Phase transitions and spectral functions from Dyson-Schwinger equations

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07th of May 2010

Work together with Axel Maas and Jens Mueller

C.F., PRL **103** (2009) 052003.

C.F. and J. A. Mueller, PRD **80** (2009) 074029.

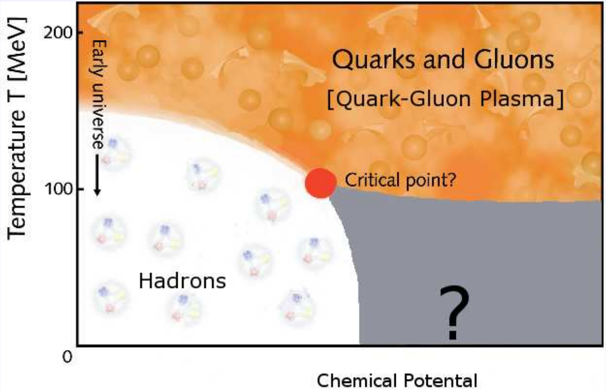
C.F., A. Maas and J. A. Mueller, arXiv:1003.1960

J. A. Mueller, C.F. and D. Nickel in preparation

Content

Content

QCD phase transitions



- Chiral limit ($m \to 0$): order parameter chiral condensate
- Static quarks ($m \to \infty$): order parameter Polyakov-loop

Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant

- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Chemical potential: no sign problem

QCD in covariant gauge

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c]\ \exp\left\{ - \int_0^{1/T} dt \int d^3x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge ($\xi = 0$) propagators in momentum space, $q = (\vec{q}, \omega_q)$:

$$D^{\text{Gluon}}_{\mu\nu}(q) = \frac{Z_T(q)}{q^2} P^T_{\mu\nu}(q) + \frac{Z_L(q)}{q^2} P^L_{\mu\nu}(q)$$

$$S^{\text{Quark}}(q) = \frac{1}{-i\,\vec{\gamma}\vec{q}\ A(q) - i\,\gamma_4\omega_n\ C(q) + B(q)}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

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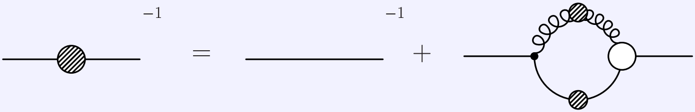
The ordinary chiral condensate



- input: gluon propagator and quark-gluon vertex
- quarks: antiperiodic boundary conditions
- Order parameter for chiral transition:

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3p}{(2\pi)^3}\; Tr_D\; S(p_{\vec{p},\omega_p})$$

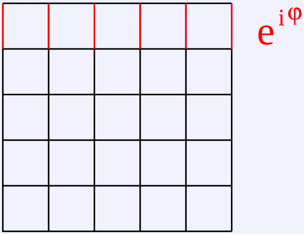
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



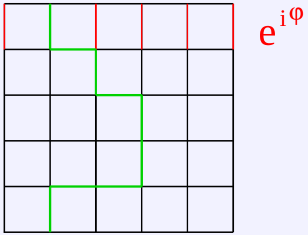
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

The dual condensate II

Relation of condensate to loops of link variables $U_\mu(x)$:

$$\langle \overline{\psi}\psi \rangle_\varphi = Tr\,[m + D_\varphi]^{-1} = \frac{1}{Vm} \sum_{l \in \mathcal{L}} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c \prod_{(x,\mu)\in l} s(l) U_\mu(x)\,.$$

- geometric series of inverse staggered Dirac operator
- winding number $n(l)$ of loop l around temporal direction



E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

The dual condensate III

Then define dual condensate Σ_n:

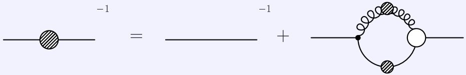
$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle \overline{\psi}\psi \rangle_\varphi$$

- $n = 1$ projects out loops with $n(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop: order parameter for center symmetry breaking
- Σ_1 is accessible with functional methods

