Magnetic equation of state in 2+1 flavour QCD: Towards the continuum limit

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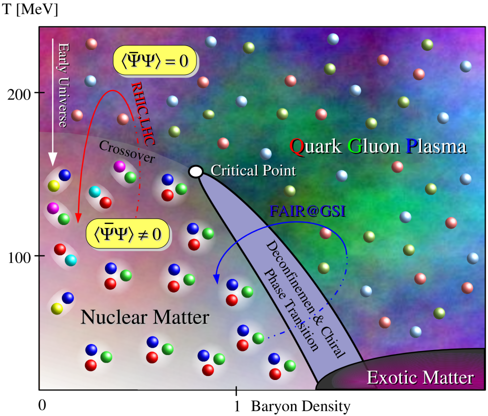
Spontaneous Chiral Symmetry Breaking in QCD

- QCD-Lagrangian exhibits chiral symmetry for $m_u, m_d, m_s \to 0$
- at low temperatures, chiral symmetry is spontaneously broken:

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$$



- at T_c chiral symmetry is restored
- order parameter is the chiral condensate:

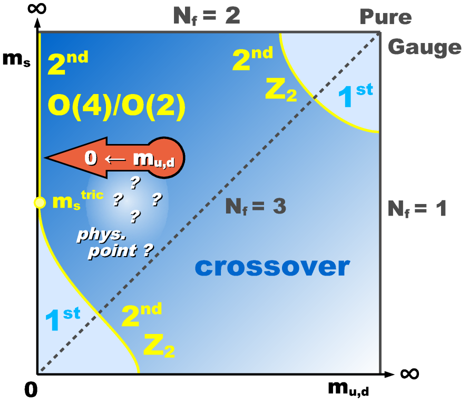
$$\langle\bar{q}q\rangle = -\frac{T}{V}\frac{\partial}{\partial m_q}\log\mathcal{Z}, \qquad \langle\bar{q}q\rangle \neq 0 \quad \text{for} \quad T < T_c$$

- quark masses m_u, m_d, m_s break chiral symmetry explicitly, but χSB still provides good approximation to low energy QCD (pions as pseudo-Goldstone bosons)

The QCD phase transitions at zero density

Columbia Plot: quark mass dependence of the order of the transition for 2+1 flavors

- at physical quark masses, a crossover is expected
- for sufficiently small quark masses (both $m_{u,d}$ and m_s) the transition is of first order
- critical lines of second order transition - limiting cases:
  $N_\text{f} = 2$: O(4) universality class
  $N_\text{f} = 3$: Ising universality class
- location of m_s^tric not known
  - below or above m_s^phys?
  (implications for nonzero density?)



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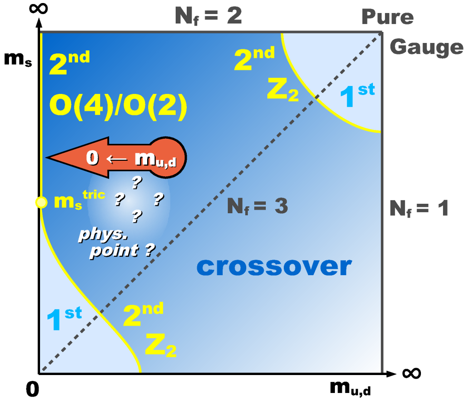

In this talk: interested in the chiral limit

lim $m_q \to 0$ at m_s fixed to physical value.

Goldstone Effect in O(N) Spin Model for d=3 and d=4

QCD at low energies may be described effectively by O(N) symmetric spin models:

- isomorphism: $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \simeq \mathrm{O}(4)$
- with $\pi^i \sim i\bar{\psi}\gamma_5 t^i \psi$, $\sigma \sim -\bar{q}q$, vector $(\vec{\pi}, \sigma)$ is O(4) invariant
- external field H corresponds to quark mass m_q, order parameter: $\Sigma = \langle \sigma \rangle$
- description is valid below and in the vicinity of the chiral phase transition

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Goldstone effect:

- $N - 1$ transverse Goldstone modes give corrections to Σ for $H \neq 0$
- calculation via chiral pert. theory from expectation value of $\langle \pi^i \pi^j \rangle$ yields:

$$d = 3: \qquad \Sigma_H = \Sigma_0 \left(1 + \frac{N-1}{8\pi} \frac{(\Sigma_0 H)^{1/2}}{F_0^3} + \mathcal{O}(H) \right)$$

$$d = 4: \qquad \Sigma_H = \Sigma_0 \left(1 - \frac{N-1}{16\pi^2} \frac{\Sigma_0 H}{F_0^4} \ln \left(\frac{\Sigma_0 H}{F_0^2 \Lambda_\Sigma} \right) + \mathcal{O}(H^2) \right)$$

[J. Gasser, H. Leutwyler - Ann. Phys. 158 (1984)]
[P. Hasenfratz, H. Leutwyler - Nucl. Phys Proc. B343 (1990)]

O(N) Critical Scaling at $T \cong T_c$

Question: Is Goldstone scaling below T_c consistent with critical scaling at T_c?

