TECHNISCHE UNIVERSITÄT DARMSTADT

QCD Phase Transitions and Green’s Functions

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Lorenz von Smekal

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HELMHOLTZ ASSOCIATION

Helmholtz Young Investigator Group
'Nonperturbative Phenomena in QCD'

HIC for FAIR
Helmholtz International Center
LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

EMMI
ExtreMe Matter Institute
Helmholtz Alliance 'Cosmic Matter in the Laboratory'

European Commission

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Contents

- Introduction: why Green's functions?
- Functional methods
- What do we know?
  (zero temperature and density)
- Applications to QCD phase transitions
  (deconfinement and chiral symmetry restoration)
- Summary and Outlook: what may we expect?

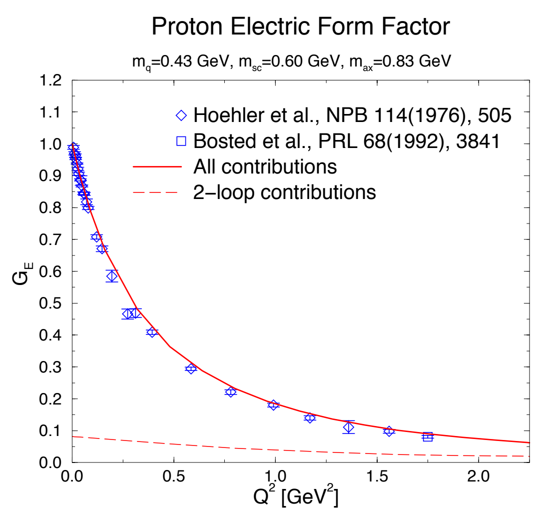
Introduction

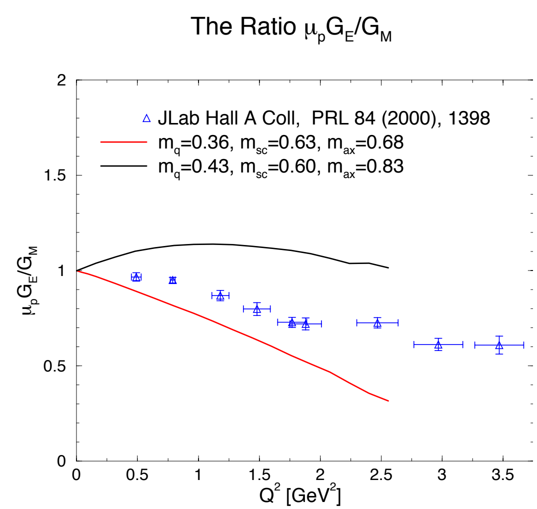
Why QCD Green's Functions?

- quantum field theoretic building blocks for hadron phenomenology

  hadron masses, decay constants, form factors, magnetic moments, charge radii etc.
  from covariant bound state equations (Bethe-Salpeter and Faddeev)

need QCD quark and gluon Green's functions as input





covariant Faddeev equation for baryons

$\longrightarrow$ **G. Eichmann, PhD Thesis, Graz (2009)**
[arXiv:0909.0703 [hep-ph]]

Oettel, Pichowsky, LvS, EPJA 8 (2000) 251
Oettel, Alkofer, LvS, EPJA 8 (2000) 553

Introduction

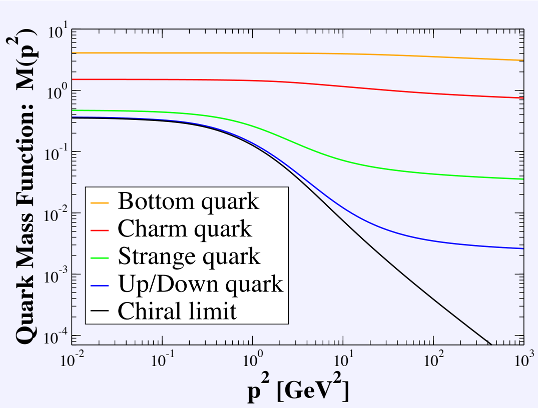
Why QCD Green's Functions?

- confinement, dynamical mass generation, chiral symmetry breaking and Goldstone's theorem

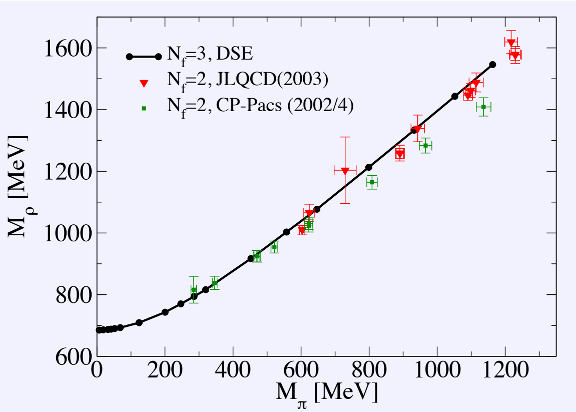
TOPCITE 500+

Alkofer & LvS, Phys. Rept. 353/5-6 (2001) 281

Fischer, J. Phys. G 32 (2006) 253



Fischer, Alkofer, PRD 67 (2003) 094020



Fischer, Watson, Cassing, PRD 72 (2005) 094025

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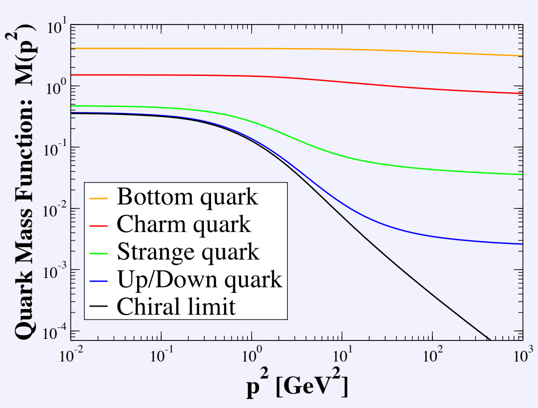
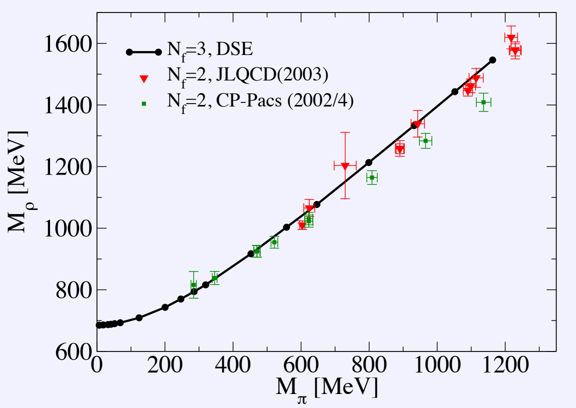




Fischer, Alkofer, PRD 67 (2003) 094020 — Fischer, Watson, Cassing, PRD 72 (2005) 094025

Here:

- chiral symmetry restoration and deconfinement transition . . .

Generalities – Partition Function

$$Z(T,V,\mu) \; = $$

$$\int \mathcal{D}[A,c,\psi]\,\delta(\partial A)\,\exp\Big\{-\int_0^{1/T} dt \int_V d^3x\,\Big[\frac{1}{4}F^2 + \bar{c}\,\partial D c + \bar{\psi}(-\not{D} + m - \mu\gamma_0)\psi\Big]\Big\}$$

Landau gauge: Lorenz condition, $\partial_\mu A_\mu = 0$

Faddeev-Popov ghosts and determinant: $\bar{c},\, c \;\leadsto\;$ Det$(-\partial_\mu D_\mu(A))$

Field strengths: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

Quark fields with a.p. b.c.'s: $\psi(1/T,\vec{x}) = -\psi(0,\vec{x})$

Note on Gribov Copies

- **Gribov, Singer (1978):**

  gauge copies unavoidable.

- **Fujikawa, Hirschfeld (1979):**

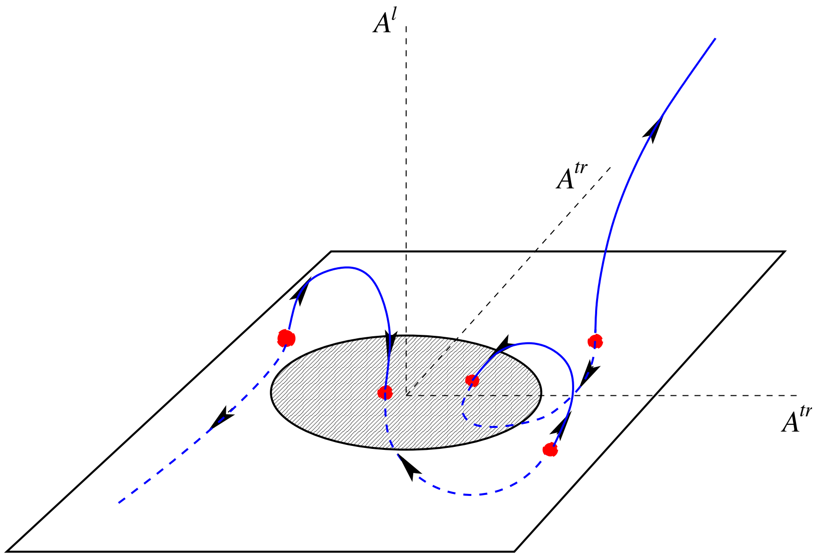
  average over copies (sign-weighted).

- **Sharpe (1984), Neuberger (1987), Schaden (1998):**

  perfect cancellation, produces 0!

- **LvS (2008):**

  can be avoided → non-perturbative Becchi-Rouet-Stora-Tyutin (BRST) symmetry (on the lattice, but with sign problem).



$\partial A = 0$ for $A = A^{tr}$, many copies!

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Non-Perturbative Tools

- **Functional Methods:**

  (a) Dyson-Schwinger Equations (DSEs)

  Partition Function → Generating Functional
  → Equations of Motion for Green's Functions

  (b) Functional Renormalisation Group (FRG)

  Effective Action → Energy-Momentum Cutoff
  → RG Flow → Wetterich Equation

  \+ chiral fermions
  \+ no sign problem
  \+ dynamical hadronisation

- **Lattice Gauge Theory**

  \+ no truncations
  \+ manifest gauge invariance

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- **Lattice Gauge Theory**

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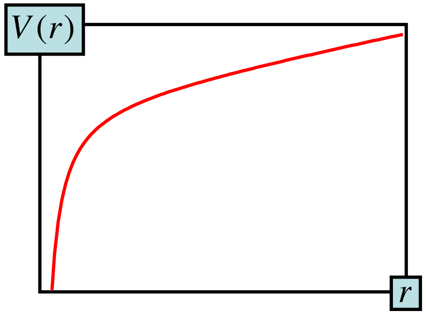
— combine & compare —

Historical Note

Historically: infrared enhanced gluon exchange,

$$D_{\text{gluon}} \propto \frac{\sigma}{p^4}\,, \quad V(\vec{r}) \propto \sigma r$$



but no – it's ghosts that dominate the long-range/infrared correlations!

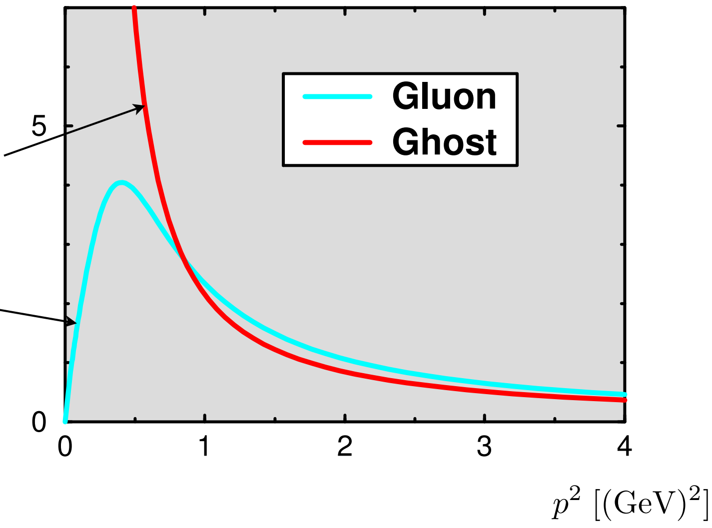
LvS, Hauck, Alkofer (1997)

2 conditions for confinement:

Kugo, Ojima (1979)

(a) avoid Higgs mechanism

(b) mass gap



→ **all physical states are color singlets.**

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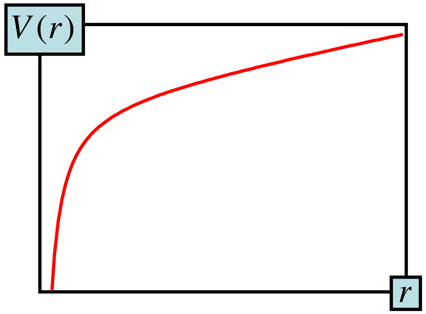

