Length scales

IR stable (QED) plasma:

quasi-particles   mean free path $x_2 \sim \frac{x_1}{e^2}$

IR unstable (QCD) plasma:

partons   hadron radius

x_1: (quasi)free particles $<$ x_2: interactions

always nonperturbative(??)   small parameter may exists

Decoherence and damping in ideal quantum gases

Classical limit

Decoherence: Diagonal reduced density matrix for macroscopic observables

Dynamics: $|m\rangle = |\phi_m\rangle \otimes |\chi_m\rangle$, $O = O \otimes 1_{env}$, $\langle O\rangle = \mathrm{Tr} O\rho$

$$\langle O(t)\rangle = \sum_{m,n} \langle n|O(t)|m\rangle p_{m,n} = \sum_{m,n} \langle n|e^{\frac{i}{\hbar}Ht} O e^{-\frac{i}{\hbar}Ht}|m\rangle p_{m,n}$$

m, n macroscopically different states, finite measuring time
$\Longrightarrow$cancellation for $m \neq n$ due to fast phase differences
Environment: $\langle \chi_m|\chi_n\rangle = \delta_{m,n}$

$$\langle O\rangle = \sum_{m,n} (\langle \chi_n| \otimes \langle \phi_n|) O (|\phi_m\rangle \otimes |\chi_m\rangle) p_{m,n} = \sum_m \langle \phi_m|O|\phi_m\rangle p_{m,m}$$

Classical and quantum probabilities:
Unique separation by the knowledge of p_n

$$\langle O\rangle = \sum_n \langle n|O|n\rangle p_n = \sum_{n,\lambda} \lambda \overbrace{\underbrace{p(\lambda|n)}_{Q} \underbrace{p(n)}_{Cl}}^{p(\lambda)}$$

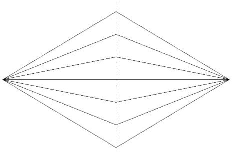
Classical limit: single λ for each n: $p(\lambda|n) = \delta_{\lambda,n}$, $Q = 1$
Simultaneous diagonalization of O and ρ

Classical limit

Consistent histories: recovery of classical probabilities

Dynamical separation of Q and C probabilities: $O = |f\rangle\langle f| \otimes 1_{env}$

$$
\begin{aligned}
1_{sys} &= \sum_j P_j(t) \\
U(T,0) &= \sum_j U(T,t)P_j(t)U(t,0)
\end{aligned}
$$



$$
\begin{aligned}
\langle i|O(T)|i\rangle &= \sum_{j,k} \langle i|U^\dagger(0,t)P_j(t)U^\dagger(t,T)|f\rangle\langle f|U(T,t)P_k(t)U(t,0)|i\rangle \\
&\to \sum_j \langle i|U^\dagger(0,t)P_j(t)U^\dagger(t,T)|f\rangle\langle f|U(T,t)P_j(t)U(t,0)|i\rangle
\end{aligned}
$$

Additive probabilities:
$\langle i|U^\dagger(0,t)P_j(t)U^\dagger(t,T)|f\rangle\langle f|U(T,t)P_k(t)U(t,0)|i\rangle \to 0$ for $j \neq k$

Classical limit

Irreversibility: The Choice

Time arrows:

1. Astrophysics: Big Bang
2. Quantum Mechanics: $\rho \rightarrow \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}$
3. Thermodynamics: $\frac{dS}{dt} \geq 0$
4. Radiation: Retarded Lienard-Wiechert potentials

Measurement:

- Enlargement: breakdown of time reversal at $\ell \sim \xi_{Q-C}$
- Selecting a single “reality”
- Producing a lasting “record”

Program

1. Classical limit
   1.1 Decoherence, consistency
   1.2 Irreversibility
2. Correlations and interactions ←
3. Double time formalism
   3.1 Classical
   3.2 Quantum
   3.3 Observables
   3.4 Equation of motion
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   4.3 Hydrodynamical equations of motion
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Correlations and interactions

Nonlinear observable as interaction

System: x, $S[x]$, Collective coordinate: $y = F(x)$

Goal: Effective dynamics of the collective coordinate

Dynamics for the collective variable: $S[x] \to S[x] + S_i[y, x]$

$$S_i[y, x] = -K \int dt [y(t) - F(x(t))]^2$$

The system dynamics is not disturbed

Interaction looking terms for nonlinear $F(x)$!