- below T_c: $\Sigma_H = c_0(T) + c_1(T) H^{1/2}$
- $T \simeq T_c$: scaling laws governed by critical exponents:

$$\Sigma_{H=0} \sim t^{\beta}, \quad \Sigma_H(t=0) \sim H^{1/\delta}, \quad t = \frac{T-T_c}{T_c}, \quad \beta,\delta = \begin{cases} 0.349, 4.780 & O(2) \\ 0.380, 4.824 & O(4) \end{cases}$$

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Critical scaling in vicinity of T_c can be described via **scaling functions**:

- rescaled scaling variable, now: $t = \frac{1}{t_0} \frac{T-T_c}{T_c}, \quad h = \frac{H}{h_0}$
- $z = t/h^{1/\beta\delta}$ invariant under rescaling $\quad t_0 \to b^{-1/\beta} t_0, \quad h_0 \to b^{-\delta} h_0$
- scaling function $f_G(z)$ describes order parameter via **magnetic equation of state:**

$$\Sigma(t,h) = h^{1/\delta} f_G(z)$$

- normalization conditions:

$$f_G(0) = 1 \qquad \text{and} \qquad \lim_{z \to -\infty} \frac{f_G(z)}{(-z)^{\beta}} = 1$$

The Magnetic Equation of State

Goldstone scaling is encoded within the O(N) scaling function $f_G(z)$

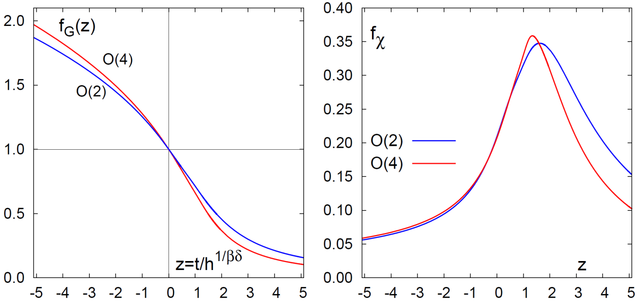
- magnetic equation of state in low temperature regime (from ϵ-expansion):

$$\Sigma(h,t) = h^{1/\delta} f_G(z), \qquad f_G(z) \simeq (-z)^{\beta}(1 + \tilde{c}_2 \beta (-z)^{-\beta\delta/2}) \qquad \text{for} \quad z \to -\infty$$

[D. J. Wallace, R, K. P. Zia - Phys. Rev. B12 (1975)]

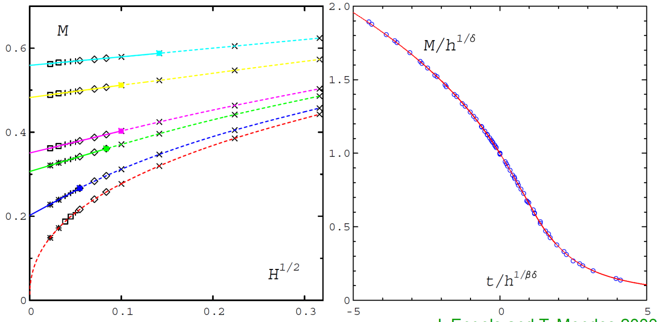
- also corresponding susceptibility $\chi \equiv \frac{\partial \Sigma}{\partial H}$ is described by scaling function $f_\chi(z)$, which is related to $f_G(z)$

$$\chi(h, t<0) \sim c_1(t) h^{-1/2} \qquad \chi(h, t=0) \sim h^{1/\delta - 1}$$



Evidence From the Lattice

- Goldstone effect in O(2)/O(4) and consistency with critical scaling numerically well established



[J. Engels, T. Mendes - Nucl. Phys Proc. Suppl. 83 (2000)]

- shown recently: our lattice data for $N_f = 2+1$ staggered fermions is well described by the scaling function $f_G(z)$ for small quark masses on a coarse lattice ($N_\tau = 4$)

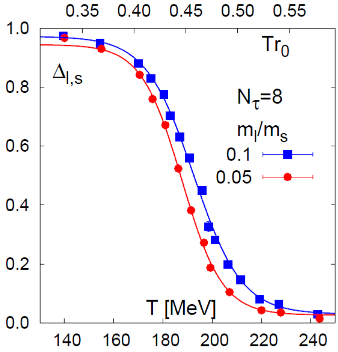
[S. Ejiri *et al* - Phys. Rev. D80 (2009)]

Motivation for Scaling Analysis in QCD

precise determination of T_c still open issue

- new $N_\tau = 8$ data: shift of chiral transition region of -5 MeV as $m_l/m_s = 1/10 \rightarrow m_l/m_s = 1/20$
- small effect: scale setting, i.e. value of $\frac{r_0}{a}(\beta)$ with Sommer scale $r_0 = 0.469(7)$ fm, $T = \frac{1}{N_\tau r_0}\frac{r_0}{a}(\beta)$
- main effect: strong quark mass dependence

[Cheng et al - Phys. Rev. D81 (2010)]



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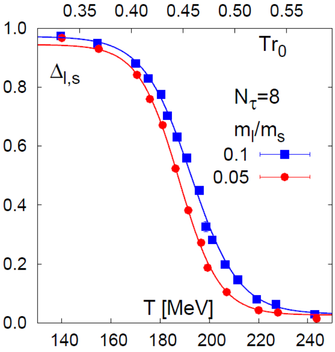


- location of m_s^{tric} crucial for influence of critical surface on physical point
- is a *chiral* CEP at finite μ really ruled out?

