Asymptotically safe gravity

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introduction

- **physics of classical gravity**

  Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

  classical action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int \sqrt{\det g}(-R(g_{\mu\nu}) + 2\Lambda)$$

  valid on length scales $\sim 10^{-2} - 10^{28}\,\mathrm{cm}$

introduction

- **physics of classical gravity**

  Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

- **physics of quantum gravity**

  Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

  Planck mass $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

  Planck time $t_{\mathrm{Pl}} \approx 10^{-44}\,\mathrm{s}$

  Planck temperature $T_{\mathrm{Pl}} \approx 10^{32}\,\mathrm{K}$

  expect modifications at energy scales $E \approx M_{\mathrm{Pl}}$

perturbation theory

- **structure of UV divergences**

  N-loop Feynman diagram $\sim \int dp\, p^{A-[G]N}$

  $[G] > 0:$ superrenormalisable

  $[G] = 0:$ renormalisable

  $[G] < 0:$ dangerous interactions

  gravity: $[g_{\mu\nu}] = 0\,, \quad [\text{Ricci}] = 2\,, \quad [G_N] = 2 - d$

  effective expansion parameter: $G_N\, p^2 \sim \frac{p^2}{M_{\text{Pl}}^2}$

perturbation theory

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- **perturbative non-renormalisability**

  gravity with matter interactions

  pure gravity (Goroff-Sagnotti term)

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

  1-loop 'running coupling'

$$G(r) = G_0 \left(1 - \frac{167}{30\pi}\frac{G_0 \hbar}{r^2}\right)$$

perturbation theory

- **effective theory for gravity** (Donoghue '94)

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  knowledge of UV completion not required

- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

perturbation theory

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- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

- **higher derivative gravity II** (Gomis, Weinberg '96)

  all higher derivative operators

  gravity 'weakly' perturbatively renormalisable

  no unitarity issues at high energies

UV fixed points

UV fixed points

- **asymptotic freedom**

  YM theory

UV fixed points

- **asymptotic freedom**

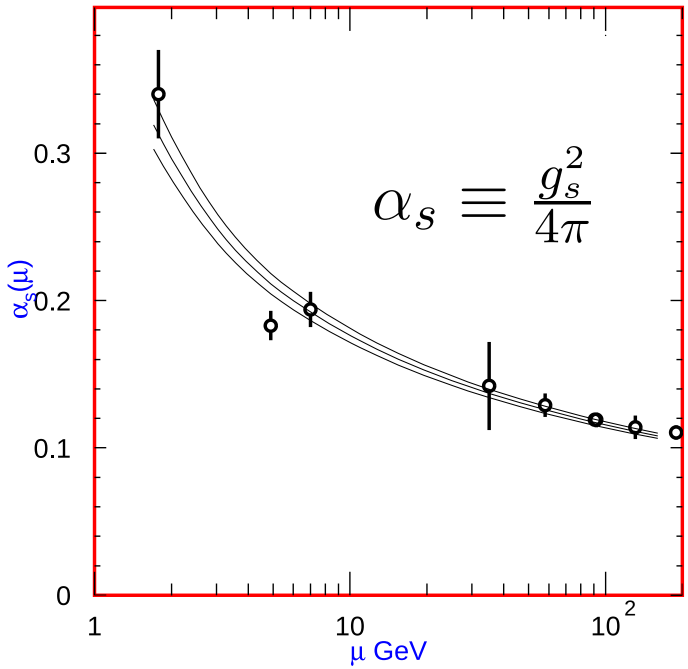
  YM theory

  running coupling

$$\frac{dg_s}{d\ln\mu} = -\frac{7g_s^3}{16\pi^2}$$

trivial UV fixed point

$$g_s = 0$$



UV fixed points

- **asymptotic freedom**

  YM theory

- **asymptotic safety** (Weinberg ’79)

  non-trivial UV fixed point for gravity

  well-defined continuum limit

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  well-defined continuum limit

  critical trajectory

  - **stable, marginal, unstable directions**

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  predictive power

    finite number of unstable directions

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  non-trivial UV fixed point for gravity

  well-defined continuum limit

  critical trajectory

  stable, marginal, unstable directions

  predictive power

  finite number of unstable directions

- **examples**

  Gross-Neveu models $(D > 2)$

  quantum gravity in $D \approx 2$

asymptotic safety

- **RG scaling of gravitational coupling**

dimensionless coupling $g(\mu) = Z_N(\mu)^{-1} \cdot G_N \cdot \mu^{D-2}$

anomalous dimension $\eta_N = -\frac{\mathrm{d}\ln Z_N}{\mathrm{d}\ln \mu}$

RG running $\frac{\mathrm{d}g}{\mathrm{d}\ln \mu} = (D - 2 + \eta_N)\, g$

(DL ’06,Niedermaier ’06)

asymptotic safety

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- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

asymptotic safety

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- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

  UV fixed point implies **weakly coupled gravity** at **high energies**

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

asymptotic safety

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  RG running $\frac{\mathrm{d}g}{\mathrm{d} \ln \mu} = (D - 2 + \eta_N)\, g$

- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

  IR fixed point implies **strongly coupled gravity** at low energies

$$\mu \to 0 : \qquad G(\mu) \to g_* \mu^{2-D} \gg G_N$$

asymptotic safety

- **RG scaling of gravitational coupling**

  dimensionless coupling $g(\mu) = Z_N(\mu)^{-1} \cdot G_N \cdot \mu^{D-2}$

  anomalous dimension $\eta_N = -\frac{\mathrm{d}\ln Z_N}{\mathrm{d}\ln\mu}$

  RG running $\frac{\mathrm{d}g}{\mathrm{d}\ln\mu} = (D - 2 + \eta_N)\, g$

- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

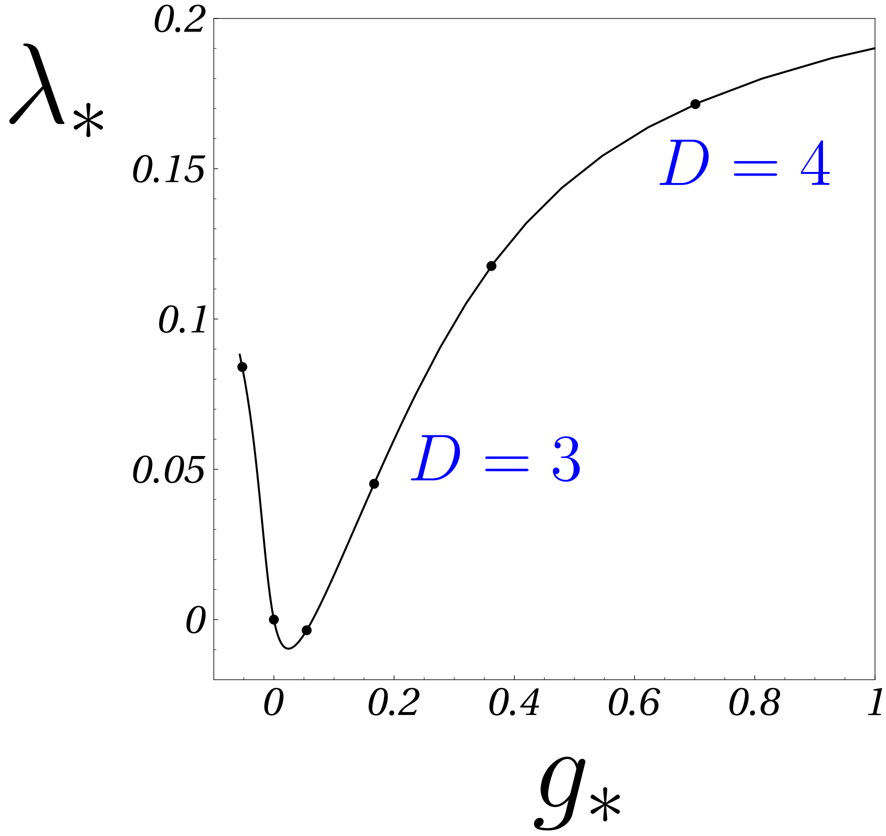
- **tools**

  discretisation: lattice technology

  continuum: **renormalisation group**

UV fixed point

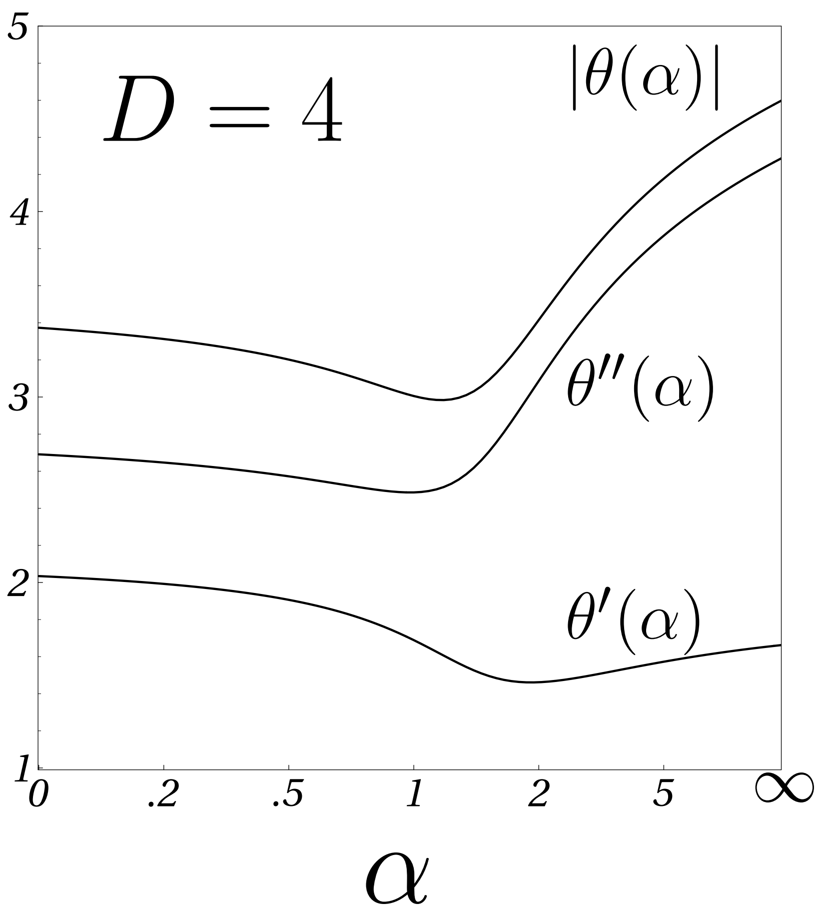
- **continuity**



- **continuous link with perturbative fixed point in $D = 2 + \epsilon$ dimensions**
- **real fixed point unique for any dimension**

