Cold Quark Matter,

or “neutron” stars to 3 loops

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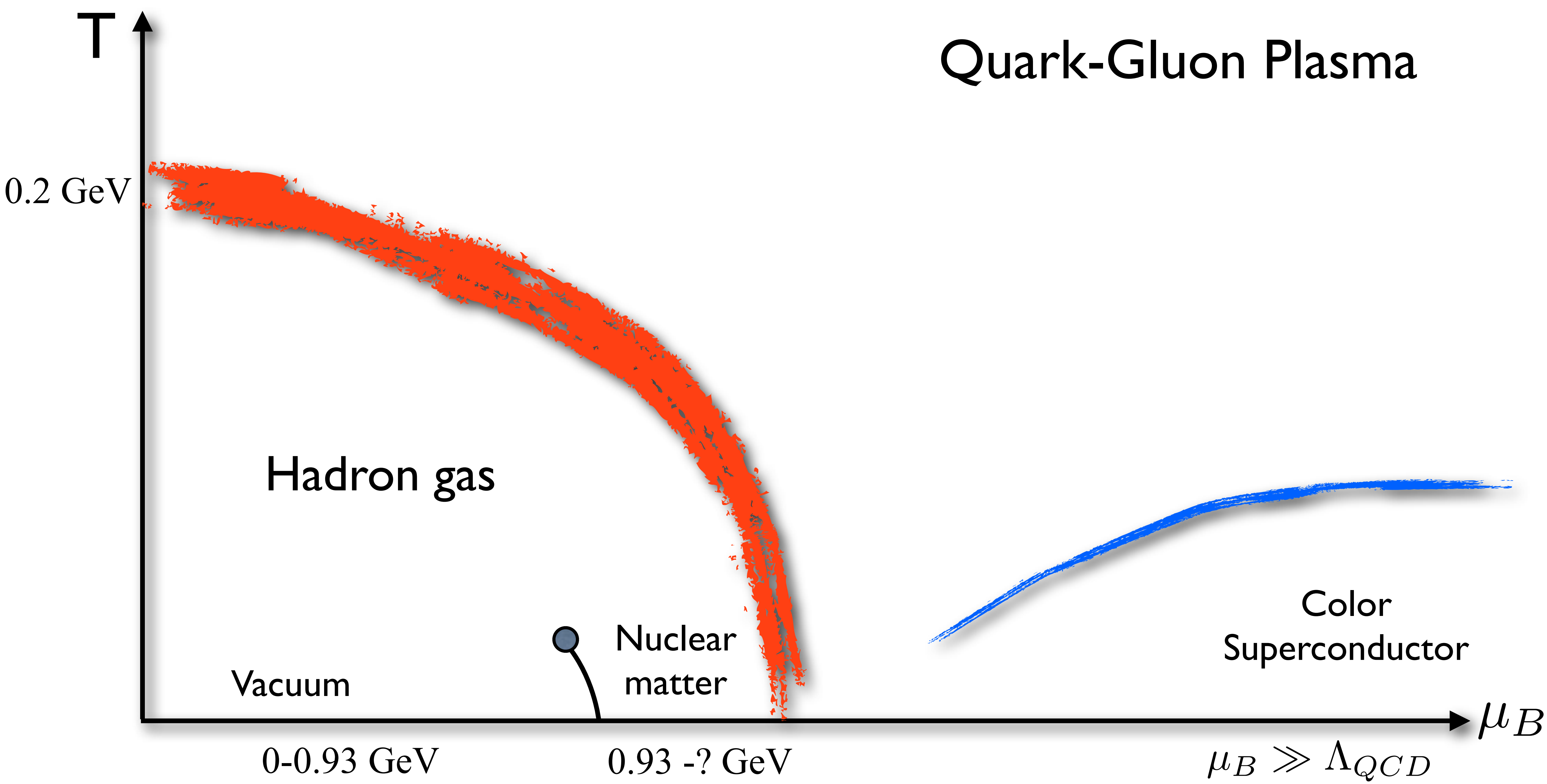
arXiv: 0912.1856

8.5. Heidelberg

Ultimate goal to understand strongly interacting matter throughout the phase diagram:



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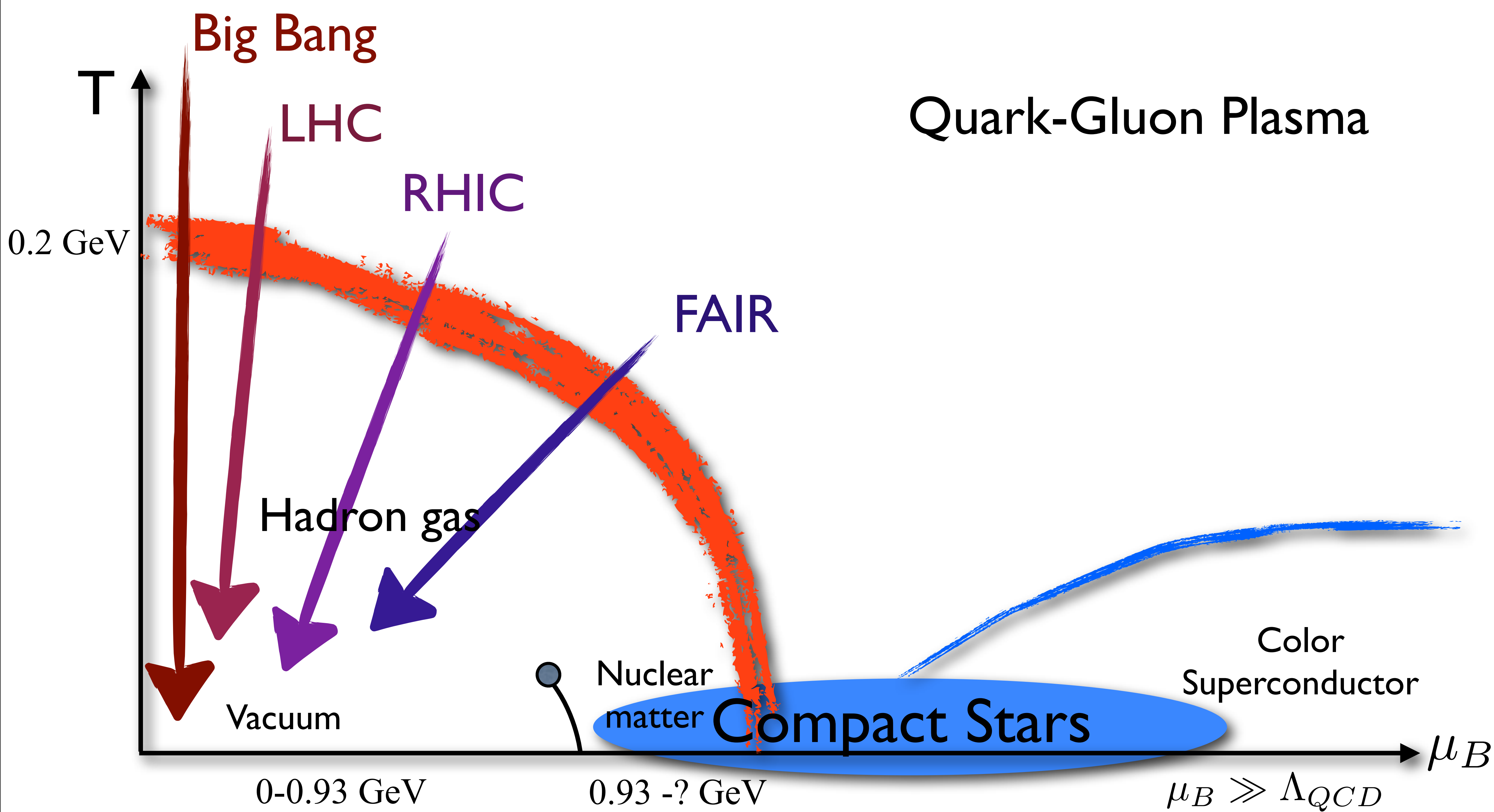
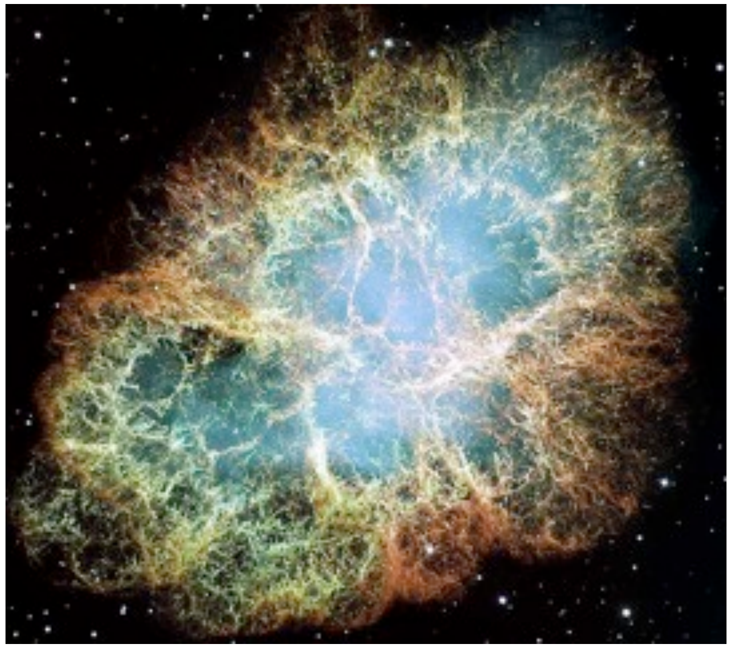
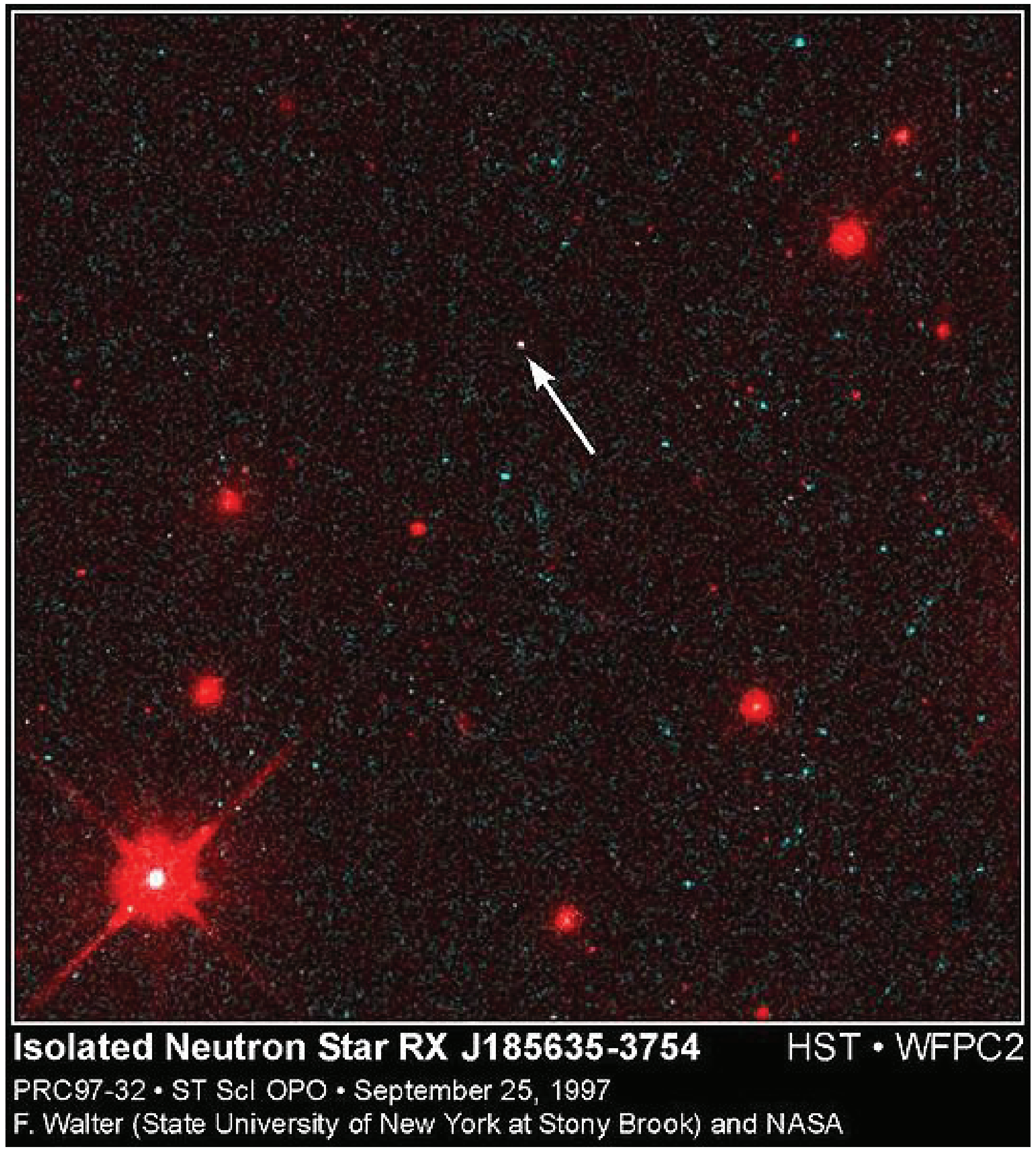