Goldstone Modes in QCD

- QCD with $N_\mathrm{f} = 2+1$ flavors: chiral condensate now in principle depends on two masses, $m_q = m_{u,d}$ and m_s
- finite temperature QCD: 3+1 dim. system with T controlled via temporal extent
- pressure of the ideal relativistic pion gas:

$$P(T, M_\pi) = \frac{N_\mathrm{f}^2 - 1}{2}\left[\frac{\pi^2 T^4}{45} - \frac{T^2 M_\pi^2}{12} + \frac{T M_\pi^3}{6\pi} - \frac{M_\pi^4}{16\pi^2}\log\frac{\Lambda}{M_\pi} + \dots\right]$$

[H. E. Haber, H. A. Weldon - Phys. Rev. Lett. 46 (1981)]

- Gell-Mann/Oakes/Renner:

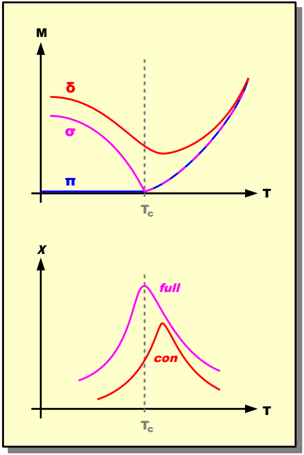
$$M_\pi = \sqrt{2 m_q B} \quad \text{with} \quad B = -\lim_{m_q \to 0} \frac{\langle 0|\bar{q}q|0\rangle}{N_\mathrm{f} F_\pi^2}$$

- chiral condensate from pressure:

$$\langle \bar{q}q \rangle = \langle 0|\bar{q}q|0\rangle \left\{1 + \frac{N_\mathrm{f}^2 - 1}{N_\mathrm{f}}\left[-\frac{(T/F_\pi)^2}{12} + \frac{1}{4\pi}\frac{T M_\pi}{F_\pi^2} + \mathcal{O}(M_\pi^2)\right]\right\}$$

Connected and Disconnected Chiral Susceptibility

Susceptibilities measure the fluctuations of the order parameter:



- in QCD: also (quark-line) connected diagrams contribute to chiral susceptiblity

$$\chi_{\text{full}} = \chi_{\text{con}} + \chi_{\text{dis}}$$

$$\chi_{\text{dis}} = N_f^2 \left(\left\langle (\bar{q}q)^2 \right\rangle - \langle \bar{q}q \rangle^2 \right)$$

$$\chi_{\text{con}} = N_f \left\langle \bar{q}(x)q(x)\bar{q}(0)q(0) \right\rangle$$

- if $\mathrm{U}_A(1)$ is effectively restored: the scalar isovector δ-meson (a_0) and the pion become mass degenerated
- to lowest order: $\chi_{\text{con}} \sim \frac{1}{M_\delta^2}$ and $\chi_{\text{full}} \sim \frac{1}{M_\sigma^2}$

[K. Rajagopal, F. Wilczek - Nucl. Phys. B399 (1993)]
[M. Marci, E. Meggiolaro - Nucl. Phys. B665 (2003)]

⇒ if $U_A(1)$ is not effectively restored at the chiral transition (in the chiral limit), the connected susceptibility cannot contribute to the scaling function!

IR Divergences of Connected and Disconnected Susceptibility

Goldstone divergences for $T < T_c$ obtained from chiral perturbation theory to one loop for N_f degenerated flavors:

- IR-Part of Connected Susceptibilities at $M_\pi \ll T \ll T_c$:

$$\chi_{\text{con}}^{\text{IR,3D}} = \frac{N_\mathrm{f}^2 - 4}{8\pi^2} \frac{T}{\sqrt{2m_q}} \left(\frac{\Sigma}{F_\pi^2}\right)^{3/2}$$

- IR-Part of Full Susceptibilities at $M_\pi \ll T \ll T_c$:

$$\chi_{\text{full}}^{\text{IR,3D}} = \frac{N_\mathrm{f}^2 - 1}{4\pi^2} \frac{T}{\sqrt{2m_q}} \left(\frac{\Sigma}{F_\pi^2}\right)^{3/2}$$

- From $\chi_{\text{con}}^{\text{IR}}$ and $\chi_{\text{full}}^{\text{IR}}$ one also gets $\chi_{\text{dis}}^{\text{IR}}$:

$$\chi_{\text{dis}}^{\text{IR,3D}} = \frac{N_\mathrm{f}^2 + 2}{8\pi^2} \frac{T}{\sqrt{2m_q}} \left(\frac{\Sigma}{F_\pi^2}\right)^{3/2}$$

i.e. $\chi_{\text{con}}^{\text{IR}} = 0$ for $N_\mathrm{f} = 2\,(+1)$

$\rightarrow$ no Goldstone effect expected for χ_{con} in the continuum

[A. V. Smilga, J. Stern - Phys. Lett. B318 (1993)]
[A. Smilga, J. J. M. Verbaarschot - Phys. Rev. D54 (1996)]