Quantum case: Indistinguishable particles, overlapping wave-functions
$\Longrightarrow$Composite operators
$\Longrightarrow$Exchange correlations, entanglement, Pauli-blocking

Program

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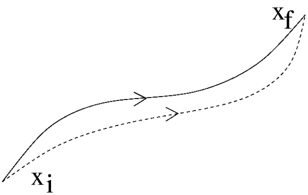
Double time formalism

Classical

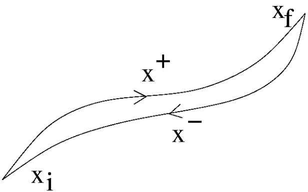
Irreversibility: Open final condition
Initial conditions only
Variational method?

No variation at the end points because $\frac{\delta S[x]}{\delta x_f} = p_f \neq 0$



CTP: $x(t) = \begin{cases} x^+(t) & 0 < t < t_f \\ x^-(t_f - t) & t_f < t < 2t_f \end{cases}$

$\Longrightarrow \frac{\delta S[x]}{\delta x_{t_f}} = 0 \quad (t_f \text{ arbitrary})$



Double time formalism

QFT: Small & large systems

J. Schwinger: CTP for QFT in Heisenberg representation
Now: (Reduced)Density matrix and irreversibility in QFT

Schwinger: Expectation values in the Heisenberg representation

$$\begin{aligned}\langle\psi(t)|O_S|\psi(t)\rangle &= \langle\psi_i|e^{i(t-t_i)H}O_S e^{-i(t-t_i)H}|\psi_i\rangle \\ &= \mathrm{Tr}[O_S \underbrace{e^{-i(t-t_i)H}\overbrace{|\psi_i\rangle\langle\psi_i|}^{\rho_i}e^{i(t-t_i)H}}_{\rho(t)}]\end{aligned}$$

Feynman: Transition amplitudes with insertion

$$\begin{aligned}\mathcal{A} &= \langle\psi_i|e^{-i(t_f-t)H}Ae^{-i(t-t_i)H}|\psi_i\rangle \\ &= \langle\psi_i|e^{-i(t_f-t_i)H}e^{i(t-t_i)H}Ae^{-i(t-t_i)H}|\psi_i\rangle \\ &\neq \langle\psi_i|e^{i(t-t_i)H}Ae^{-i(t-t_i)H}|\psi_i\rangle \quad \text{for} \quad H|\psi_i\rangle\neq|0\rangle\end{aligned}$$

Goal: Path integral and the perturbation expansion in the Heisenberg representation for the (reduced) density matrix

Double time formalism

Generating functional

$A(t)$ for vertices and observables, $\quad \hat{j} = \begin{pmatrix} j^+ \\ j^- \end{pmatrix}$

$$
\begin{aligned}
e^{\frac{i}{\hbar}W[\hat{j}]} &= \mathrm{Tr}\, T[e^{-\frac{i}{\hbar}\int dt(H(t)-j^+(t)A(t))}]\rho_i\, T^*[e^{\frac{i}{\hbar}\int dt(H(t)+j^-(t)A(t))}] \\
&= \mathrm{Tr}[\bar{T}[e^{\frac{i}{\hbar}\int dt(H(t)+j^-(t)A(t))}e^{-\frac{i}{\hbar}\int dt(H(t)-j^+(t)A(t))}]\rho_i]
\end{aligned}
$$

Expectation values rather than transition amplitudes
Reduplication of the degrees of freedom
$\Longrightarrow$friction and retarded interactions

Double time formalism

Generating functional

Path integral:

$$
\begin{aligned}
e^{\frac{i}{\hbar}W[\hat{j}]} &= \mathrm{Tr}\, T[e^{-\frac{i}{\hbar}\int dt(H(t)-j^{+}(t)x(t))}]\rho_i T^*[e^{\frac{i}{\hbar}\int dt(H(t)+j^{-}(t)x(t))}] \\
&= \int D[\hat{x}] e^{\frac{i}{\hbar}S[x^{+}]-\frac{i}{\hbar}S[x^{-}]+\frac{i}{\hbar}S_{BC}[\hat{x}]+\frac{i}{\hbar}\int dt\hat{j}(t)\hat{x}(t)} \\
&= \int D[\hat{x}] e^{\frac{i}{\hbar}S[\hat{x}]+\frac{i}{\hbar}\int dt\hat{j}(t)\hat{x}(t)}
\end{aligned}
$$

CTP: Closed Time Path $x^{\pm}(t_f) = x_f$
OTP: Open Time Path $x^{\pm}(t_f) = x_f^{\pm}$