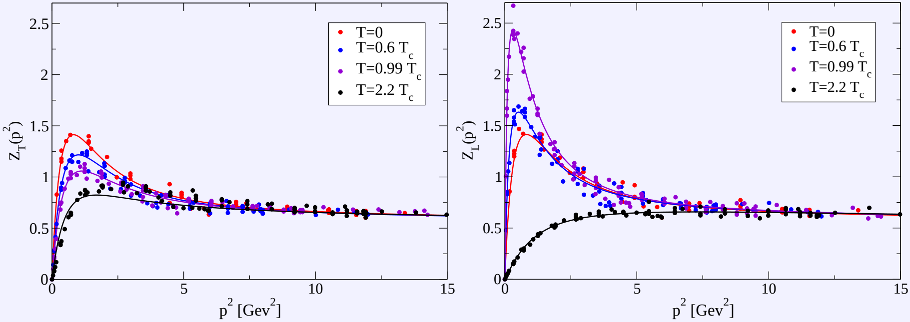
  C.F., PRL **103** (2009) 052003

C. Gattringer, PRL **97**, 032003 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** 094007 (2008).
F. Synatschke, A. Wipf and K. Langfeld, PRD **77**, 114018 (2008).
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, arXiv:0908.0008 [hep-ph]

T-dependent gluon

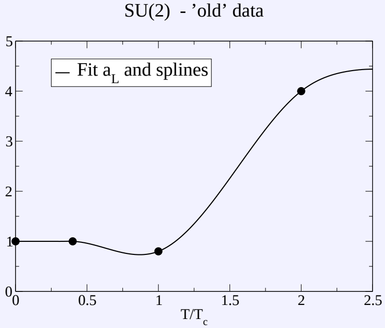


- T-dependent gluon propagator from lattice data



C.F., Maas and Mueller, arXiv:1003.1960

T-dependent gluon screening mass (old!)



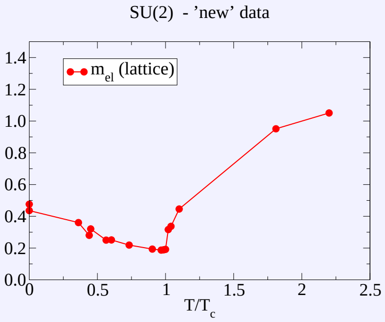
Cucchieri, Maas, Mendes, PRD75 (2007)

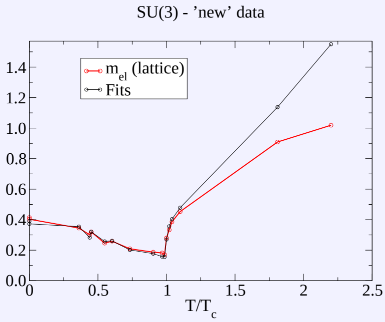
C.F., PRL **103** (2009) 052003.

C.F. and J. A. Mueller, PRD **80** (2009) 074029.

$$Z_{T,L}(q,T) = \frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left\{\left(\frac{c}{q^2+\Lambda^2 a_{T,L}(T)}\right)^2 + \frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^\gamma\right\}$$

T-dependent gluon screening mass (new!)



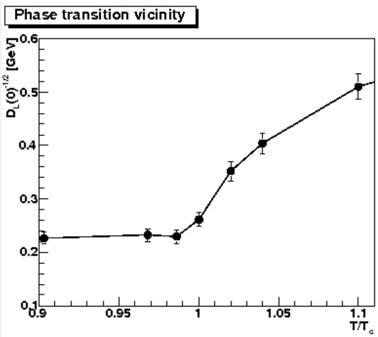


C.F., Maas, Mueller, arXiv:1003.1960

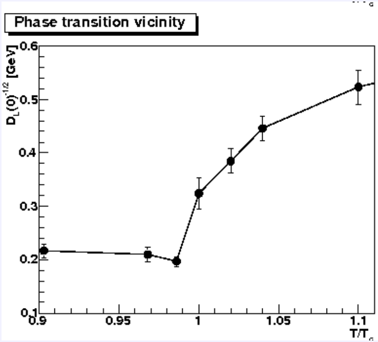
$$Z_{T,L}(q,T) = \frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left\{\left(\frac{c}{q^2+\Lambda^2 a_{T,L}(T)}\right)^{b_{T,L}(T)} + \frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^{\gamma}\right\}$$