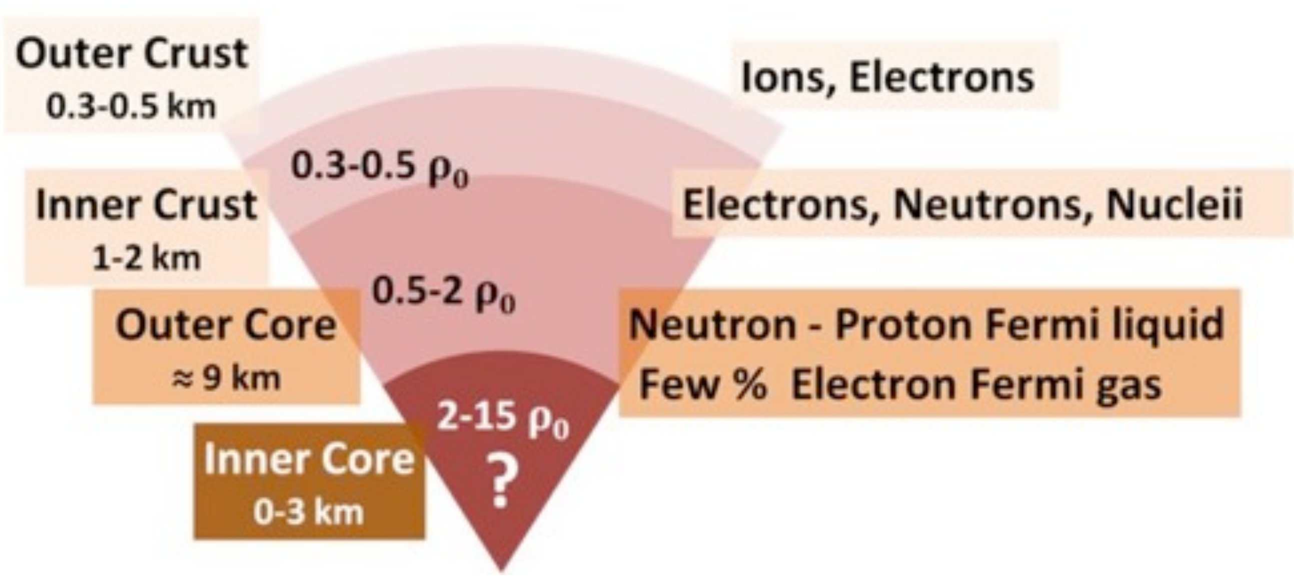
Compact Stars:

Results from a core collapse of a star with $M \gtrsim 10 M_\odot$

- Masses $\lesssim 2.0 M_\odot$
- Radii $\sim 15\text{km}$
- $T < 10^6\text{K} < \text{KeV} \ (T_0 \sim 30\text{MeV})$
- $n \lesssim 15\rho_0 \quad (\rho_0 = 0.16\text{fm}^{-3})$







Outline:

- Introduction
- Computation of the grand potential to three loops with m_s
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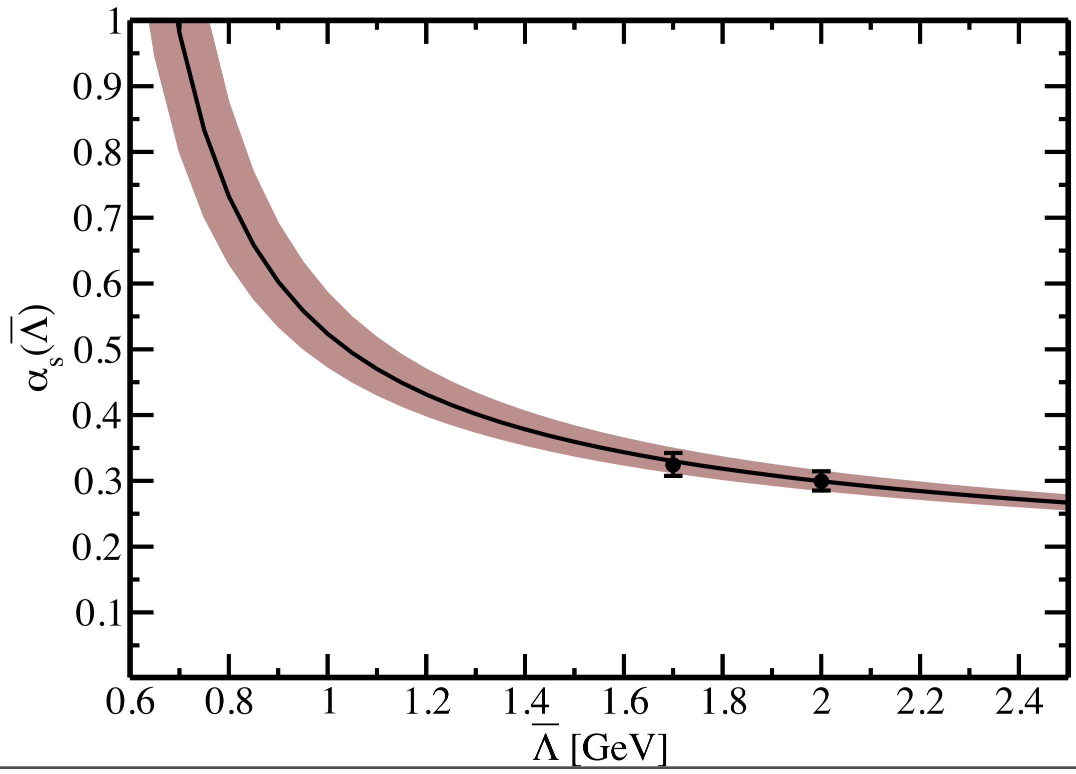
  - Resort to approximations ($T = 0$):
    - At low densities ($\rho_B < 0.16$ fm^{-3}, $\mu_B \sim$ 1GeV): Quantum many-body theory. Dynamics of nucleons, hyperons, etc..
    - At (asymptotically) high densities:

$$\alpha_s(\mu) \sim 1/\log(\mu^2)$$

⟶ Perturbation theory



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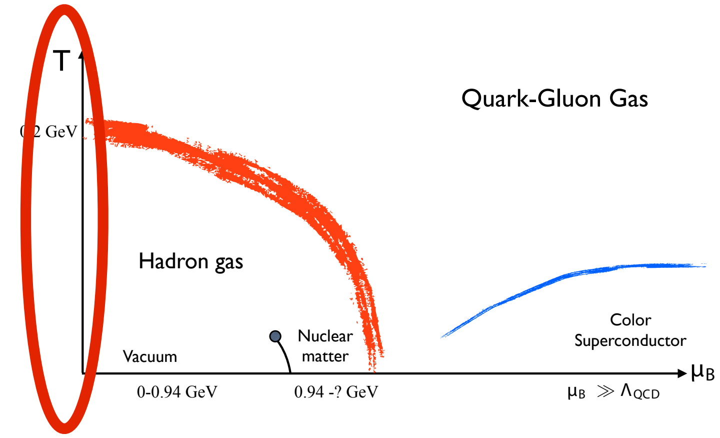
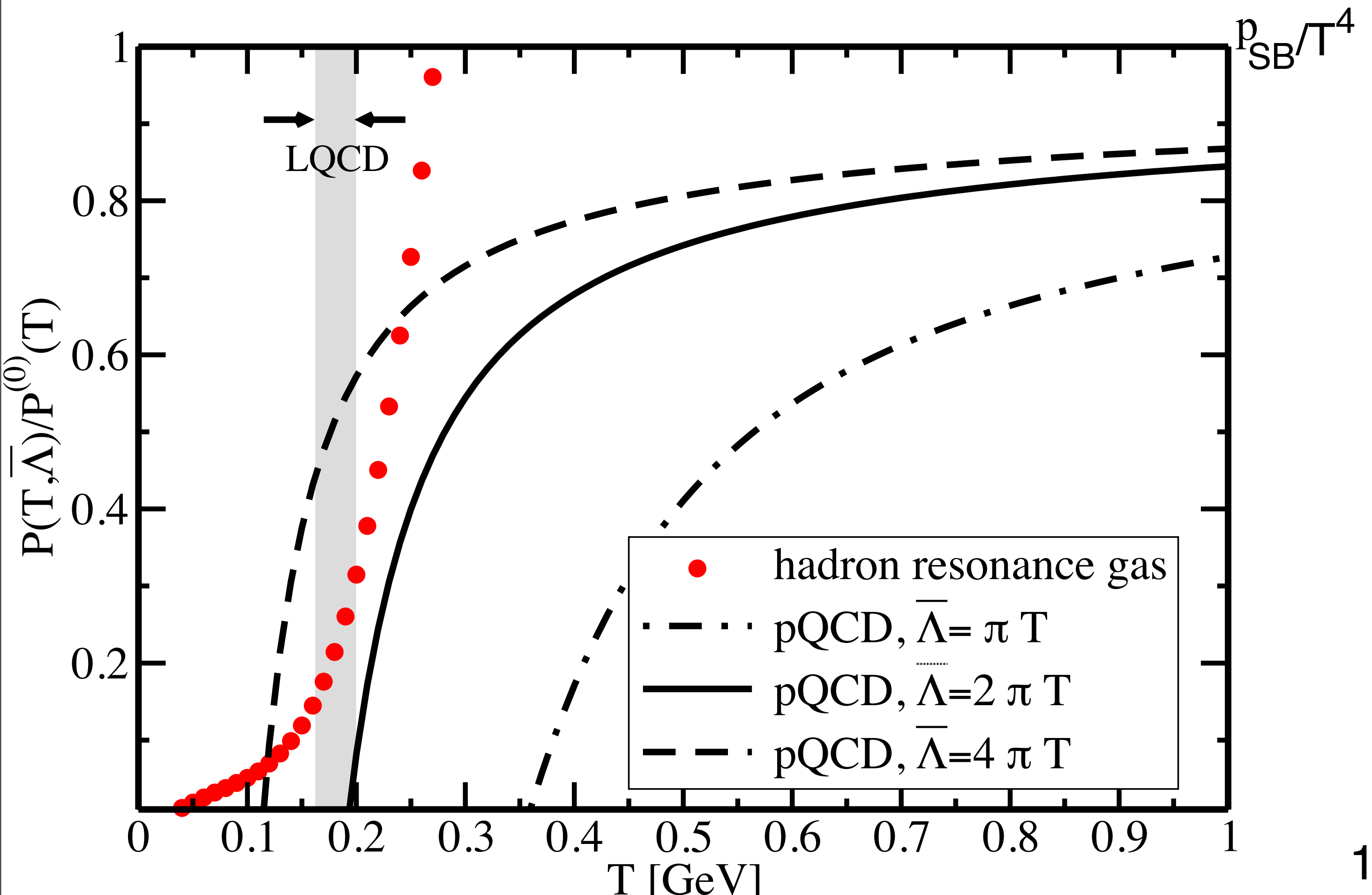
$\longrightarrow$ Perturbation theory