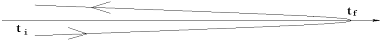
Entanglement, mixed state:

$$
S[\hat{x}] = S_0[x^{+}] - S_0[x^{-}] + S_{ent}[x^{+}, x^{-}]
$$

Double time formalism

Expectation values

Operator insertion:



Two ways: $A = \frac{\delta W[j^+, j^-]}{\delta j^+}_{|j^+=-j^-} = \frac{\delta W[j^+, j^-]}{\delta j^-}_{|j^+=-j^-}$

Parametrization: $j^{\pm} = \frac{j}{2}(1 \pm \kappa) \pm \bar{j}$

$$\hat{j}\hat{x} = j \underbrace{\left(\frac{1+\kappa}{2} x^+ + \frac{1-\kappa}{2} x^- \right)}_{x} + \bar{j} \underbrace{(x^+ - x^-)}_{x^a}$$

$\bar{j}$: physical, drive adiabatically to $|\Psi_i\rangle$

j: book keeping device, set to zero at the end

averages are independent of t_f

Problem: $\kappa = 0$ (Keldysh): $W^*[j^+, j^-] = -W[-j^-, -j^+]$

$\Longrightarrow W^*[j, \bar{j}] = -W[-j, \bar{j}]$

$\Longrightarrow$ No $\mathcal{O}(j^2)$ real part!

Double time formalism

Effective action

Equation of motion for expectation values

$$\Gamma[x, x^a] = \Re W[j, \bar{j}] - \int dt[j(t)x(t) + \bar{j}(t)x^a(t)].$$

Legendre transformation: $x = \dfrac{\delta \Re W[j, \bar{j}]}{\delta j}, \quad x^a = \dfrac{\delta \Re W[j, \bar{j}]}{\delta \bar{j}}$

Inverse transformation: $\underbrace{j = -\dfrac{\delta \Gamma[x, x^a]}{\delta x}, \quad \bar{j} = -\dfrac{\delta \Gamma[x, x^a]}{\delta y^a}}_{\text{Equations of motion}}$

$x = \dfrac{1+\kappa}{2}\langle x^+\rangle + \dfrac{1-\kappa}{2}\langle x^-\rangle = \langle x^\pm\rangle$ physical expectation value

$x^a = \langle x^+\rangle - \langle x^-\rangle$ decoherence

off diagonal density matrix elements

Physical variable only:

$$\Gamma[x] = \Re W[j, \bar{j}] - \int dt j(t)x(t), \qquad j = -\frac{\delta \Gamma[x]}{\delta x}$$

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Ideal gas

Free parameters:
Dispersion relation: $\epsilon_{\mathbf{k}} = \hbar\omega_{\mathbf{k}}$
Exchange statistics: $\xi = \pm 1$
Spin degeneracy factor: n_s

Action:

$$S[\psi^\dagger, \psi] = \int d^4x \psi^\dagger_x \left[i\hbar\partial_t + \frac{\hbar^2}{2m}\Delta + \mu \right] \psi_x$$

Observables: Density-current vector

$$J^\mu_x = (\rho_x, \mathbf{j}_x) = \psi^\dagger C^\mu_x \psi, \qquad (C^\mu_x)_{yz} = \delta_{y,x}\delta_{z,x} \left(1, -\frac{i\hbar}{2m}(\boldsymbol{\nabla}_z - \boldsymbol{\nabla}_y) \right)$$

Ideal gas

Propagators: Generating functional

External source a^μ as a local Galilean boost: $p \longrightarrow p - a$:

$$\begin{aligned} e^{\frac{i}{\hbar}W[\hat{a}]} &= \mathrm{Tr}\, T[e^{-\frac{i}{\hbar}\int_x (H_x - a^+_{\mu x} J^\mu_x)}]\rho_i \bar{T}[e^{\frac{i}{\hbar}\int_x (H_x + a^-_{\mu x} J^\mu_x)}] \\ &= \int D[\hat{\psi}]D[\hat{\psi}^\dagger] e^{\frac{i}{\hbar}\sum_{\sigma\sigma'} \psi^{\dagger\sigma}((G^{-1})^{\sigma\sigma'} + \delta^{\sigma\sigma'} a^\sigma C)\psi^{\sigma'}}, \end{aligned}$$

$$\hat{\psi} = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}, \qquad \hat{a} = \begin{pmatrix} a^+ \\ a^- \end{pmatrix}$$