universality

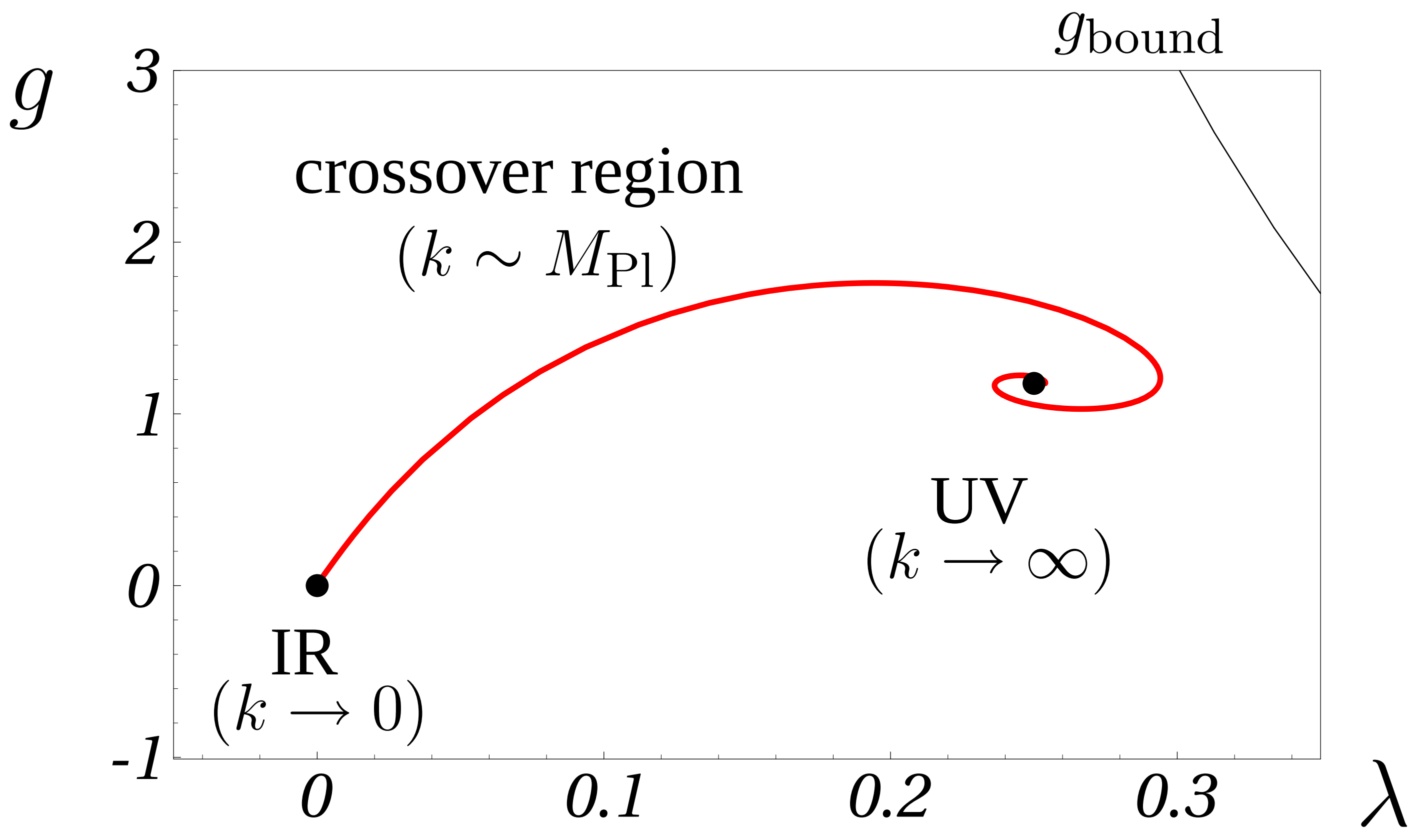
- **scaling exponents**



- **universal eigenvalues at criticality** $\theta = \theta' + i\theta''$
- **Landau-de Witt gauge is RG fixed point** (DL, Pawlowski '98)
- **consistent for all** $\alpha \in [0, \infty]$
- **large-**α **behaviour correct**
- θ **consistent with Regge lattice simulations** (Hamber '00)

