Confinement and infrared propagators in lattice Landau gauge

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Institute for Theoretical Physics
Research Training Group "Simulational Methods in Physics"

Delta meeting, Heidelberg
May 8, 2010

Talk mainly based on

- A. Maas, J. M. Pawlowski, D.S., A. Sternbeck, L. von Smekal:
  Strong-coupling study of the Gribov ambiguity in lattice Landau gauge
  0912.4203 [hep-lat], Eur. Phys. J. C, in press
- J. M. Pawlowski, D.S., A. Sternbeck, L. von Smekal:
  work in preparation (on free b.c.)
- and also (marginally)
  J. M. Pawlowski, D.S., I.-O. Stamatescu:
  Lattice Landau gauge with stochastic quantisation
  0911.4921 [hep-lat], Nucl. Phys. B 830:291, 2010

Outline

Outline

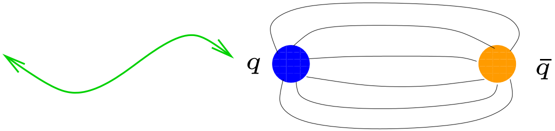
Confinement and infrared propagators

Signatures of confinement

- Confining quark-antiquark potential (gauge-invariant)
  - demands an explanation
- Infrared behavior of gluon & ghost propagator (gauge-dependent)
  - may provide an explanation
    - ★ Gribov–Zwanziger scenario
      – related to confinement via topological defects
  - here: Landau gauge, $\partial_\mu A_\mu = 0$

$$\langle AA \rangle$$

$$\langle c\bar{c} \rangle$$



Gluon & ghost propagator

Basic task

Gauge fixing

↓

extract infrared Yang–Mills propagators

Question

Is the IR behavior of

- scaling type or
- decoupling type?

Gluon & ghost propagator

Landau gauge, $\partial_\mu A_\mu = 0$

- Gluon propagator:

$$\langle AA \rangle \propto \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D_{\mathrm{gl}}(q^2)$$

- Ghost propagator:

$$\langle c\bar{c} \rangle \propto -\delta^{ab} D_{\mathrm{gh}}(q^2)$$

- Dressing function $\equiv q^2 \cdot D_{\mathrm{gl/gh}}(q^2)$

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  ...related to Faddeev-Popov operator

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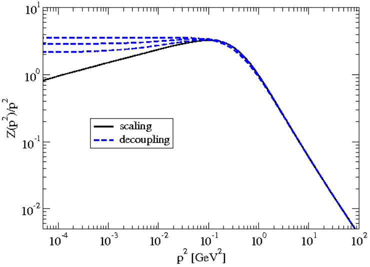
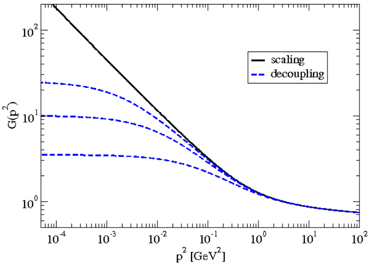
IR propagators: continuum solutions

One-parameter family of solutions from FRG & DSE (figs. from Fischer et al. '08)

Gluon propagator D_{gl}



Ghost dressing function $q^2 D_{\text{gh}}$



Infrared exponents

$$\lim_{q^2 \to 0} D_{\text{gl/gh}}(q^2) \propto \frac{1}{(q^2)^{\kappa_{A/C}+1}}$$

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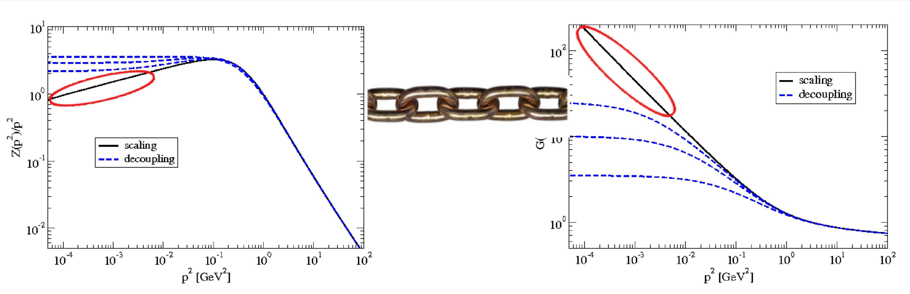

Infrared exponents

(Lerche/von Smekal '02, Zwanziger '01, Pawlowski et al. '03)

$$\lim_{q^2\to 0} D_{\text{gl/gh}}(q^2) \propto \frac{1}{(q^2)^{\kappa_{A/C}+1}}$$

if scaling: $\kappa_A = -2\underbrace{\kappa_C}_{=:\kappa} + \frac{d-4}{2}$

IR gluon & ghost propagator

Confinement mechanism ⇒ IR behavior

Confinement scenarios
(Gribov-Zwanziger, Kugo-Ojima) $\xrightarrow[\text{global BRST}]{}$ infrared scaling

Problem

- Lattice results: No scaling – rather decoupling ($d = 3, 4$)

IR behavior ⇒ confinement

- Scaling solution and decoupling solutions are confining

(Braun/Gies/Pawlowski '07)

- At issue:
  - Confinement mechanism
  - Global BRST invariance   scaling: ✓   decoupling: ✗

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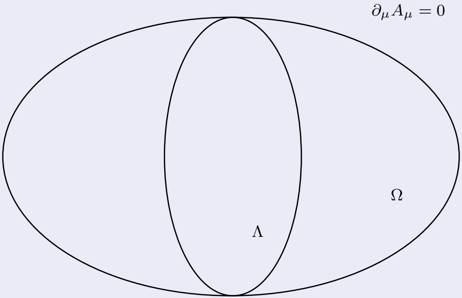
Gribov problem

Relevant regions in configuration space



- G.f. $\rightsquigarrow -\partial\mathcal{D} > 0$: 1st Gribov region Ω
- Still multiple gauge copies (Gribov '78, Singer '78)

  Gribov ambiguity
- Unique copy: Fundamental modular region Λ

  NP-hard optimization problem

Gribov problem

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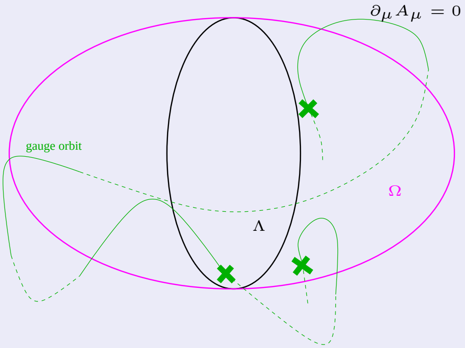

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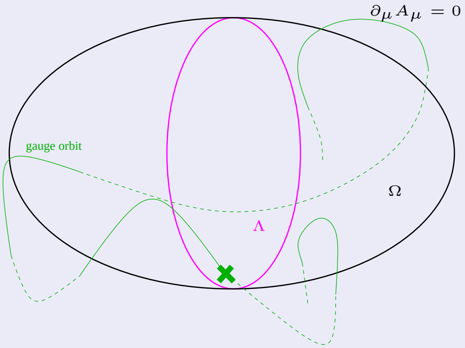

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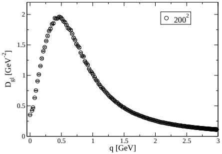
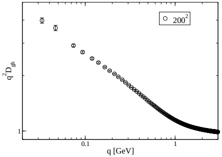
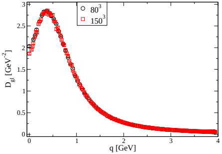
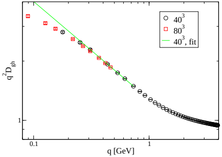
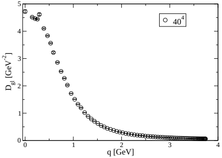
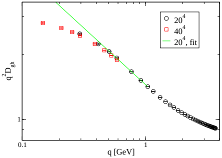
  - ⛔ NP-hard optimization problem

Standard lattice scenario *here:* from stochastic gauge fixing (Pawlowski/DS/Stamatescu '09)

	gluon prop.	ghost d.f.	scaling
$d = 2$			✓
$d = 3$			✗
$d = 4$			✗

- here: quenched $SU(2)$
- $N_c = 3$ (Bogolubsky et al. '07, Cucchieri et al. '07, Oliveira et al. '07, Sternbeck et al. '07) or dyn. fermions (Ilgenfritz et al. '06) } not crucial

Outline

Strong-coupling limit $\beta = 0$

Motivation

- $$\frac{2N_c}{g^2} = \beta \to 0 \qquad \hat{=} \qquad a \to \infty$$

  $\Rightarrow$ IR behavior visible at large aq
- 4D: Conformal behavior (Sternbeck/von Smekal '08)
- Here: $d = 2$ & 3
  Interpretation under debate
  - position 1 (Cucchieri/Mendes '09):
    "lattice sees
    - *decoupling in $d > 2$,*
    - *scaling in $d = 2$"*
  - position 2 (Maas/Pawlowski/DS/Sternbeck/von Smekal '09):
    "issue not settled"

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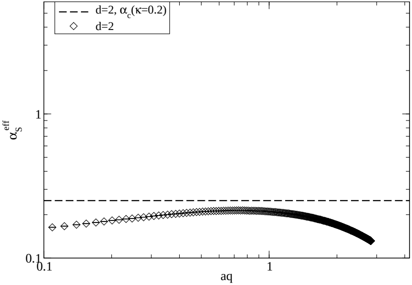
Running coupling

eff. running coupling

$$\alpha_S^{\text{eff}} = \frac{g^2}{4\pi} q^{d+2} D_{\text{gl}} D_{\text{gh}}^2$$

Running coupling, two dimensions



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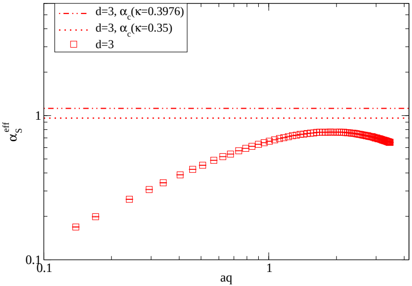
cp. with continuum predictions for α_c

(Lerche/von Smekal '02)

peak at 85% } of α_c in $d = \{ 2$

Running coupling, three dimensions



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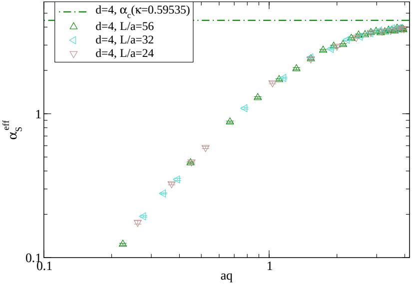
peak at 70% } of α_c in $d = \{ 3$

Running coupling, four dimensions



← 4d data: Sternbeck/von Smekal '08

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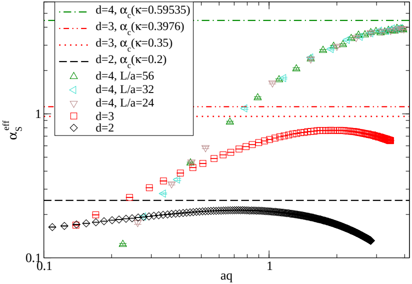
peak at 90% $\Big\}$ of α_c in $d = \Big\{$ 4

Running coupling



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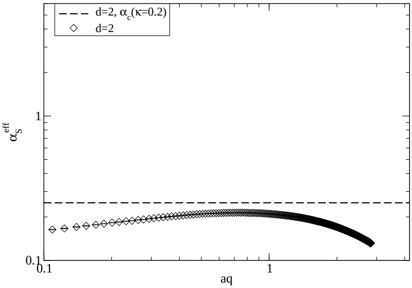
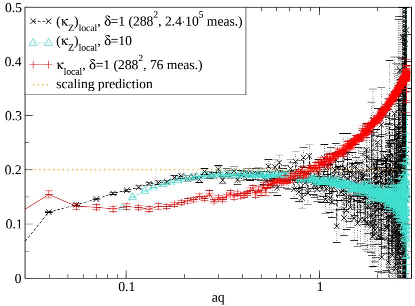
$$\text{peak at} \begin{matrix} 85\% \\ 70\% \\ 90\% \end{matrix} \Bigg\} \text{ of } \alpha_c \text{ in } d = \begin{cases} 2 \\ 3 \\ 4 \end{cases}$$