The inverse CTP propagator:

$$\hat{G}^{-1} = \begin{pmatrix} G_0^{-1} & 0 \\ 0 & -G_0^{-1*} \end{pmatrix} + G^{-1}_{\mathrm{BC}},$$

$$G^{-1}_{0\ x,x'} = \left(i\hbar\partial_t + \frac{\hbar^2}{2m}\Delta + \mu + i\epsilon \right) \delta_{x,x'},$$

Ideal gas

Propagators: Generating functional

$$
\begin{aligned}
W[\hat{a}] &= i\xi\hbar \mathrm{Tr} \ln(\hat{G}^{-1} + \hat{a}^{\mu} C_{\mu}) \\
&= i\xi\hbar \left[\mathrm{Tr} \ln \hat{G}^{-1} - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \mathrm{Tr}[\hat{G}(\hat{a}C)]^n \right] \\
\tilde{G}_{\tilde{x}_1,\ldots,\tilde{x}_n} &= (-1)^{n+1} \frac{i\xi\hbar}{n!} \sum_{\pi \in S^n} \mathrm{Tr}[\hat{G} C_{x_{\pi(1)}} \cdots \hat{G} C_{x_{\pi(n)}}],
\end{aligned}
$$

π: permutation of n objects

Connected Green functions with arbitrary large number of legs (nonlinear composite operators)

"Interactions" without small parameter: Entanglement

Ideal gas

Retarded, advanced propagators

$$W^{(2)}[\hat{a}] = \hat{J}_{\rm gr}\hat{a} - \frac{1}{2}\hat{a}\tilde{G}\hat{a}$$

with

$$\begin{aligned} J_{\rm gr}^{\sigma\mu} &= i\xi\hbar\mathrm{Tr}[\hat{G}C_x^{\sigma\mu}] = (\sigma n_0, \mathbf{0}) \\ \tilde{G}_{xx'}^{(\sigma\mu)(\sigma'\mu')} &= i\xi\hbar\mathrm{Tr}[\hat{G}^{\sigma'\sigma}C_x^{\mu}\hat{G}^{\sigma\sigma'}C_{x'}^{\mu'}]. \end{aligned}$$

$T[AB] + T^*[AB] = \xi BA + AB \Longrightarrow$ Three real functions:

$$\tilde{G}_{xx'}^{(\sigma\mu)(\sigma'\nu)} = \begin{pmatrix} \tilde{G}_{xx'}^{n\mu\nu} & -\tilde{G}_{xx'}^{f\mu\nu} \\ \tilde{G}_{xx'}^{f\mu\nu} & -\tilde{G}_{xx'}^{n\mu\nu} \end{pmatrix} + i\tilde{G}_{xx'}^{i\mu\nu}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Retarded, advanced Green functions: $\tilde{G}^{\substack{r\\a}} = \tilde{G}^n \pm \tilde{G}^f$

Ideal gas

Symmetries of the two point function

Fourier space: $X = \tilde{G}^{\sigma\sigma'}, \tilde{G}^n, \tilde{G}^f$ or $\tilde{G}^i$, $q^\mu = (\omega, \mathbf{q})$

$$X_q = \int dt d^3x e^{i\omega t - i\mathbf{x}\mathbf{q}} X_{(t,\mathbf{x}),(0,\mathbf{0})},$$

Rotational invariance: $q = |\mathbf{q}|$, $L = \mathbf{q} \otimes \mathbf{q}/\mathbf{q}^2$ and $T = 1 - L$

$$X_q^{\mu\nu} = \begin{pmatrix} X_{\omega,q}^{tt} & \mathbf{q} X_{\omega,q}^{ts} \\ \mathbf{q} X_{\omega,q}^{st} & L X_{\omega,q}^{L} + T X_{\omega,q}^{T} \end{pmatrix}$$

Current conservation: $\partial_t \rho + \nabla \mathbf{j} = 0$, $\mathbf{n} = \mathbf{q}/q$

$$X_q = \begin{pmatrix} X_{\omega,q}^{tt} & -\mathbf{n}\frac{\omega}{q} X_{\omega,q}^{tt} \\ -\mathbf{n}\frac{\omega}{q} X_{\omega,q}^{tt} & T X_{\omega,q}^{T} - L\frac{\omega^2}{q^2} X_{\omega,q}^{tt} \end{pmatrix}$$