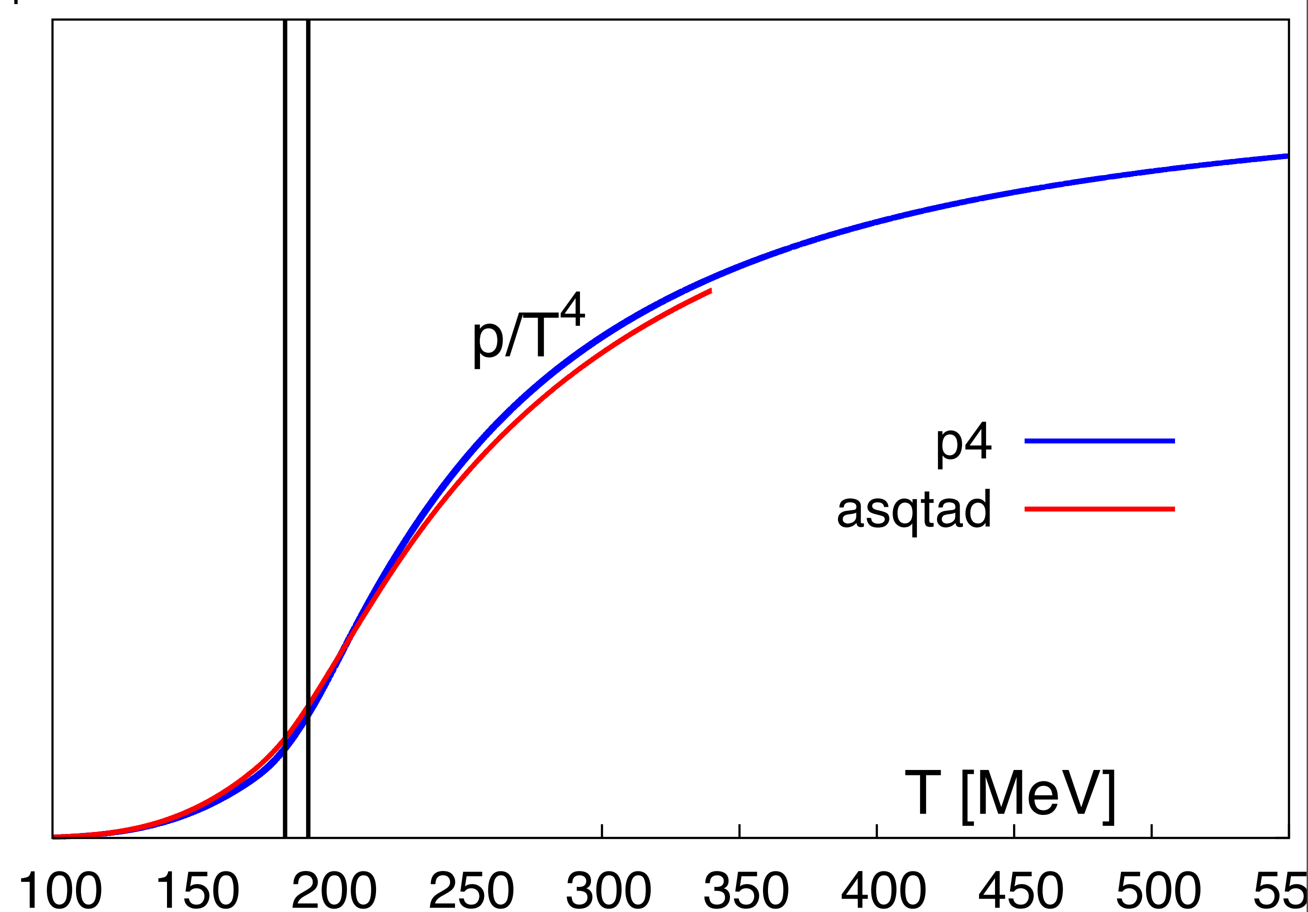
- Strategy: Interpolate between high and low densities to get a unified description of EoS.
  - Relies on not having exotic phase between!

Inspiration from finite-T case:



Hot Quark Matter (μ=0)



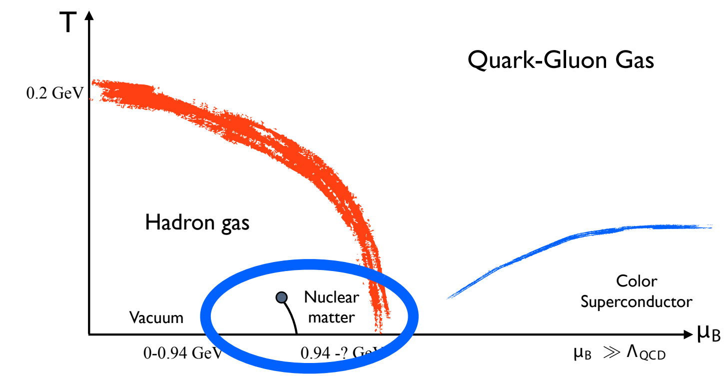


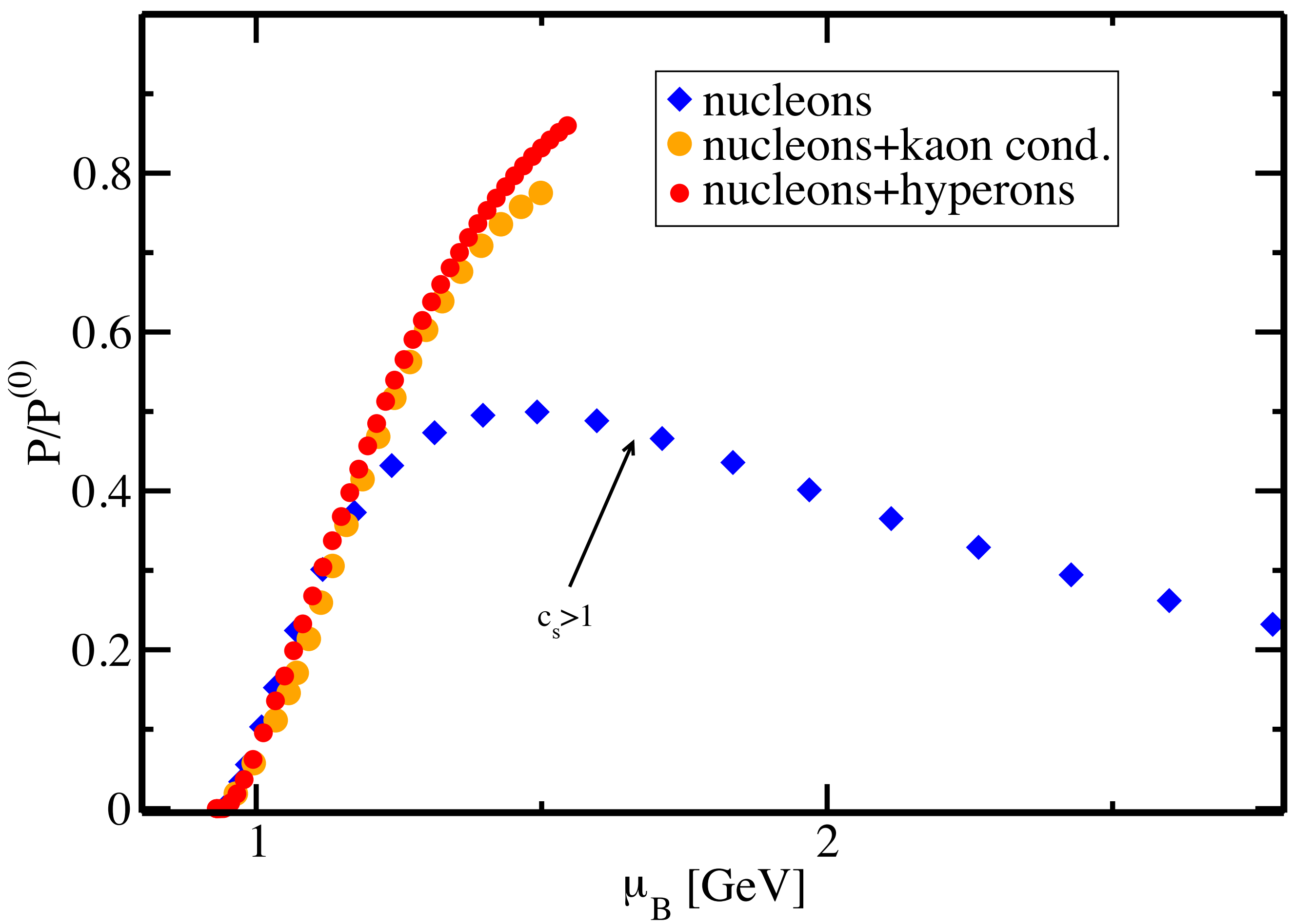
- Low-T : Hadron resonance gas
- High-T : Resummed perturbation theory ($\alpha_s^{5/2}$)

Lattice

Bazavov et al. 0903.4379

At low densities:





Nucleons (=APR **nucl-th/9804027**):

- Input:
  - 2-body: Argonne v_{18}
  - 3-body: Urbana IX

- Variational Chain Summation:
  - EoS for PNM and SNM

- Interpolate the correct neutron/proton fraction using Skyrme-model ansatz

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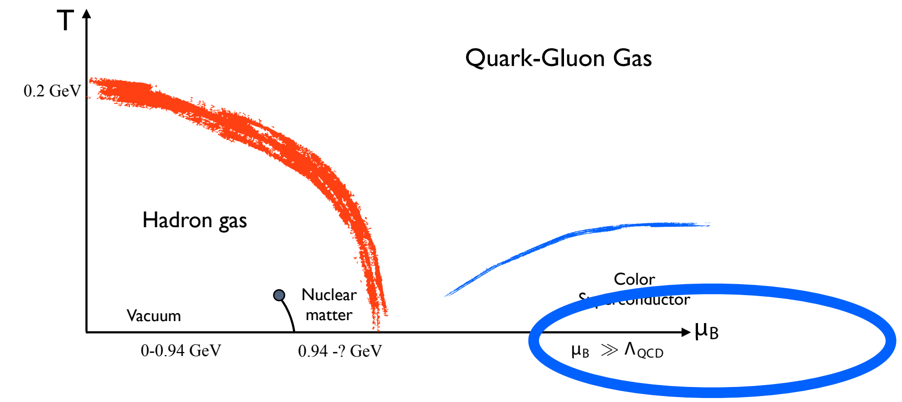
Perturbative evaluation of the EoS:



- Thermodynamics defined by the grand potential

$$\Omega(\mu_u, \mu_d, \mu_s, m_s) = -T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{\rm QCD}}$$

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i \gamma_0)\psi_i.$$

- All thermodynamical quantities derived from $\Omega(\mu_u, \mu_d, \mu_s, m_s)$

$$pV = -\Omega(\mu_u, \mu_d, \mu_s, m_s)$$

$$n_i = -\partial_{\mu_i}\Omega(\mu_u, \mu_d, \mu_s, m_s)$$

$$\varepsilon = -p + n_i \mu_i$$

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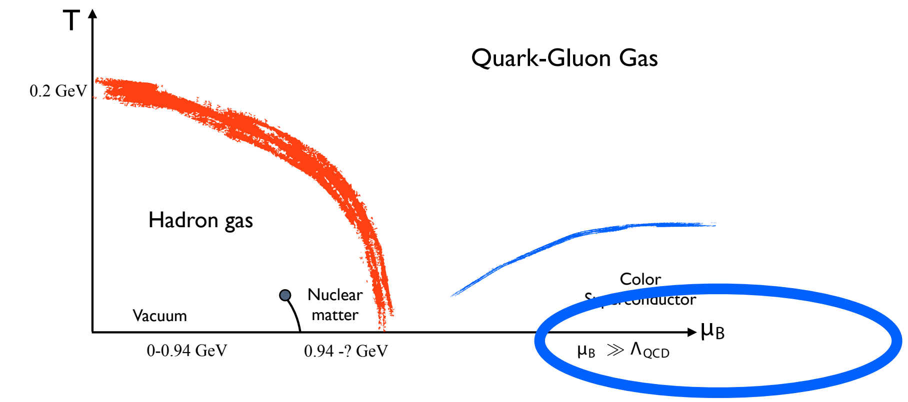