Running coupling & local exponents





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with κ_Z def. via scaling rel.

$$\kappa_A = -2\kappa_Z + \frac{d-4}{2}$$

local exponents

$$\left.\begin{matrix}\kappa \\ \kappa_Z\end{matrix}\right\} = \text{ghost exp. from } \left\{\begin{matrix}\text{ghost} \\ \text{gluon}\end{matrix}\right. \quad \text{data}$$

locally from similar momenta q_i, $q_{i+\delta}$

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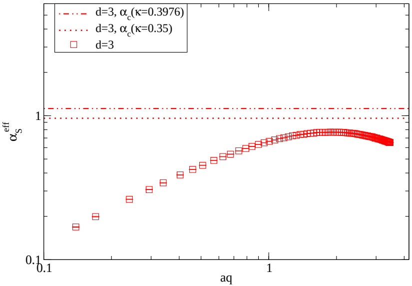

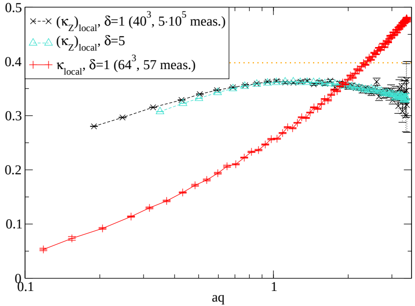

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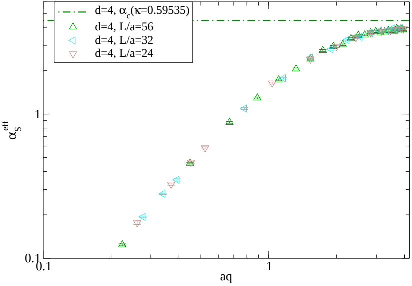

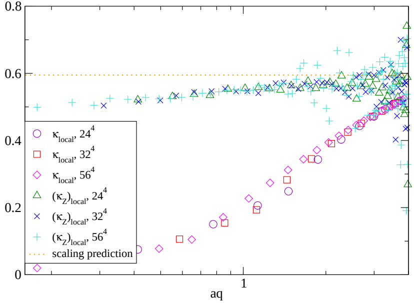

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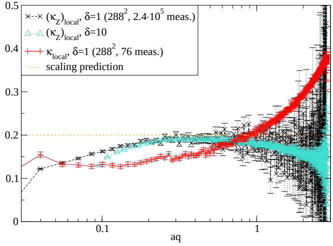
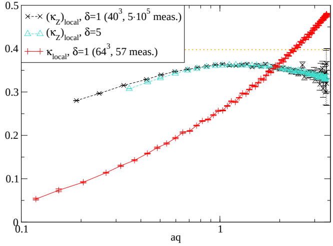
Two vs. three dimensions





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  - local κ & κ_Z

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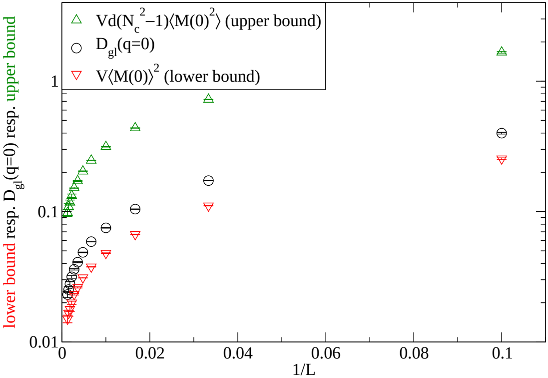

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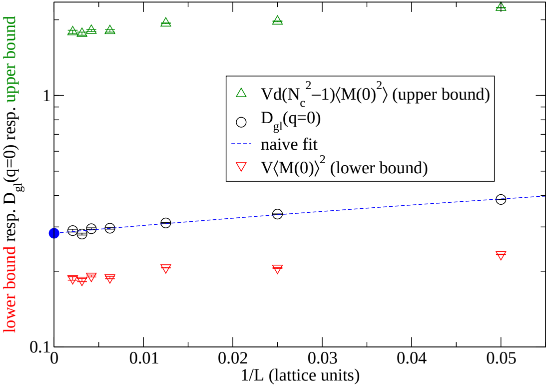

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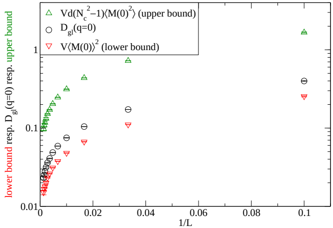

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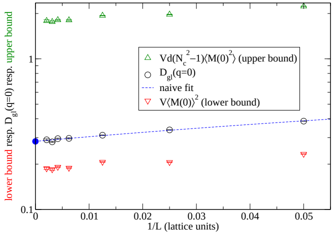

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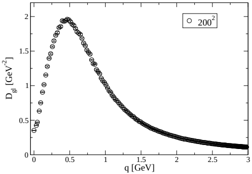
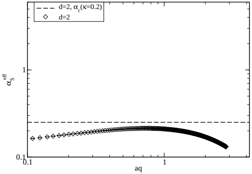
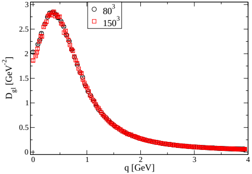
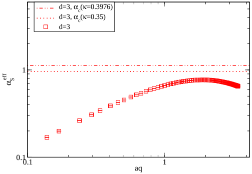
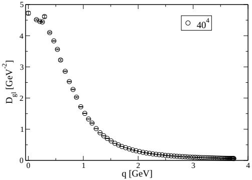
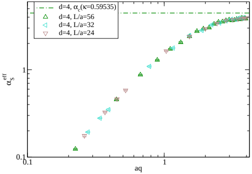
Upshot of standard gauge fixing at $\beta = 0$

$d = 2$, 3 and 4

- no uniform scaling at all aq
- possible scaling window
  - moves towards larger aq from $d = 2$ to $d = 4$
  - same pattern at finite β

Room for speculation . . .

	gluon prop., finite β	running coupling, $\beta = 0$
$d = 2$		
$d = 3$		
$d = 4$		

Outline

Impact of Gribov ambiguity

Basic idea of 'Landau max-B' gauge (Maas '09)

- Functional methods $\rightsquigarrow$ one-parameter family of solutions

  (Fischer et al. '08, Boucaud et al. '08)

- Analogous lattice procedure
  - $$\text{parameter } B := \frac{D_{gh}(q_{min})}{D_{gh}(Q)}$$
  - for each config'
    - fix n_{copy} times to Landau gauge
    - choose the copy with

      $B = \max$ (here)

- Here: first simulations at $\beta = 0$, in $d = 2, 3$

  finite β: Maas '09 ...

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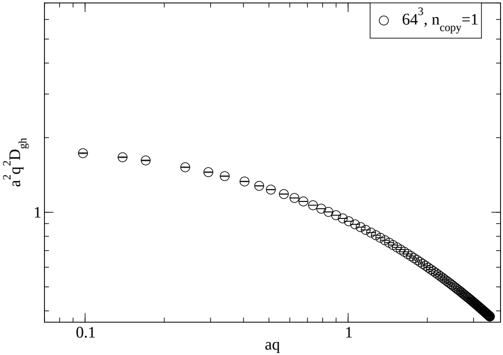
      $B = \max$ (here)

- Here: first simulations at $\beta = 0$, in $d = 2, 3$

  finite β: Maas '09 ...

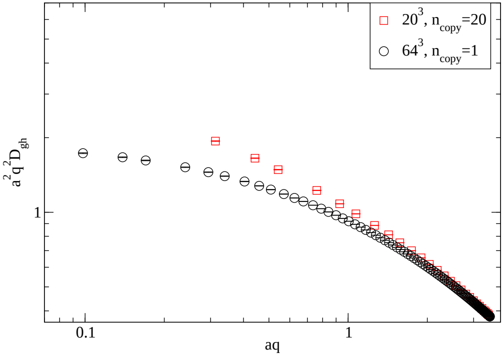
Impact of Gribov ambiguity



$d = 3$: ghost dressing function

- increase n_{copy} ...

Impact of Gribov ambiguity



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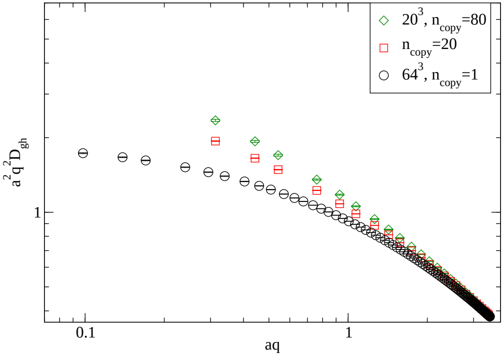

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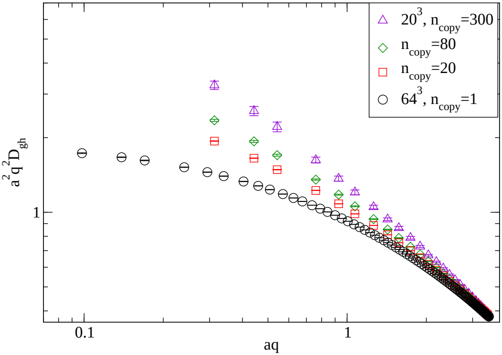

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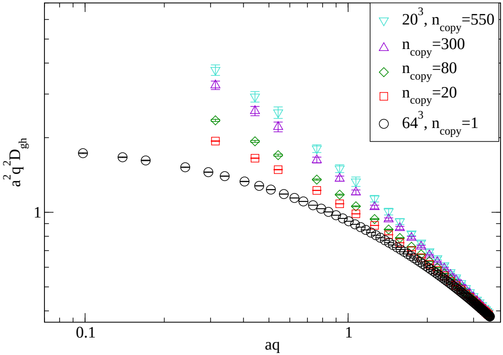
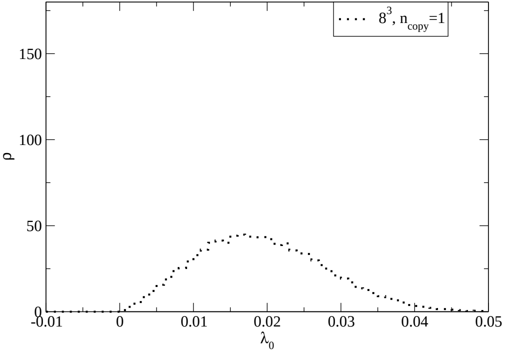


$d = 3$: ghost dressing function

- increase n_{copy} ...

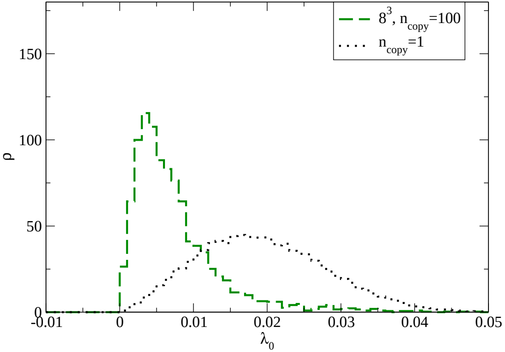
  $\rightsquigarrow$ strong effect

Impact of Gribov ambiguity



- FPO spectrum . . .

