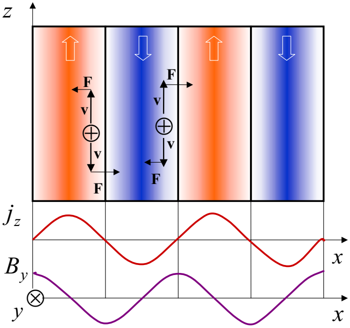

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- Charges get 'trapped'
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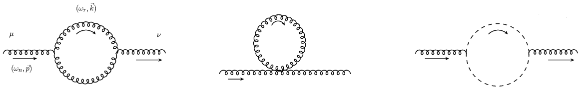
In contrast, particles with momenta $\vec{p} \cdot \vec{q} \neq 0$ have a stabilising effect.

Instabilities from HTL- calculations

Compute self-energy in HTL[1]- approximation:

$\Pi_{\mu\nu} =$ 

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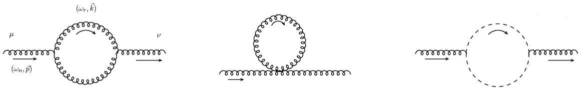

With momentum space particle distribution $f(\vec{k})$ and $\vec{v} := \vec{k}/|\vec{k}|$):

$$\Pi_{ij}(p) \simeq -g^2 N(d-2) \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{f(\vec{k})}{|\vec{k}|}(v_i v_j + g_{ij}) + \frac{\partial f}{\partial k^l}(\vec{k}) \frac{p_l v_i v_j}{p_0 + \vec{v}\cdot\vec{p}} \right\}$$

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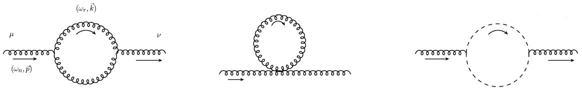
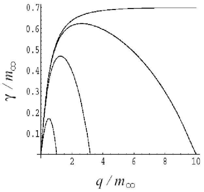


Then look for zeros $\Omega(\vec{p}) = \omega + i\gamma$ in the inverse gluon propagator $G^{-1}_{\mu\nu}(p) = G^{-1}_{0,\mu\nu}(p) - \Pi_{\mu\nu}(p)$ and find growth rates:



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Outline of the talk

Objectives

Long-term aim: Understand QCD- thermalization from first principles within appropriate approximations.

More concrete:

- Find out how requirements for hydrodynamics are fulfilled
- Understand the early stages of the thermalization process (isotropization, prethermalization)
- Verify / falsify the scenario of instability driven isotropization

Approach

- Study the classical statistical limit of pure SU(2)- gauge theory
- Static geometry, i. e. no expansion
- Lattice discretization
- Anisotropic initial conditions

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- Anisotropic initial conditions

Advantages:

- Classical statistical approximation reliable for high occupation numbers
- Controlled approximation of the underlying field theory
- Conceptually clear limit, no modelling

Implementation

Use common lattice discretization scheme:

Link variables: $U_{x,\mu} := e^{igaA_\mu(x)}$

Plaquette variables: $U_{x,\mu\nu} := U_{x,\mu} U_{(x+\hat{\mu}),\nu} U^{-1}_{(x+\hat{\nu}),\mu} U^{-1}_{x,\nu}$

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Dynamics from Wilson- lattice action in Minkowski-spacetime (Ambjorn et al. NPB 353) :

$$
\begin{aligned}
S[U] = & -\beta_0 \sum_x \sum_i \left\{ \frac{1}{2\mathrm{tr}\,\mathbb{1}} \left(\mathrm{tr}\, U_{x,0i} + \mathrm{tr}\, U^\dagger_{x,0i} \right) - 1 \right\} \\
& + \beta_s \sum_x \sum_{\substack{i,j \\ i<j}} \left\{ \frac{1}{2\mathrm{tr}\,\mathbb{1}} \left(\mathrm{tr}\, U_{x,ij} + \mathrm{tr}\, U^\dagger_{x,ij} \right) - 1 \right\}
\end{aligned}
$$

$$
\beta_0 := \frac{2\gamma \mathrm{tr}\,\mathbb{1}}{g_0^2}\,,\ \beta_s := \frac{2\mathrm{tr}\,\mathbb{1}}{\gamma g_s^2}\,,\ \gamma := \frac{a_s}{a_t}
$$

Variation w. r. t. spatial links $\Rightarrow$ Equations of motion
Variation w. r. t. temporal links $\Rightarrow$ Gauss constraint
We use temporal axial gauge, i. e. $A_0 \equiv 0$.