Taste Breaking

- Lee-Sharpe Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{\text{cont}} + a^2 \sum_i C_i \mathcal{O}_i(U)$$

- taste-index: $t \in \{P, A, T, V, I\}$ corresponds to taste channels defined via Euclidean gamma matrices. $\xi_t \in \{\gamma_5, i\gamma_\mu\gamma_5, i\gamma_\mu\gamma_\nu, \gamma_\mu, 1\}$
- impact of taste breaking on meson masses:

$$M^2_{f,f',t} = B(m_f + m_{f'}) + a^2 \Delta_t$$

with taste-violations: $\Delta_P = 0$ and $\Delta_t \neq 0$ for $t \neq P$

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- impact of taste breaking on meson masses:

$$M_{f,f',t}^2 = B(m_f + m_{f'}) + a^2 \Delta_t$$

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remaining axial U(1) symmetry even for finite lattice spacing

resulting in a pseudo-Goldstone boson $M_{\pi,5}$ in the pseudo taste channel
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- taste violation contribution for $N_\mathrm{f} = 2+1$ ($m_q \rightarrow 0$, a fixed):

$$\chi^{\mathrm{con,IR}}_{\mathrm{S}\chi\mathrm{PT}} = B^{\mathrm{IR}}_{a_0}(0) \sim \frac{1/16}{M_{\pi,5}}$$

$$\chi^{\mathrm{dis,IR}}_{\mathrm{S}\chi\mathrm{PT}} = B^{\mathrm{IR}}_{f_0}(0) - B^{\mathrm{IR}}_{a_0}(0) \sim \frac{1/16}{M_{\pi,5}}$$

Setup of Lattice Calculations

preliminary data of the RBC-Bielefeld collaboration for $N_{\rm f} = 2+1$:

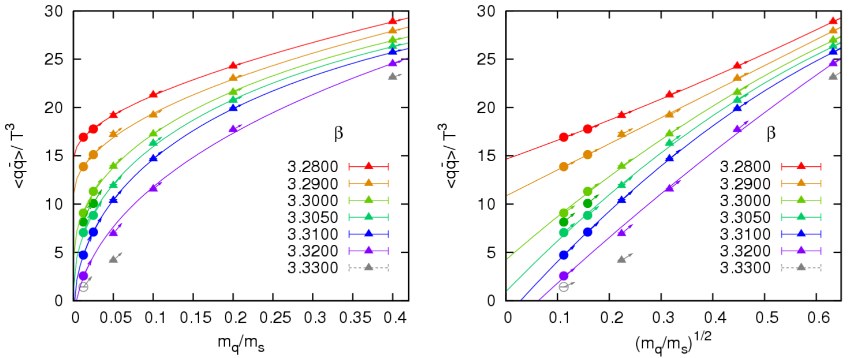
- fermion action: p4fat3 (fattening reduces taste breaking)
- susceptibilities measured with up to 20 random vectors (noisy estimator)
- statistics (measurements separated by 10 trajectories):

lattice dim.	m_q/m_s	statistics	lattice dim.	m_q/m_s	statistics
$32^3 \times 4$	1/80	$\mathcal{O}(20000)$			
$32^3 \times 4$	1/40	$\mathcal{O}(20000)$			
$16^3 \times 4$	1/40	$\mathcal{O}(30000)$	$32^3 \times 8$	1/40	just started
$16^3 \times 4$	1/20	$\mathcal{O}(40000)$	$32^3 \times 8$	1/20	$\mathcal{O}(20000)$
$16^3 \times 4$	1/10	$\mathcal{O}(40000)$	$32^3 \times 8$	1/10	$\mathcal{O}(30000)$
$16^3 \times 4$	1/5	$\mathcal{O}(40000)$	$32^3 \times 8$	1/5	$\mathcal{O}(30000)$
$16^3 \times 4$	2/5	$\mathcal{O}(40000)$			
$\beta = 3.2800$,	...	3.3300	$\beta = 3.4800$,	...	3.5400

- with m_s fixed to physical value, $m_s a = 0.065$ for $N_\tau = 4$ and $m_s a = 0.024$ for $N_\tau = 8$ ($\rightarrow M_{s\bar{s}} \simeq 669$ MeV) we find:
  - $m_q/m_s = 1/20$: $\rightarrow$ $M_{\pi,5} \simeq 150$ MeV
  - $m_q/m_s = 1/80$: $\rightarrow$ $M_{\pi,5} \simeq 75$ MeV
- thermodynamic limit well under control, for smallest mass: $M_{\pi,5} N_\sigma \simeq 3$
- in particular: no evidence for finite size scaling $\rightarrow$ crossover region