Impact of Gribov ambiguity



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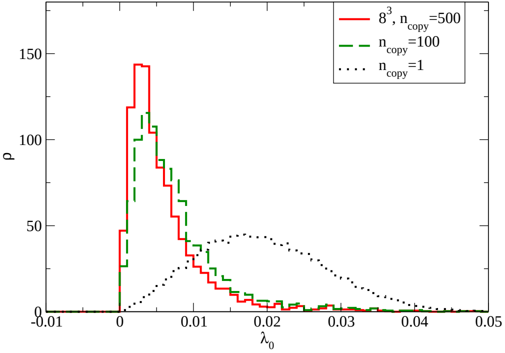
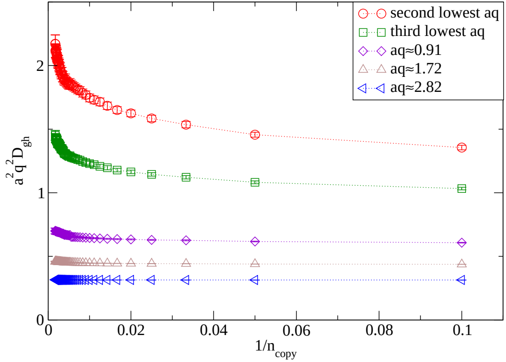


- FPO spectrum . . .
  changes accordingly:
  closer to Gribov horizon

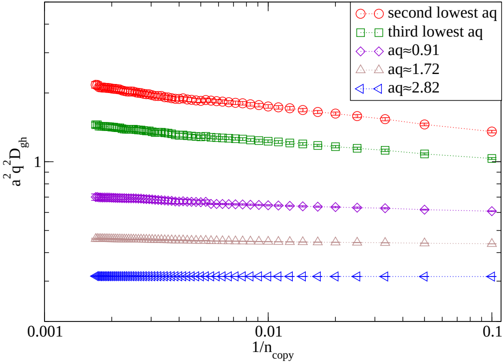
Impact of Gribov ambiguity



$d = 2$: ghost dressing function vs. n_{copy}^{-1}

- No ‘saturation’ at 600 copies

Impact of Gribov ambiguity



$d = 2$: ghost dressing function vs. n_{copy}^{-1}

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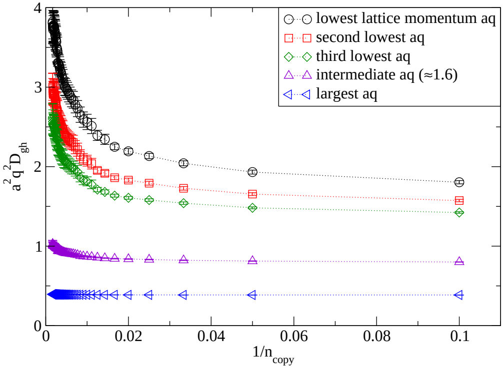
(log-log plot)

Impact of Gribov ambiguity



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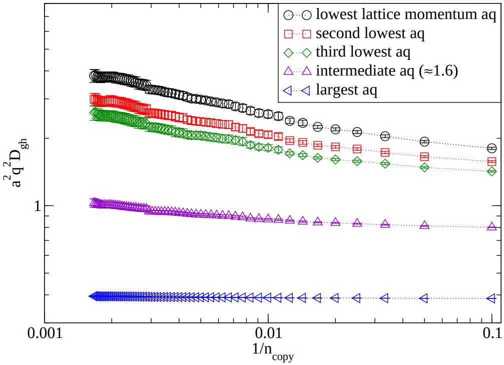


$d = 3$: ghost dressing function vs. n_{copy}^{-1}

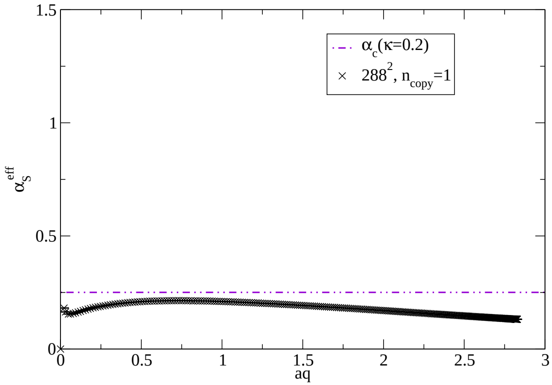
- No 'saturation' at 600 copies (log-log plot)

  $\Rightarrow$ No $n_{\text{copy}} \to \infty$ extrapolation possible

Impact of Gribov ambiguity on running coupling

- $\alpha_S^{\text{eff}} \propto D_{\text{gh}}^2 D_{\text{gl}}$
- Large impact on ghost
- Small impact on gluon

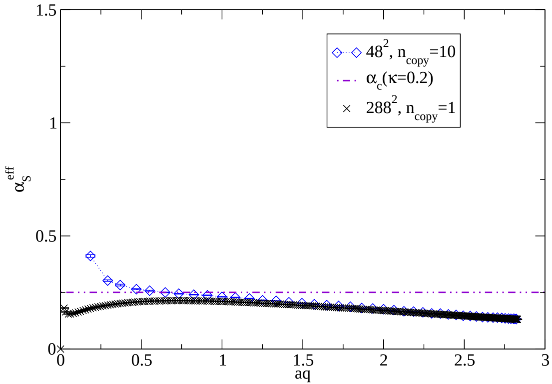
Impact of Gribov ambiguity on running coupling



$d = 2$

Large Gribov copy effect on running coupling

Impact of Gribov ambiguity on running coupling



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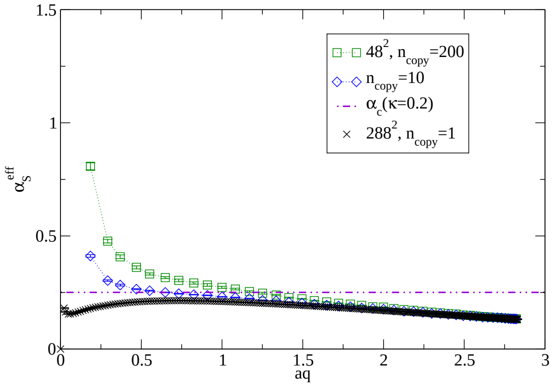

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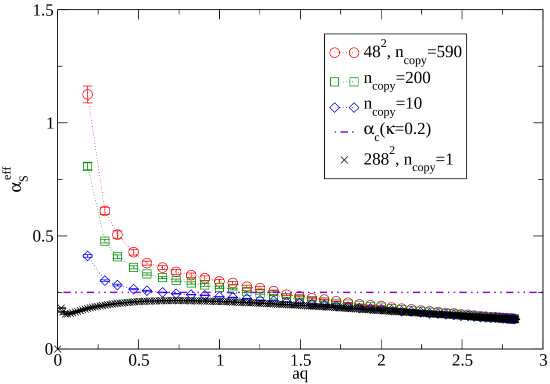

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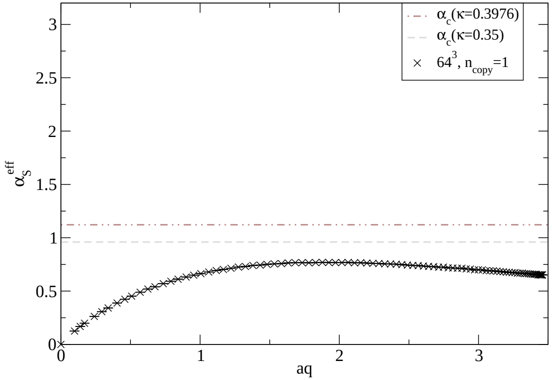

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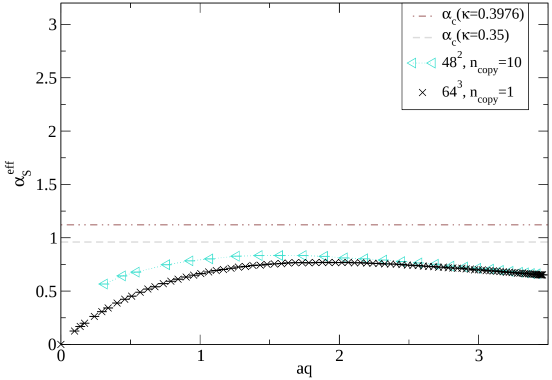

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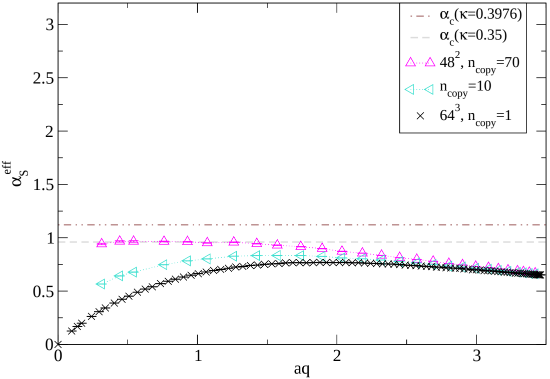

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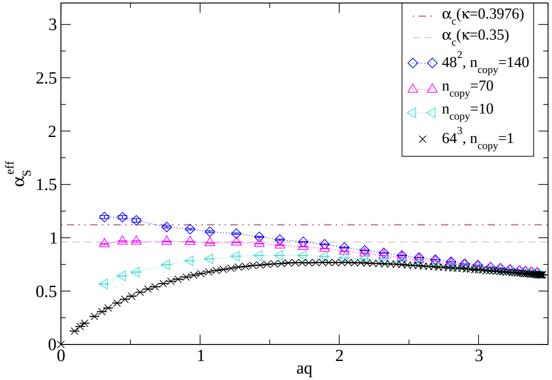

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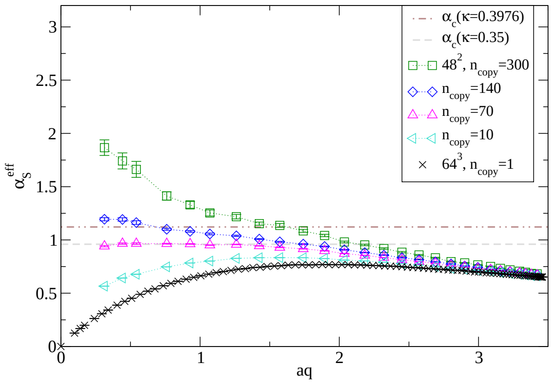

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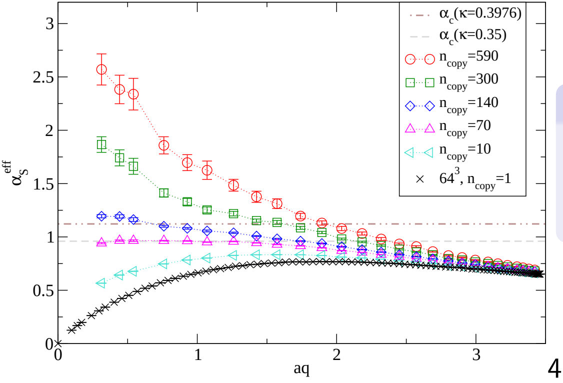


$d = 3$

Large Gribov copy effect on running coupling

$\Rightarrow$ even 'over-scaling'

Impact of Gribov ambiguity

Interpretation

- Gribov ambiguity more severe than expected
  - Should also hold at finite β
- No IR solution type (scaling vs. decoupling) excluded
  - Stronger & diff. Gribov copy effect than by 'global maximization'

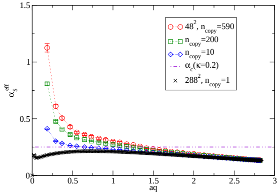
    (e.g. Bogolubsky et al. '05, '07, '09, Bornyakov et al. '08, '09)

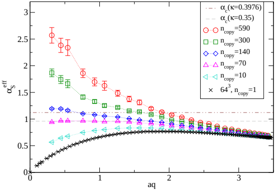
    - ★ $\mathcal{O}(100\%)$ vs. 10% for ghost

      gluon: small effect vs. 20%
    - ★ 'over-scaling' vs. decoupling

      but: 'over-scaling' ruled out in continuum

      (uniqueness proof: Fischer et al. '06, '09, Huber et al. '07)





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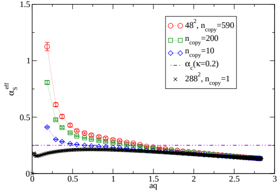

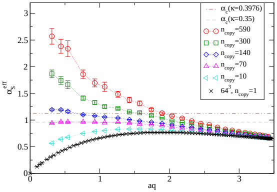

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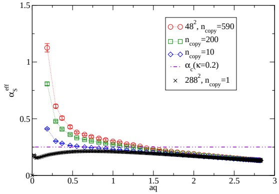
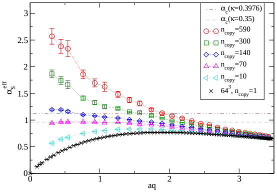
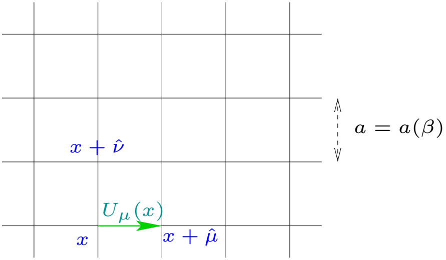




Outline

Free boundary conditions



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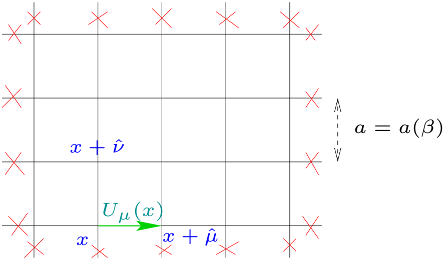

Links at boundary vanish $\rightsquigarrow$ 'finite lattice'

Free boundary conditions: Why & how?