Initial conditions

Compute e. g. correlators as

$$\langle A(t,\vec{x})A(t',\vec{y})\rangle = \int \mathcal{D}A(t=0)\,\mathcal{D}\dot{A}(t=0)\,P[A(0),\dot{A}(0)]\,A(t,\vec{x})A(t',\vec{y})$$

with $P[A(0),\dot{A}(0)]$ such that

$$\langle A^a_j(t=0,\vec{k})A^a_j(t=0,-\vec{k})\rangle \sim C\exp\Big\{-\frac{k_x^2+k_y^2}{2\Delta_x^2}-\frac{k_z^2}{2\Delta_z^2}\Big\}\,,$$

$$\Delta_x \gg \Delta_z \;\; \text{(anisotropy)}$$

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$$\Delta_x \gg \Delta_z \;\; \text{(anisotropy)}$$

N.B.:

- Set $\vec{E}(t=0)\equiv 0$ ($\rightarrow$ Gauss constraint fulfilled)
- Distribution $\delta(k_z)-$ like on the lattice
- Amplitude C determined from the energy density

Fixing the units

Need to relate the lattice spacing a_s to physical units.

Do this using the energy density:

$$\epsilon = \hat{\epsilon} \cdot a_s^{-4} \;\Rightarrow\; a_s = \sqrt[4]{\frac{\hat{\epsilon}}{\epsilon}}$$

- Assumption: $g_0 = g_s = g = 1$.
- For $g \neq 1$, $a_s \propto 1/\sqrt{g}$, i. e. 'mild' dependence.
- Take ϵ from the literature (Gyulassy, McLerran, NPA 750) .

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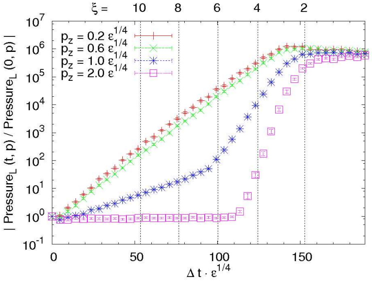
Phenomenology of instabilities

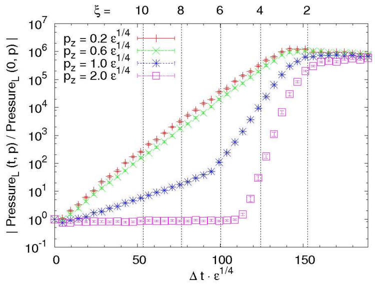


$$T_{33}(t,\vec{x}) = \frac{1}{2}\,\mathrm{tr}\left[B_1(t,\vec{x})^2 + B_2(t,\vec{x})^2 - B_3(t,\vec{x})^2 + \text{ same for } \vec{E}\,\right]$$

Then plot $\left|\dfrac{T_{33}(t,p_z\hat{z})}{T_{33}(t=0,p_z\hat{z})}\right|$.

Bulk anisotropy: $\xi(t) := \log_{10}\left\{\dfrac{\sum_{\vec{p}}(p_x^2+p_y^2)\left(\sum_{j=1}^3\sum_{a=1}^3 |\tilde{A}_j^a(t,\vec{p})|^2\right)}{\sum_{\vec{q}} q_z^2\left(\sum_{k=1}^3\sum_{b=1}^3 |\tilde{A}_k^b(t,\vec{q})|^2\right)}\right\}$

Phenomenology of instabilities

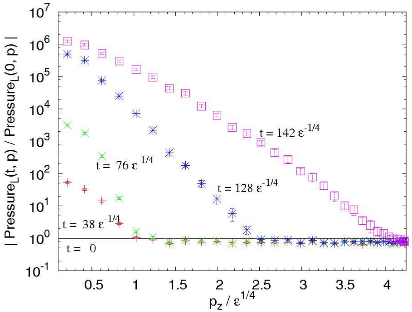


Find *primary* & *secondary* instabilities, qualitatively similar to parametric resonance in scalar field theory.

- Times are very large ($\epsilon^{-1/4} \sim 0.4\,\mathrm{fm/c}$)
- High degree of anisotropy till $\Delta t \simeq 120\epsilon^{-1/4}$
- Physical zero point of time somewhat arbitrary
- $\rightsquigarrow$ consider growth rates
- Will see that secondary instabilities are driven by fluctuations

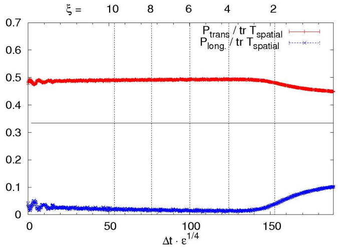
Phenomenology of instabilities (II)

A slightly different perspective:



- Narrow band of low-momentum modes unstable initially
- Find sth. similar to a "cascade" (Arnold, Moore, PRD 73) or an "avalanche" (Dumitru, Nara, Strickland, PRD 75) to the UV at intermediate times

Isotropization?