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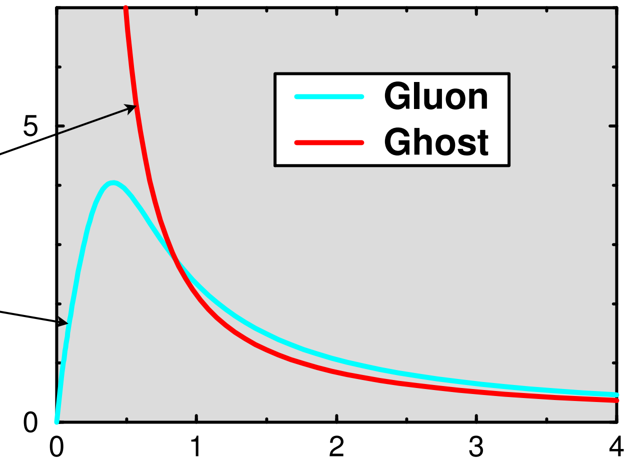


⟶ **all physical states are color singlets.**

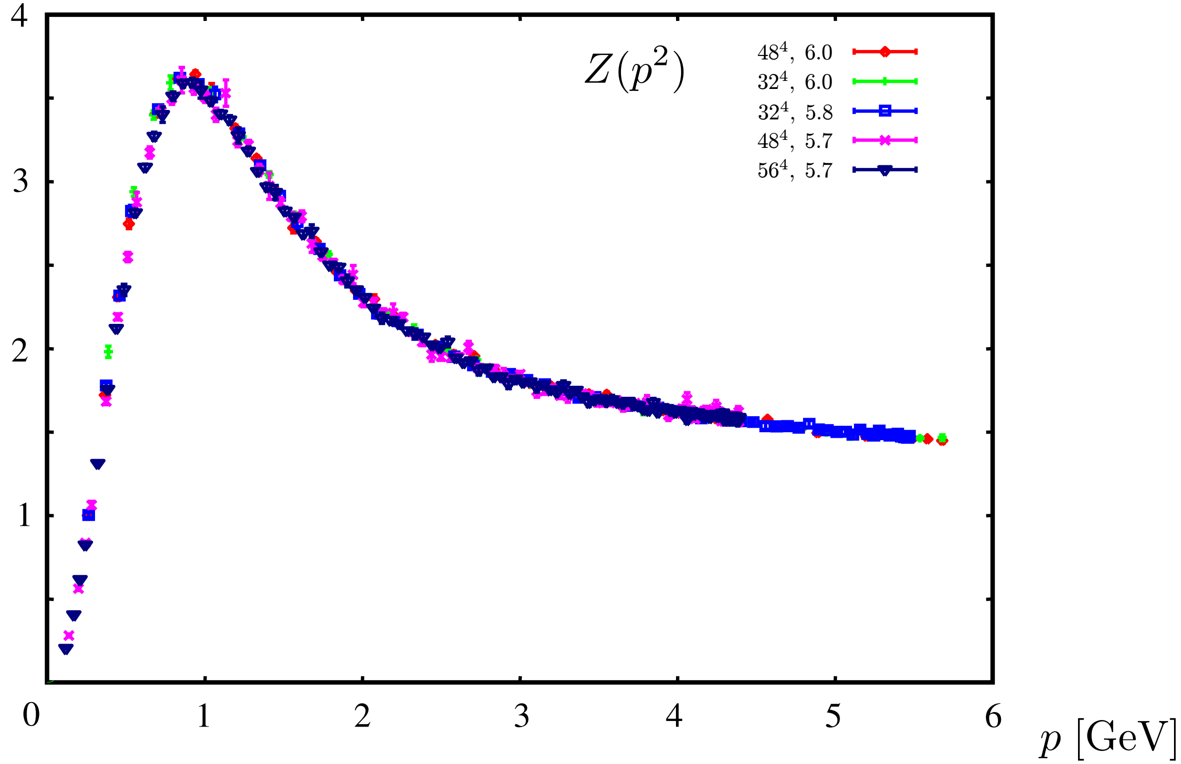
F. Coester:

"Farbloser Quark zusammengeleimt von Gespenstern mit farbigem Leim"

Gluon Propagator

Pure Yang-Mills, $T = 0$

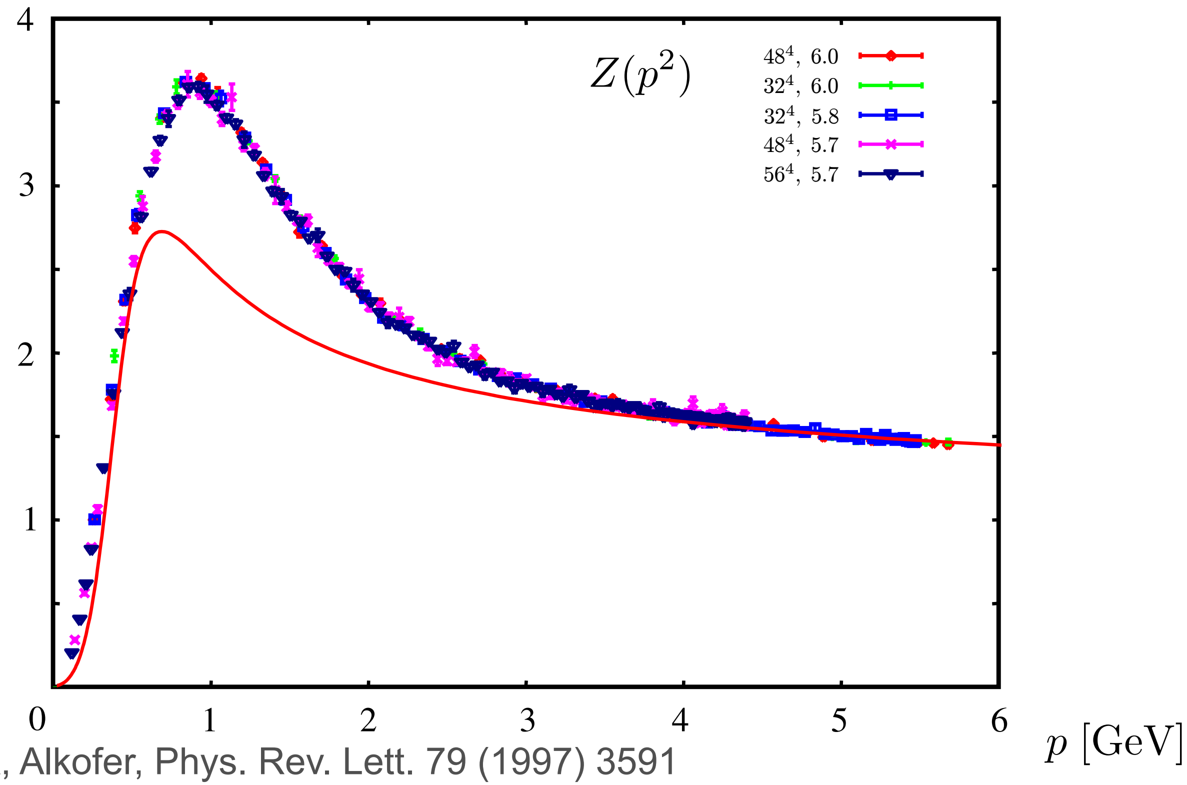
Sternbeck et al., PoS LAT2006, 76



$$\langle A_\mu(x) A_\nu(y) \rangle : \qquad D_{\mu\nu}(p) = \frac{Z(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

Gluon Propagator

Sternbeck et al., PoS LAT2006, 76



LvS, Hauck, Alkofer, Phys. Rev. Lett. 79 (1997) 3591

Gluon Propagator

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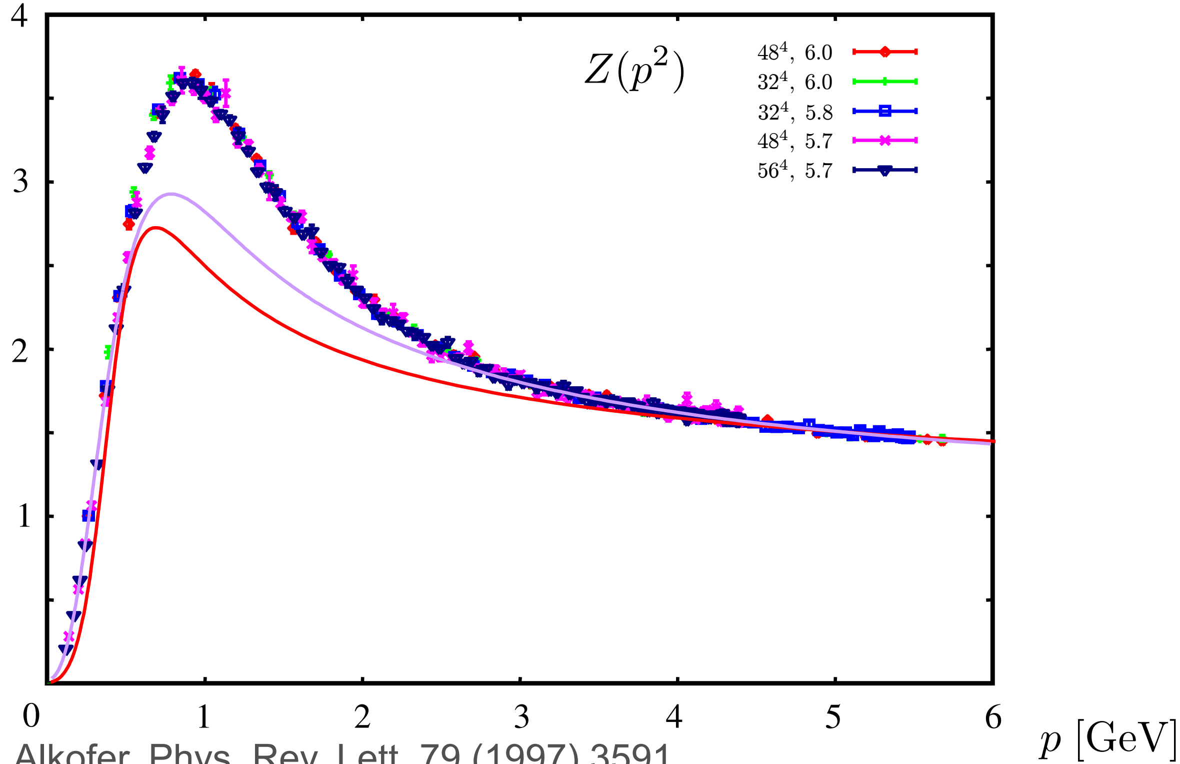

LvS, Hauck, Alkofer, Phys. Rev. Lett. 79 (1997) 3591

Lerche, LvS, Phys. Rev. D 65 (2002) 125006

Fischer, Alkofer, Phys. Rev. D 65 (2002) 094008

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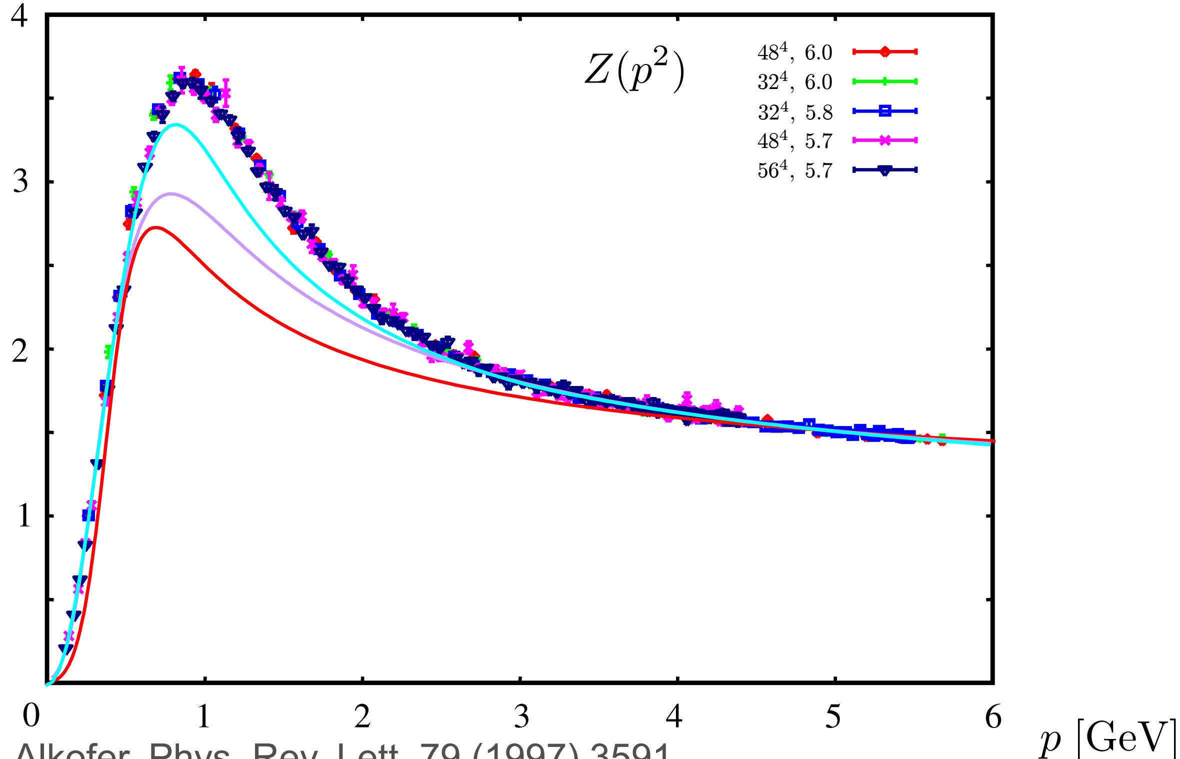

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Pawlowski, Litim, Nedelko, LvS, Phys. Rev. Lett. 93 (2004) 152002; Pawlowski (2006), unp.