- Free b.c. & Landau gauge ⇒ $\boxed{D_{\mathrm{gl}}(q=0)=0}$ (Schaden/Zwanziger '94)
  - Maybe obtain scaling?!
- Caveat: Non-periodicity
  ⇒ modify def. of D_{gl}

'Center def.':

$$D_{\mathrm{gl}}(q) \propto \sum_{a,\mu} \sum_{x} \langle A_\mu^a(x) A_\mu^a(c) \rangle \cos(q \cdot (x-c))$$

- factor of V lacking

required

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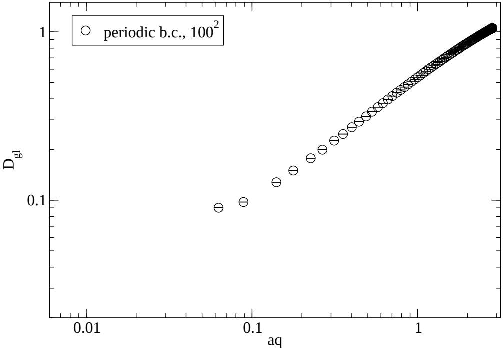
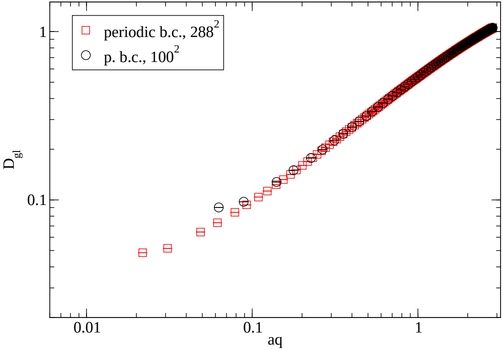
Gluon propagator from free b. c., two dimensions



$\beta = 0$ **$d = 2$**

- Periodic b.c.

Gluon propagator from free b. c., two dimensions



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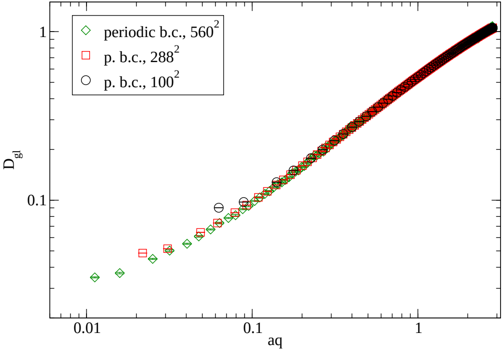

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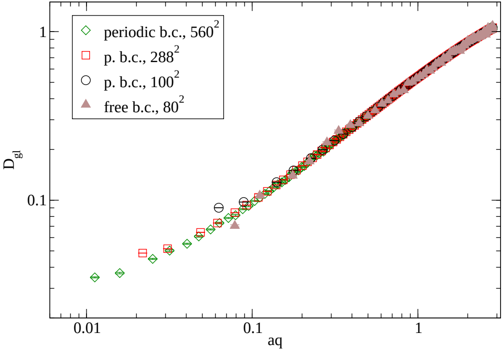

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- Periodic b.c.
- vs. free b.c.

(typically 500.000 config's)

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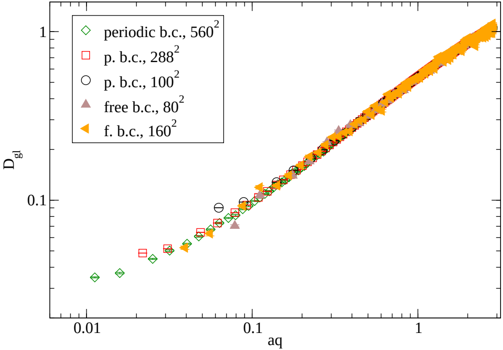

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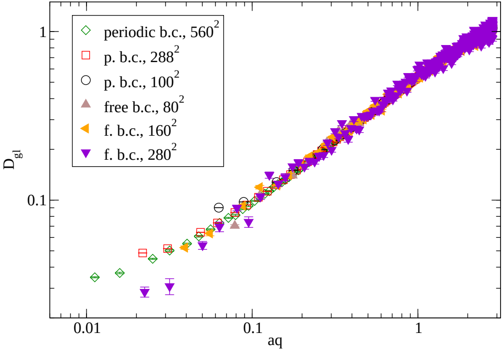

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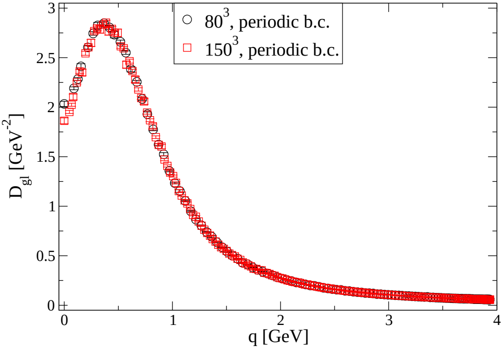
(typically 500.000 config's)

- ▶ consistent with same $V \to \infty$ limit

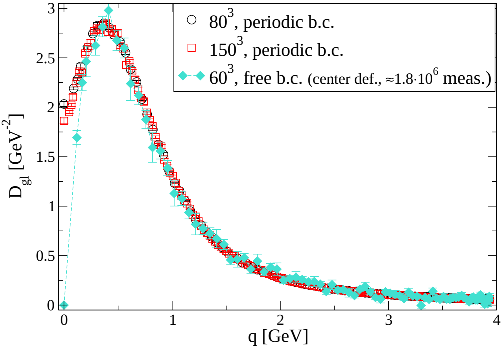
Gluon propagator from free b. c., three dimensions



finite coupling $d = 3$

Periodic b. c. . . .

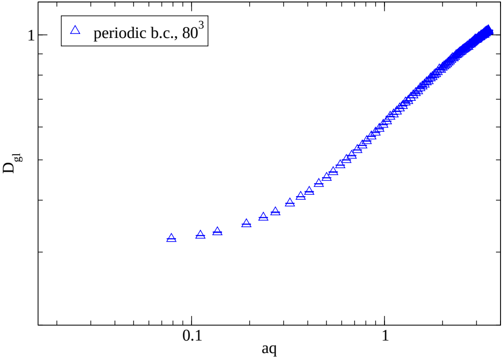
Gluon propagator from free b. c., three dimensions



finite coupling $d = 3$

Periodic vs. free b. c.

Gluon propagator from free b. c., three dimensions



$\beta = 0$ $d = 3$

Gluon prop. with periodic b. c.

. . .

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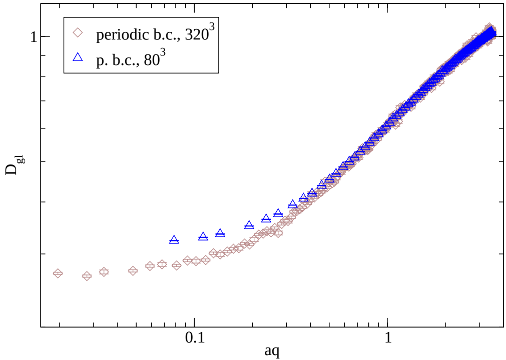
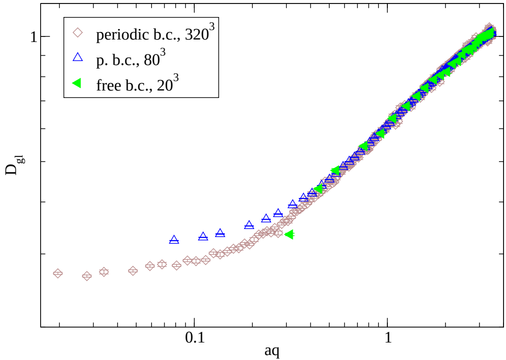


$\beta = 0$ $d = 3$

Gluon prop. with periodic b. c. decreases with V

Gluon propagator from free b. c., three dimensions



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Gluon prop. with free b. c. ...

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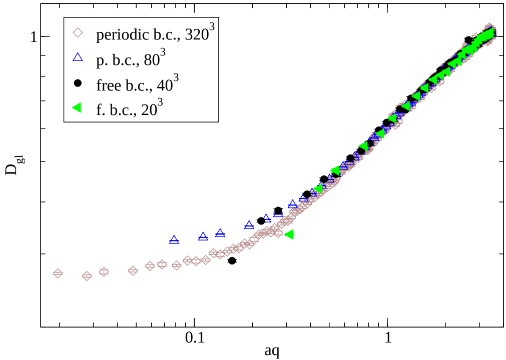

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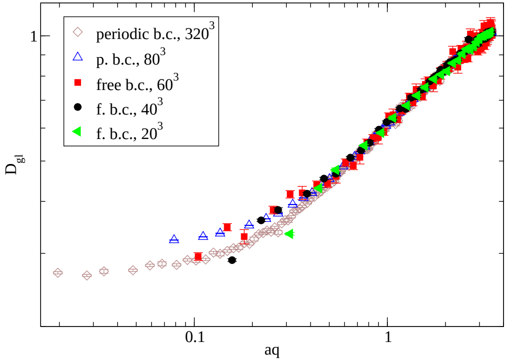


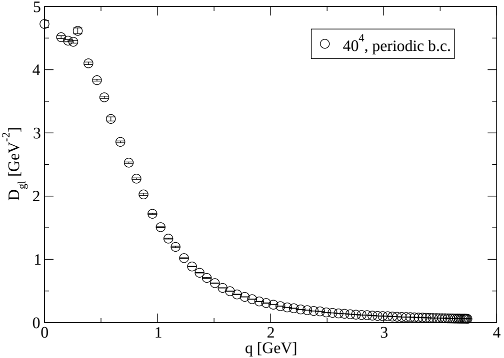
$\beta = 0$ $d = 3$

$V \to \infty$: Free b. c. prop. not below periodic b. c. prop.

- $\Rightarrow$ no uniform scaling

60^3: 10^6 config's
40^3: $2.5 \cdot 10^6$ config's

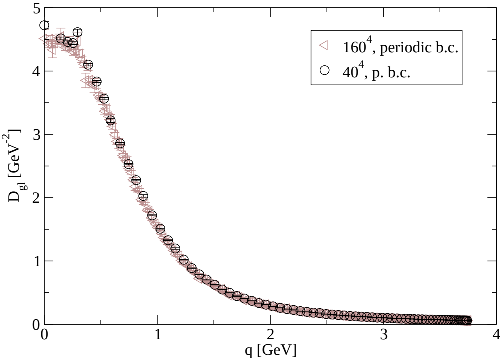
Gluon propagator from free b. c., four dimensions



Finite coupling $d = 4$

- Periodic b.c.

Gluon propagator from free b. c., four dimensions



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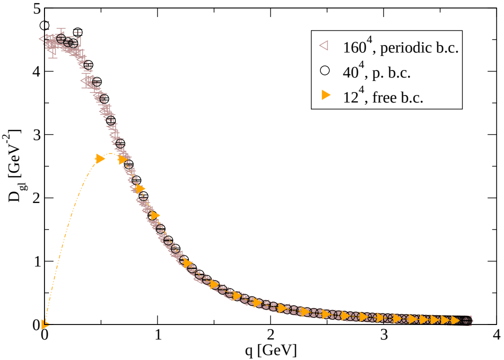
- Periodic b.c.:
  rel. small volume effect

  $V = (34\ \text{fm})^4$

Gluon propagator from free b. c., four dimensions



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- Free b.c.