T-dependent gluon screening mass at T_c

SU(2)



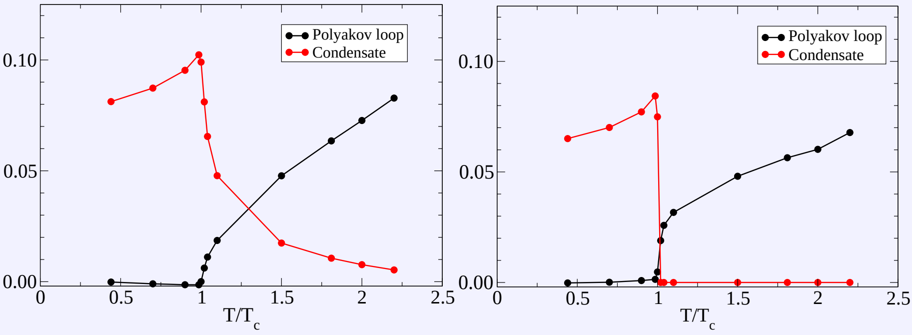
SU(3)



C.F., Maas, Mueller, arXiv:1003.1960

- first or second order ? refined calculation necessary!

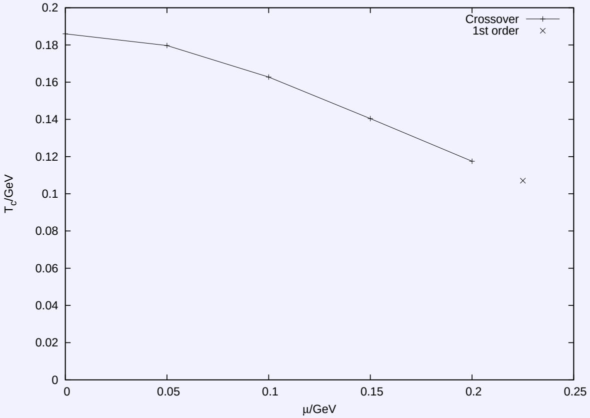
Transition temperatures



C.F., Maas, Mueller, arXiv:1003.1960.

- similar transition temperatures for chiral and deconfinement
- SU(2): $T \approx 305$ MeV SU(3): $T \approx 270$ MeV
- increasing chiral condensate due to electric screening masses

A first exploration of the phase diagram



C.F., Luecker, Mueller, work in progress

Content

Framework

- Vital input into transport approaches
- Computation of quark-loop contribution of dilepton production

  Braaten, Pisarski, Yuan, PRL **64**, 1990

Idea: Fit spectral representation to quark propagator

F. Karsch and M. Kitazawa, PRD **80**, 056001 (2009).
F. Karsch and M. Kitazawa, PLB **658**, 45 (2007).

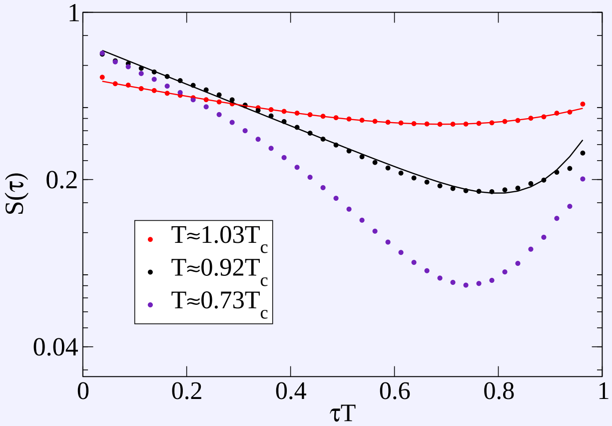
$$S(\omega_p, \vec{p}) = \int d\omega' \, \frac{\rho(\omega', \vec{p})}{\omega_p - \omega'}$$