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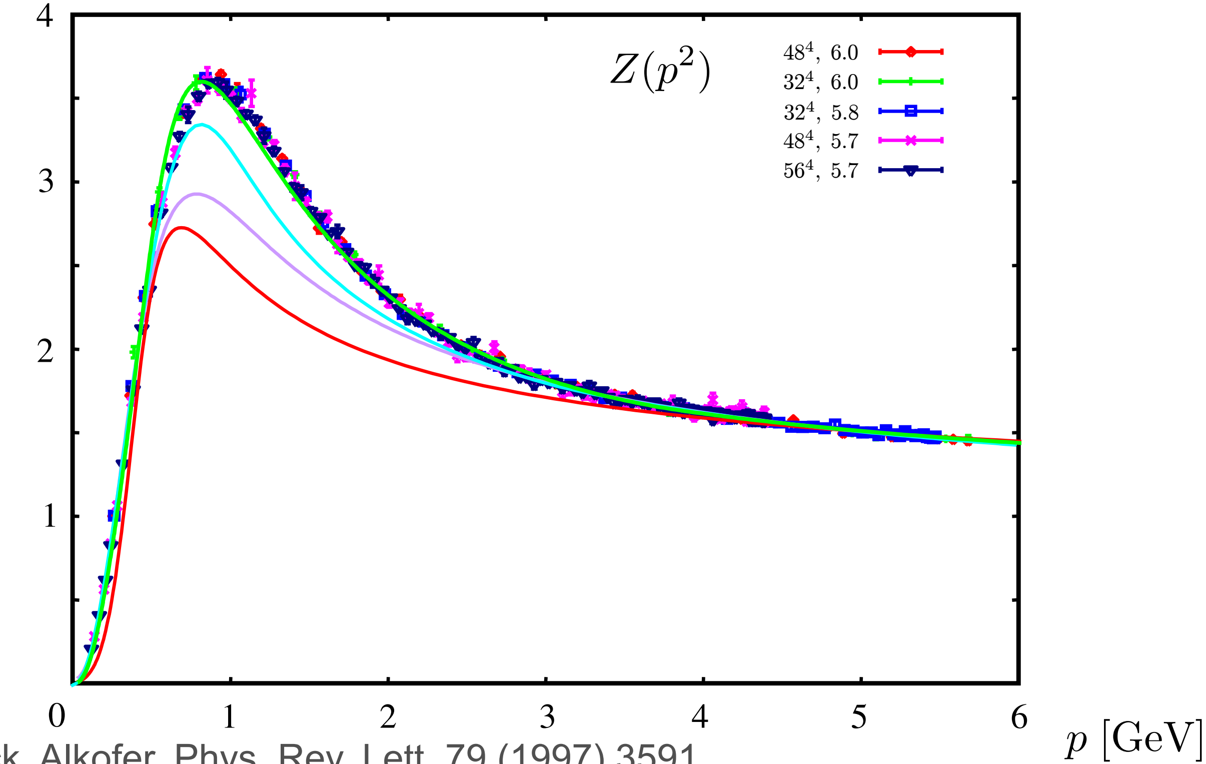

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Pawlowski, Litim, Nedelko, LvS, Phys. Rev. Lett. 93 (2004) 152002; Pawlowski (2006), unp.

Fischer, Maas, Pawlowski, Annals Phys. 324 (2009) 2408; Pawlowski, in prep.

Landau gauge QCD Propagators

- **Since 1997, DSEs, FRGEs, Stochastic Quantisation, Lattice Simulations:**

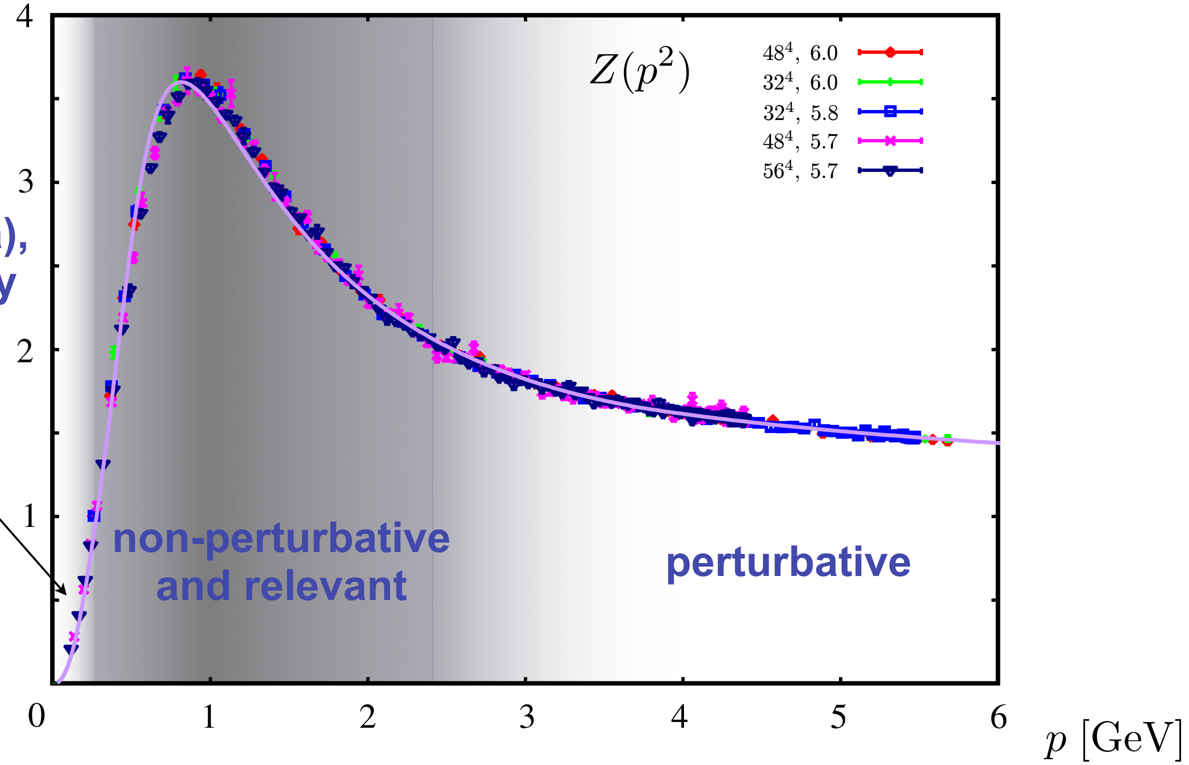
Alkofer, Aguilar, Binosi, Bicudo, Bloch, Boucaud, Bogolubsky, Bornyakov, Bowman, Braun, Cucchieri, De Soto, Dudal, Fischer, Gies, Gracey, Huber, Ilgenfritz, Langfeld, Leinweber, Leroy, Litim, Llanes-Estrada, Nakamura, Natale, Nedelko, Maas, Mendes, Micheli, Mitrjushkin, Müller-Preußker, Oliveira, Papavassiliou, Pawlowski, Pene, Petreczky, Quandt, Reinhardt, Rodriguez-Quintero, Schwenzer, Silva, Skullerud, Sorella, Stamatescu, Sternbeck, Vandersickel, Verschelde, LvS, Wambach, Williams, Zwanziger, ...

- **Numerical solutions & analytic infrared asymptotics**

(infrared-scaling, confinement criteria, Gribov ambiguity ...).

Gluon Propagator

Gribov ambiguity, of fundamental interest (mass gap, Kugo-Ojima), but phenomenologically not relevant!



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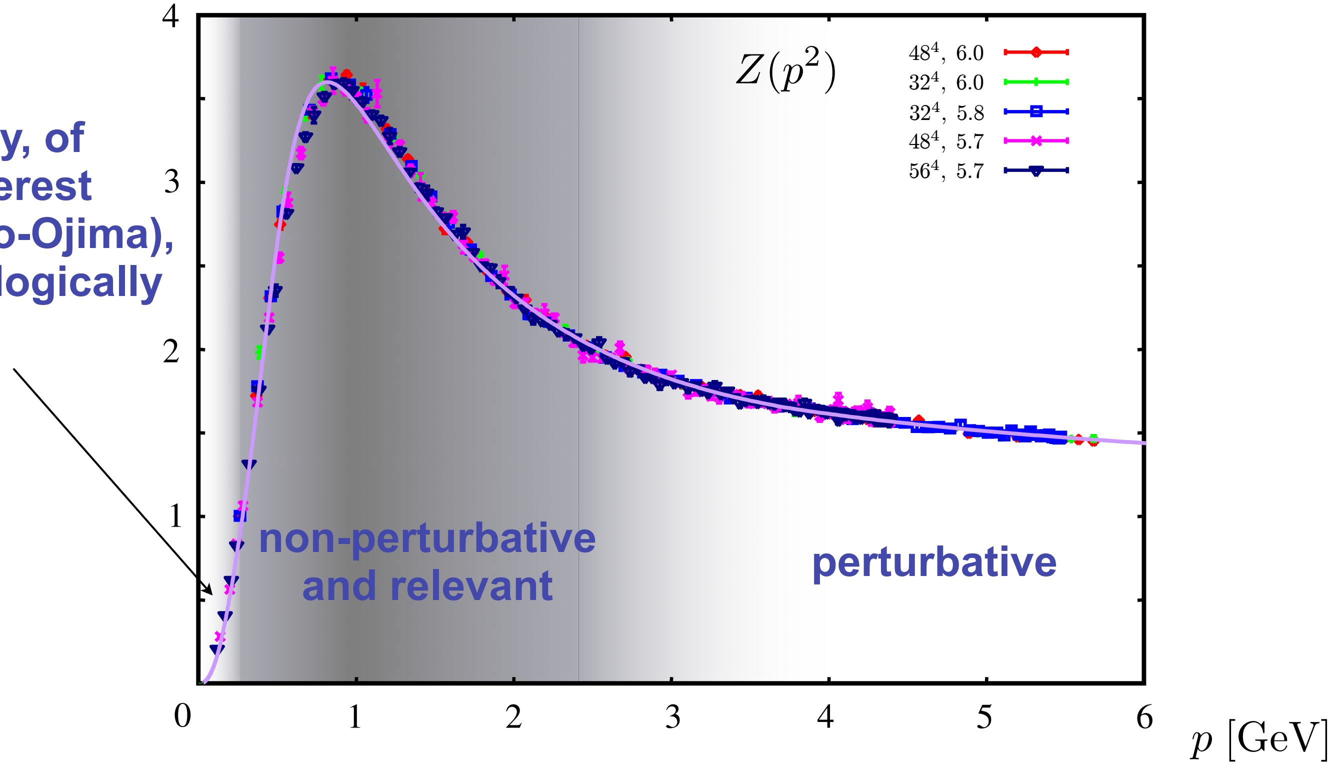

- Combine functional methods (FRGEs + DSEs).
- Compare with lattice simulations where possible.