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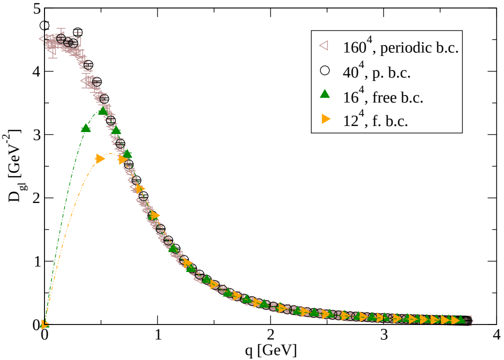

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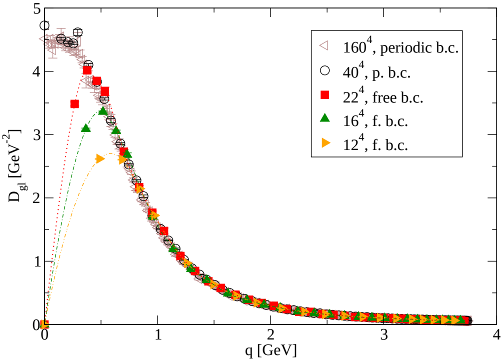

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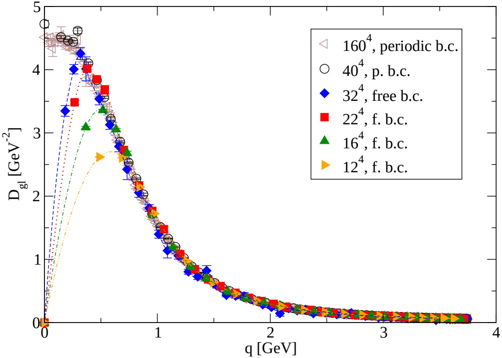
- Periodic b.c.:
  rel. small volume effect
  $V = (34\text{ fm})^4$
- Free b.c.

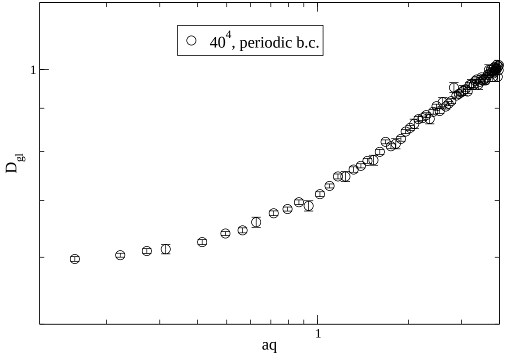
Gluon propagator from free b. c., four dimensions



Finite coupling $d = 4$

- Periodic b.c.:
  rel. small volume effect
  $V = (34\ \mathrm{fm})^4$
- Free b.c.:
  much stronger volume effect

Gluon propagator from free b. c., four dimensions



$\beta = 0$ $\quad d = 4$

- Periodic b.c. ...

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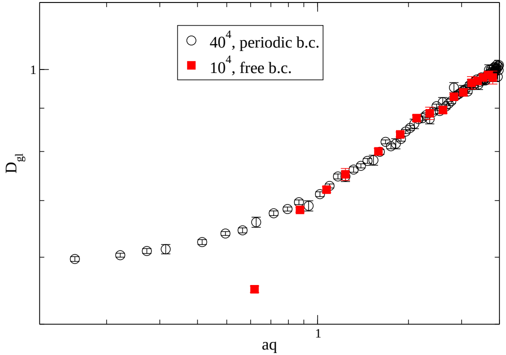

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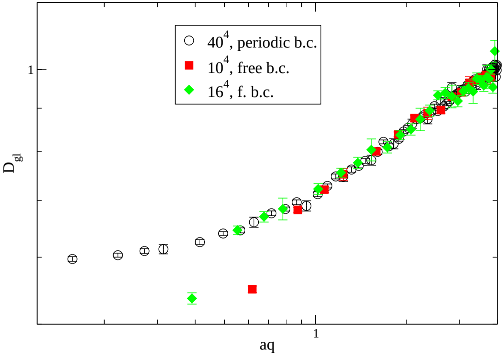

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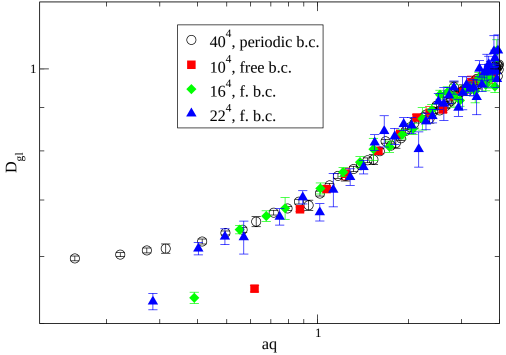


$\beta = 0$ $d = 4$

- Periodic b.c. ...
- Free b.c.
  - convergence against same result

22^4: > 400.000 config's

Outline

Summary

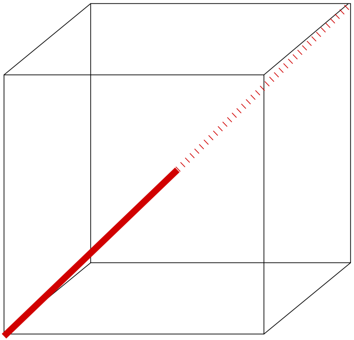
- Strong-coupling limit
  - Scaling branch
  - but no uniform scaling –
    not even in $d = 2$ (!)
- 'max-B' gauge
  - Huge effect of Gribov ambiguity
    - ★ Surprising 'over-scaling' in finite volumes
    - ★ No subset of solutions can be excluded yet
- Free b. c.
  - D_{gl} IR suppressed on moderate V
  - but 'decoupling branch' restored in $V \to \infty$ limit ($d = 3, 4$)

Outlook

- Current results surprise and raise new questions
  - $V \to \infty$ and $n_{\text{copy}} \to \infty$ limit of max-B gauge
  - Max-B gauge at finite β (Maas '09 ...
- BRST invariant lattice formulation ... (von Smekal et al. '07, '08 ...)

Outline

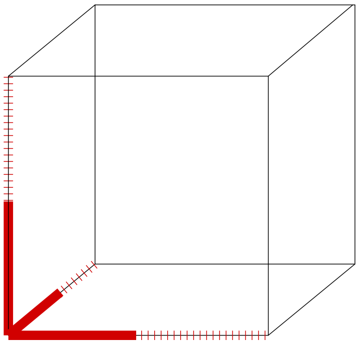
Discretization artifacts at $\beta = 0$



Idea

- Usually: cylinder cut $\rightsquigarrow$ momenta near diagonal
- Now cp. with on-axis momenta
- $\Rightarrow$ Assess effect of breaking rotational invariance

Discretization artifacts at $\beta = 0$



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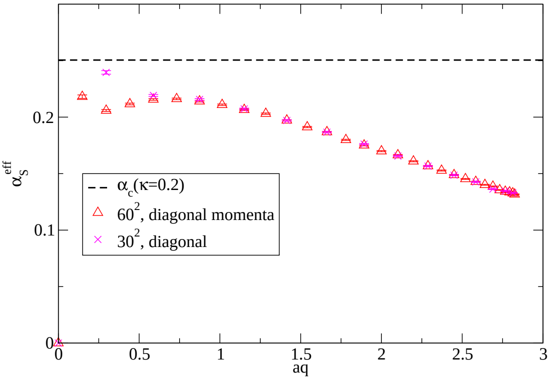
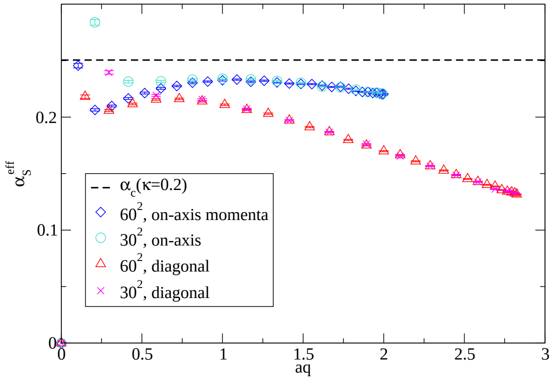


$d = 2$

- Diagonal mom.

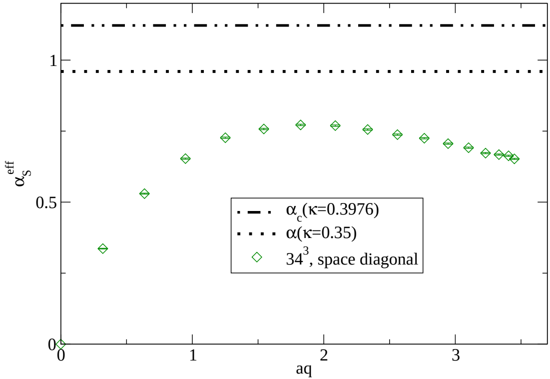
Discretization artifacts at $\beta = 0$



$d = 2$

- Diagonal mom.
- On-axis mom.
  - closer to scaling behav.

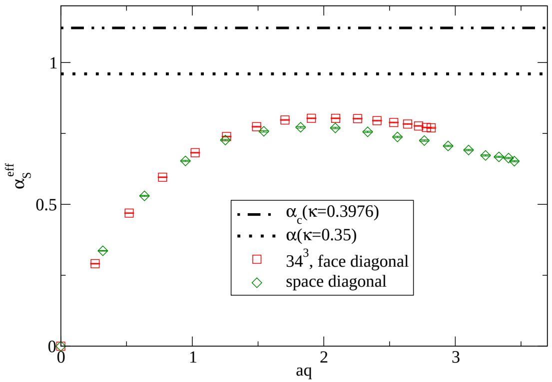
Discretization artifacts at $\beta = 0$



$d = 3$

- Space-diag. mom.

Discretization artifacts at $\beta = 0$



$d = 3$

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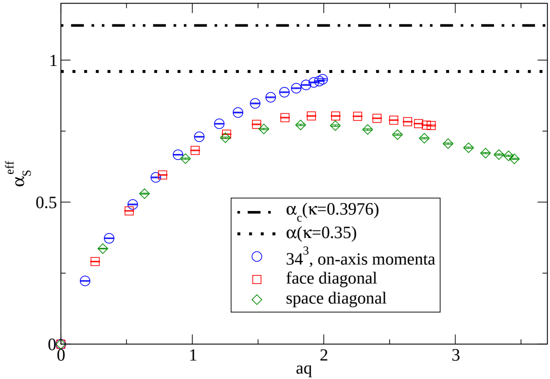

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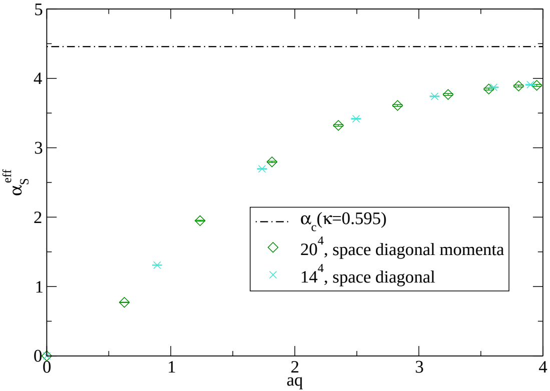

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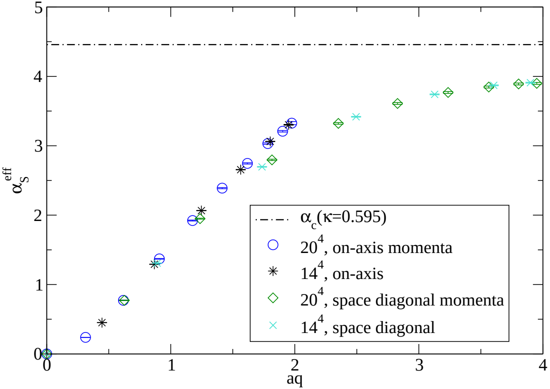
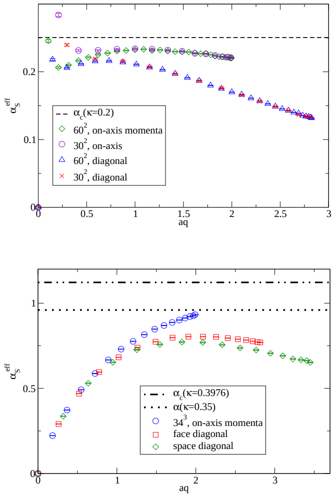


$d = 4$

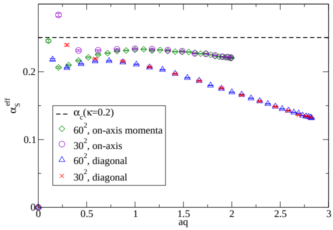
- Space-diag. mom.
- On-axis mom.
  - Effect weaker at larger d

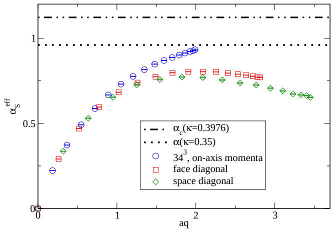
Discretization artifacts at $\beta = 0$



- $\Rightarrow$ Discretization problem at large aq
- Now for a stronger effect: Gribov copies ...