flow from UV to IR

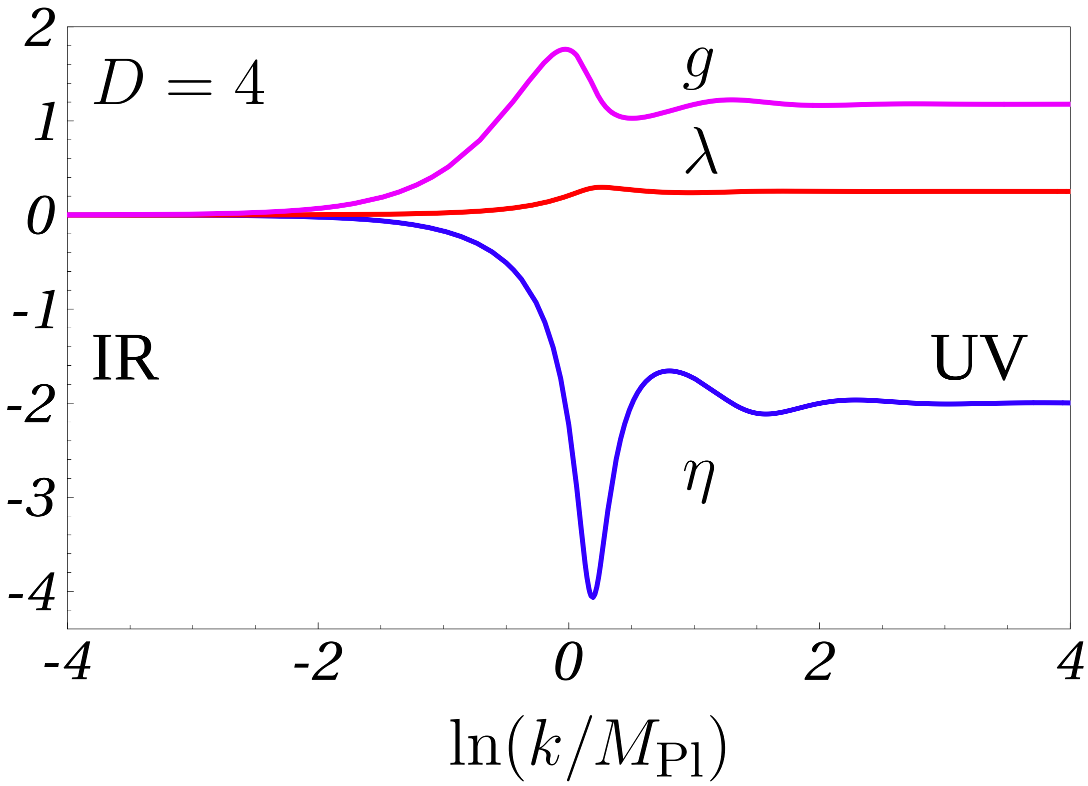
- **separatrix in four dimensions**



flow trajectories

- **cross-over behaviour**

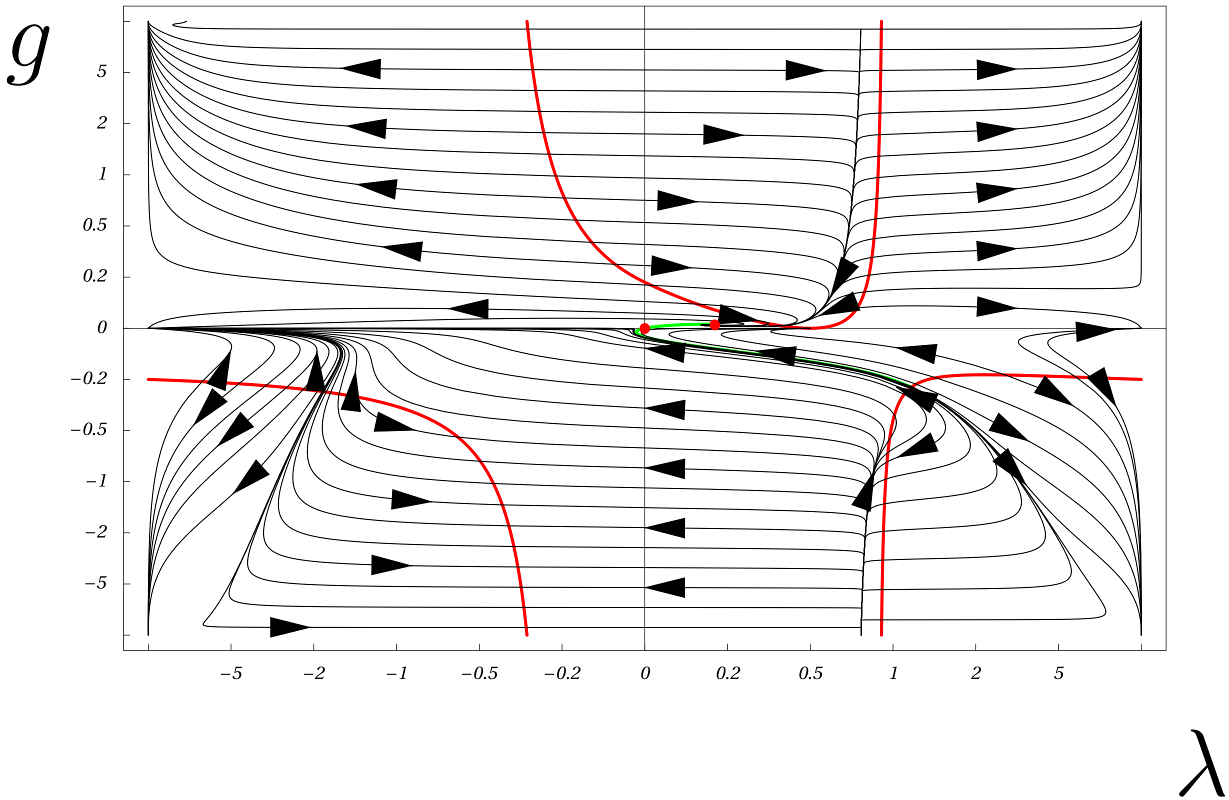
  integrated flow with $\sqrt{g}R$, $\sqrt{g}$



phase diagram

- **full flow**

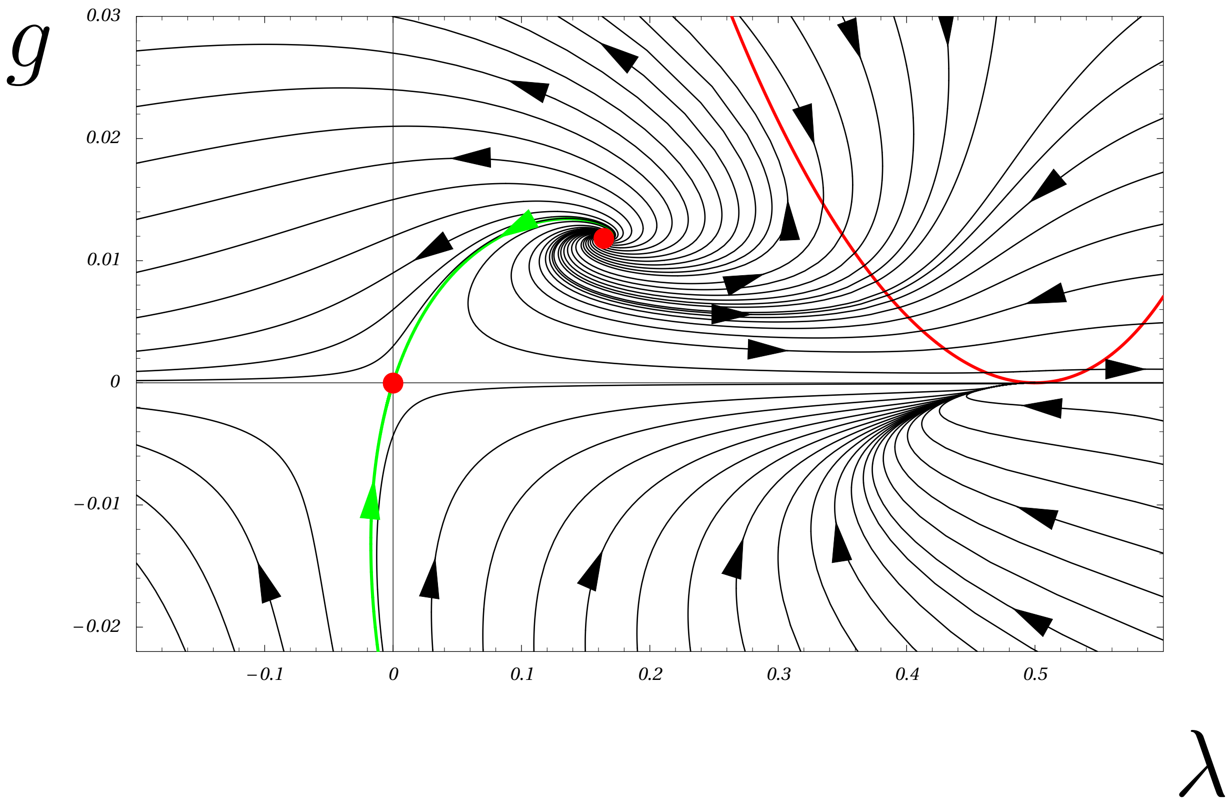
  4D integrated flow with $\sqrt{g}R$, $\sqrt{g}$



phase diagram

- **full flow – vicinity of fixed points**

  4D integrated flow with $\sqrt{g}R$, $\sqrt{g}$



more curvature invariants

- **extensions including** $\sqrt{g}$ **and** $\sqrt{g}(R)^i,\ \mathrm{i} = 1, \cdots, n$.