Running Coupling

- **From Landau gauge gluon and ghost propagators:**

$$\alpha_S^{\mathrm{MM}}(p^2) = \frac{g^2}{4\pi} Z(p^2) G^2(p^2) \xrightarrow{p^2 \to 0} 2.97 \quad \textbf{(infrared-scaling)}$$

LvS, Hauck, Alkofer, Annals Phys. 267 (1998) 1

Lerche, LvS, Phys. Rev. D 65 (2002) 125006

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LvS, Hauck, Alkofer, Annals Phys. 267 (1998) 1

Lerche, LvS, Phys. Rev. D 65 (2002) 125006

- **"MiniMOM" scheme**

  precise definition, known to 4 loops

  LvS, Maltman, Sternbeck, Phys. Lett. B 681 (2009) 336

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LvS, Hauck, Alkofer, Annals Phys. 267 (1998) 1

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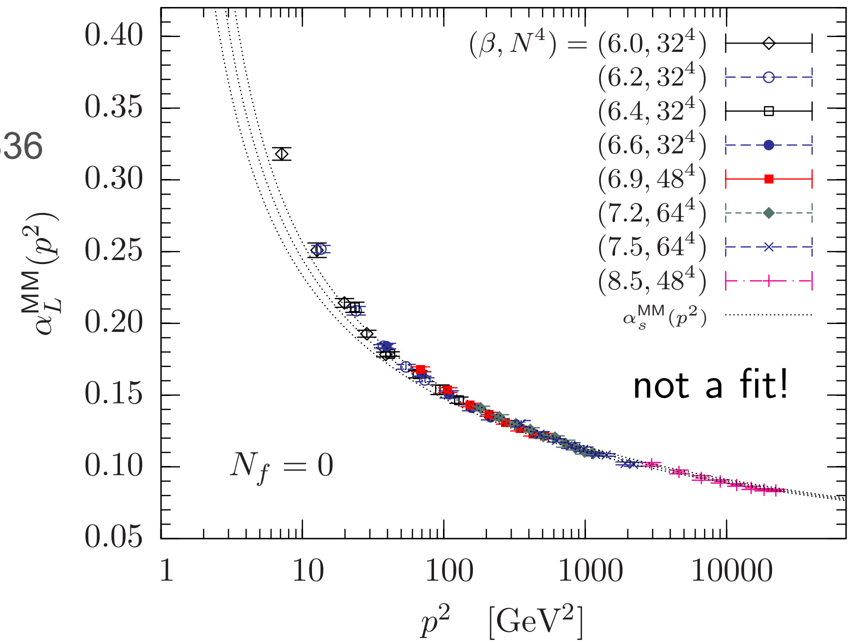
LvS, Maltman, Sternbeck, Phys. Lett. B 681 (2009) 336

- **Lattice determination of α_s**

Sternbeck et al. (LvS), PoS LAT2009, 210

$$r_0 \Lambda^{(0)}_{\overline{MS}} = 0.62(1), \qquad r_0 \Lambda^{(2)}_{\overline{MS}} = 0.60(3)(2)$$



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Deconfinement Transition

- **Pure gauge theory with static quarks:** $P(\vec{x}) = \mathcal{P} \exp \left\{ ig \int_0^{1/T} dt A_0(t, \vec{x}) \right\}$

  and (global) Z_3 center symmetry: $P(\vec{x}) \to Z P(\vec{x}) \,,$

  $$Z \in \{1, \, \exp 2\pi i/3, \, \exp 4\pi i/3\}$$

- **Order parameter:**

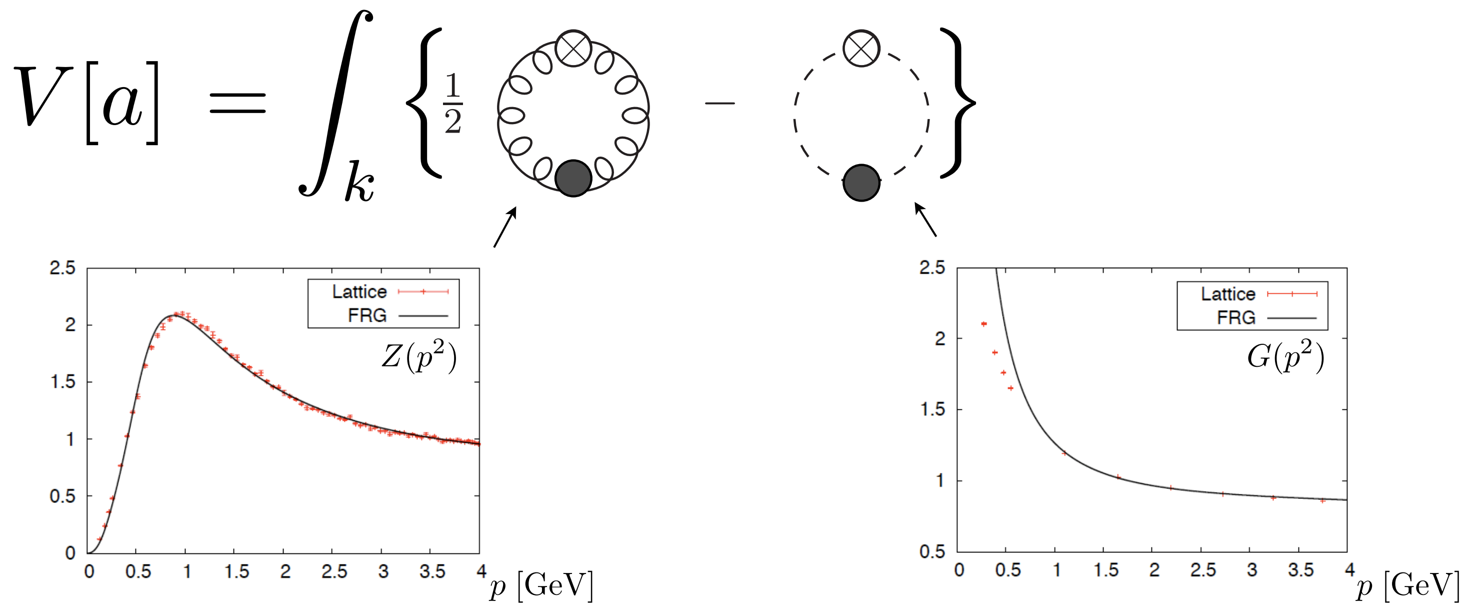
$$\Phi \; = \; \frac{1}{3} \left\langle \mathrm{tr}\, P(\vec{x}) \right\rangle \; \sim \; e^{-F_q/T}$$

$$\to \begin{cases} \Phi = 0, \; \text{confined, (center) symmetric, disordered} \\ \Phi \neq 0, \; \text{deconfined, spontan. broken } Z_3 \text{, ordered} \end{cases}$$

Effective Potential

Braun, Gies, Pawlowski, Phys. Lett. B 684 (2010) 262

- **Constant background:** $a = \frac{1}{T}\langle g A_0^3 \rangle \,, \quad \Phi[a] = \frac{1}{3}\left(1 + 2\cos\frac{a}{2}\right) ,$



with: $p_0 = 2\pi T\,(n - \nu\, a/4\pi)\,,$

$$\nu = \{0, \pm 1, \pm 2\}$$

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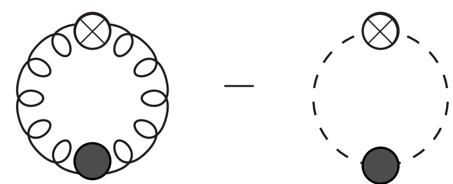
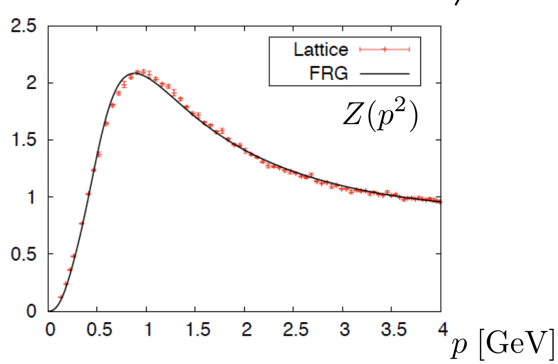
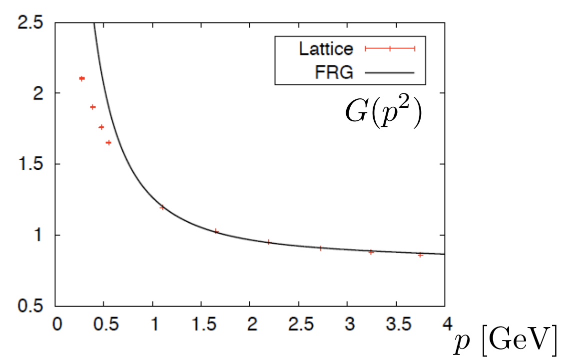
- **Constant background:** $a = \frac{1}{T}\langle g A_0^3 \rangle \,, \quad \Phi[a] = \frac{1}{3}\left(1 + 2\cos\frac{a}{2}\right) ,$

$$V[a] = \int_k \left\{ \frac{1}{2} \;[\text{gluon loop}]\; - \;[\text{ghost loop}] \right\}$$







with: $p_0 = 2\pi T\,(n - \nu\, a/4\pi)\,,$

$$\nu = \{0, \pm 1, \pm 2\}$$

- **Up to terms** $\mathcal{O}(V''[a])$

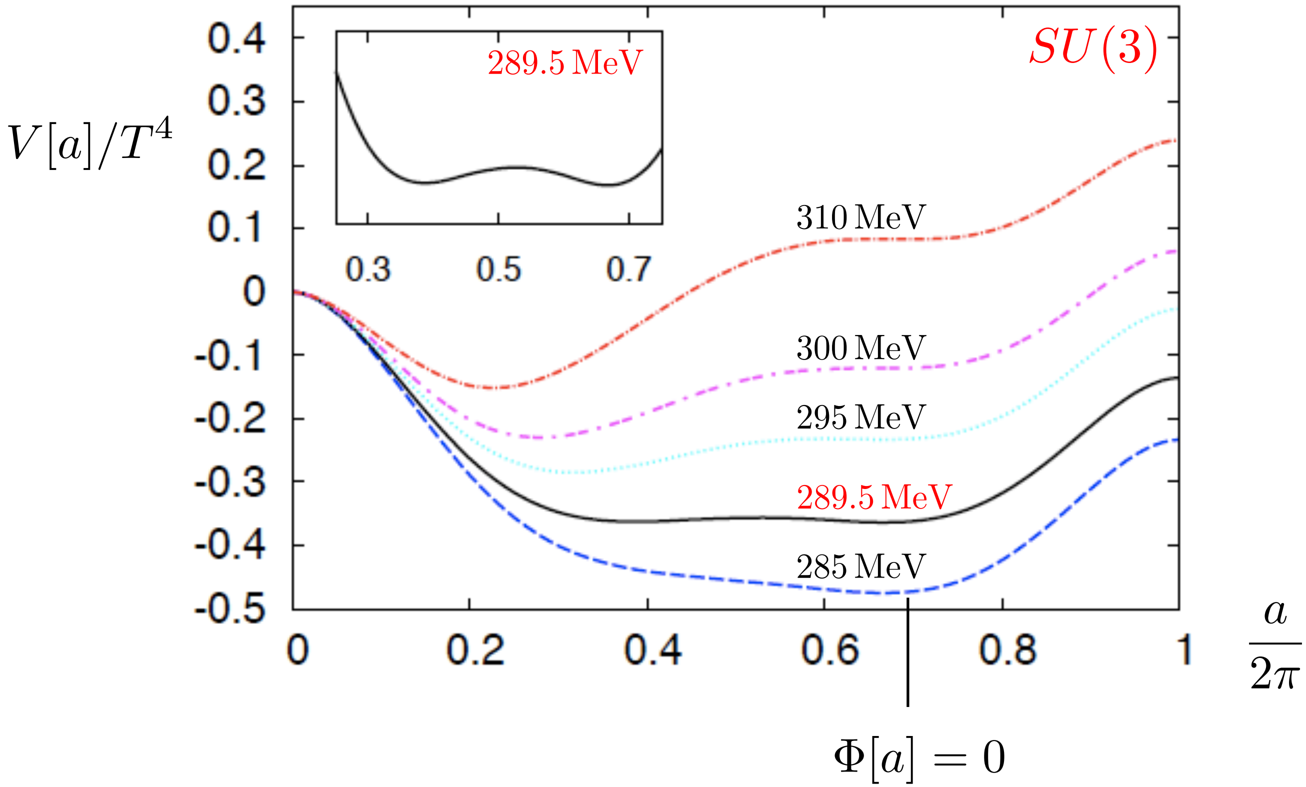
(suppressed at T_c)

Effective Potential

$$T_c = 289.5 \pm 10\,\mathrm{MeV}\,,$$

Braun, Gies, Pawlowski, Phys. Lett. B 684 (2010) 262

$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$ **(consistent with lattice average)**