Discretization artifacts at $\beta = 0$





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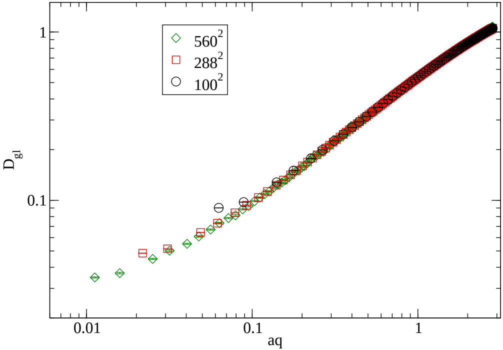
Strong-coupling limit: Two dimensions



Ghost propagator

- plane-wave source ⇝ precise result ($\kappa \approx 0.37$)
- local κ
  - monotonically rising
  - in general $\kappa \neq \kappa_Z$
    ⇒ no scaling
- cp. with point source

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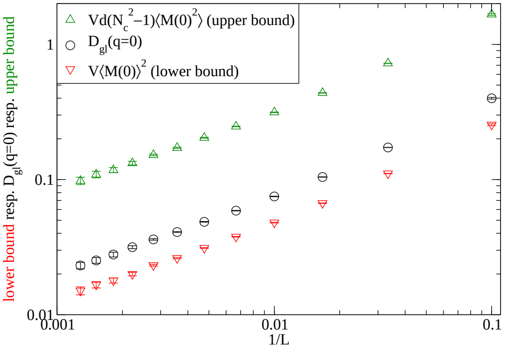

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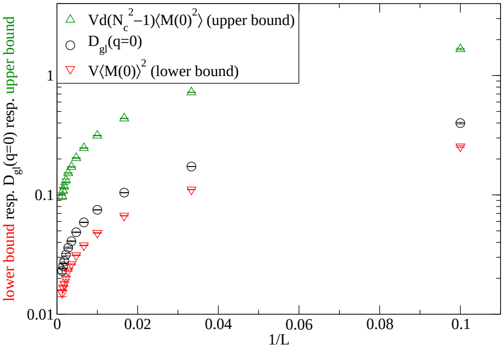

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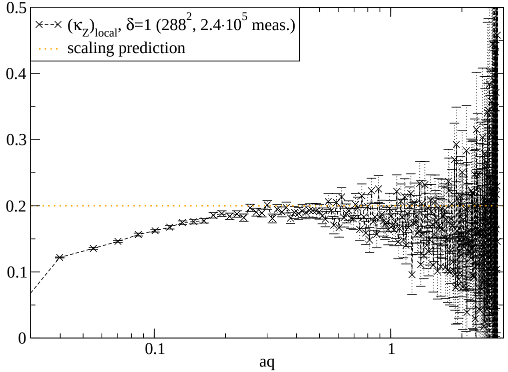

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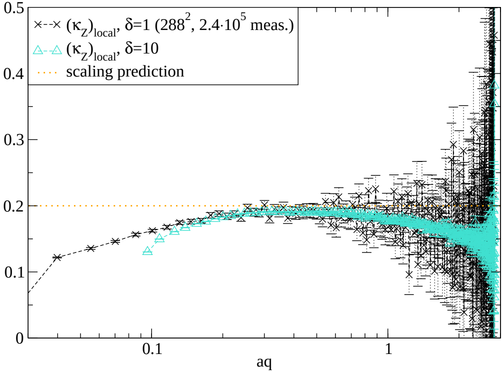

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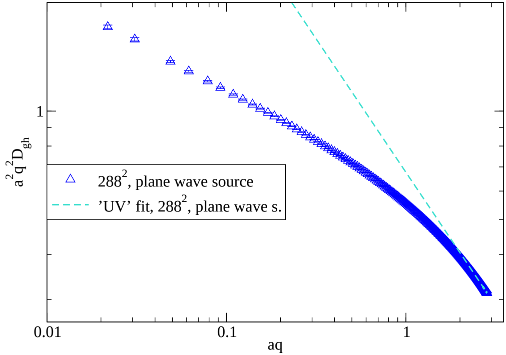

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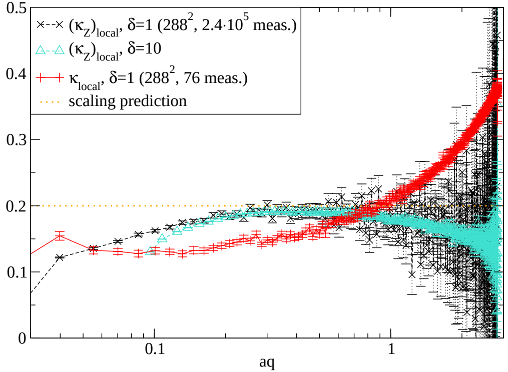

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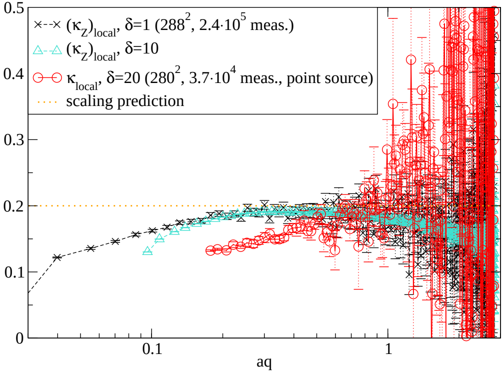
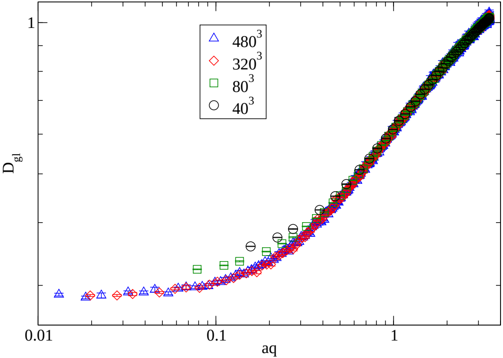


Strong-coupling limit: Three dimensions



Gluon propagator

- . . . at different V
- finite V behavior of $D_{\mathrm{gl}}(0)$
  - decoupling branch survives in $V \to \infty$ limit
- local κ_Z
  - qualitatively similar to 2D case

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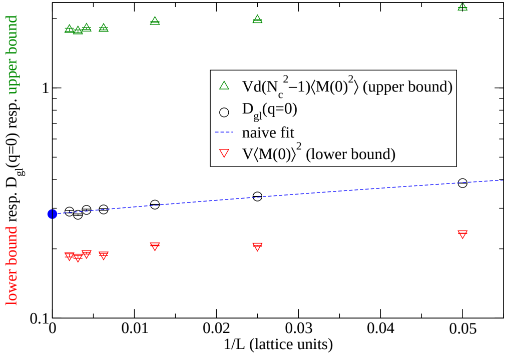

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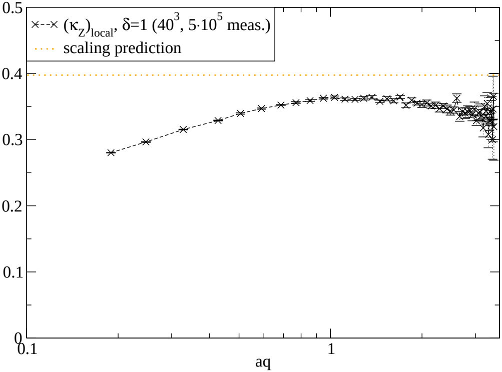

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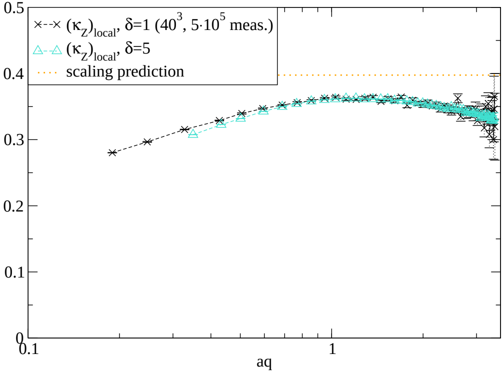

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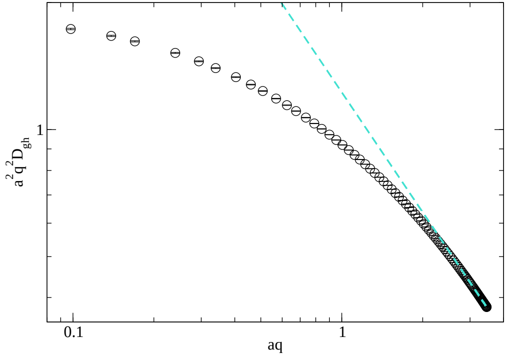

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  - $\kappa \neq \kappa_Z$
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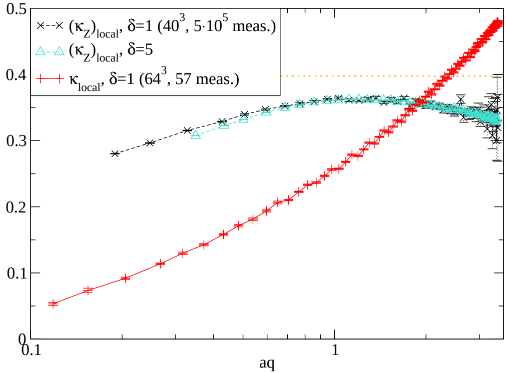

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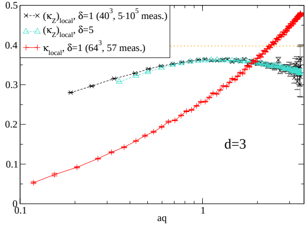

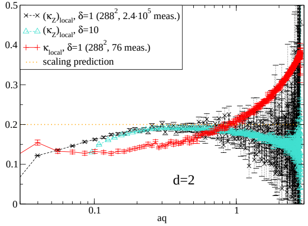

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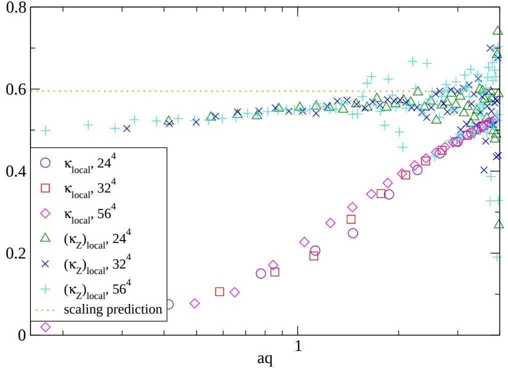


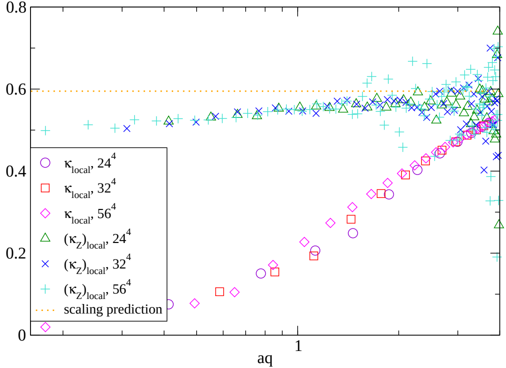
local κ & κ_Z

- additional analysis of previous 4D data

  (Sternbeck/von Smekal '08)

- relation to effective running coupling ...