Bulk pressure does not isotropize!

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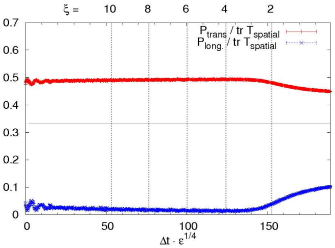

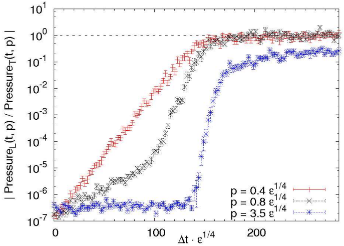

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Compute $\left| \frac{T_{||}(t, \vec{p}_{||})}{T_{\perp}(t, \vec{p}_{\perp})} \right|_{|\vec{p}_{||}|=|\vec{p}_{\perp}|}$

Fields become isotropic in the IR- regime ($p \lesssim \epsilon^{1/4}$);

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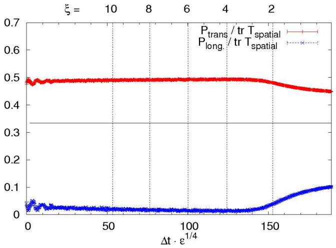

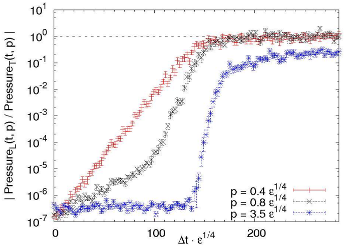

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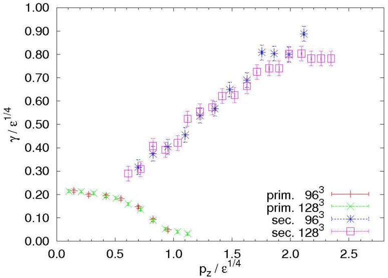
$\rightsquigarrow$ "bottom-up isotropization"

Growth rates & time scales

Growth rates for $|A(t,\vec{p})|^2$ ($\sim$ particle number):



ϵ	$1/\gamma_{\max}^{(\mathrm{pr})}$	$1/\gamma_{\max}^{(\mathrm{sec})}$
30 GeV/fm^3	1.0 fm/c	0.3 fm/c
1 GeV/fm^3	2.6 fm/c	0.8 fm/c

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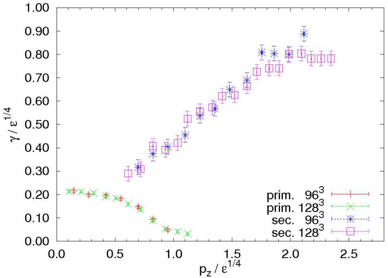

- $\gamma^{-1} \sim 1$ fm/c, ok!
- However, need $\sim 4\gamma^{-1}$
- Rôle of secondaries depends strongly on initial conditions
- IR becomes isotropic first

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Fluctuation effects & secondaries

Correlation function $F^{ab}_{\mu\nu}(x,y) := \langle\, A^a_\mu(x) A^b_\nu(y)\,\rangle$ obeys a 2PI- evolution equation:

$$[D_0^{-1}]^\gamma_\mu F_{\gamma\nu}(x,y) = \int_{t_0}^{x^0} dz \Pi^\gamma_{(\rho)\mu}(x,z)\, F_{\gamma\nu}(z,y) - \int_{t_0}^{y^0} dz \Pi^\gamma_{(F)\mu}(x,z)\, \rho_{\gamma\nu}(z,y)$$

(ρ : Poisson bracket) (Berges, PRD 70)

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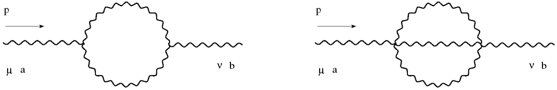
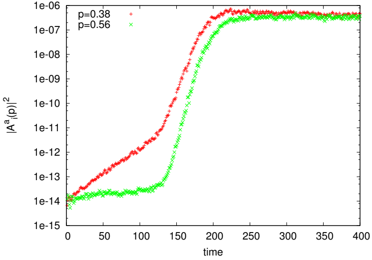
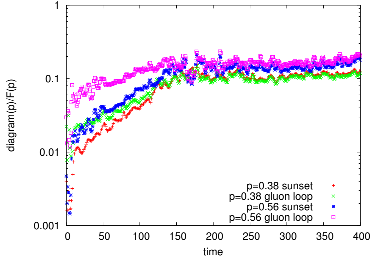
Try to identify times when certain diagrams make $\mathcal{O}(1)$-contributions to the self-energy $\Pi_{\mu\nu}$.