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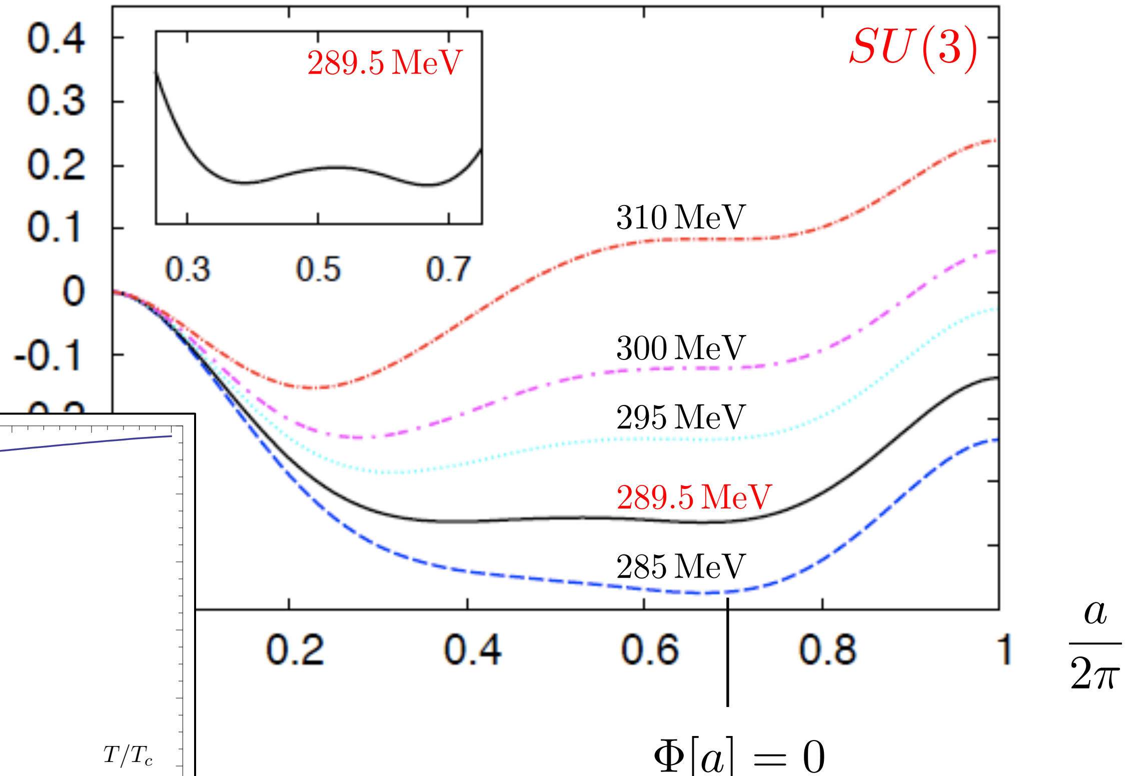
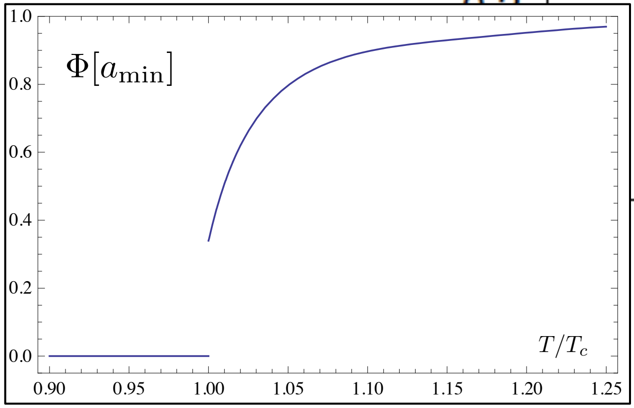


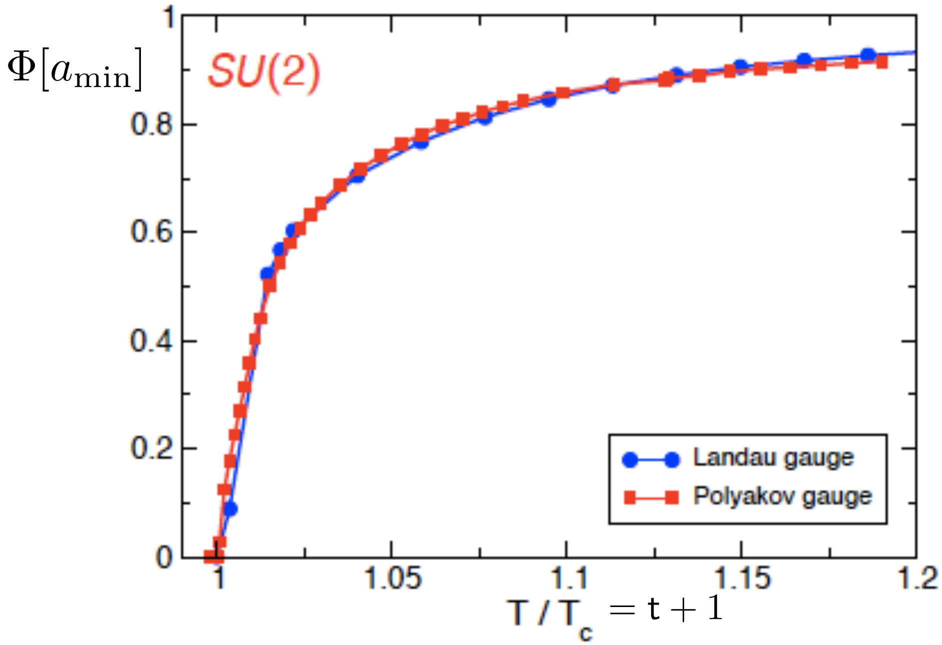


Order Parameter

- **Polyakov gauge:**

Marhauser, Pawlowski, arXiv:0812.1144

$T_c = 305^{+40}_{-55}\mathrm{MeV}$



$$V[a] = -\int_k \mathrm{flow}\left[V''[a], \alpha_s^{\mathrm{MM}}(k)\right]$$

- **critical exponent from screening mass:**

$$V''[a_{\min}] \propto |t|^{2\nu}, \quad \nu = 0.65(2) \qquad [\nu_{\mathrm{Ising}} = 0.63]$$

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Finite Temperature Propagators

- **Pure (Landau) gauge:**

$$D_{\mu\nu}(p) = \frac{Z_{\mathsf{T}}(p^2)}{p^2} P^{\mathsf{T}}_{\mu\nu} + \frac{Z_{\mathsf{L}}(p^2)}{p^2} P^{\mathsf{L}}_{\mu\nu}$$

$$T = 0 : \qquad Z_{\mathsf{T}} = Z_{\mathsf{L}} = Z$$

$$T > 0 : \qquad Z_{\mathsf{T}} \lesssim Z$$

- **Lattice & DSEs:**

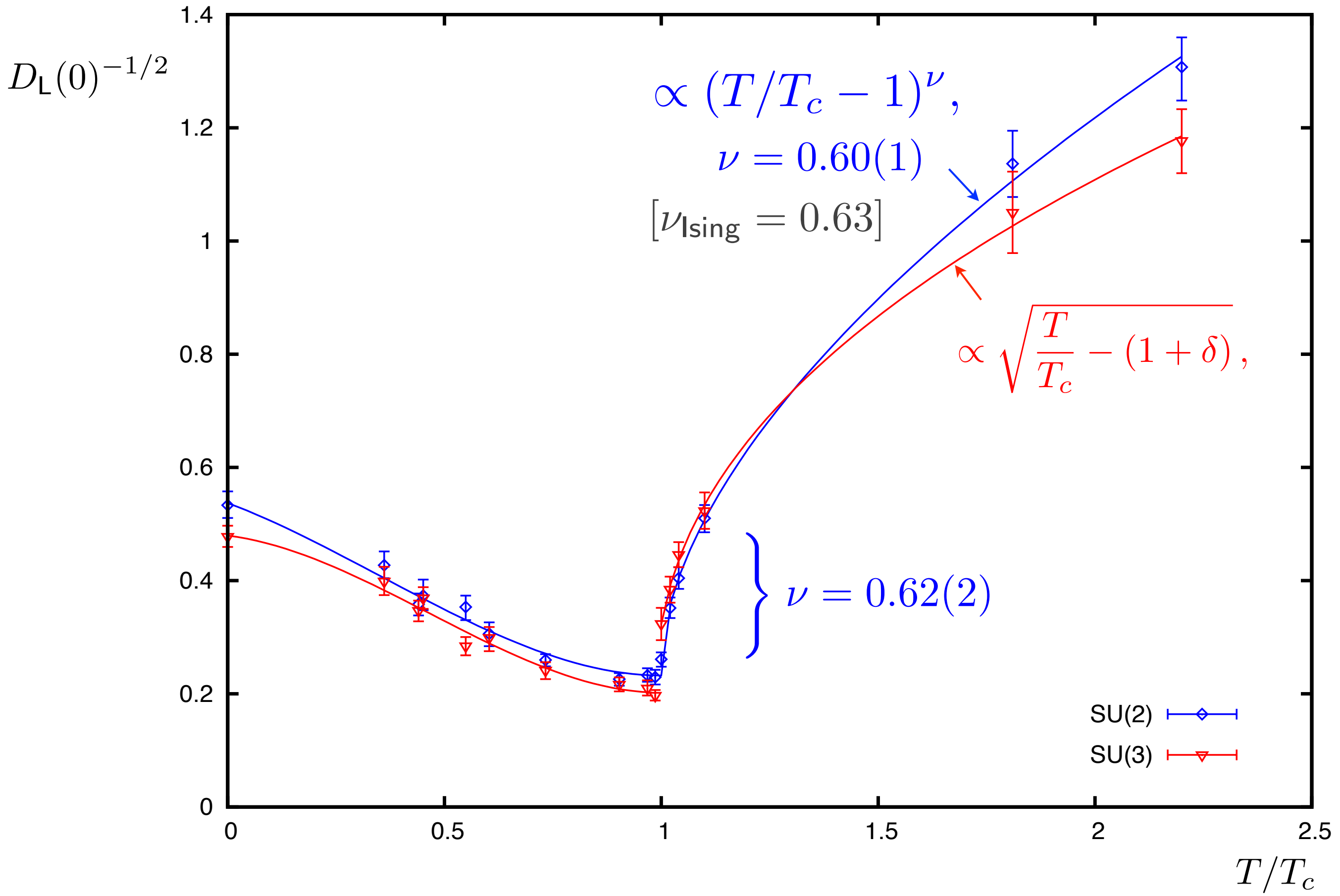
Cucchieri, Karsch, Petreczky, Phys. Lett. B 497 (2001) 80
Cucchieri, Karsch, Petreczky, Phys. Rev. D 64 (2001) 036001
Maas, Wambach, Grüter, Alkofer, EPJC 37 (2004) 335
Maas, Wambach, Alkofer, EPJC 42 (2005) 93
Cucchieri, Maas, Mendes, Phys. Rev. D 75 (2007) 076003
Maas, arXiv:0911.0348

- **Running coupling, FRG:**

Braun, Gies, JHEP 06 (2006) 024
Braun, Gies, Phys. Lett. B 645 (2007) 53
Marhauser, Pawlowski, arXiv:0812.1144

3-dim. fixed-point behavior for $p < T$,
only weakly T-dependent otherwise,
N_f dependence

Electric Screening Mass



- **finite volume**
- **Gribov copies**

- **critical behavior?**
  (well established)

Maas, LvS, in progress

Data: Maas, private communication
Fischer, Maas, Müller, arXiv:1003.1960

Deconfinement & Chiral Symmetry Restoration

— quenched —

Applications II

Deconfinement & Chiral Symmetry Restoration

— quenched —

- **Allow quarks with b.c.'s:** $\psi(1/T, \vec{x}) = -e^{2\pi i\theta}\psi(0, \vec{x})$

- **in observables:** $\mathcal{O}(\theta) = \langle O[\psi_\theta] \rangle$

  e.g., θ-dependent quark condensate, mass function, density . . .

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Deconfinement & Chiral Symmetry Restoration

— quenched —

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- **in observables:** $\mathcal{O}(\theta) = \langle O[\psi_\theta] \rangle$

  e.g., θ-dependent quark condensate, mass function, density . . .

- **observe:**

  $\mathcal{O}(\theta + 1) = \mathcal{O}(\theta)$, always $\rightsquigarrow$ Fourier series

  $\mathcal{O}(\theta + 1/3) = \mathcal{O}(\theta)$, in Z_3-symmetric confined phase

Dual Order Parameters

- **Order parameter for confinement:**

$$\widetilde{\mathcal{O}} = \int_0^1 d\theta \; \mathcal{O}(\theta)\, e^{-2\pi i\theta}$$

, dual quark condensate, dual mass, density . . .