Strong-coupling limit: Four dimensions



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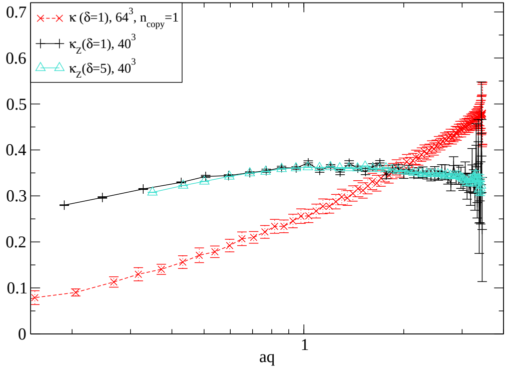
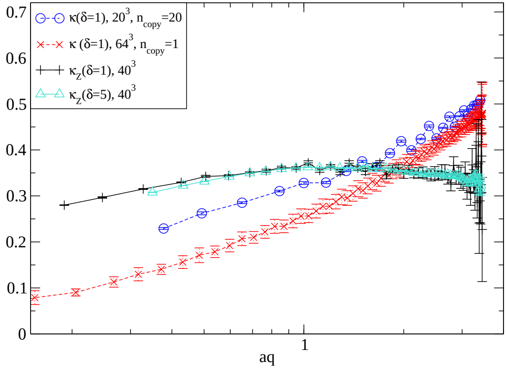
Impact of Gribov ambiguity



- increase n_{copy} . . .

Impact of Gribov ambiguity



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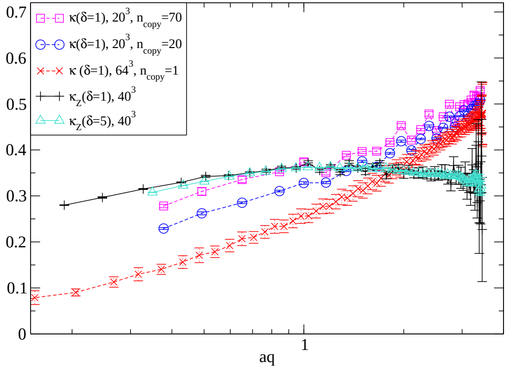
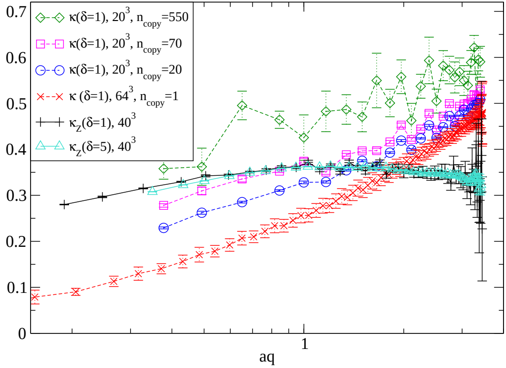


- increase n_{copy} . . .

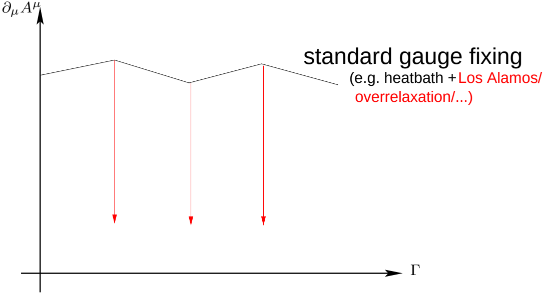
  (≈ scaling can be obtained . . .)

Impact of Gribov ambiguity



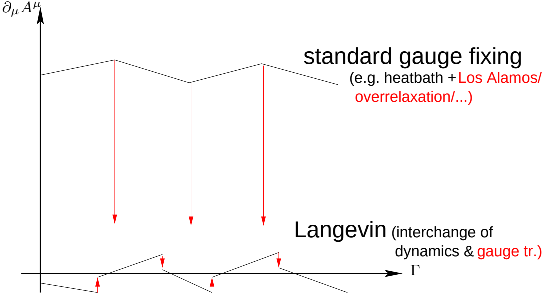
- increase n_{copy}
  $\rightsquigarrow$ 'over-scaling'

Stochastic gauge fixing: Intuitive picture



(cp. Nakamura/Saito/Sakai ’04)

Stochastic gauge fixing: Intuitive picture



Langevin eq. incl. gauge force (Zwanziger '81)

more local than standard g. f.

$\Rightarrow$ possibly different sampling of config' space

(cp. Nakamura/Saito/Sakai '04)

Propagators from stochastic gauge fixing

Bottom line

Confirmation of

standard lattice results . . .

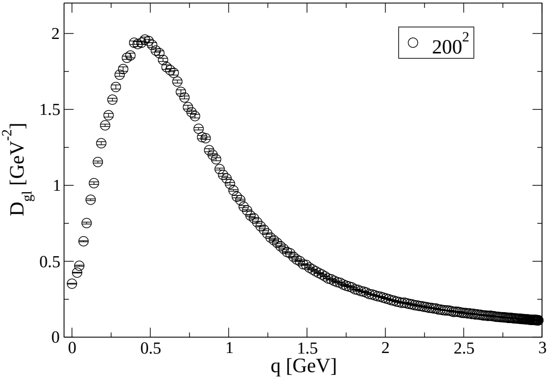
2D: scaling, 3&4D: decoupling

e.g.

- 2D: Maas '07
- 3D: Cucchieri/Mendes '03
- 4D: Sternbeck et al. '05, '06, Bogolubsky et al. '07, '08, '09, Cucchieri/Mendes '07 . . .

. . . with alternative gauge fixing method

Propagators from stochastic gauge fixing



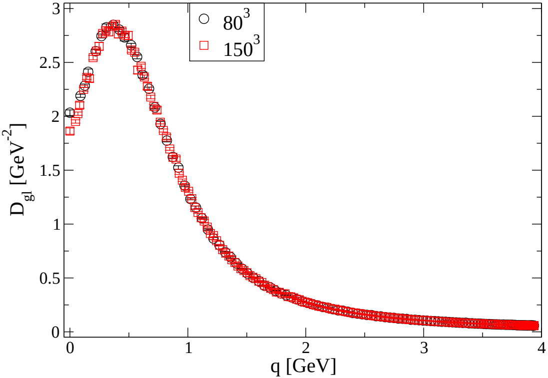
Gluon propagator

2D: compatible with IR scaling

- $\kappa \approx 0.2$, as expected

But: $\beta = 0$ results raise new questions

Propagators from stochastic gauge fixing



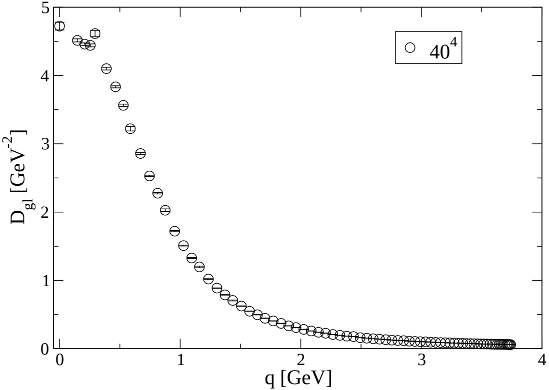
Gluon propagator

3D: Peak, but IR finite

$\rightsquigarrow$ decoupling

But: at $\beta = 0$, 3D similar to 2D

Propagators from stochastic gauge fixing

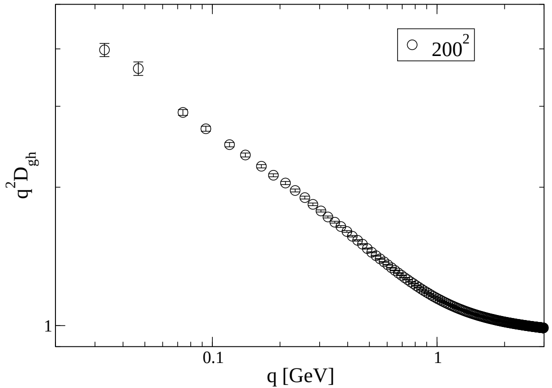


Gluon propagator

4D: IR finite

$\rightsquigarrow$ decoupling

Propagators from stochastic gauge fixing



Ghost dressing function

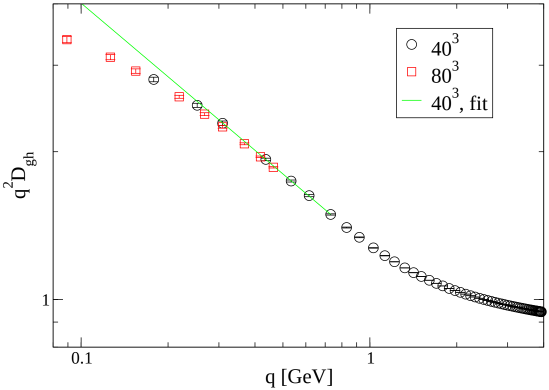
2D: IR divergent

$\kappa \approx 0.2$

- as expected for scaling
- consistent with gluon data

But: different picture at $\beta = 0$

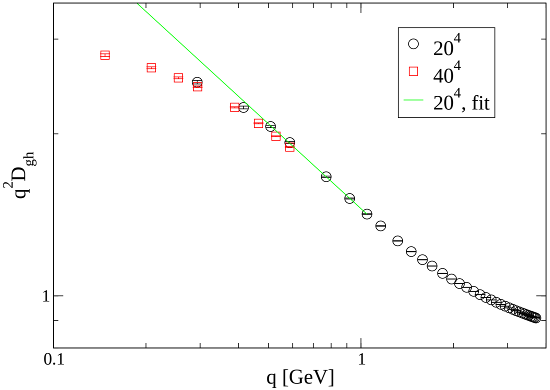
Propagators from stochastic gauge fixing



Ghost dressing function

3D: tends towards decoupling

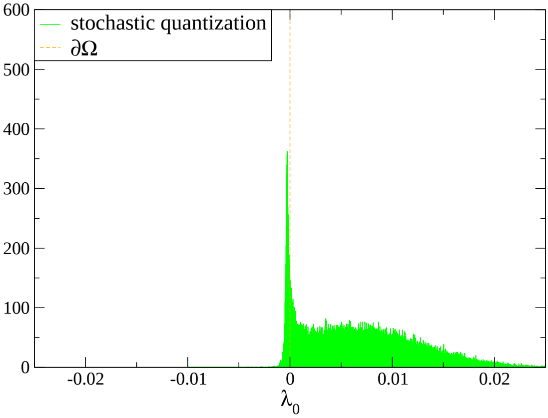
Propagators from stochastic gauge fixing



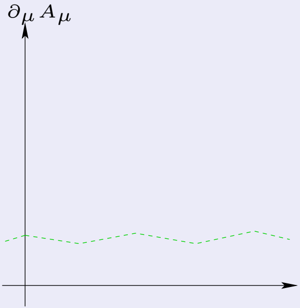
Ghost dressing function

4D: tends towards decoupling

Faddeev-Popov operator spectrum



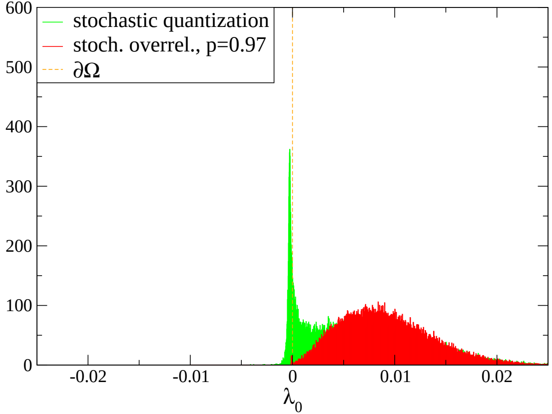
imperfect g. f.