n	θ'	θ''	θ_2	
1	$1.1 - 2.3$	$2.5 - 7.0$	$-$	(Lauscher, Reuter '01)
1	$1.4 - 2.0$	$2.4 - 4.3$	$-$	(DL '03)
1	$1.5 - 1.7$	$3.0 - 3.2$	$-$	(Fischer, DL '06)
1	2.4	2.2	$-$	(Codello, Percacci, Rahmede '07)
2	$2.1 - 3.4$	$3.1 - 4.3$	$8.4 - 28.8$	(Lauscher, Reuter '02)
2	1.4	2.8	25.6	(DL '07)
2	1.7	3.1	3.5	(DL '07)
2	1.4	2.3	26.9	(Codello, Percacci, Rahmede '07)

more curvature invariants

- **extensions including** $\sqrt{g}$ **and** $\sqrt{g}(R)^i$, $\mathrm{i} = 1, \cdots, n$.

n	θ'	θ''	θ_2	θ_3	θ_4'	θ_4''	θ_6	θ_7	θ
1	2.38	2.17							
2	1.38	2.32	26.9						
3	2.71	2.27	2.07	-4.23					
4	2.86	2.45	1.55	-3.91	-5.22				
5	2.53	2.69	1.78	-4.36	-3.76	-4.88			
6	2.41	2.42	1.50	-4.11	-4.42	-5.98	-8.58		
7	2.51	2.44	1.24	-3.97	-4.57	-4.93	-7.57	-11.1	
8	2.41	2.54	1.40	-4.17	-3.52	-5.15	-7.46	-10.2	-12.

(Codello, Percacci, Rahmede '07, Machado, Saueressig '07)

extensions

- **higher derivative gravity**

  1-loop (Codello, Percacci ’05, Niedermaier ’09)

  beyond 1-loop, matter (Benedetti, Machado, Saueressig ’09, DL, Rahmede ’10)

extensions

- **higher derivative gravity**
- **higher dimensions**

  Einstein-Hilbert, extensions (Fischer, DL ’05)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**

  large N expansion (Percacci ’05)

  minimally coupled (Percacci, Perini ’05, Narain, Percacci ’09, Narain, Rahmede ’09)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**

  1-loop (Robinson, Wilczek ‘05, Pietrykowski ‘06, Toms ‘07, Ebert, Plefka, Rodigast ‘08)

  beyond 1-loop (Manrique, Reuter, Saueressig ’09)

  beyond 1-loop, and fully coupled system (Folkerts, DL, Pawlowski ’09)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**
- **dynamical ghosts**

  de Donder gauge (Groh, Saueressig ’10)

  Landau-deWitt gauge (Eichhorn, Gies, ’10)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**
- **dynamical ghosts**
- **conformally reduced gravity**

  leading order (Reuter, Weyer '08)

  next-to-leading order (Machado, Percacci '09)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**
- **dynamical ghosts**
- **conformally reduced gravity**
- **consistency with lattice simulations**

  simplicial gravity / Regge calculus (Hamber ’00, Hamber, Williams ’04)

  causal dynamical triangulations (Ambjorn, Jurkiewicz, Loll et. al. ‘04, ’05)

extensions

- **higher derivative gravity**
- **higher dimensions**
- **matter fields**
- **Yang-Mills gravity**
- **dynamical ghosts**
- **conformally reduced gravity**
- **consistency with lattice simulations**
- **phenomenology**

cosmology, black holes

Bonanno, Reuter ’01

Weinberg ’09, Falls, DL, Raghuraman ’10

LHC phenomenology

Fischer, DL ’06

Hewett, Rizzo ’07, DL, Plehn ’07, Koch ’07, DL ’08

Falls, Hiller, DL ’10

gravitational scattering

Gerwick, DL, Plehn ’10, Brinckmann, Hiller, DL ’10

extensions

- higher derivative gravity
- higher dimensions
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- dynamical ghosts
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- phenomenology

  ⋮

quantum gravity and black holes

- **Schwarzschild solution**

  Schwarzschild metric

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Omega^2_{d-2}$$

  classical lapse function

$$f = 1 - \frac{G_N\,M}{r^{d-3}}$$

  classical Schwarzschild radius

$$r_{\rm cl} = (G_N\,M)^{1/(d-3)}$$

quantum gravity and black holes

- **RG improved black holes** Falls, DL, Raghuraman (ERG '08, 1002.0260 [hep-th])

running gravitational coupling

$$G_N \to G(r)\,, \quad f_{\rm cl}(r) \to f_{\rm imp}(r) = 1 - \frac{G(r)\,M}{r^{d-3}}$$