$\big(Z_3$-symmetry, confinement $\Rightarrow \widetilde{\mathcal{O}} = 0 \big)$

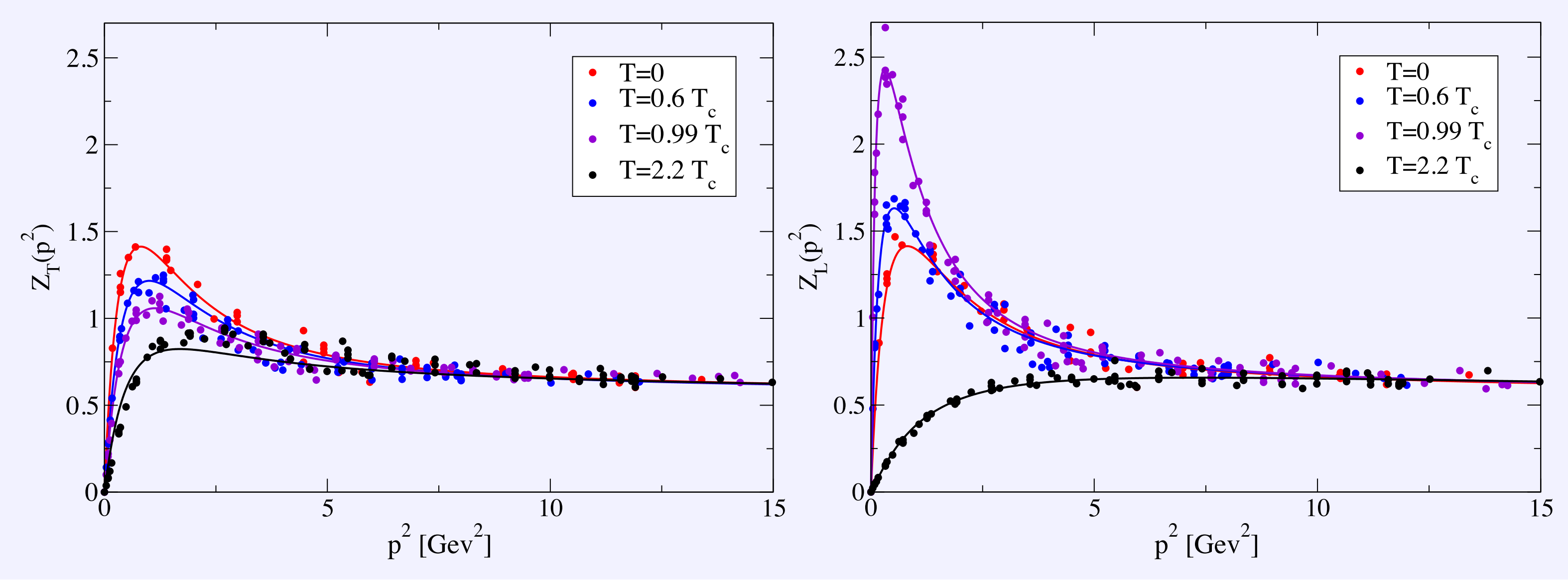
- **Originally from lattice**

Gattringer, Phys. Rev. Lett. 97 (2006) 032003
Synatschke, Wipf, Wozar, Phys. Rev. D 75 (2007) 114003
Synatschke, Wipf, Langfeld, Phys. Rev. D 77 (2008) 114018
Bilgici, Bruckmann, Gattringer, Hagen, Phys. Rev. D 77 (2008) 094007

T-dependent Gluon Propagator

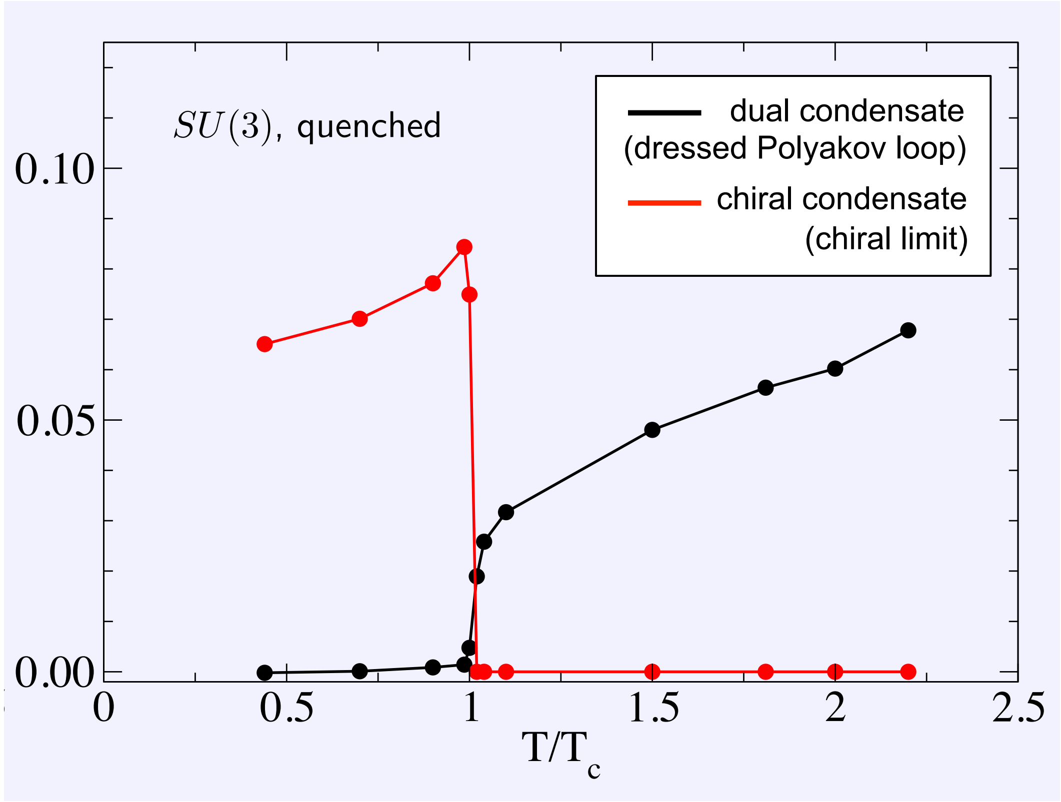
- **Input for quark DSE:**

Fischer, Phys. Rev. Lett. 103 (2009) 052003
Fischer, Müller, Phys. Rev. D 80 (2009) 074029



Fischer, Maas, Müller, arXiv:1003.1960

Chiral & Deconfinement Transition(s)



$T_{\text{chiral}} \approx T_{\text{conf}} \approx 277$ MeV

Fischer, Maas, Müller, arXiv:1003.1960

⟶ **Christian Fischer's talk**

Applications III

Full Dynamical QCD

— at zero & imaginary chemical potential —

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Applications III

Full Dynamical QCD

— at zero & imaginary chemical potential —

- **Quarks with b.c.'s:** $$\psi(1/T, \vec{x}) = -e^{2\pi i\theta}\psi(0, \vec{x})$$

- **but observables:** $$\mathcal{O}_\theta(\theta) = \big\langle O[\psi_\theta] \big\rangle_\theta$$

observables in different theories, built on θ-dependent ground states

Applications III

Full Dynamical QCD

— at zero & imaginary chemical potential —

- **Quarks with b.c.'s:** $\psi(1/T, \vec{x}) = -e^{2\pi i\theta}\psi(0, \vec{x})$
- **but observables:** $\mathcal{O}_\theta(\theta) = \left\langle O[\psi_\theta] \right\rangle_\theta$

  observables in different theories, built on θ-dependent ground states
- **observe:** $\mathcal{O}(\theta + 1/3) = \mathcal{O}(\theta)$ **now always!**

  Roberge, Weiss, Nucl. Phys. B 257 (1986) 734
- **imaginary chemical potential:** $\mu = 2\pi i T\theta$

Full Dynamical QCD

- **Landau gauge Yang-Mills with background, compare:**

(a) $a_0 = \frac{1}{T} \langle g A_0^3 \rangle_{\theta=0}$, breaks Roberge-Weiss symmetry,
$\rightarrow$ dual observables, deconfinement transition (dual condensate, dual density . . .)

(b) $a_\theta = \frac{1}{T} \langle g A_0^3 \rangle_\theta$, quark condensate, mass, density, f_π,. . . , order parameters for chiral phase transition in QCD_θ (but no dual order parameters)

- **include $N_f = 2$ dynamical quarks**

  massless, chiral limit

- **coupled to mesons**

  dynamical hadronisation

c.f., quark meson model:

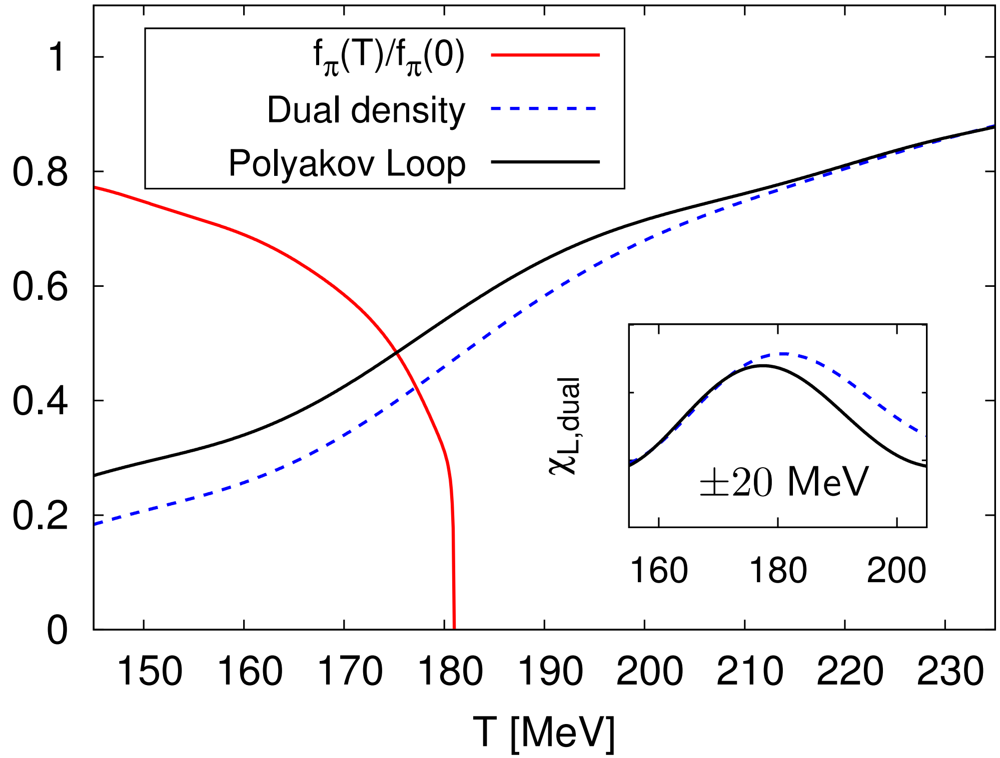
Berges, Tetradis, Wetterich, Phys. Rept. 363 (2002) 223

Schaefer, Wambach, Phys. Part. Nucl. 39 (2008) 1025

Full Dynamical QCD, $N_f = 2$, chiral limit

(a) $a_0 = \frac{1}{T} \langle g A_0^3 \rangle_{\theta=0} \,, \quad \mu = 0$

Braun, Haas, Marhauser, Pawlowski, arXiv:0908.0008



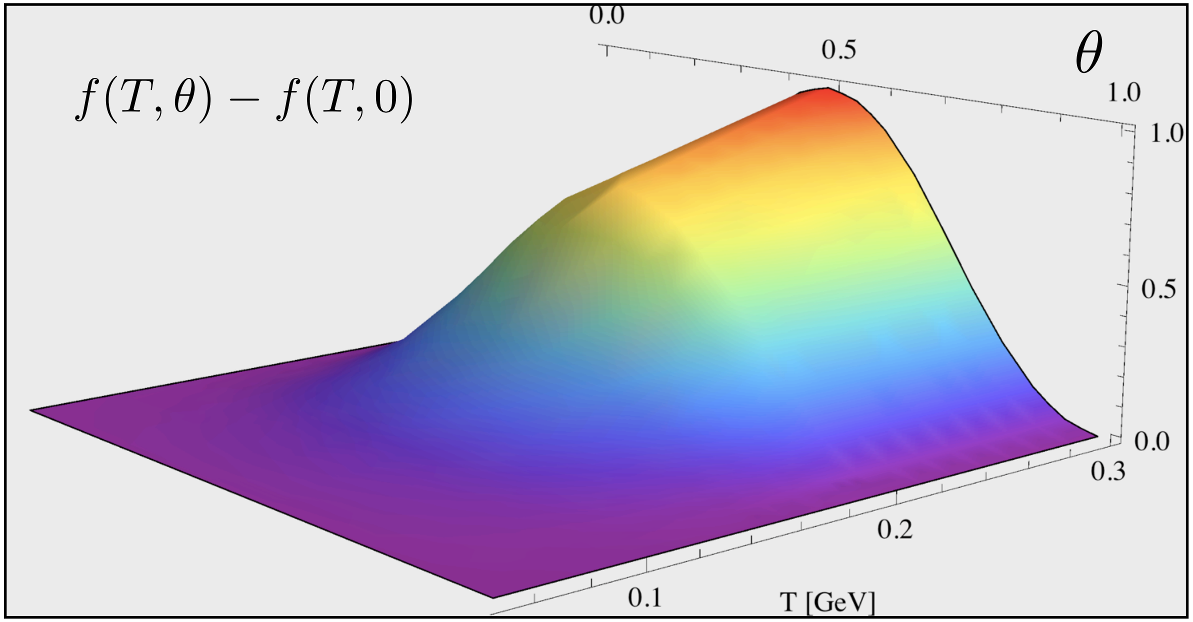
$T_{\text{chiral}} \approx T_{\text{conf}} \approx 180$ MeV