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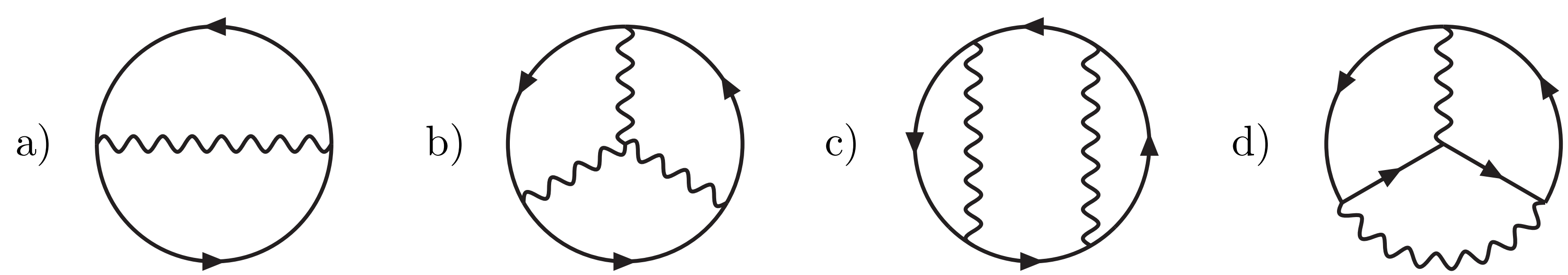
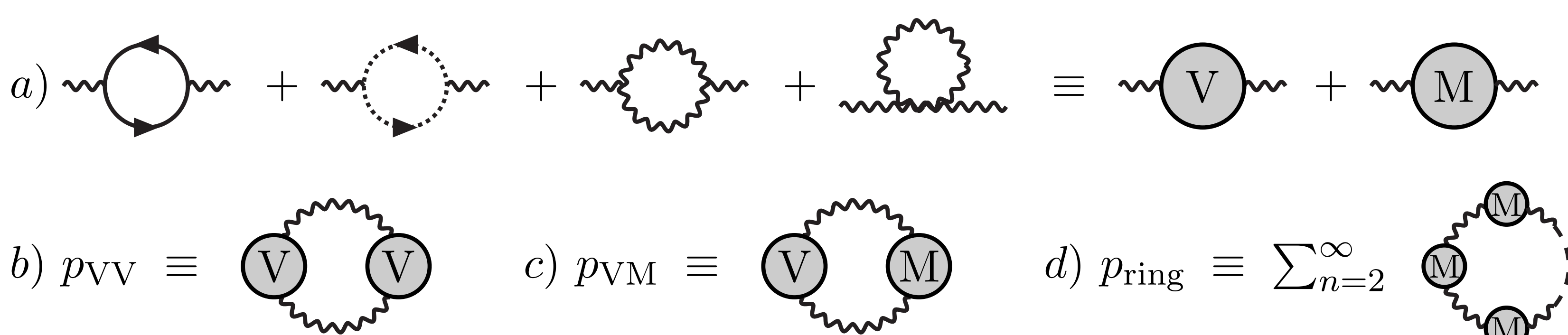
- $\Omega(\mu_u,\mu_d,\mu_s,m_s)$ available through 1PI diagrams:





Physical EoS at high density:

- Electric neutrality and beta-equilibrium fix all but one chemical potentials:

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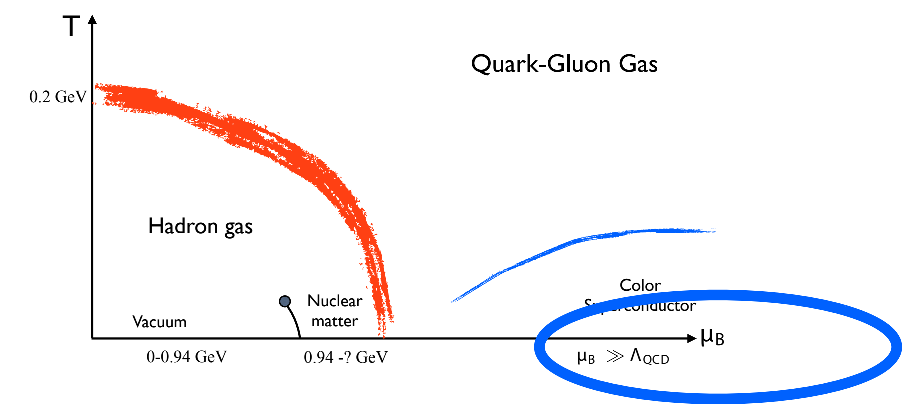
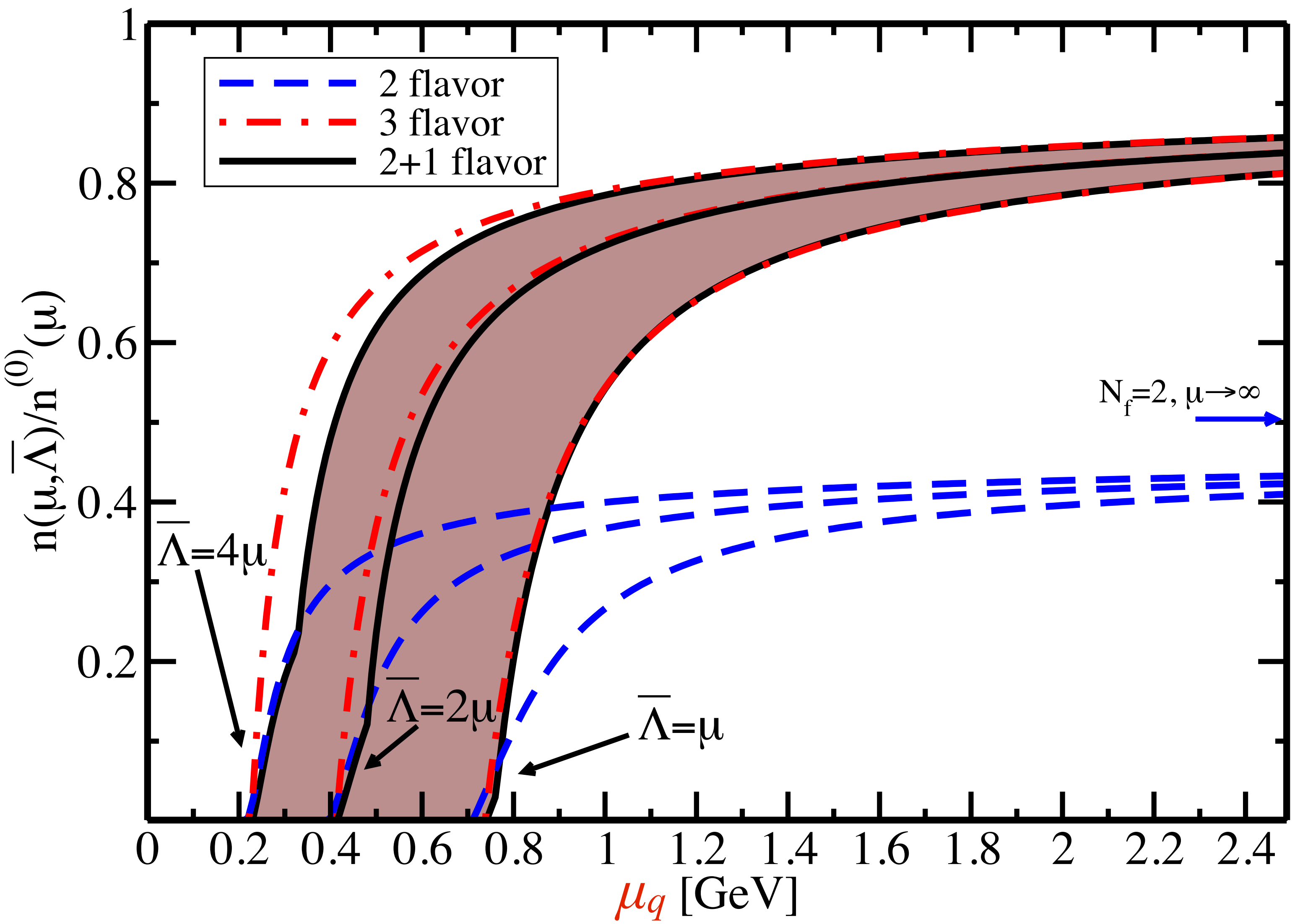
- The theory has a pairing instability → Non-perturbative term in $p(\mu)$

$$p(\mu) = p^{(\mathrm{pert})}(\mu, \bar{\Lambda}) - B + \frac{\Delta^2 \mu_B^2}{3\pi^2}$$

With Δ=0,...,100 MeV ($\Delta << k_F$)
And vary B for all allowed values

At high densities:





$$n(\mu_B,\bar{\Lambda}) = \partial_{\mu_B} p(\mu_B,\bar{\Lambda})$$

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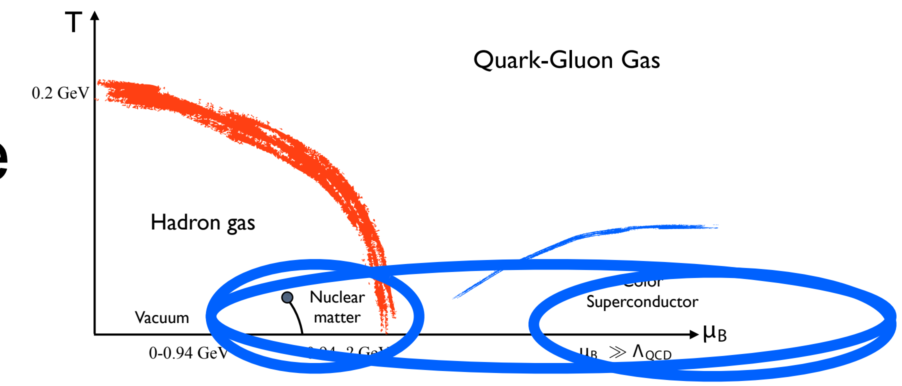
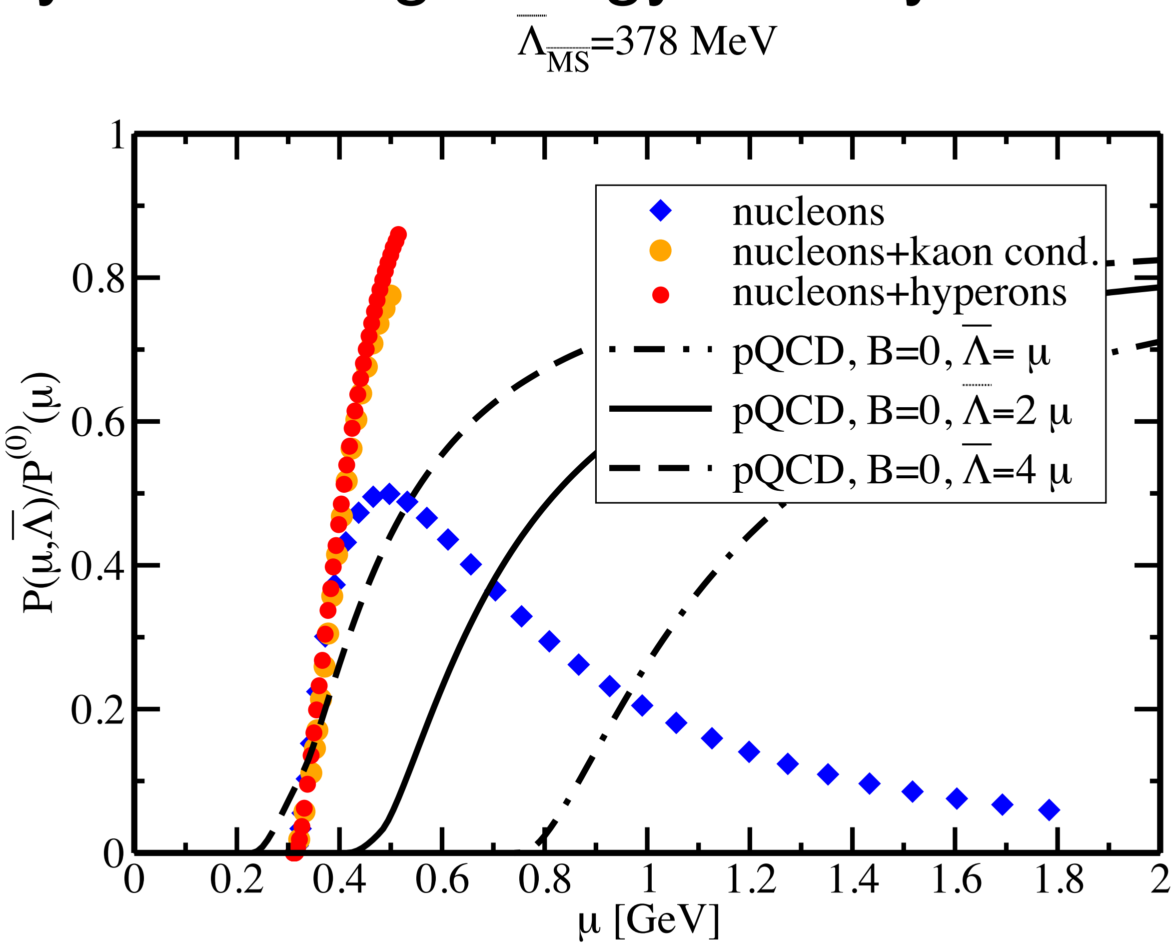
If there are no exotic phases, there will be a phase transition between hadronic and quark matter phases at some μ_{pt}