improved Schwarzschild radius r_s from

$$f_{\rm imp}(r_s) = 0$$

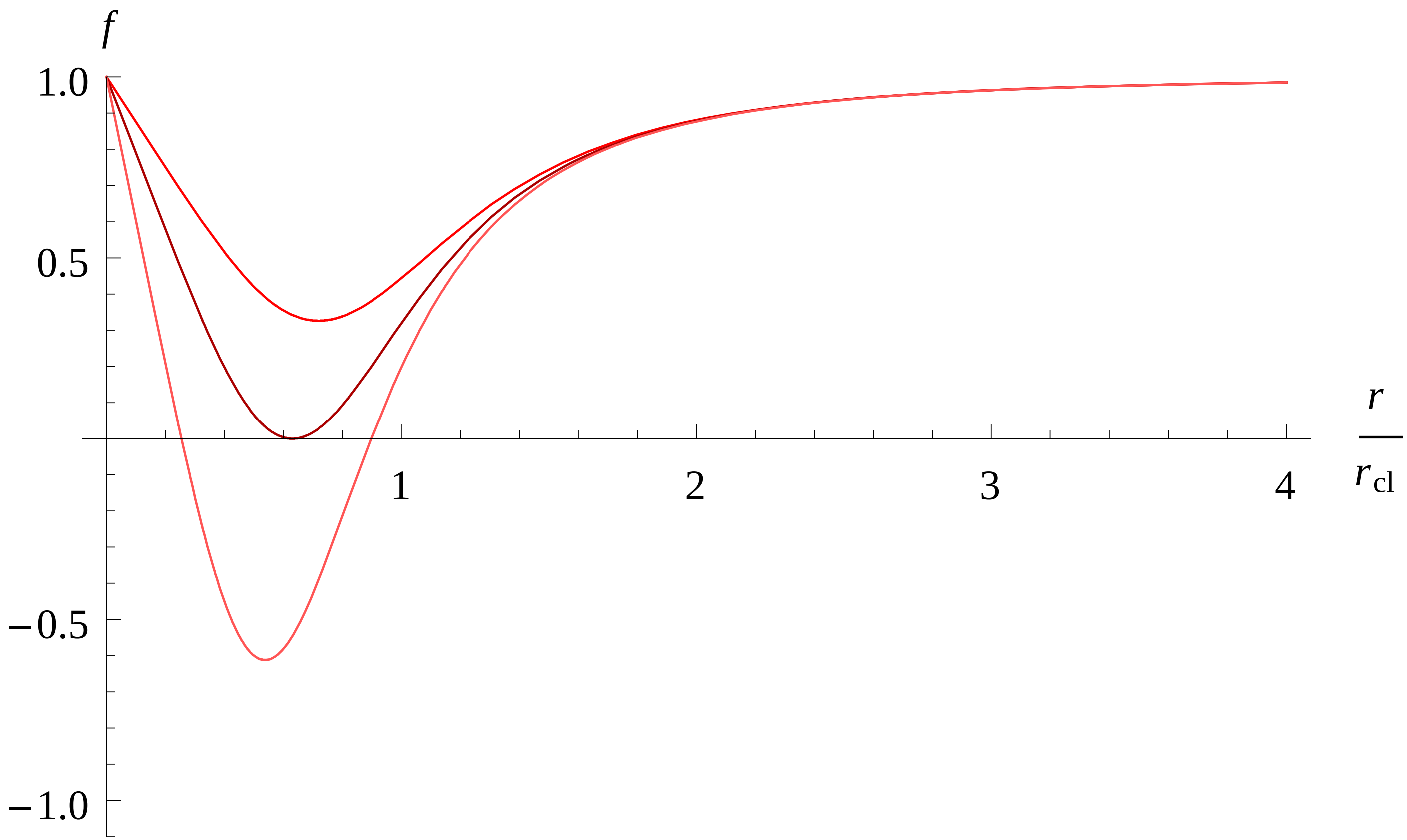
critical black hole mass M_c from

$$d - 3 = \left.\frac{\partial \ln G(r)}{\partial \ln r}\right|_{r=r_c(M_c)}$$

quantum gravity and black holes

- **RG improved black holes** — Falls, DL, Raghuraman (ERG '08, 1002.0260 [hep-th])

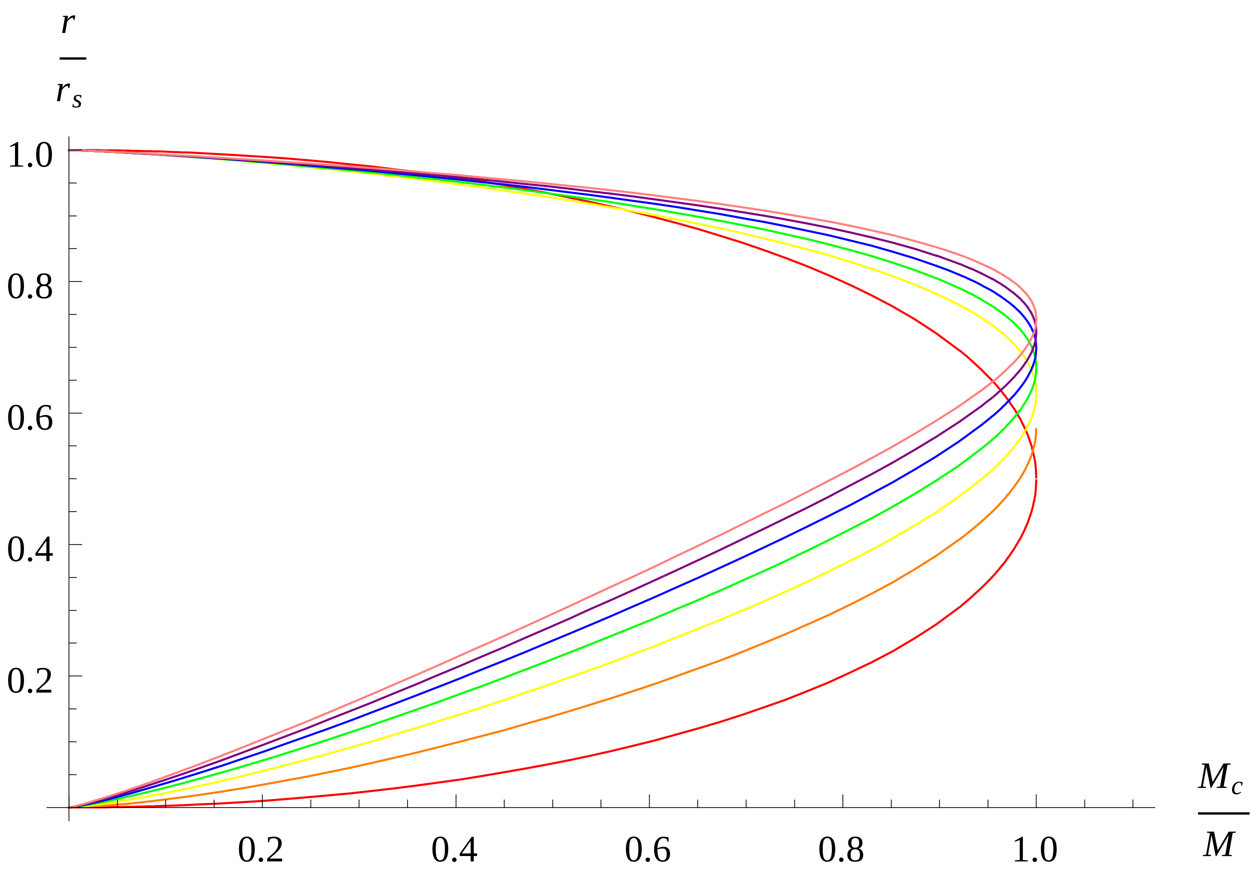
  metric, dependence on M **(D=6)**



quantum gravity and black holes

- **RG improved black holes** Falls, DL, Raghuraman (ERG ’08, 1002.0260 [hep-th])