Ideal gas

Loop integral for the two point function

$$\hat{\tilde{G}}^{\mu\nu} = \begin{pmatrix} L & iS^- \\ -iS^- & -L \end{pmatrix}^{\mu\nu} - iS^{+\mu\nu} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

$$L_q^{\mu\nu} = n_s \hbar \int \frac{d^3k}{(2\pi)^3} n_{\mathbf{k}} P \left[\frac{F^{\mu\nu+}(\mathbf{k},\mathbf{q})}{\omega - \frac{\hbar q^2}{2m} - \frac{\hbar \mathbf{kq}}{m}} - \frac{F^{\mu\nu-}(\mathbf{k},\mathbf{q})}{\omega + \frac{\hbar q^2}{2m} + \frac{\hbar \mathbf{kq}}{m}} \right]$$

$$R_q^{\pm\mu\nu} = n_s \pi \hbar \int \frac{d^3k}{(2\pi)^3} \delta(\pm\omega - \omega_{\mathbf{k}+\mathbf{q}} + \omega_{\mathbf{k}}) n_{\mathbf{k}} (1 + \xi n_{\mathbf{k}+\mathbf{q}}) F^{\mu\nu\pm}(\mathbf{k},\mathbf{q})$$

$$F^{\pm\mu\nu}(\mathbf{k},\mathbf{q}) = \begin{pmatrix} 1 & \mp\frac{\hbar}{m}\left(\mathbf{k} + \frac{\mathbf{q}}{2}\right) \\ \mp\frac{\hbar}{m}\left(\mathbf{k} + \frac{\mathbf{q}}{2}\right) & \frac{\hbar^2}{m^2}\left(\mathbf{k} + \frac{\mathbf{q}}{2}\right) \otimes \left(\mathbf{k} + \frac{\mathbf{q}}{2}\right) \end{pmatrix}$$

with $S^\pm = R^+ \pm R^-$.

Gaussian approximation:

$$e^{\frac{i}{\hbar}W[\hat{a}]} = e^{\frac{i}{\hbar}\hat{J}_{\rm gr}\hat{a} - \frac{i}{2\hbar}\hat{a}\tilde{G}\hat{a}} = \int D[\hat{J}] e^{\frac{i}{\hbar}S_B[\hat{J}] + \frac{i}{\hbar}\hat{J}\hat{a}}$$

Effective bare action:

$$S_B[\hat{J}] = \frac{1}{2}(\hat{J} - \hat{J}_{\rm gr})\tilde{G}^{-1}(\hat{J} - \hat{J}_{\rm gr})$$

$$\Re\hat{\tilde{G}}^{-1} = \frac{1}{L^2 + S_-^2}\begin{pmatrix} L & iS^- \\ -iS^- & -L \end{pmatrix}, \quad \Im\hat{\tilde{G}}^{-1} = \frac{S^+}{L^2 + S_-^2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Ideal gas

Hydrodynamical equations of motion

Effective action:

$$\kappa\Gamma[J] = J\tilde{G}^{n-1}(\tilde{G}^{r}\bar{a} + J_{\mathrm{gr}}) - \frac{1}{2}J\tilde{G}^{n-1}J$$

Equation of motion:

$$\tilde{G}^{r-1}(J - J_{\mathrm{gr}}) = \bar{a}$$

Current conservation: $\Longrightarrow j^{\mu} = (n, \mathbf{j}) \rightarrow (n, \mathbf{j}^{T})$

Dimensionless variables:

$$k_{gas} = \begin{cases} k_F & \textit{fermions} \\ \frac{\sqrt{mk_B T}}{\hbar} & \textit{bosons} \end{cases}, \quad Q = \frac{q}{k_{gas}}, \quad z = \frac{v_{\mathrm{probe}}}{v_{\mathrm{gas}}} = \frac{\omega/k}{\hbar k_{gas}/m}$$

Hydrodynamical regime: $Q, z \ll 1$

Ideal gas

Hydrodynamical equations of motion: Fourier space

Equation of motion:

$$-\frac{n_s m k_F}{4\pi^2}\bar{a}^0 = \left(a_0 + a_z \frac{m}{\hbar k_{gas}}\frac{i\omega}{k} + \frac{a_{kk}}{k_{gas}^2}k^2\right) n,$$

$$-\frac{n_s m k_F}{4\pi^2}\frac{\hbar^2 k_F^2}{m^2}\bar{\mathbf{a}}^T = \left(b_0 + b_z \frac{m}{\hbar k_{gas}^2}\frac{i\omega}{k} + \frac{b_{kk}}{k_{gas}^2}k^2\right) \mathbf{j}^T.$$

Fermions at finite density, vanishing temperature:

$$a_0 = \frac{1}{2}, \quad a_z = \frac{\pi}{4}, \quad a_{kk} = \frac{1}{24}, \quad b_0 = \frac{3}{2}, \quad b_z = \frac{9\pi}{4}, \quad b_{kk} = \frac{39}{128}$$