$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
by SQ

$\Delta^a \hat{=} \partial_\mu A^a_\mu$

Faddeev-Popov operator spectrum



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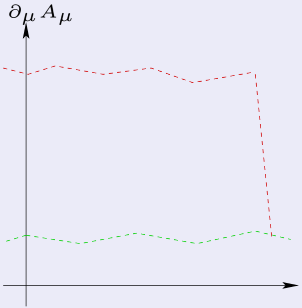
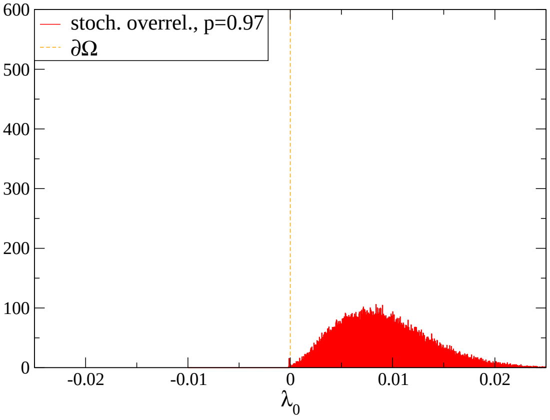


$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$

by SQ vs. stoch. overrel.

after heatbath

Faddeev-Popov operator spectrum



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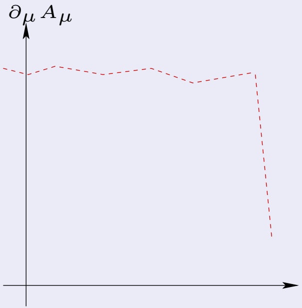
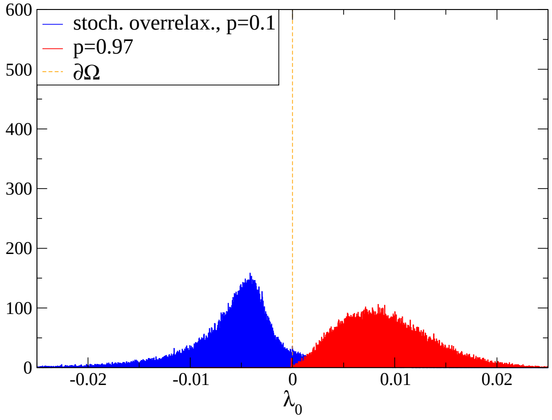
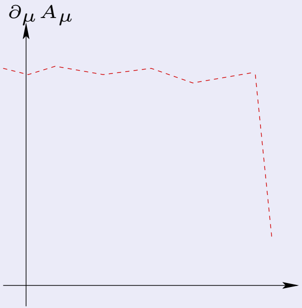


$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
by stoch. overrel. after heatbath at different overrel. parameters

Faddeev-Popov operator spectrum



imperfect g. f.



$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
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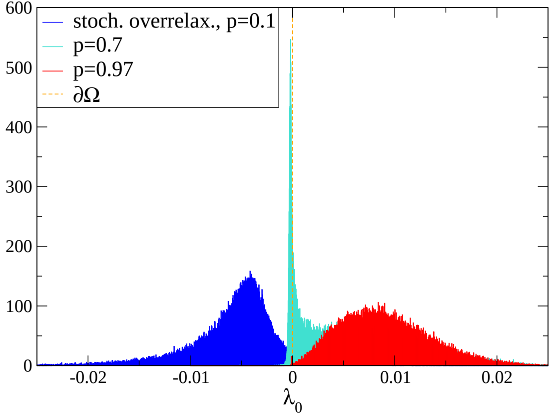

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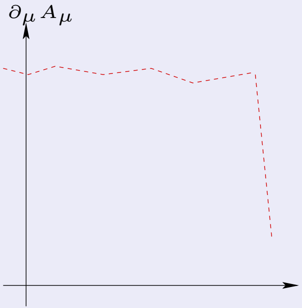

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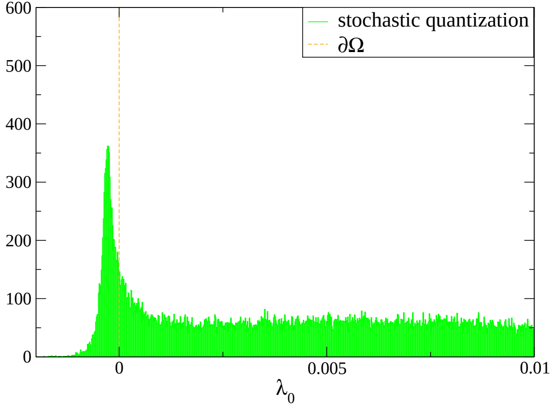

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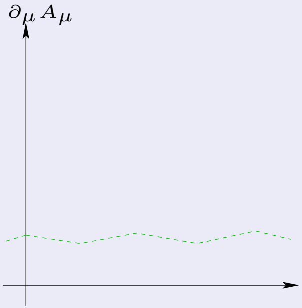
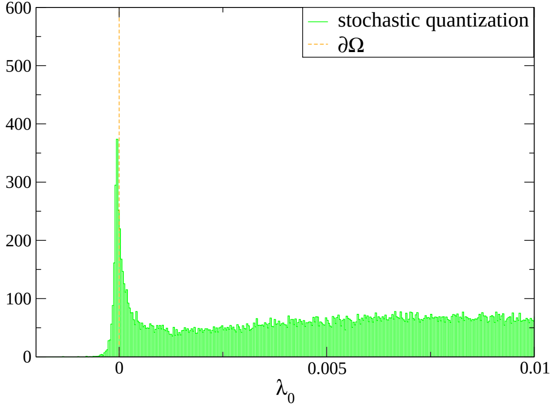


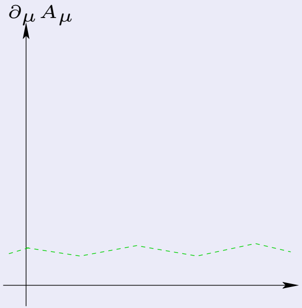
$\Delta^2 \in \mathcal{O}\left(10^{-3}\right)$
by SQ

$\Delta^a \hat{=} \partial_\mu A^a_\mu$

Faddeev-Popov operator spectrum

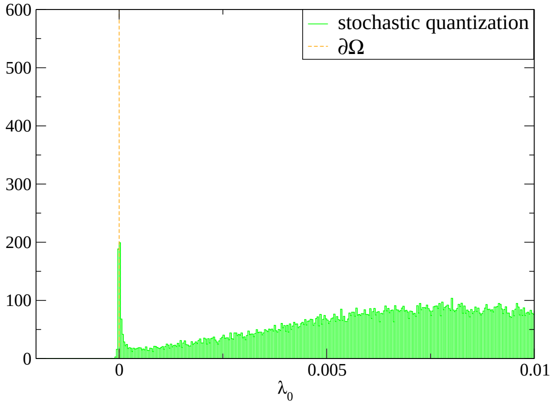


$\Delta^2 \longrightarrow 0$



$\Delta^2 \approx 3 \cdot 10^{-5}$
by SQ

Faddeev-Popov operator spectrum



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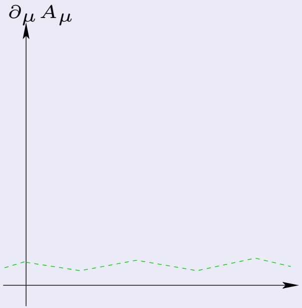
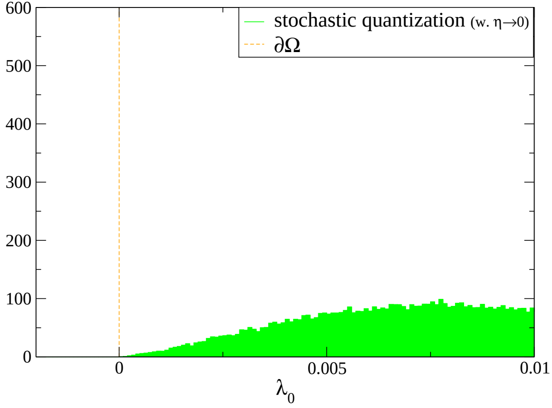


$\Delta^2 \approx 1.5 \cdot 10^{-6}$
by SQ

Faddeev-Popov operator spectrum



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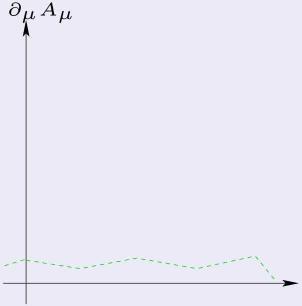


$\Delta^2 < 10^{-15}$

by SQ & step size $\to 0$

Faddeev-Popov operator spectrum

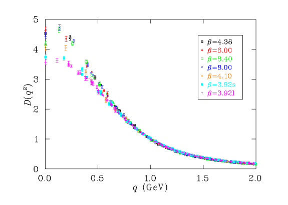
Upshot

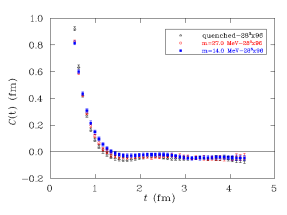
Peak disappears for sufficiently small Δ^2

Importance of IR asymptotics

What κ may tell about confinement

- IR asymptotics of $D_{\text{gl}}(q^2)$, $D_{\text{gh}}(q^2)$
  $\rightsquigarrow$ Test predictions of confinement scenarios
- E.g.: Violation of reflection positivity by IR non-divergent gluon propagator (lattice: Bowman et al. '07 (4D), Cucchieri et al. '05 (3D))

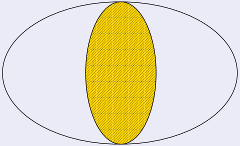




How to sample the configuration space

Possible requirements on the algorithm

Find either of the following:



- Fundamental modular region Λ (free of Gribov copies)
  $\hat{=}$ NP-hard optimization problem
- Entire Gribov region Ω with Faddeev-Popov weight
  - (Conjecture: Ω & Λ equiv. for expectation values (Zwanziger '04))
  - Restriction to Ω automatically (by g.f.)
    precise distribution nontrivial
- Bias towards Gribov horizon $\partial\Omega$ (Gribov–Zwanziger sc.: config's near $\partial\Omega \cap \partial\Lambda$ account for confinement)

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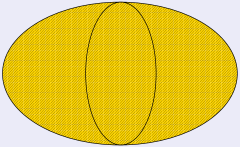

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    - ★ Faddeev-Popov operator is Hessian of $\int d^4x |^{\omega}A|^2$, which is minimized by numerical Landau g. f.;

  precise distribution nontrivial
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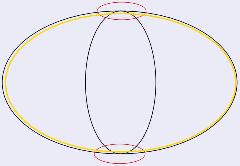

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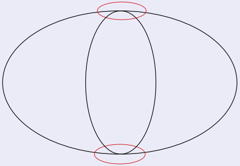
Confinement scenarios

Gribov–Zwanziger scenario (Gribov '78, Zwanziger '94, '04)

- Basic idea: Configurations in vicinity of $\partial\Omega(\cap\partial\Lambda)$ account for confinement of gluons.



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  - (Situation less clear due to e^{-S})
- Implications depend on gauge. . .

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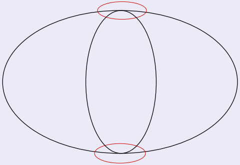

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- Implications depend on gauge...
  - In Landau gauge:
    IR vanishing of gluon propagator (horizon condition).
    ('IR slavery' scenario of *quark* confinement, $D \sim 1/q^4$, obsolete.)

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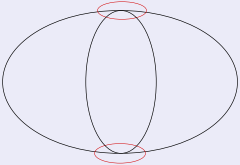

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    Ghost dressing function divergent in the IR.

Confinement scenarios

Kugo–Ojima scenario (Kugo/Ojima '79, review: Nakanishi/Ojima '90)

- Confinement by BRST quartet mechanism
  - $\approx$ Gupta-Bleuler in QED, but applies also to *transversal* gluons
- Integral part of Kugo–Ojima criterion:
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  $\Longleftrightarrow$

  1. mass gap &
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Confinement scenarios and Green's functions

Implications in a nutshell

- Kugo–Ojima
  - (BRST quartet mechanism, $\hat{\approx}$ Gupta-Bleuler in QED)

  $\kappa > 0$
- Gribov–Zwanziger (stronger implications)
  - (confinement by config's close to 1st Gribov horizon)

  $\boxed{\kappa > 0}$ for ghost, $\kappa > 1/2$ for gluon (i.e. $\boxed{\kappa_A < -1}$)

  I.e.:
  - Ghost dressing function IR divergent
  - Gluon propagator IR vanishing