$\longrightarrow$ **Lisa Haas' talk**

Full Dynamical QCD, $N_f = 2$, chiral limit

Braun, Haas, Marhauser, Pawlowski, arXiv:0908.0008

- **free energy difference:**



change boundary conditions of confined quarks at no cost

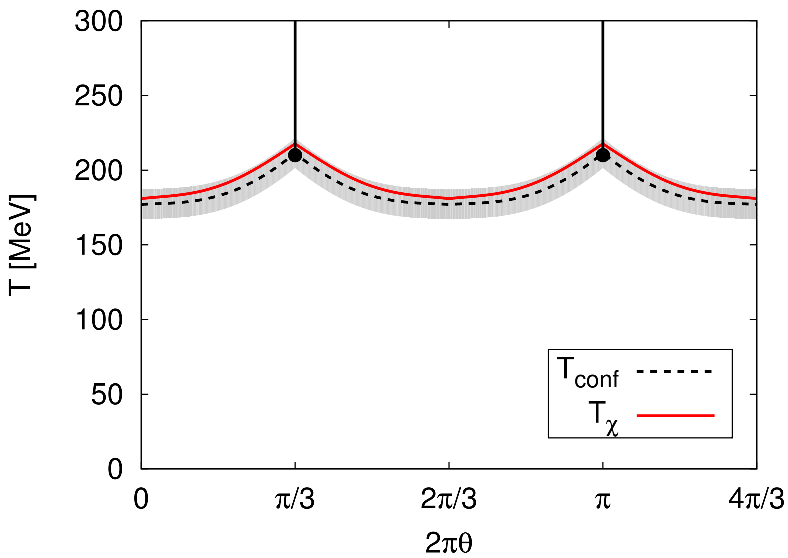
QCD$_\theta$ – imaginary chemical potential

- **Originally from lattice**
  no sign poblem

de Forcrand, Philipsen, Nucl. Phys. B 642 (2002) 290
D' Elia, Lombardo, Phys. Rev. D 67 (2003) 014505
Kratochvila, de Forcrand, Phys. Rev. D 73 (2006) 114512
Wu, Luo, Chen, Phys. Rev. D 76 (2007) 034505

- **Here**
  (b) $\mathrm{QCD}_\theta\,, \quad \mu = 2\pi i T\theta$

Braun, Haas, Marhauser, Pawlowski, arXiv:0908.0008



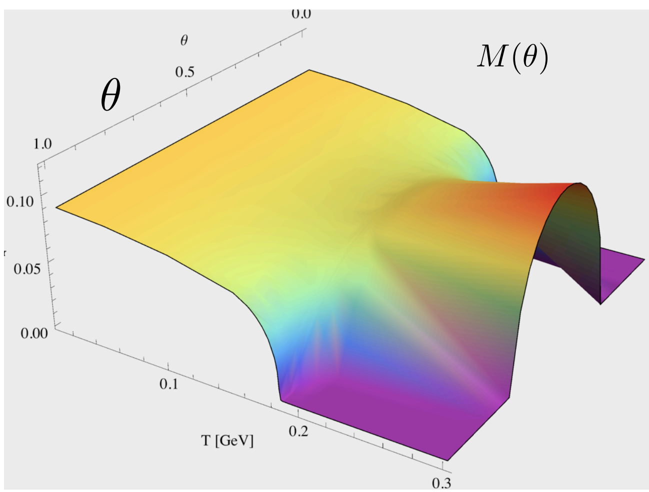
$\longrightarrow$ **Lisa Haas' talk**

Dual Observables vs. Chiral Transition in QCD$_\theta$

- **Quark mass parameter**

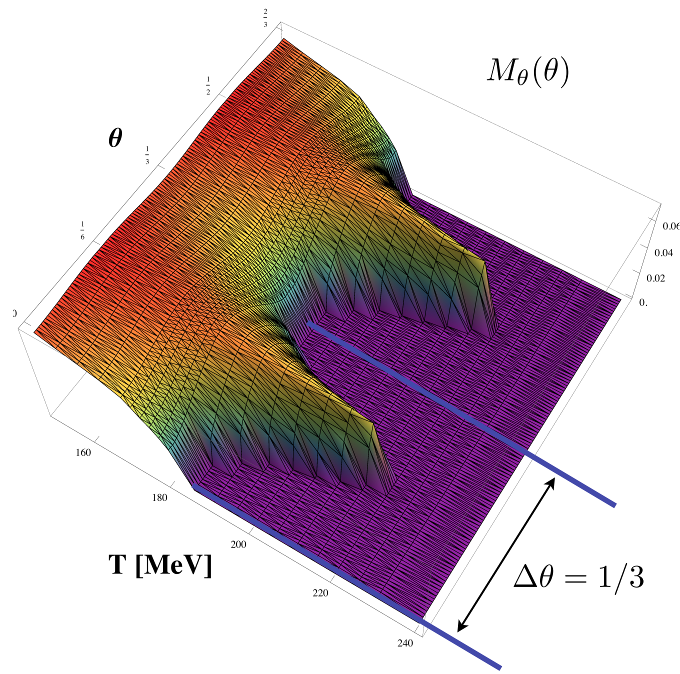
(a) dual mass, confinement

$$\widetilde{M} = \int_0^1 d\theta \; M(\theta)\, e^{-2\pi i\theta}$$



(b) mass in QCD$_\theta$, chiral symmetry breaking

$$M_\theta(\theta + 1/3) = M_\theta(\theta)$$



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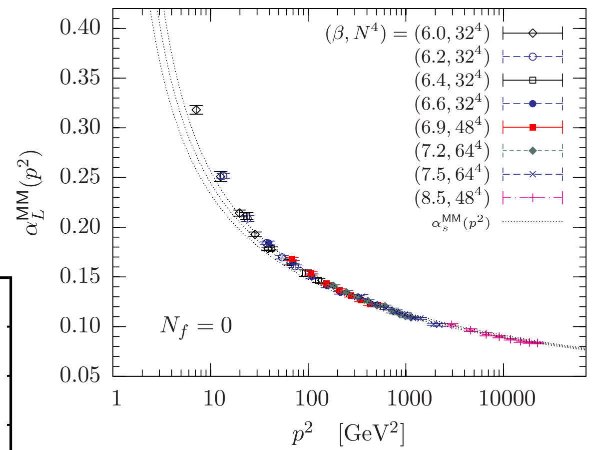
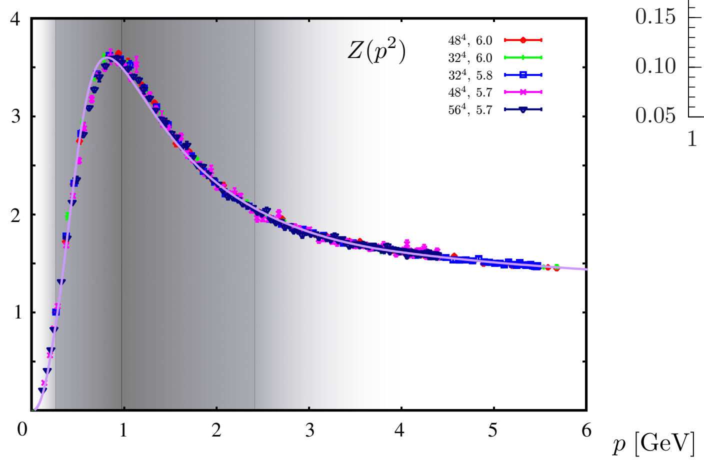
Summary & Outlook

- **Propagators and Running Coupling of Landau Gauge QCD**

  very well understood at T=0,
  increasingly well at finite T





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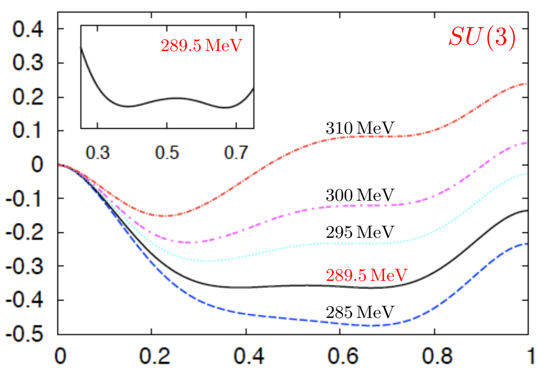

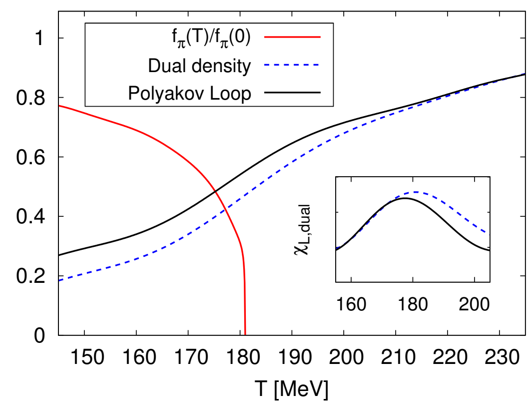

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Thank You!

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Extra Sides

Some Additional Material

Dyson-Schwinger Equations

Generating functional:

$$Z[j] = \langle \exp(j,\phi) \rangle$$

$\longrightarrow$ **Green's functions**

$(T = 0 \text{ here})$

$\langle A_\mu(x) A_\nu(y) \rangle$:

$$D_{\mu\nu}(p) = \frac{Z(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

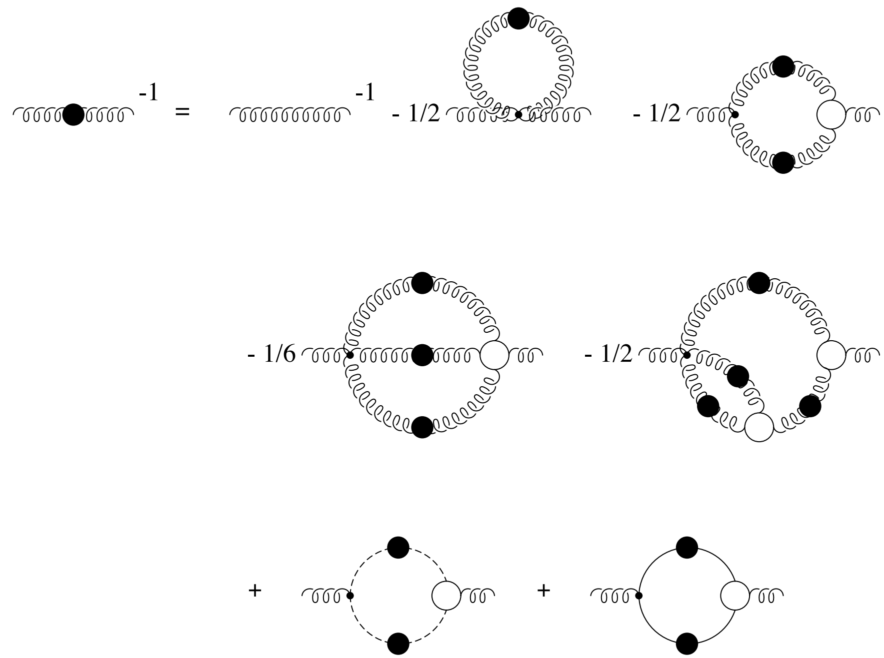
$\langle c(x) \bar{c}(y) \rangle$:

$$D_G(p) = -\frac{G(p^2)}{p^2}$$

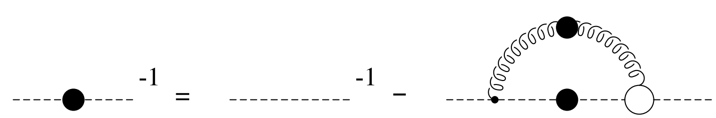
$\langle \psi(x) \bar{\psi}(y) \rangle$:

$$S(p) = \frac{Z_\psi(p^2)}{i\not{p} + M(p^2)}$$

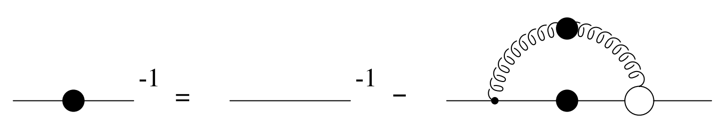
glue:



ghost:



quark:



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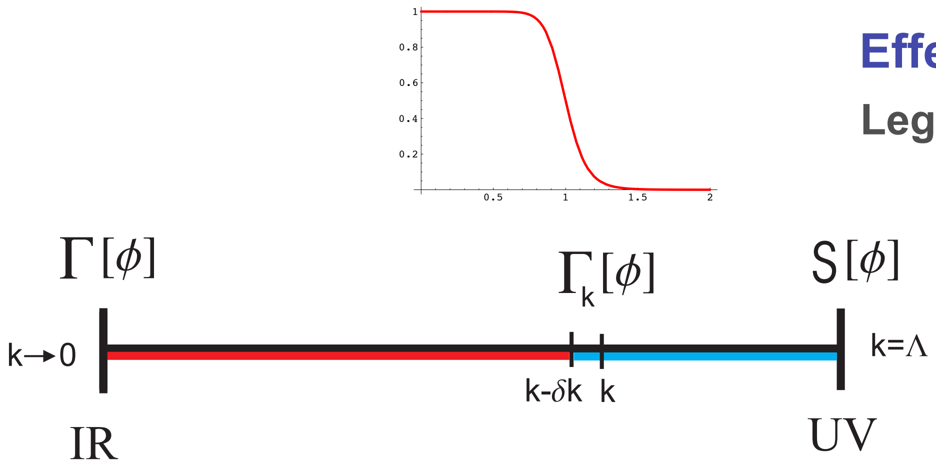
Functional RG (Flow) Equations