Multiplying by k and going into real space:

Ideal gas

Hydrodynamical equations of motion: Real space

$$
\begin{aligned}
\frac{a_z m}{\hbar k_{gas}^2}\partial_t n_{\mathbf{x}} &= \left(\frac{a_{qq}}{k_{gas}^2}\Delta - a_0\right)\frac{1}{k_{gas}}\int d^3y\,\frac{[(\mathbf{y}\boldsymbol{\nabla}_y)^2 - 2\mathbf{y}\boldsymbol{\nabla}_y]n_{\mathbf{y}}}{2\pi^2(\mathbf{x}-\mathbf{y})^4} \\
&\quad - \frac{n_s m}{4\pi^2}\int d^3y\,\frac{[(\mathbf{y}\boldsymbol{\nabla}_y)^2 - 2\mathbf{y}\boldsymbol{\nabla}_y]\bar{a}^0_{\mathbf{y}}}{2\pi^2(\mathbf{x}-\mathbf{y})^4} \\
\frac{b_z m}{\hbar k_{gas}^2}\partial_t \mathbf{j}^T_{\mathbf{x}} &= \left(\frac{b_{qq}}{k_{gas}^2}\Delta - b_0\right)\frac{1}{k_{gas}}\int d^3y\,\frac{[(\mathbf{y}\boldsymbol{\nabla}_y)^2 - 2\mathbf{y}\boldsymbol{\nabla}_y]\mathbf{j}^T_{\mathbf{y}}}{2\pi^2(\mathbf{x}-\mathbf{y})^4} \\
&\quad - \frac{n_s m}{4\pi^2}\int d^3y\,\frac{[(\mathbf{y}\boldsymbol{\nabla}_y)^2 - 2\mathbf{y}\boldsymbol{\nabla}_y]\mathbf{a}^T_{\mathbf{y}}}{2\pi^2(\mathbf{x}-\mathbf{y})^4}.
\end{aligned}
$$

Reminiscent of Navier-Stokes equation

Viscosity?

Program

Damping

Viscous flow

Equation of motion:

$$\bar{a}^0 = \frac{n_k}{-L^{tt} + iS^{tt-}}, \qquad \bar{\mathbf{a}}^T = \frac{\mathbf{j}^T}{-L^T + iS^{T-}}$$

Solution: $a^0 = a^0(x)$, $\bar{\mathbf{a}} = \mathbf{z}\bar{a}(x)$, $\mathbf{j} = \mathbf{z}j_z(x)$

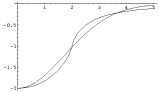
$$n_{\mathbf{k}} = \frac{n_s m k_F}{4\pi^2}\left[-1 + \left(\frac{1}{Q} - \frac{Q}{4}\right)\ln\left|\frac{2-Q}{2+Q}\right|\right]\bar{a}^0_{\mathbf{k}},$$

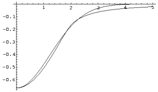
$$\mathbf{j}^T_{\mathbf{k}} = \frac{n_s m k_F}{4\pi^2}\frac{\hbar^2 k_F^2}{m^2}\left[-\frac{5}{12} + \frac{Q^2}{16} + \frac{1}{Q}\left(1 - \frac{Q^2}{4}\right)^2 \ln\left|\frac{2-Q}{2+Q}\right|\right]\bar{\mathbf{a}}^T_{\mathbf{k}}.$$

Gaussian approximation:

$$n_{\mathbf{k}} \approx -\frac{n_s m k_F}{4\pi^2} 2 e^{-\frac{Q^2}{6}}\bar{a}^0_{\mathbf{k}},$$

$$\mathbf{j}^T_{\mathbf{k}} \approx -\mathbf{z}\frac{n_s m k_F}{4\pi^2}\frac{\hbar^2 k_F^2}{m^2}\frac{2}{3}e^{-\frac{Q^2}{3}}\bar{\mathbf{a}}^T_{\mathbf{k}}.$$





Damping

Viscous flow

External source:

$$\bar{a}^0_{\mathbf{x}} = \frac{u_d}{(2\pi\ell_{\rm ext}^2)^{3/2}} e^{-\frac{x^2}{2\ell_{\rm ext}^2}}, \qquad \bar{a}^T_{\mathbf{x}} = \frac{u_c}{(2\pi\ell_{\rm ext}^2)^{3/2}} e^{-\frac{x^2}{2\ell_{\rm ext}^2}}.$$