Diagrams & secondaries





Upper panel:

$$|\,A(t,\vec{p})\,|^2$$

Lower panel:

$$\left|\,\frac{\text{diagram}(\vec{p})}{F(\vec{p})}\,\right|$$

$\vec{p}$ chosen as for gauge field in upper panel.

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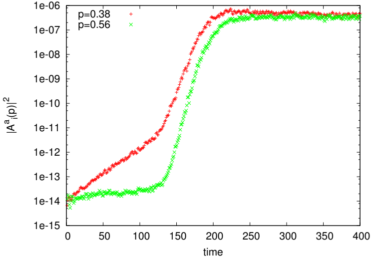

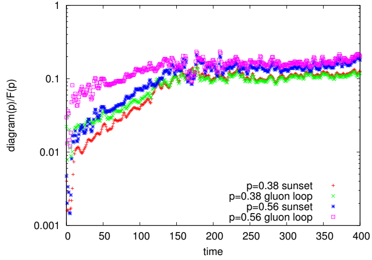

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Lower panel:

$$\left|\, \frac{\text{diagram}(\vec{p})}{F(\vec{p})} \,\right|$$

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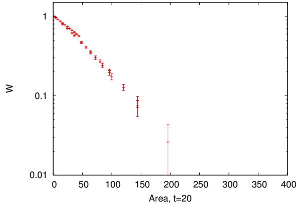
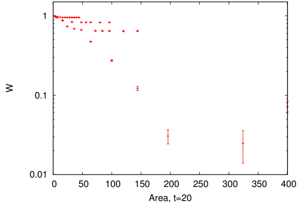
Onset of secondaries coincides with fluctuation effects becoming large, analogous to parametric resonance in scalar theories.

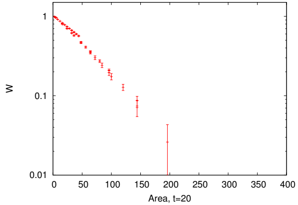
Spatial Wilson loop

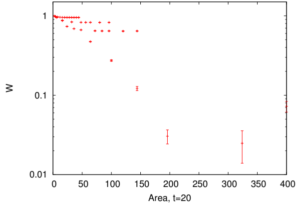




Spatial Wilson- loops at early times ($\simeq 10\,\epsilon^{-1/4}$) in the transverse (top) and mixed plane (bottom).

Spatial Wilson loop





Spatial Wilson- loops at early times ($\simeq 10\,\epsilon^{-1/4}$) in the transverse (top) and mixed plane (bottom).

- See area law in the transverse plane
- Physics non-perturbative
- Later, longitudinal loops obey area law, too.

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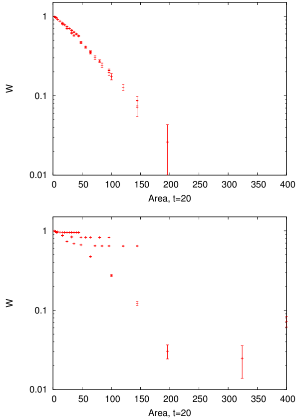

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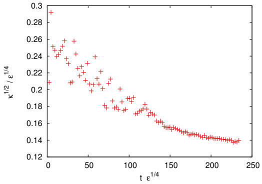
- See area law in the transverse plane
- Physics non-perturbative
- Later, longitudinal loops obey area law, too.
- Do not see convergence at late times yet:



'String tension' vs. time.

Outline of the talk

Summary

- Have studied the classical statistical limit of SU(2)-gauge theory
- Confirmed existence of instabilities for anisotropic initial conditions
- Low-momentum sector becomes isotropic first
- Identified secondary instabilities as driven by fluctuations
- Growth rates still seem to be too small

Outlook

Conceivable resolutions of the thermalization puzzle:

- Fermions could speed up the thermalization, in particular in the UV
- Going to SU(3) might change growth rates.
- Initial conditions are unrealistic or miss out some important features.

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Outlook on future work:

- Check what happens in SU(3) (only minor changes expected, though)
- Apply 2PI- techniques, including fermions

Thanks for your attention.