  improved Schwarzschild radii, various dimension



BH production at the LHC

- **semi-classical**

  semi-classical production cross section

$$\hat{\sigma} = \pi r_{\rm cl}^2 (M = \sqrt{s}) \times \theta(\sqrt{s} - M_{\rm min})$$

production cross section at the LHC $\quad pp \longrightarrow$ **final state**

$$\sigma = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \, f_i(x_1) \, f_j(x_2) \, \hat{\sigma}(q_i q_j \longrightarrow \textbf{final state})$$

parton distribution functions from **CTEQ61**
evaluated at $Q^2 = M_{\rm BH}^2$.

BH production at the LHC

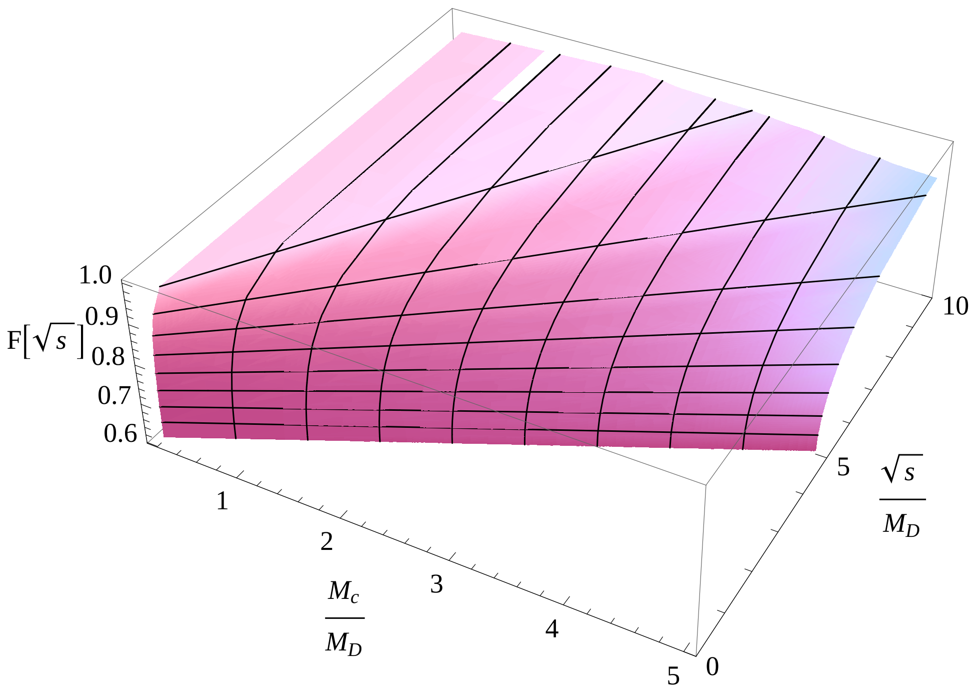
- **renormalisation group**

Falls, DL, Raghuraman 1002.0260 [hep-th]

quantum corrected production cross section

$$\hat{\sigma} \longrightarrow \hat{\sigma} = F(\sqrt{s}) \times \pi r_{\rm cl}^2(M = \sqrt{s}) \times \theta(\sqrt{s} - M_c)$$