Solution:

$$n_x = -2\frac{n_s m k_F}{4\pi^2}\frac{u_d}{(2\pi\ell_{\rm flow}^2)^{3/2}} e^{-\frac{x^2}{2\ell_{\rm flow}^2}},$$

$$\mathbf{j}^T_x = -\mathbf{z}\frac{2}{3}\frac{n_s m k_F}{4\pi^2}\frac{\hbar^2 k_F^2}{m^2}\frac{u_c}{(2\pi\ell_{\rm flow}^2)^{3/2}} e^{-\frac{x^2}{2\ell_{\rm flow}^2}}.$$

Collective phenomenon (diffusion):

Source $\ell_{\rm ext} \Longrightarrow$ Flow $\ell_{\rm flow} = \sqrt{\ell_{\rm ext}^2 + \frac{1}{3k_F^2}}$ $\quad (k_F \Longrightarrow$ Pauli-blocking)

$\bar{a}^T_{\mathbf{x}}$: space dependent Galilean boost spreads $\Longrightarrow$ shear viscosity

Damping

Physical origin: Landau damping for quantum systems

Plane waves: pointer states, no further decoherence
Damping: wave packets $\Longrightarrow$spread
loss of fine tuned phase relations
Inhomogeneity: Ballance between reflection from barriers and spread
(like for stationnary states)

Classical plasma: Dephasing in the Fourier representation
of the retarded Lienard-Wiechert potential

Dissipation: Irreversible spread of energy
needs separation of scales and
dimensional terms in the Hamiltonian (interaction)
yields increasing entropy

Program

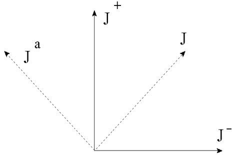
1. Classical limit
   1.1 Decoherence, consistency
   1.2 Irreversibility
2. Correlations and interactions
3. Double time formalism
   3.1 Classical
   3.2 Quantum
   3.3 Observables
   3.4 Equation of motion
4. Ideal gas
   4.1 Propagators
   4.2 Effective current dynamics
   4.3 Hydrodynamical equations of motion
5. Damping
6. Irreversibility, decoherence $\leftarrow$

Irreversibility, decoherence

Path integral for the reduced density matrix:

$$e^{\frac{i}{\hbar}W[\hat{a}]} = \int D[\hat{J}]e^{\frac{i}{\hbar}S_B[\hat{J}]+\frac{i}{\hbar}\hat{J}\hat{a}}$$

$J = \frac{1}{2}(J^+ + J^-) \Longrightarrow$probability distribution of J
$J^a = J^+ - J^- \Longrightarrow$decoherence of J



Contribution of a quasi particle to the Q-C crossover:

- $\Im S[J, J^a]$ decoherence, consistency
  $\uparrow$
- $\Im S[J^+, J^-]$ irreversibility
  $\uparrow$

Irreversibility, decoherence

Quasi particles

$$S_B[\hat{J}] = \frac{1}{2}(\hat{J} - \hat{J}_{\rm gr})\tilde{G}^{-1}(\hat{J} - \hat{J}_{\rm gr})$$

$$\Re\hat{\tilde{G}}^{-1} = \frac{1}{L^2 + S_-^2}\begin{pmatrix} L & iS^- \\ -iS^- & -L \end{pmatrix} \qquad \Im\hat{\tilde{G}}^{-1} = \frac{S^+}{L^2 + S_-^2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

- **Quasi particles:** $\Re(L_{\omega,\mathbf{q}} + iS^+_{\omega,\mathbf{q}}) = L_{\omega,\mathbf{q}} = 0$ (Lindhard)
- **Inverse life-time:** $\tau^{-1} = -\Im(L + iS^+) = -\frac{S^+}{L^2 + S_-^2}$
- **Decoherence:** Gaussian distribution $\frac{\Delta p}{p_{max}} = -\frac{S^+}{L^2 + S_-^2}$