Goldstone Scaling of Chiral Condensate

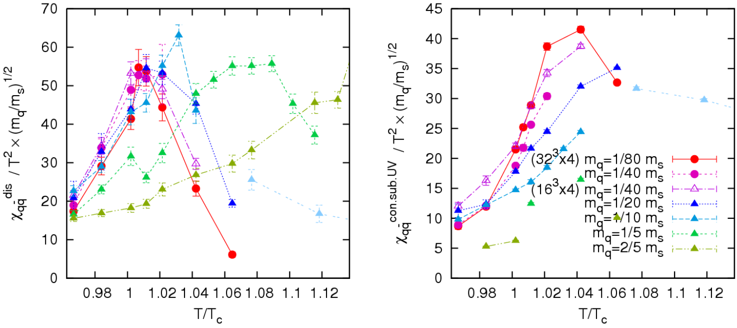
- data indicate Goldstone Effect below T_c
- fit ansatz $\langle \psi\bar{\psi} \rangle = a + b \left(\frac{m_q}{m_s}\right)^{1/2} + c \left(\frac{m_q}{m_s}\right)$ describes data well
- prediction of chiral limit can be improved a lot by taking into account taste breaking



chiral condensate over quark mass, arrows indicate full susceptibility as its mass derivative

Goldstone Scaling of Chiral Susceptibilities

- disconnected part: Goldstone effect extends into peak region
- connected part (corrected for UV-divergent constant): also Goldstone scaling found



rescaled chiral susceptibility - left: disconnected part, right: connected part

Remark on Goldstone Fits

- usually: first go to continuum limit, then to chiral limit (not feasable)
- or: understand the chiral limit for finite lattice spacing systematically
- indeed, we observe agreement with predictions of staggered chiral perturbation theory!

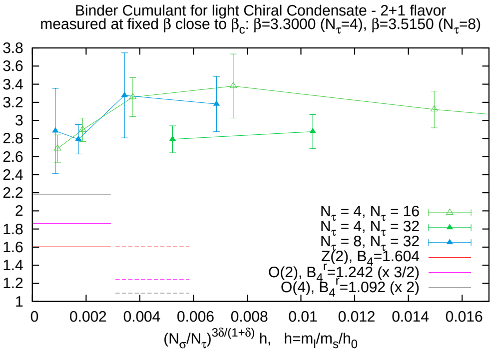
Binder Cumulant

From the cumulants

$$M_k = \frac{\partial^k}{\partial h^k} \mathcal{F}_s(h, V^{-1})$$

with $\lambda = L^d$, one obtains for the Binder Cumulant:

$$B_4(z_h) = \frac{f_h^{(4)}(z_h)}{f_h^{(2)}(z_h)}, \qquad z_h = L^{d a_h} h$$



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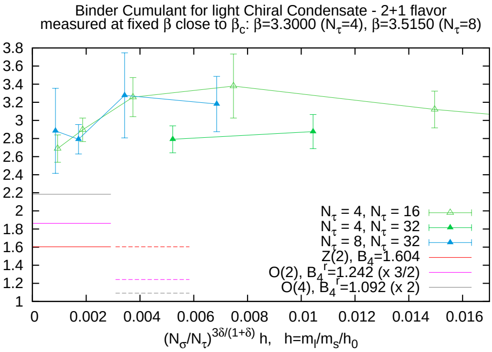


Note:

- B_4 is an RG-invariant number at $h = 0$, and independent of volume $V = L^d$
- simulations performed at $h > 0$, criticality only reached in chiral limit
- universal value of B_4 altered by group integration factors as O(N) field is always aligned to longitudinal magnetization (here $s^{||} \sim \bar{q}q$)

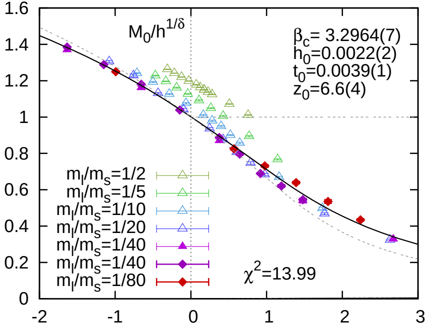
$$B_4^r = \frac{\left\langle s^{||4}\right\rangle}{\left\langle s^{||2}\right\rangle^2} = \frac{\left\langle |\vec{S}|^4\right\rangle \int_{-1}^{1} dx x^4 f_N(x)}{\left\langle |\vec{S}|^2\right\rangle^2 \left(\int_{-1}^{1} dx x^2 f_N(x)\right)^2} = B_4 \times \begin{cases} 1 & \text{for} \quad N=1 \\ \frac{3}{2} & \text{for} \quad N=2 \\ 2 & \text{for} \quad N=4 \end{cases}$$

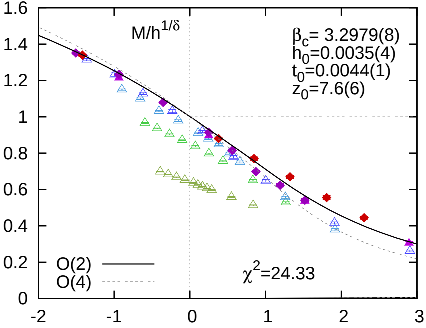
$$f_N(x) = c_N \int_0^{\pi} d\theta_{N-1} \sin^{N-2}\theta_{N-1}\delta(x - \cos\theta_{N-1})$$