- Extrapolate the low- and high-density EoS to the intermediate region
- Catalog all possible self consistent EoS's $(B(\mu_{pt}), \bar{\Lambda})$
  - Equal pressure at phase transition
  - Monotonically increasing energy density



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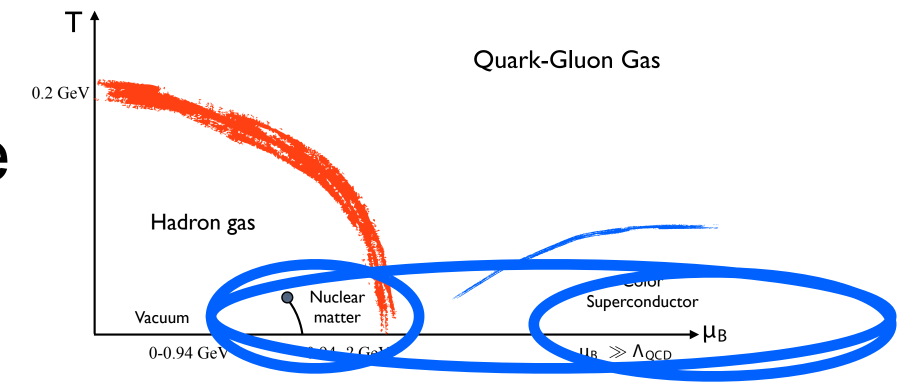
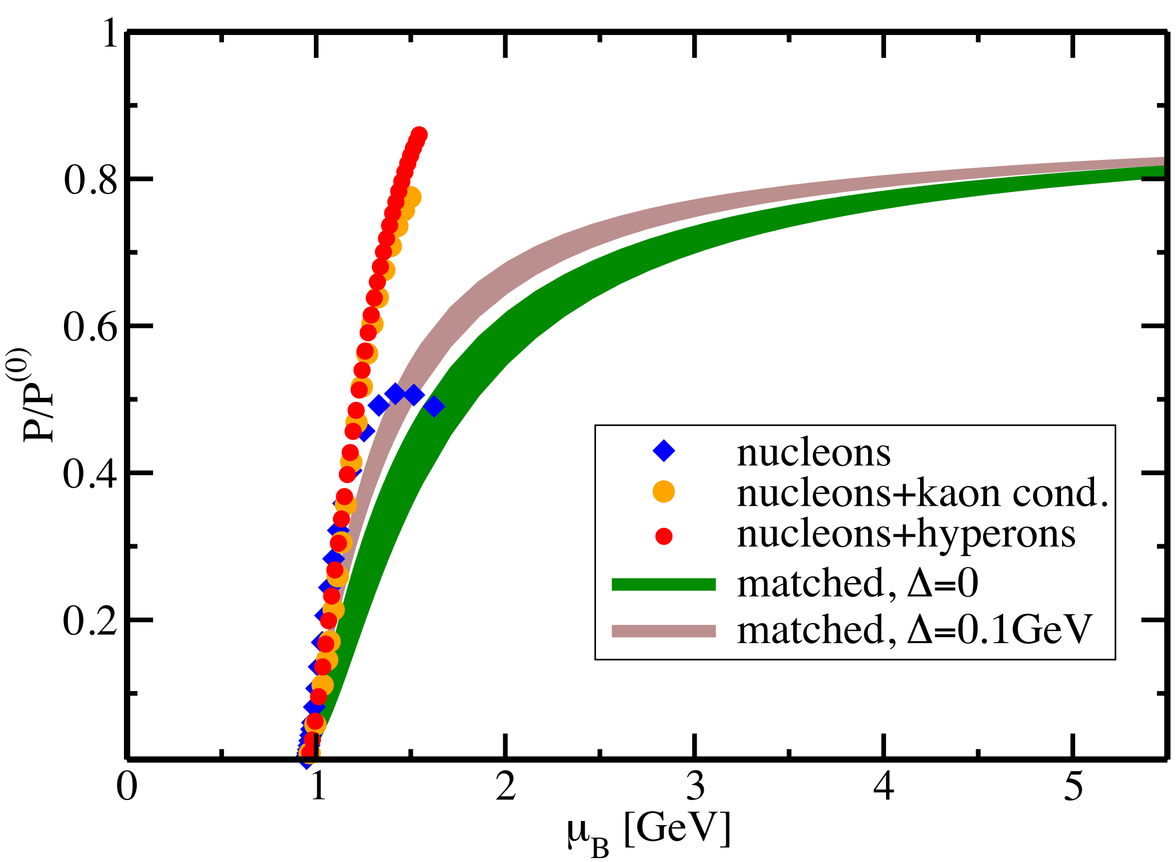


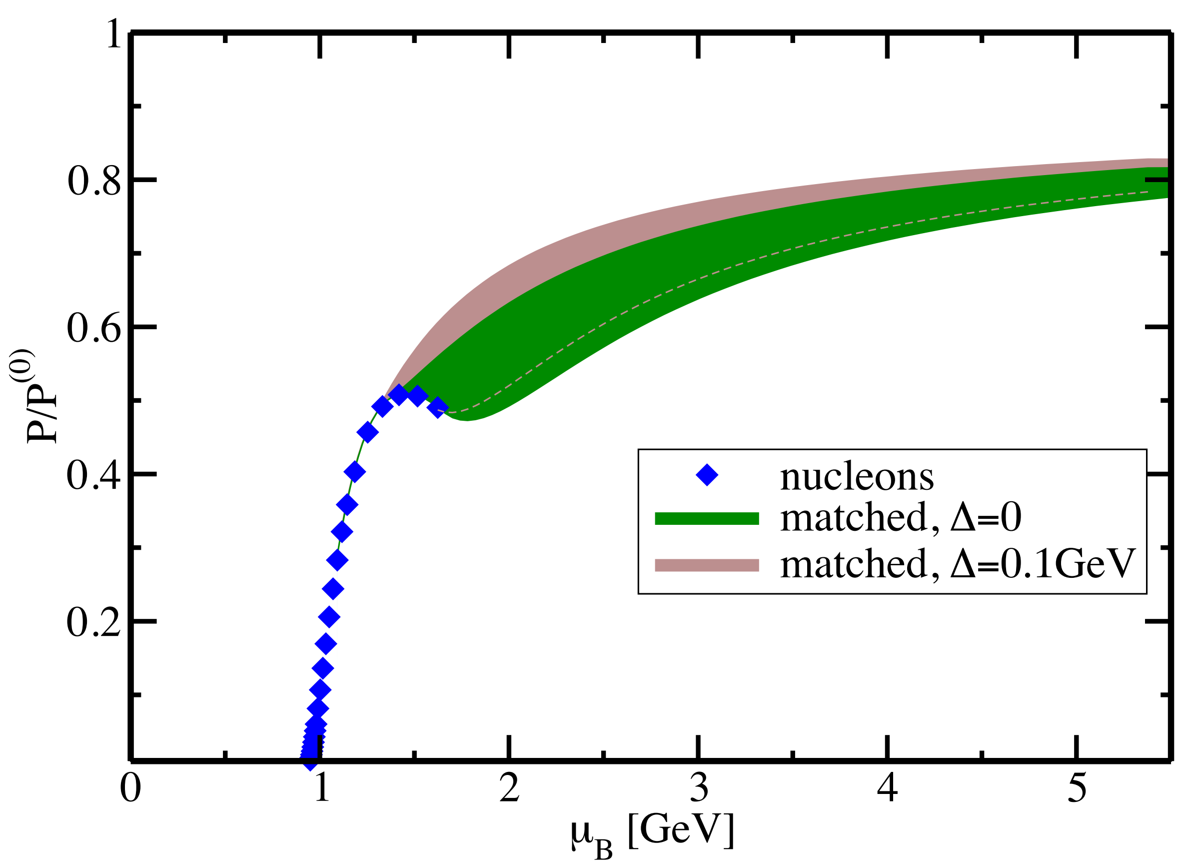
- Matching possible in two disjoint regions:

Case I ('low μ') Matching



$$0.16\text{fm}^{-3} < \rho_B^c \lesssim 0.32\text{fm}^{-3}$$

Case II ('high μ') Matching



$$\rho_B^c > 0.64\text{fm}^{-3}$$

Represents the best educated guess available for the true EoS on full μ-range

The location of μ_{pt} not available via perturbation theory, only via matching.

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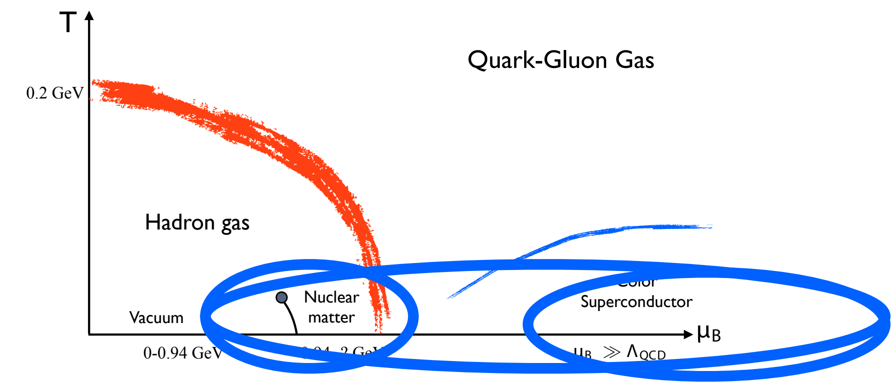


Strange quark matter hypothesis:

If the energy per baryon in quark matter is less than

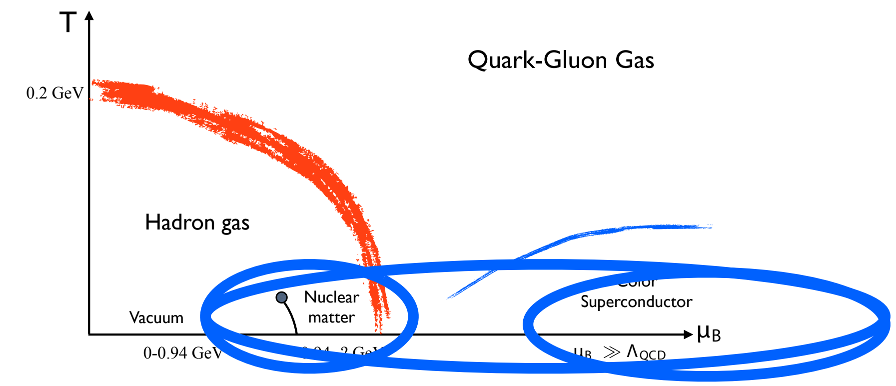
$$E/A = 3\mu_c = 0.93\text{GeV} \qquad {}^{56}\text{Fe}$$

then quark matter is the true ground state → Nuclear matter metastable.