Use ansatz for spectral function:

$$\rho(\omega) = Z_1 \; \delta(\omega - E_1) + Z_2 \; \delta(\omega + E_2)$$

Pseudoparticles: Quark and Plasmino

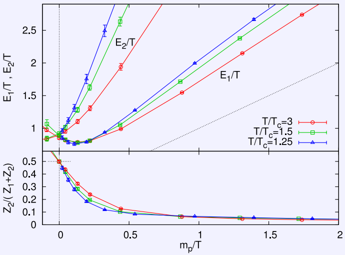
Results I



- $T > T_c$: Two pole ansatz works: quark and plasmino
- $T < T_c$: Positivity violations; two pole fit does not work at all

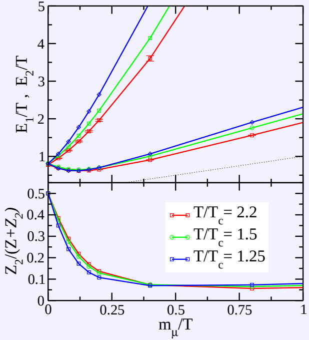
Results II

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

DSE

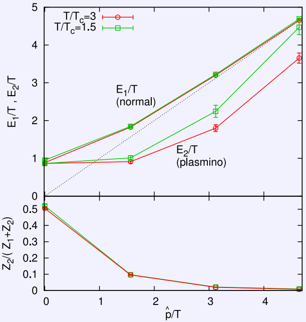


Mueller, C.F., Nickel in preparation

- Qualitative agreement with lattice results
- Large quark masses: plasmino disappears

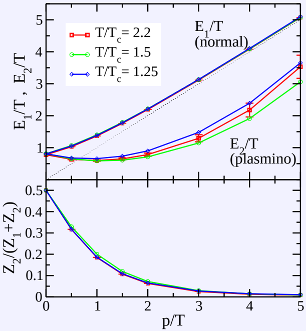
Results III: Dispersion Relation

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

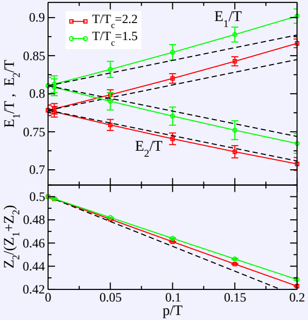
DSE

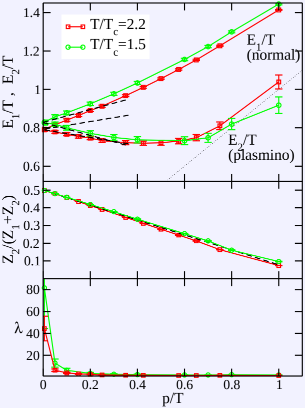


Mueller, C.F., Nickel in preparation

- Min for Plasmino at $p \neq 0$
- Plasmino enters spacelike region -> include continuum in fit

Results IV: Dispersion Relation - Details





- Dashed lines: HTL-result of slope at $p \to 0$.
- Include continuum part (Landau damping) into fits:

$$\rho_\pm(p_0,p) = 2\pi\left[Z_1\delta(p_0 \mp E_1) + Z_2\delta(p_0 \pm E_2)\right] + \lambda\left(1 - \frac{p_0^2}{p^2}\right) e^{-p_0^2}\,\Theta\left(1 - \frac{p_0^2}{p^2}\right)$$

Mueller, C.F., Nickel in preparation

Summary and outlook

Summary:

- Temperature dependent gluon propagator:
  characteristic behaviour of electric screening mass at T_c
- Similar T_c from dressed Polyakov-loop calculated from DSEs
- Similar T_c from positivity violations of quark
- Similar chiral T_c from ordinary quark condensate

Outlook:

- Unquenching
- Finite chemical potential beyond mean field
- Thermodynamic observables

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TECHNISCHE UNIVERSITÄT DARMSTADT

HELMHOLTZ GEMEINSCHAFT

GSI

HIC for FAIR

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

EMMI

Helmholtz-Alliance: Extremes of density and temperature; cosmic matter in the laboratory

Gluon and Quark-Gluon-Vertex

Ansatz for Quark-Gluon-Vertex:

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right) .$$