Magnetic Equation of State for Chiral Condensates

- determination of the critical temperature T_c and the normalization constants h_0, t_0
- O(2) slightly preferred, however, by reparametrizing $z \to 1.2z$, O(2) moves on O(4) scaling functions almost indistinguishable
  $\to$ not possible to discriminate O(2) from O(4) here
- $z_0 = t_0 h_0^{-1/\beta\delta}$ is invariant under rescaling (h_0, t_0 are not!)
- un-subtracted and subtracted condensate (to remove UV-div. $\sim m_l/a^2$) are fitted:

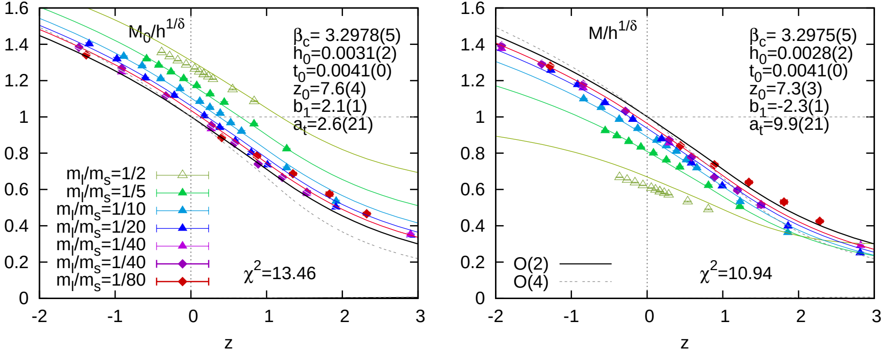
$$M_0 = m_s \langle \bar{q}q \rangle_l / T^4, \qquad M = m_s(\langle \bar{q}q \rangle_l - m_l/m_s \langle \bar{q}q \rangle_s)/T^4$$





Magnetic Equation of State for Condensates, Deviations from Scaling

- quark masses with $m_q/m_s \leq 1/20$ well described by scaling function
- for $m_q/m_s > 1/20$ scaling deviations become substantial
- fit ansatz for scaling deviations: $M(t,h) = f_T(t,h)|t|^\beta + +a_t(T-T_c)H + b_1 H$
- problem for $N_\tau = 8$ data: fits without scaling deviations not possible



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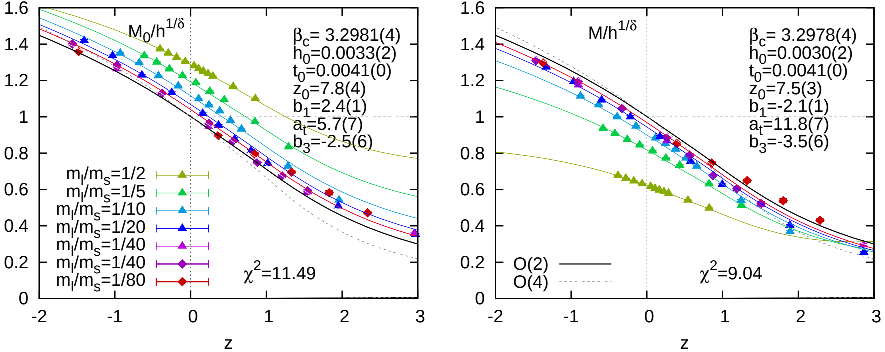

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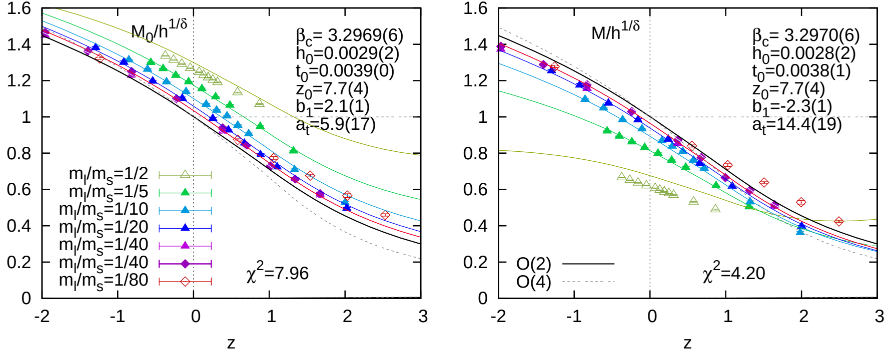
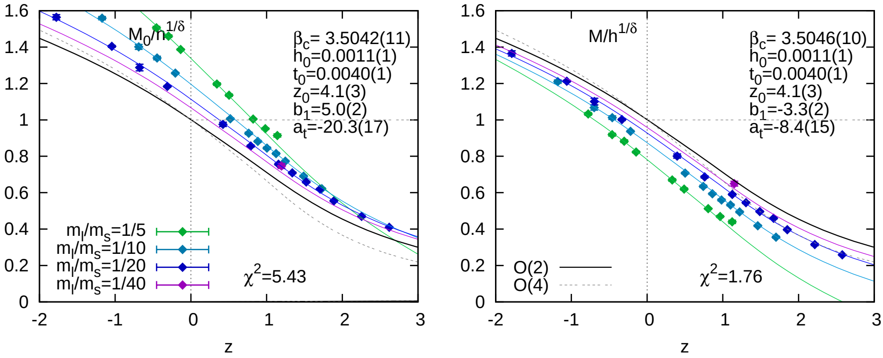
- fit ansatz for scaling deviations: $M(t,h) = f_T(t,h)|t|^{\beta} + a_t(T-T_c)H + b_1 H$
- problem for $N_\tau = 8$ data: fits without scaling deviations not possible