Life time:

- Nucleons → 2 Flavor quark matter:
  - Equilibration through Strong interactions → $t_{eq} \sim 1/\Lambda_{QCD}$
  - Short lived. Ruled out by “experiment”
- Nucleons → strange quark matter:
  - Equilibration through Weak interactions $t_{eq} \sim 10^{60}$ years for $A > 6$
  - Adding d.o.f’s increases pressure → more likely to be stable
  - Experimentally plausible, lets find out what the theory says!

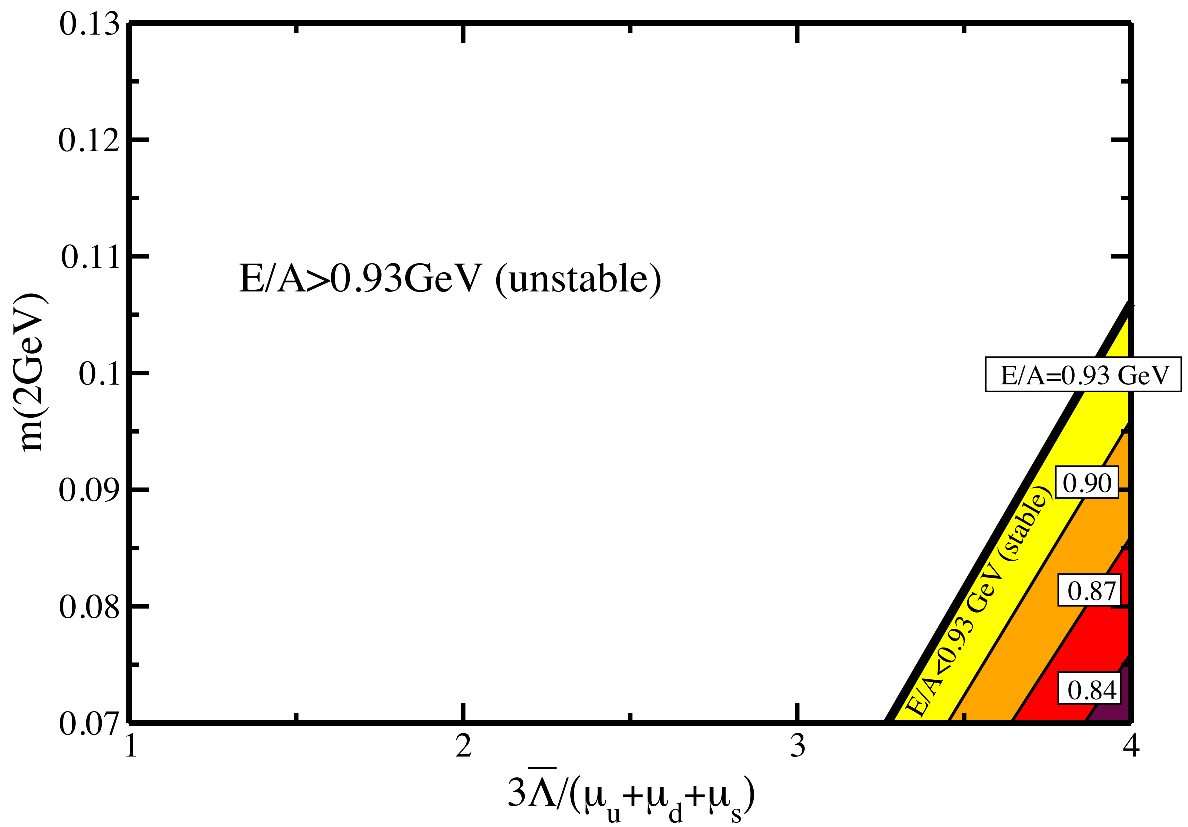
Strange quark matter hypothesis:



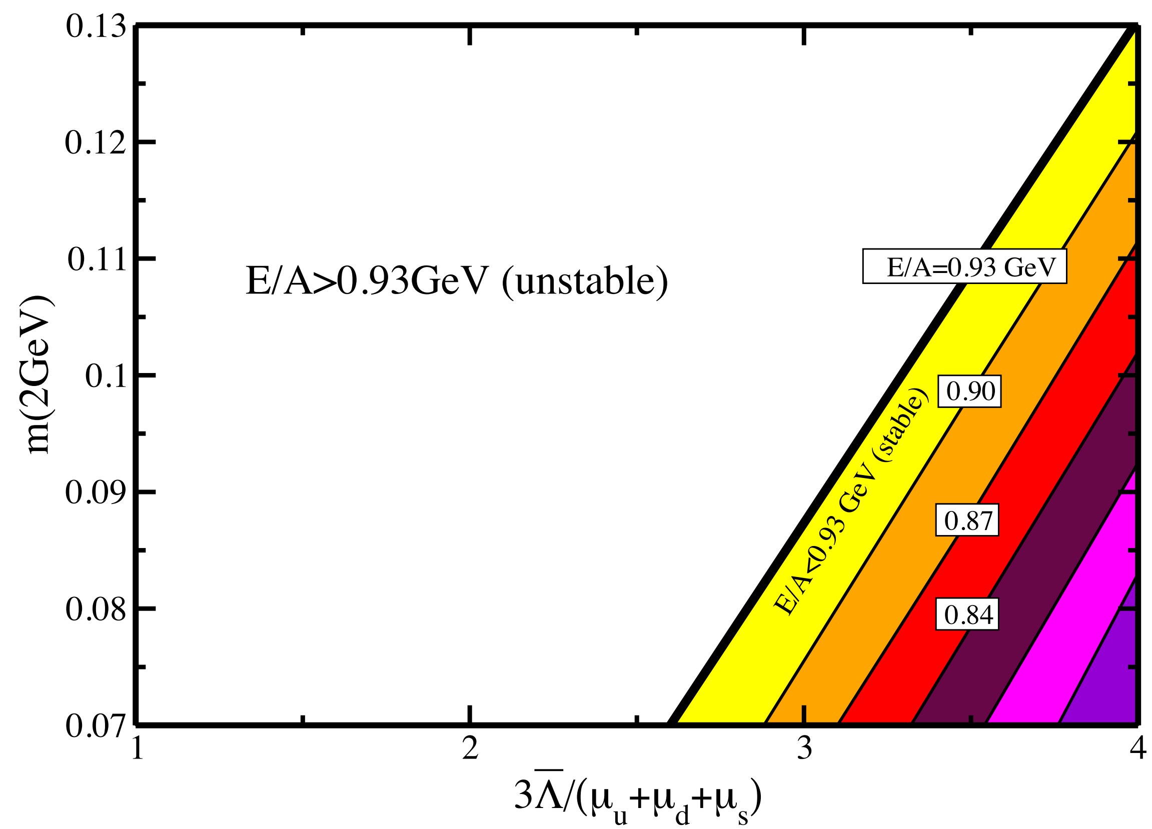
Strategy: Find out if SQM stable in the parameter space (B, $\bar{\Lambda}$) with

- $n_s > 0$, quark mass essential!
- $E/A > 0.93$ GeV

Normal Quark matter, $\Delta=0$, $\Lambda_{\overline{MS}}=0.378$GeV



CSC, $\Delta=100$ MeV, $\Lambda_{\overline{MS}}=0.378$GeV



- Parameter space very hostile, adding CSC makes bit more plausible
  - Absolutely stable SQM disfavored but not ruled out

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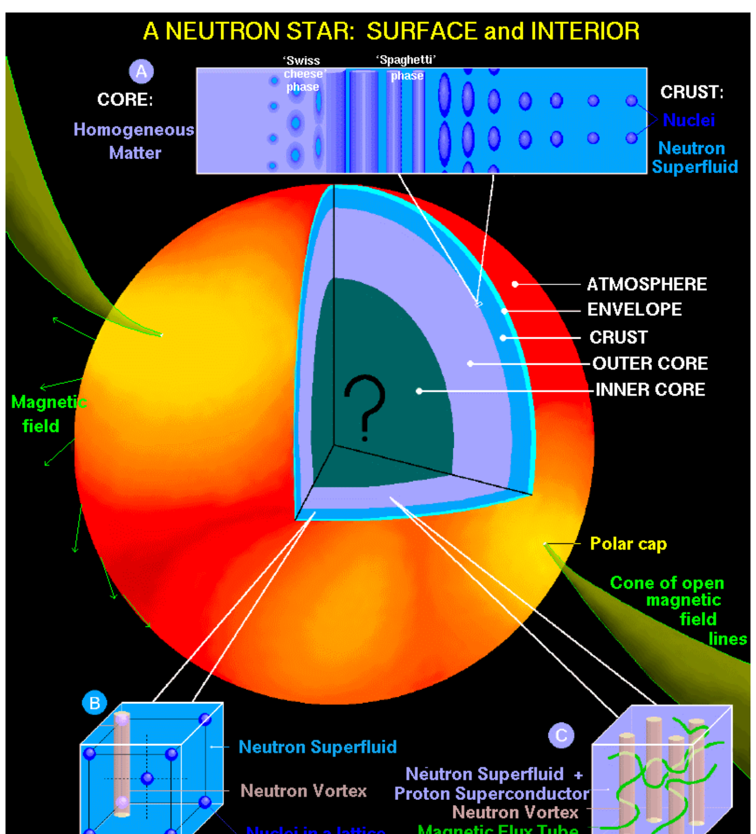
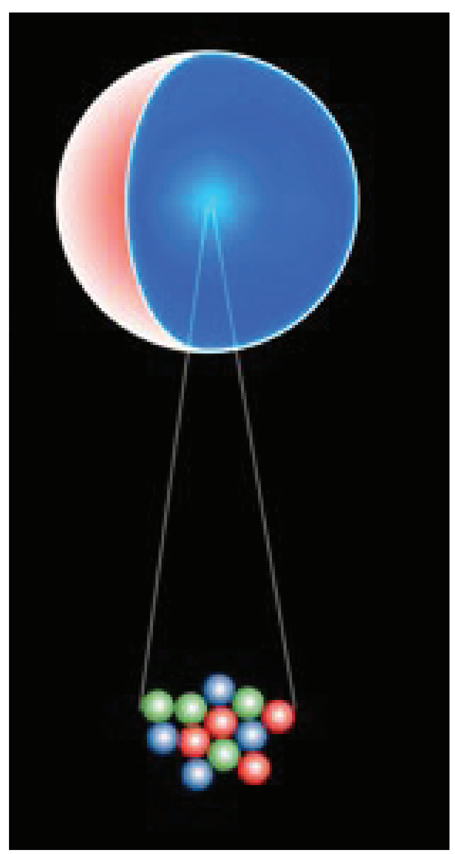
Compact stars:

- Compact stars formed of degenerate matter form a sequence in M-R plane (unlike white dwarfs)
- The M-R relation is very sensitive to the EoS
- M-R relation solved from the TOV-equations:

$$
\begin{aligned}
dM(r) &= 4\pi r^2 \varepsilon(r) dr, \\
dP(r) &= -\frac{G\left(P(r)+\varepsilon(r)\right)\left(M(r)+4\pi r^3 P(r)\right)}{r\left(r-2GM(r)\right)} dr\,,
\end{aligned}
$$