new form factor F

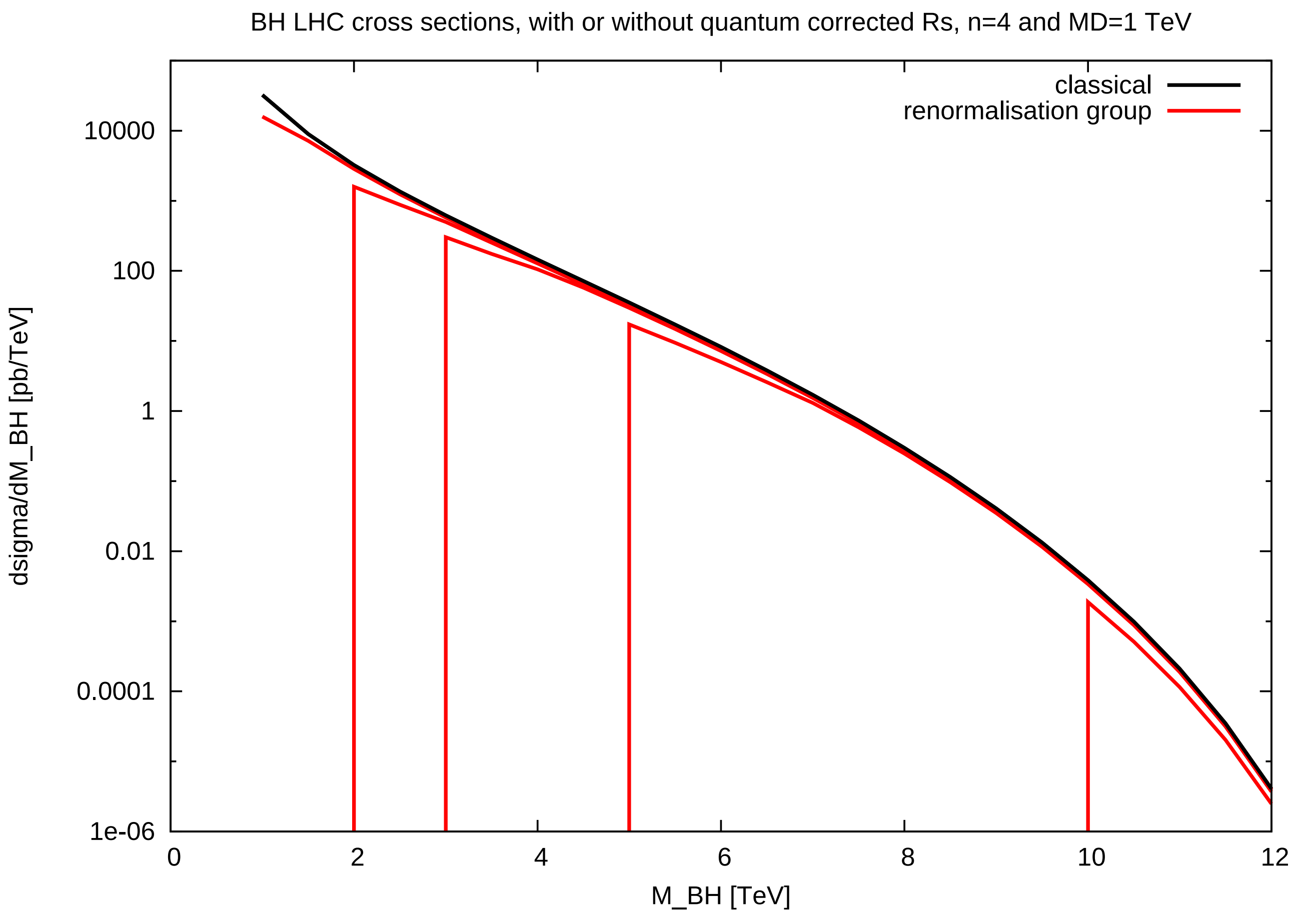


BH production at the LHC

- **semi-classical vs renormalisation group**

Falls, Hiller, DL (Pascos '09)

$n = 4$ **extra dimensions**



unitarity bounds

J. Brinkmann, G. Hiller, DL ('09)

- **Higgs-Higgs elastic scattering**

  extra dimensions, gravity-mediated, KK modes

  effective theory study X.G. He ('00)
  partial wave decomposition:

$$M(s,t) = 16\pi \sum_J (2J+1)\, a_J(s)\, P_J(\cos\theta), \quad t = t(\cos\theta)$$

$$\sigma \approx 16\pi \frac{|a_0(s)|^2}{s}, \quad a_0(s) = \frac{1}{16\pi} \frac{1}{s - 4m_h^2} \int_{4m_h^2 - s}^{0} dt\, M(s,t)$$

  optical theorem, unitarity bound

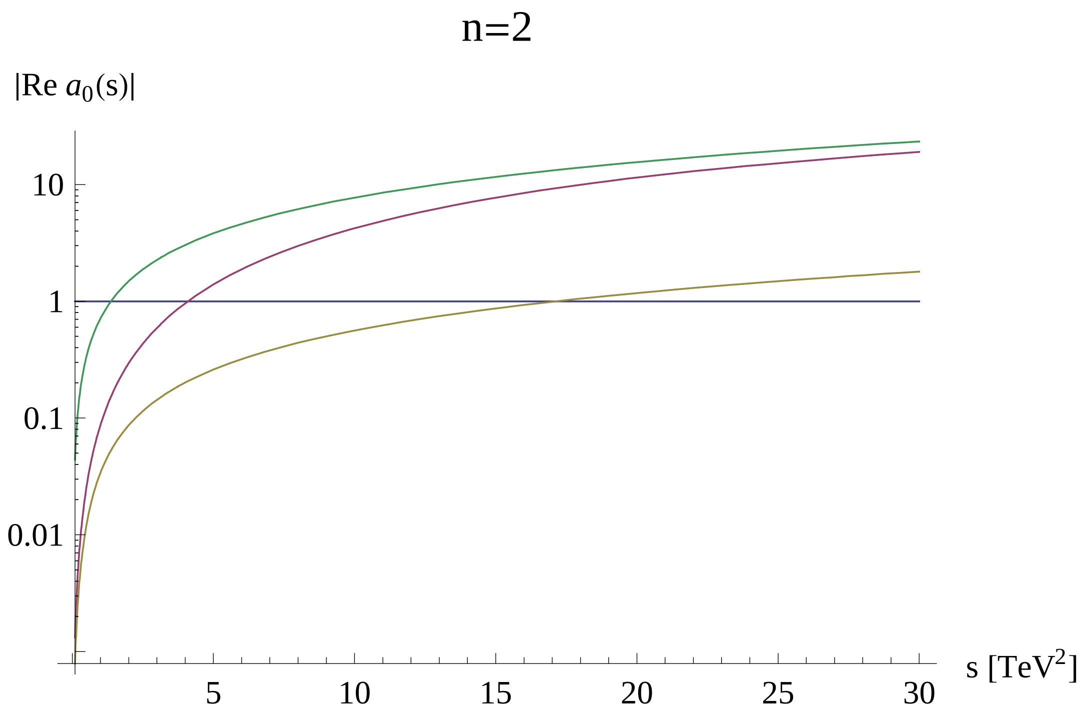
$$|a_0(s)| \leq 1$$

unitarity bounds

- **results** $\quad |a_0(s)| \to c_n \frac{s}{M_D}$

effective theory: valid for $s < M_D^2$

RG study: $\quad c_n \ll 1 \quad$ J. Brinkmann, G. Hiller, DL ('09)



parameter: $m_h = 0, n = 2, M_D = 1\text{TeV}, \Lambda_X = 1, 5, \frac{1}{5}\text{TeV}$

conclusions

- **asymptotically safe gravity**

  fascinating idea, tools are available

  results are promising

conclusions

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- **asymptotically safe phenomenology**

  cosmology, physics of black holes

  particle physics models, signatures at colliders

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- **challenges**

  complete theory of Planck scale physics

  lattice $\leftrightarrow$ continuum RG $\leftrightarrow$ PT$\leftrightarrow$ other