Magnetic Equation of State for Condensates, Deviations from Scaling

- question: is the range $1/20 \leq m_q/m_s$ enough to determine β_c, t_0 and h_0?
- yes, for $N_\tau = 4$ β_c and z_0 is recovered within errors
- assumption: should then also work for $N_\tau = 8$

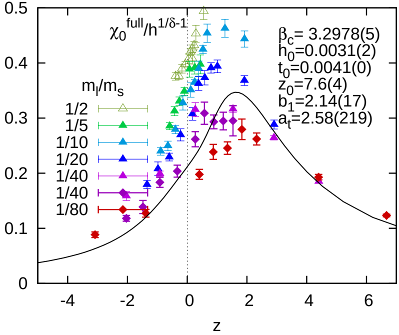


- z_0 is specific to QCD and in the continuum should only depend on m_s
- on the lattice: cut-off dependence should be seen in z_0, due to O(2) → O(4)

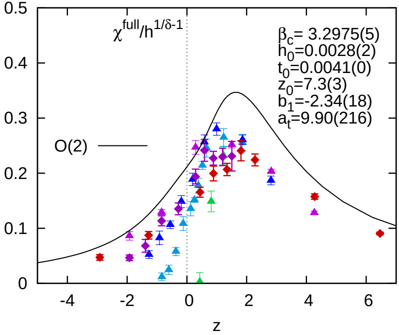
Magnetic Equation of State for Susceptibilities

- full susceptibilities quite well described by $f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta}\frac{d}{dz}f_G(z)\right)$
- in principle, O(2) and O(4) scaling should be distinguishable in f_χ for lattice data, however, statistics might not be sufficient
- $N_\tau = 4$

usual susceptibility



susceptibility for subtracted condensate



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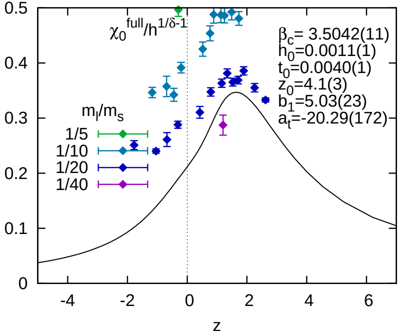

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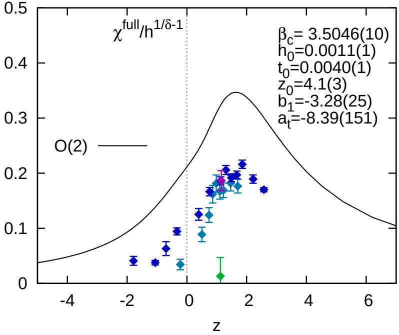

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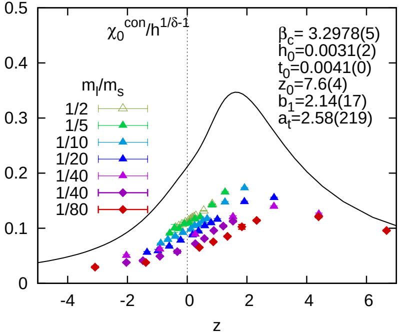
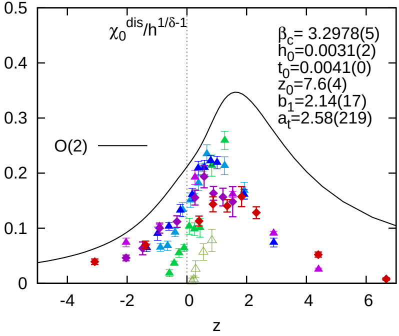
- connected and disconnected part, $N_\tau = 4$:

connected susceptibility



disconnected susceptibility



- connected part should not contribute substantially to the scaling function, but lattice data are somewhat ambiguous

Magnetic Equation of State for Susceptibilities

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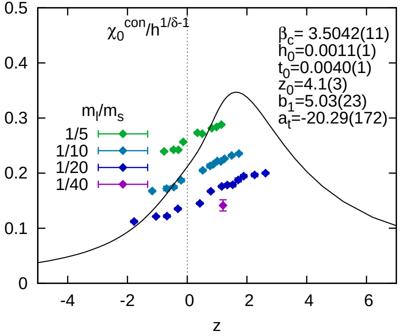
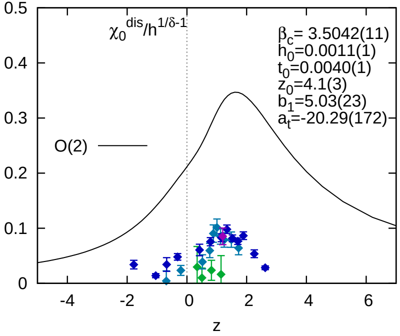
- in principle, O(2) and O(4) scaling should be distinguishable in f_χ for lattice data, however, statistics might not be sufficient
- connected and disconnected part, $N_\tau = 8$:

renormalized order parameter



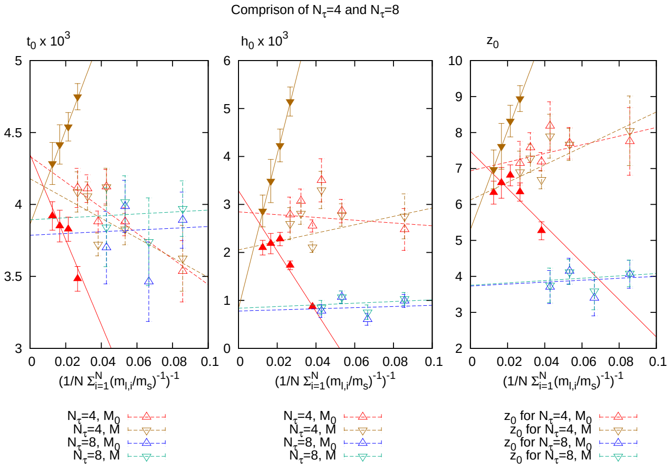
un-renormalized order parameter



- stronger contribution of connected part due to UV-divergent constant ca^2, but becomes suppressed in chiral limit

Determination of z_0

- z_0 decreases in the continuum limit



Consequences for Determination of T_{pc} for physical quark masses

now believed:

- for staggered fermions, the pseudocritical temperature T_{pc} of the physical point (m_l, m_s) is strongly influenced by critical scaling (deviations from scaling still small)
- z_0 together with

  peak position of f_χ: $z_p = 1.56(10)$ for O(2), $z_p = 1.33(5)$ for O(4)

  allows to determine the pseudocritical line
- with approximation $m_q/m_s \sim 0.52(M_{\pi,5}/M_K)^2$:

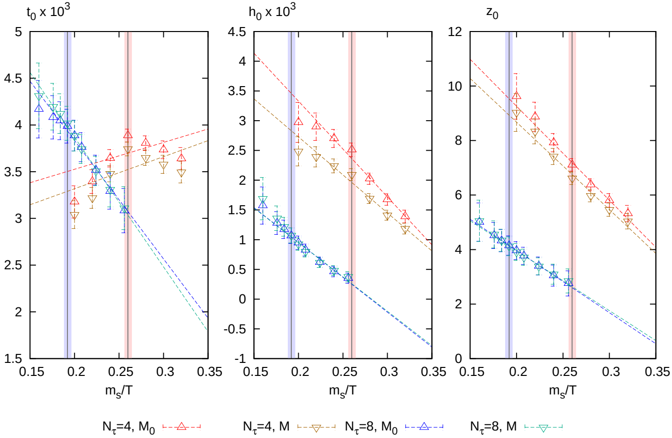
$$\frac{T_{pc}(M_{\pi,5}) - T_c}{T_c} = 0.68 \frac{z_p}{z_0} \left(\frac{M_{\pi,5}}{M_K} \right)^{2/\beta\delta}$$

- dependence of T_{pc} on $M_{\pi,5}$ is weak:
  $N_\tau = 4: \quad 0.68 z_p/z_0 \sim 0.15(5), \qquad N_\tau = 8: \quad 0.68 z_p/z_0 \sim 0.20(8)$
- interesting whether z_0 remains finite in continuum limit
- decrease might be related to multiplicity of Goldstone modes as O(2) $\rightarrow$ O(4)

Dependence of z_0 on the strange quark mass:

reweighting in the strange quark mass, quark mass ratio fixed:

- z_0 increases for decreasing m_s (both $N_\tau = 4$ and $N_\tau = 8$)
  $\rightarrow$ weaker dependence of T_{pc} on $M_{\pi,5}$
- might be related to transition to Ising-like scaling



Conclusions and Outlook

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- evidence for Goldstone and critical scaling seen in $\langle\bar{q}q\rangle$ and $\chi^{\rm dis}$
- Goldstone scaling also seen in $\chi^{\rm con}$, unphysically induced by lattice artifacts
- evidence for O(2) or O(4) critical scaling up to physical quark masses $m_q/m_s \leq 1/20$, both for $N_\tau = 4$ and $N_\tau = 8$
  - $N_\tau = 4$: O(2) scaling slightly preferrred
  - $N_\tau = 8$: O(4) scaling slightly preferrred
- exclusion of Z_2 scaling subtle issue, due to additional parameter m_q^c
- BUT: deviations from Goldstone scaling expected for Z_2 (here, f_G does NOT incorporate Goldstone scaling)
- interesting quantity: QCD-invariant $z_0(m_s, a^2)$

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Outlook:

- $N_\tau = 8$ data are continuously improved in statistics, smaller masses will not be possible in near future
- in preparation: determination of slope $\frac{\partial^2}{\partial \mu_B^2} T_{pc}(\mu_B)\Big|_{\mu_B=0}$ for $N_\tau = 8$
- also: clarify whether the connected part might contribute to the scaling function, carefull analysis of $\mathrm{U_A}(1)$-induced quark mass dependence in χ^{con} above T_c needed