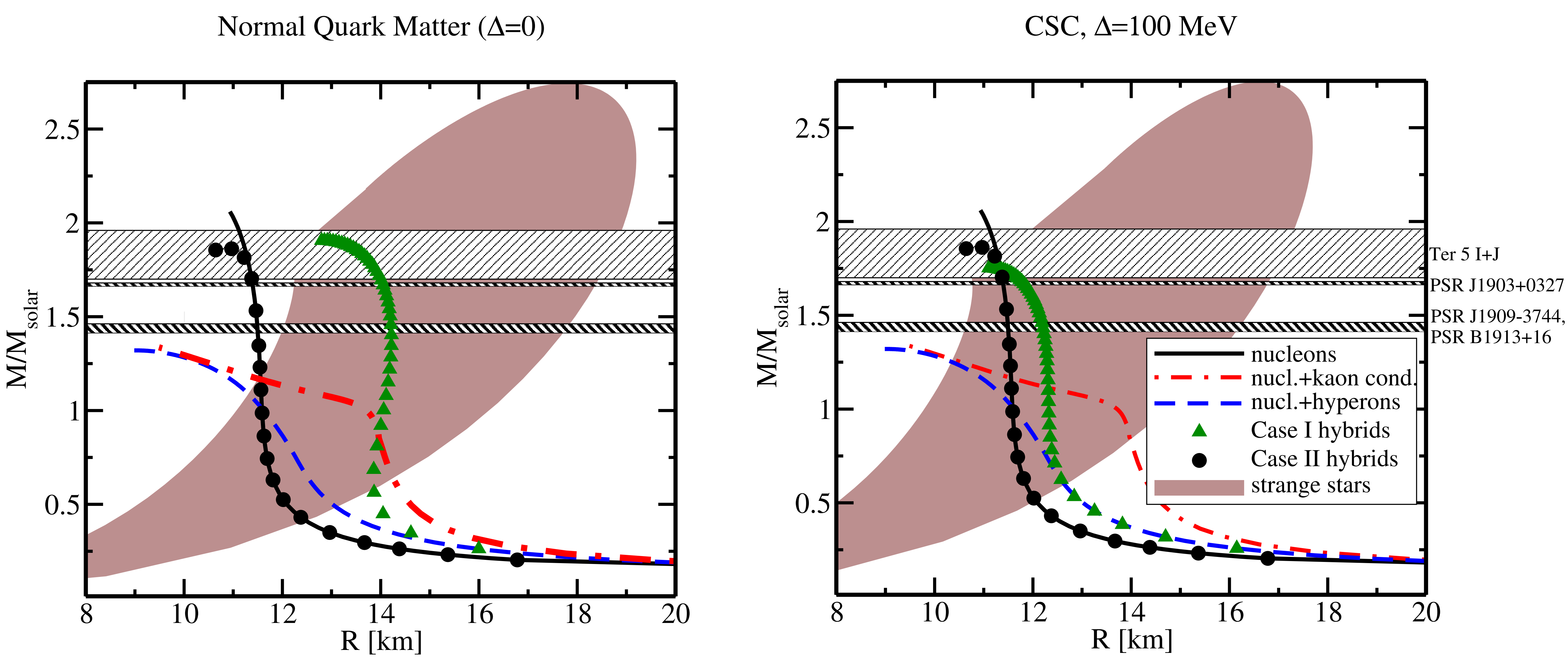
- Takes $\varepsilon(p)$ as input





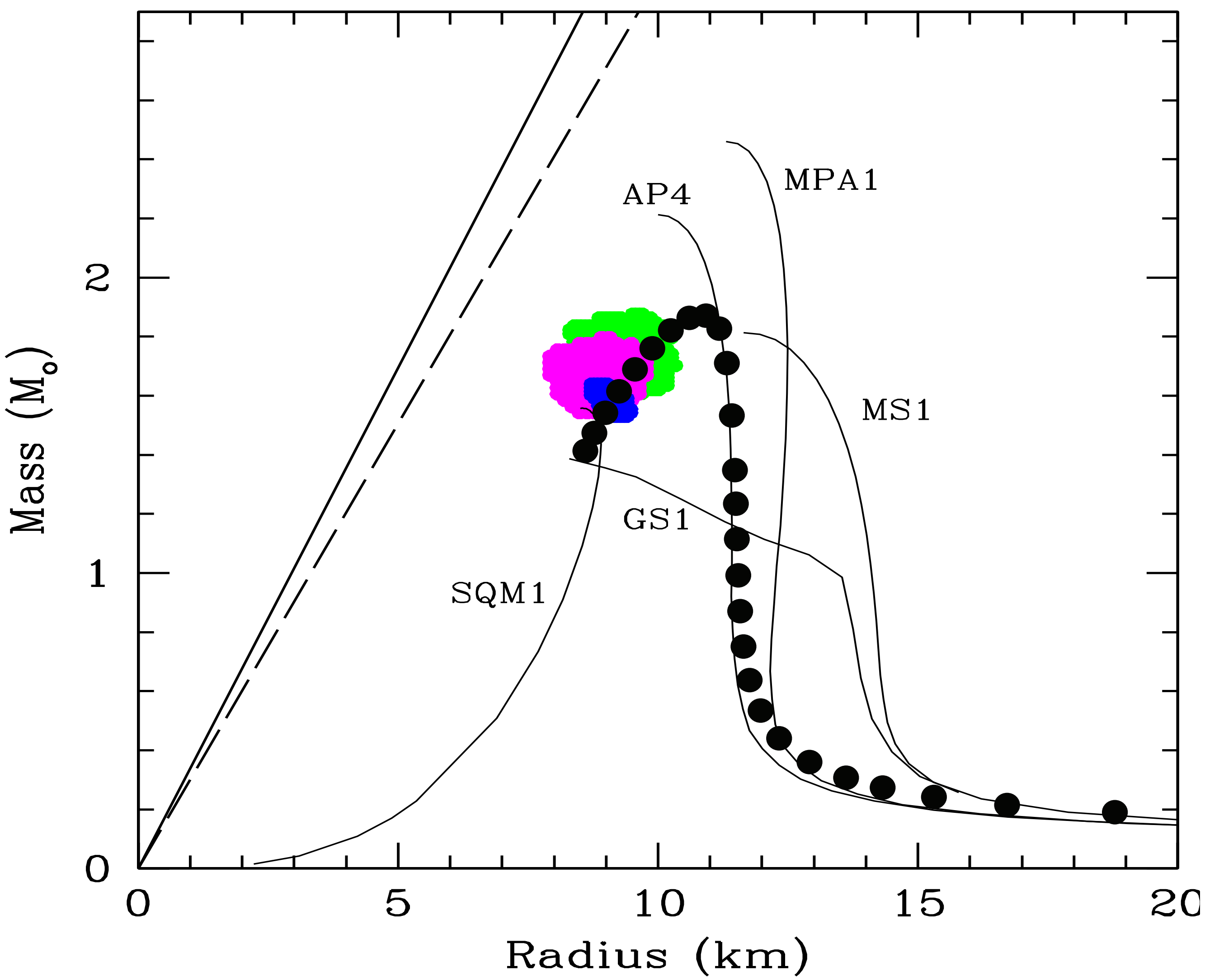
Let's consider compact stars made of:

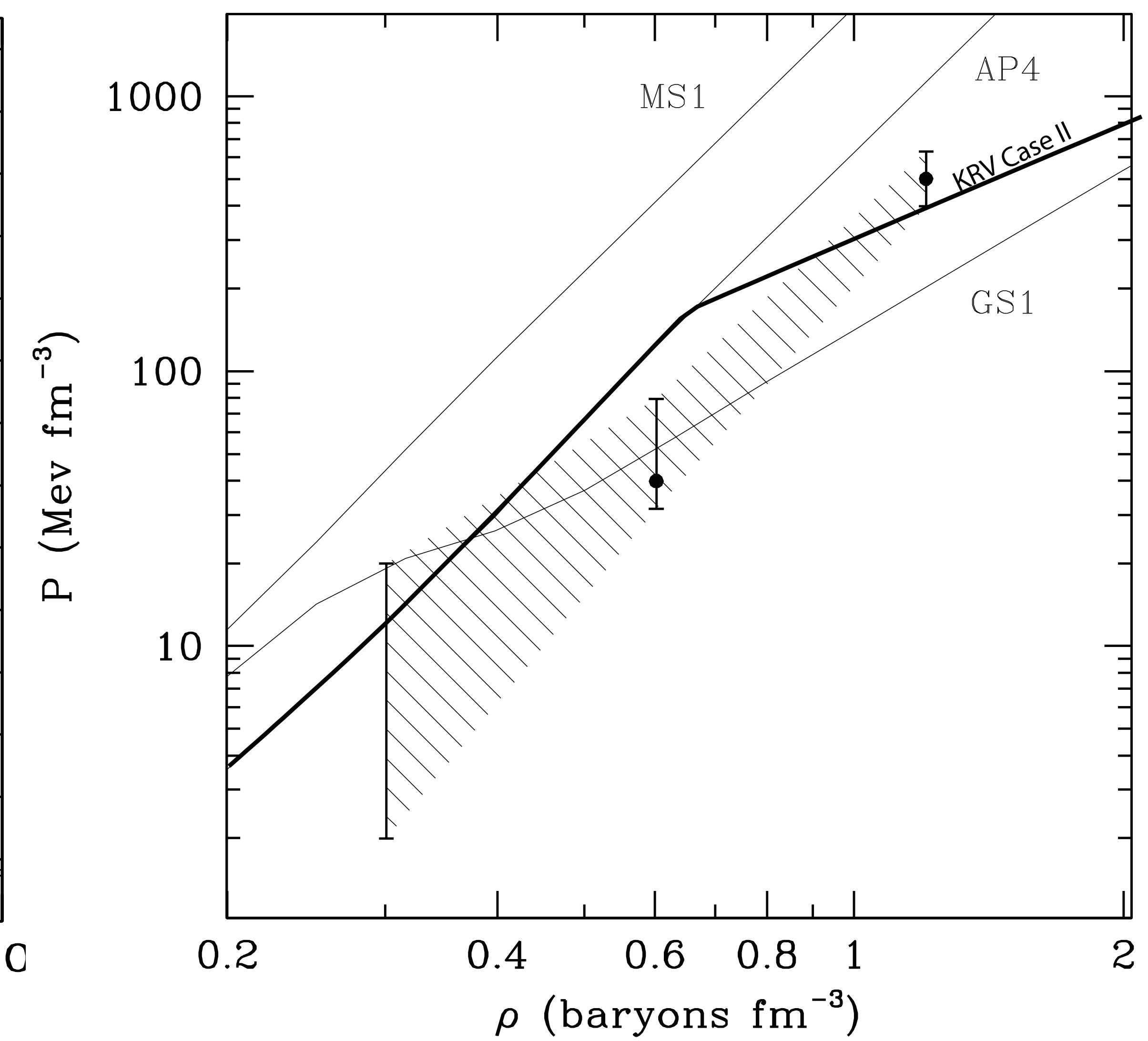
- Pure Nuclear matter
- Pure Quark matter
- Hybrid stars with
  - large quark core with thin nucleonic crust (case I)
  - small quark core with thick nucleonic crust (case II)



- Effect of CSC very small.
- Hyperonic/Kaonic EoS ruled out
- For neutron/hybrid stars $M_{\text{max}} \sim 2M_{\text{solar}}$
- Cannot exclude very massive strange stars
    - Dense quark stars ruled out

New observations from February:





- Case II agrees with the data better than any of the standard EoS
- Overestimates the radius
  - → Accounting for the (possible) 2-component admixture in the transition:
    - Reduces radius, doesn't affect the maximum mass.
    - Smoothens EoS around transition
- Superconductivity reduces radius → Improve the treatment of CFL

Conclusions:

- The grand potential of QCD at finite density with finite m_s computed to α_s^2.
  - Needed to create new perturbation theory machinery to overcome technical challenges
- Modeled the EoS in full range of μ_B (three logical possibilites):
  - Hadron / quark matter transition:
    - Realistic description for full range of μ_B
  - Absolutely stable strange quark matter
    - is disfavored but not ruled out
    - ...but an observation of a $M > 2M_{\mathrm{solar}}$ would be a strong evidence in the opposite direction.
  - Exotic (non-CSC) phases between hadrons and quark matter
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Our Case II matching seems to perform better than any of the standard EoS in describing the recent experimental data.