Irreversibility, decoherence

Quasi particles

$$
\begin{aligned}
\Re S^B &= \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \\
&\left[\frac{L_q^{tt} n_q^* n_q^a - iS_{\omega,q}^{tt-} n_q^* n_q^a}{(1+z^2)[(L_q^{tt})^2 + (S_q^{tt-})^2]} + \frac{L_q^T \mathbf{j}_q^{T*} \mathbf{j}_q^{Ta} - iS_q^{T-} \mathbf{j}_q^{T*} \mathbf{j}_q^{Ta}}{(L_q^T)^2 + (S_q^{T-})^2}\right] \\
\Im S^B &= \frac{1}{4}\int \frac{d^4q}{(2\pi)^4} \\
&\left[\frac{S_q^{tt+} n_q^{a*} n_q^a}{(1+z^2)[(L_q^{tt})^2 + (S_q^{tt-})^2]} + \frac{S_q^{T+} \mathbf{j}_q^{Ta*} \mathbf{j}_q^{Ta}}{(L_q^T)^2 + (S_q^{T-})^2}\right]
\end{aligned}
$$

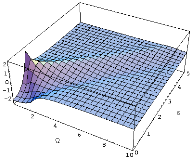
Irreversibility, decoherence

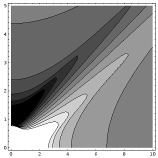
Numerics

Lindhard function $\tilde{L}^{tt}$:

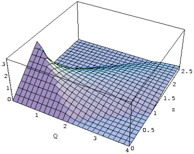
Roots $\downarrow$

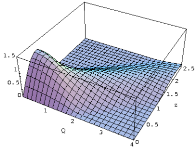
$$L^{tt} = \frac{n_s m k_F}{4\pi^2}\tilde{L}^{tt}$$
$$Q = \frac{q}{k_F}$$
$$z = \frac{m\omega}{\hbar q k_F}$$





Imaginary part or off-diagonal part $\tilde{R}^{tt+}$





$$R^{tt} = \frac{n_s m k_F}{4\pi^2}\tilde{R}^{tt} \qquad R^{T} = \frac{n_s m k_F^3}{4\pi^2}\tilde{R}^{T}$$

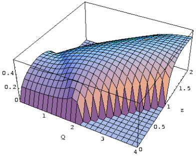
Irreversibility, decoherence

Numerics

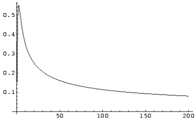
Absolut strength of decoherence:

$$(D^{tt})^2 = \left|\frac{\hbar\langle n^* n\rangle}{\langle n^* n^a\rangle\langle n^{a*} n\rangle}\right| = \frac{|S^{tt+}|}{2(1+z^2)[(L^{tt})^2+(S^{tt-})^2]},$$

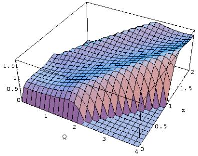
$$(D^T)^2 = \left|\frac{\hbar\langle \mathbf{j}^{T*}\mathbf{j}^T\rangle}{\langle \mathbf{j}^{T*}\mathbf{j}^{Ta}\rangle\langle \mathbf{j}^{Ta*}\mathbf{j}^T\rangle}\right| = \frac{|S^{T+}|}{2[(L^T)^2+(S^{T-})^2]},$$



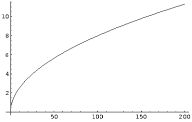
D^{tt}



$z = \frac{Q}{2}$ $\left(\hbar\omega = \frac{\hbar^2 q^2}{2m}\right)$



D^T



$z = \frac{Q}{2}$

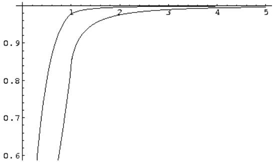
$Q = \frac{q}{k_F}$, $z = \frac{m\omega}{\hbar q k_F}$

Classical features at $Q \gg 1$ for free particles only

Relative strength of decoherence:

$$(R^{tt})^2 = \left|\frac{\langle n^* n\rangle^2}{\langle n^* n^a\rangle\langle n^{a*} n\rangle}\right| = \frac{(S^{tt+})^2}{(L^{tt})^2 + (S^{tt-})^2},$$

$$(R^T)^2 = \left|\frac{\langle \mathbf{j}^{T*}\mathbf{j}^T\rangle^2}{\langle \mathbf{j}^{T*}\mathbf{j}^{Ta}\rangle\langle \mathbf{j}^{Ta*}\mathbf{j}^T\rangle}\right| = \frac{(S^{T+})^2}{(L^T)^2 + (S^{T-})^2},$$



$R^T \uparrow \quad T^{tt} \downarrow$

$z = \frac{Q}{2}$ $(\hbar\omega = \frac{\hbar^2 q^2}{2m})$

Summary

1. “Interactions” from exchange correlations

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2. QFT for the quantum-classical crossover

Summary

1. "Interactions" from exchange correlations
2. QFT for the quantum-classical crossover
3. Irreversibility, decoherence, consistency and damping in ideal gas