Effective action: $\Gamma[\phi_j] = (j,\phi_j) - \ln Z[j]$

Legendre transform

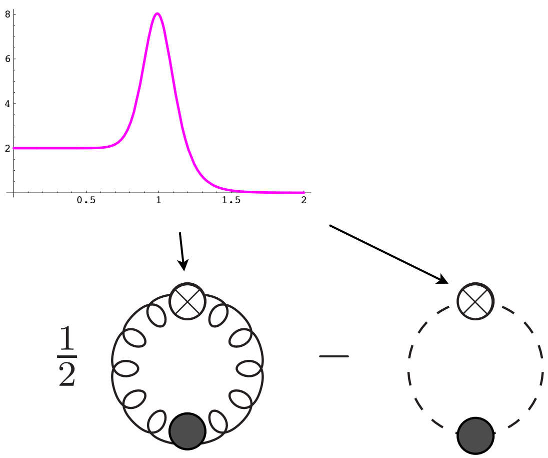
⟶ **1PI vertex functions**

$\Gamma^{(n)}(x_1, \ldots x_n)$

⟶ **free energy with**

$\phi_j = \langle \phi \rangle_j$

$$k\partial_k \Gamma_k[\phi] \;=\; \frac{1}{2}\; (\ldots) \;-\; (\ldots)$$

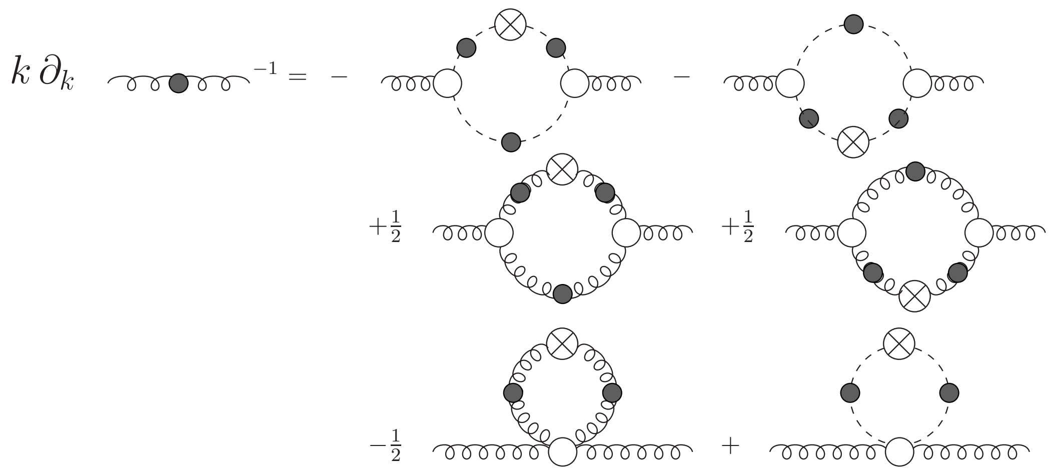


Wetterich, Phys. Lett. B 301 (1993) 90

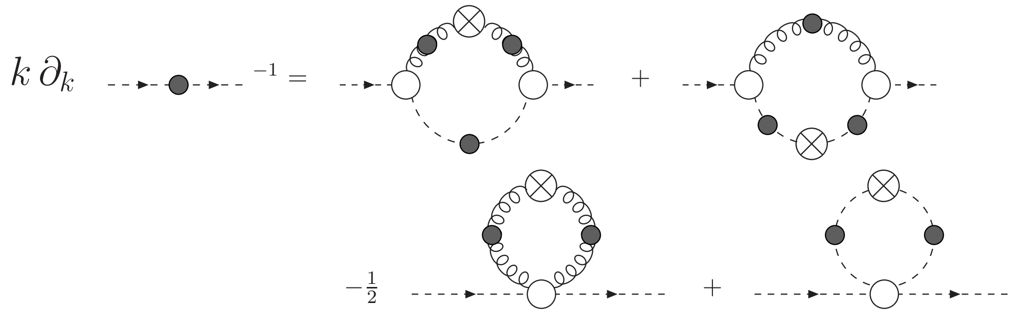
Functional RG (Flow) Equations

Landau gauge QCD propagators,

glue:



ghost:



Generating Functional and Effective Action

Fields: $\phi = \{A, c, \bar{c}, \psi, \bar{\psi}\}$

Introduce sources: $Z[j] = \langle \exp(j, \phi) \rangle \longrightarrow$ **Green's functions**

$\langle \phi(x)\phi(y) \rangle, \ \langle \phi(x)\phi(y)\phi(z) \rangle, \ \ldots$

Effective action: $\Gamma[\phi_j] = (j, \phi_j) - \ln Z[j] \longrightarrow$ **1PI vertex functions**

Legendre transform

$\Gamma^{(n)}(x_1, \ldots x_n)$

$\longrightarrow$ **free energy with**

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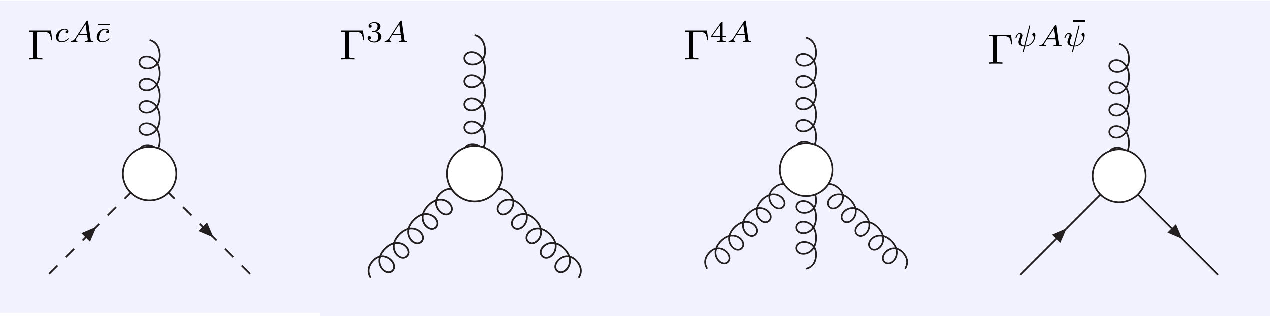
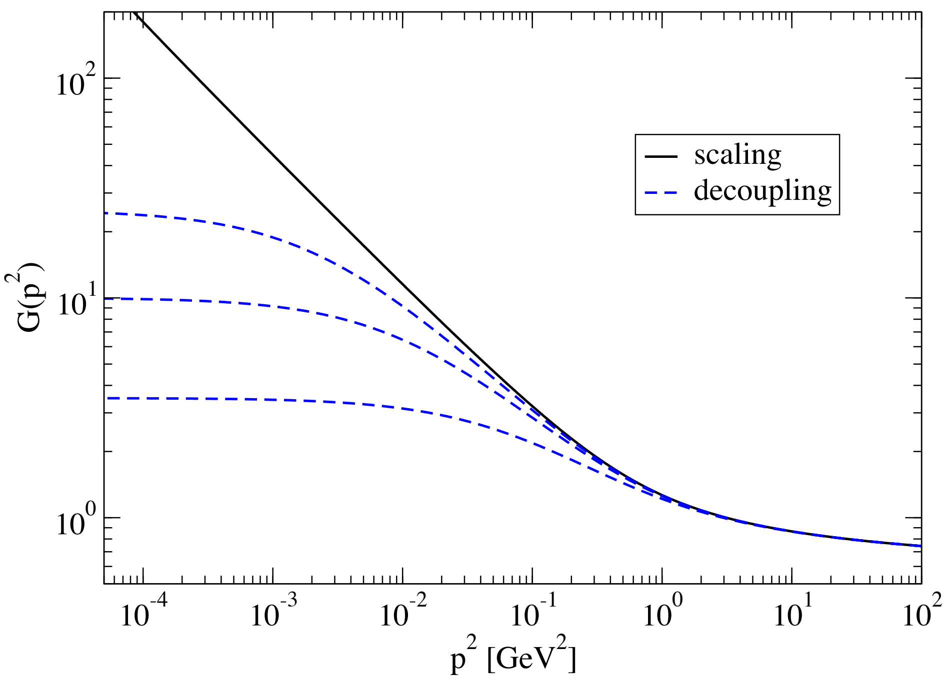
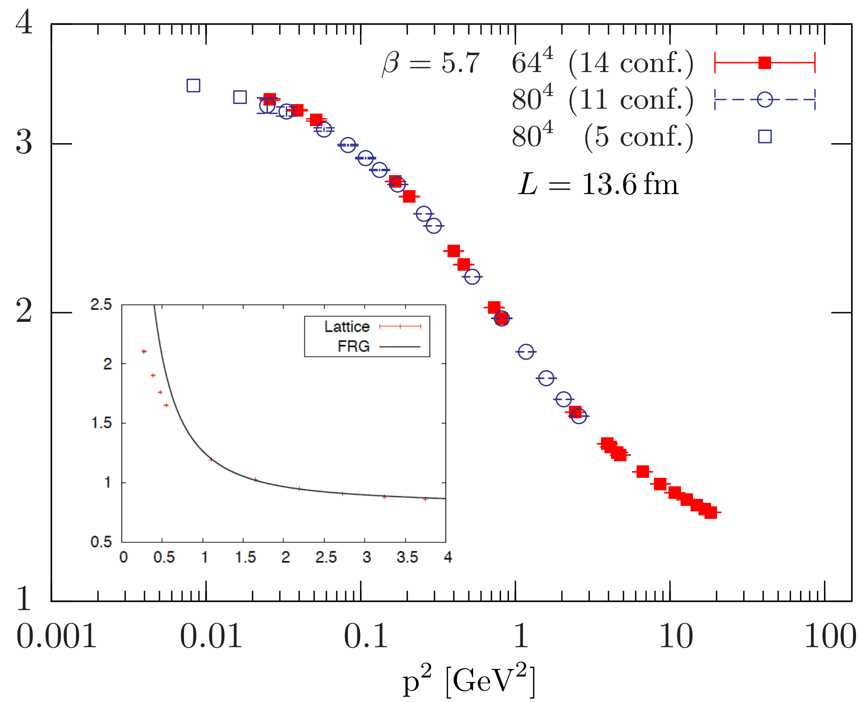


Ghost Propagator

Bogolubsky et al., Phys. Lett. B 676 (2009) 69



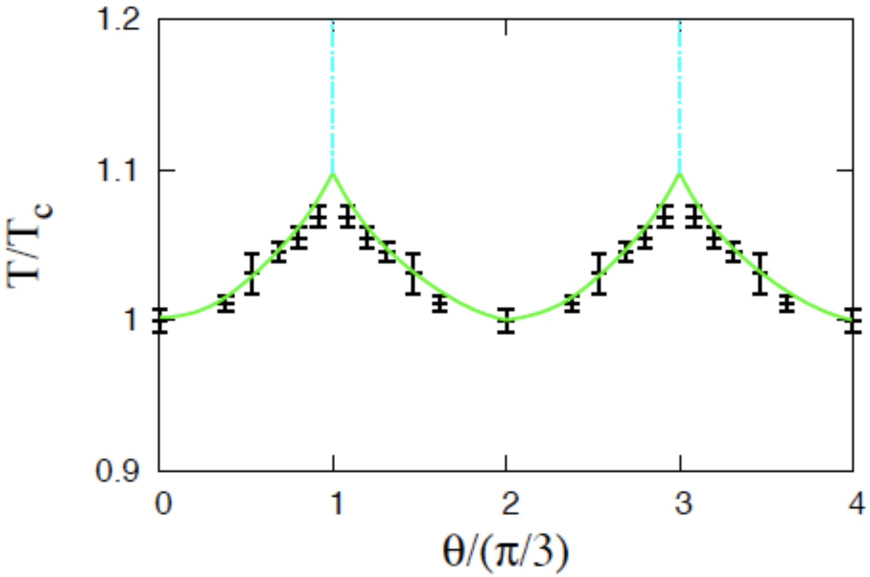


Fischer, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

Maas, arXiv:0907.5185

Imaginary Chemical Potential

- **Polyakov-NJL model:**



Sakai, Kashiwa, Kouno, Matsuzaki, Yahiro, Phys. Rev. D 79 (2009) 096001

Lattice data: Wu, Luo, Chen, Phys. Rev. D 76 (2007) 034505