Outlook:

- Improve the modeling of the effects of CSC:
  - By computing the mismatch of the fermi spheres
  - Assessing the different possibilities for CSC: CFL, 2CS...

- Improve the perturbative calculation:
  - $\alpha_s^2 \log(\alpha_s)$: Only ring diagrams involved
  - α_s^3: Major undertaking

- Improve the astrophysical modeling:
  - Two-component mixtures of hadronic and quark matter
  - Moment of inertia, glitches
  - Neutron star oscillations
  - Rotating stars, r-modes
  - Cooling rates and transport effects

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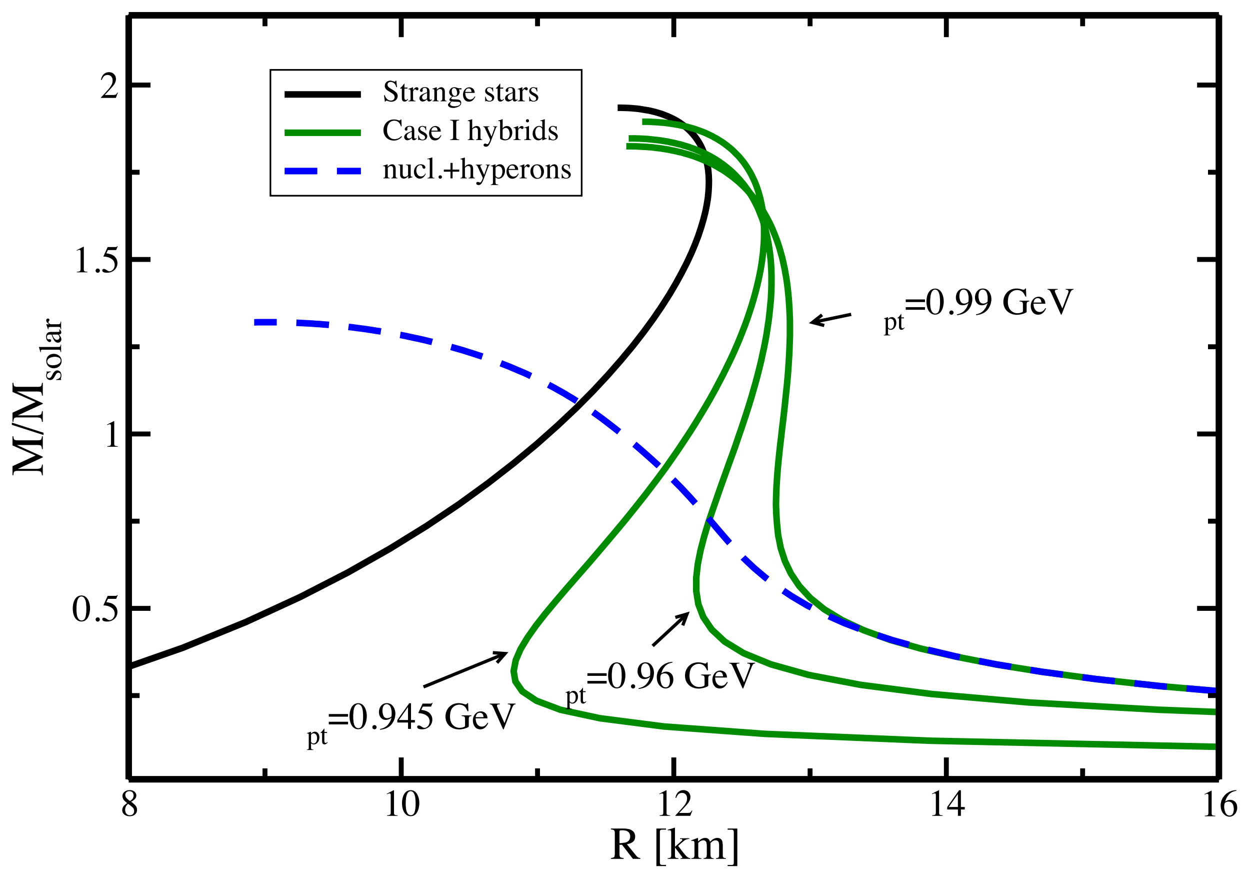
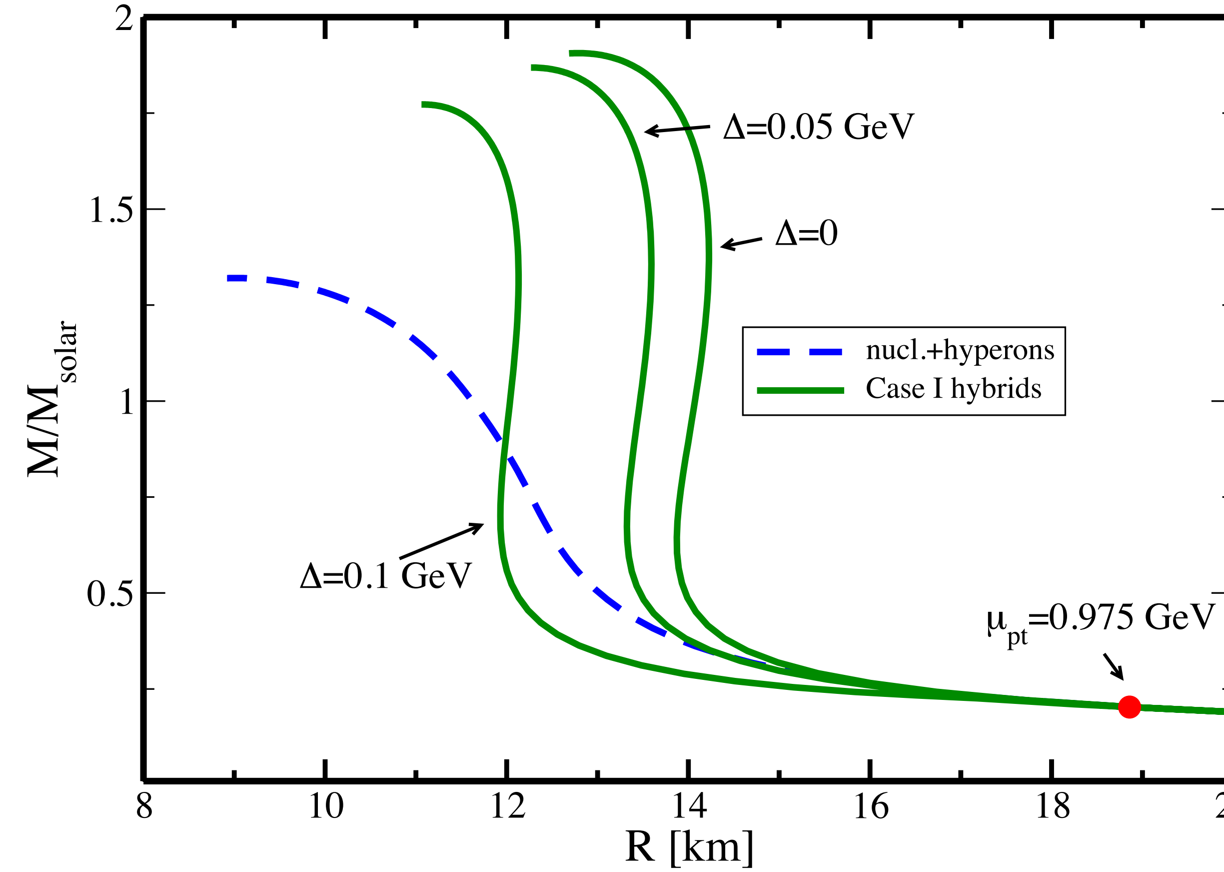
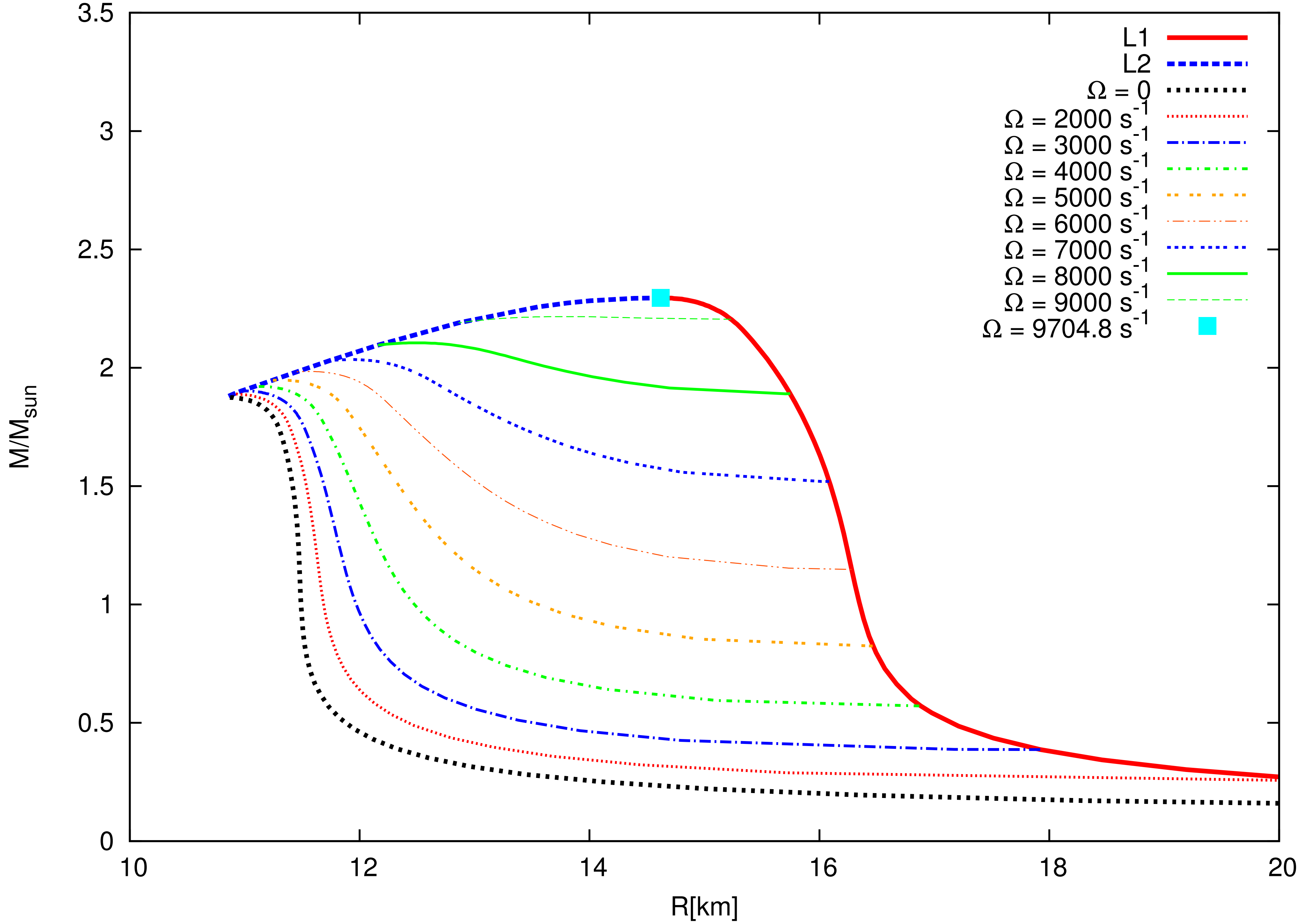
and of course: the observations are advancing very fast,
new data expected to come anytime!!

Effect of the matching density and CSC:





Effect of rotation to the MR-curves (Case II matching)



PRELIMINARY!!! With Bin Wu