String Breaking and Confinement in G2–Gauge Theory

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- Polyakov loop

$$P(\vec{x}) = \text{tr}\mathcal{P}(\vec{x}), \quad \mathcal{P}(\vec{x}) = \frac{1}{N}\text{tr}\left(\exp\ i\int_0^\beta A_0(\tau,\vec{x})\,d\tau\right)$$

order parameter for confinement - deconfinement transition

- *center transformation*:
non-periodic gauge transformation, periodic in the adjoint reps

$$g(\tau+\beta,\boldsymbol{x}) = zg(\tau,\boldsymbol{x}),\ z\in\mathcal{Z}$$

- Polyakov loop transforms: $P(\vec{x}) \to zP(\vec{x}) \to$ order parameter
- $\mathcal{Z}$ realized in the confined phase, $\langle P\rangle = 0$
- broken in the deconfined phase, P near a center element
- deconfinement phase transition $=$ $\mathcal{Z}$-breaking transition

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- free energy of static $q\bar{q}$ pair

$$\langle P(\boldsymbol{x})\bar{P}(\boldsymbol{y})\rangle \propto e^{-\beta F(\boldsymbol{x}-\boldsymbol{y})} \stackrel{|\boldsymbol{x}-\boldsymbol{y}|\to\infty}{\longrightarrow} \langle P\rangle\langle \bar{P}\rangle$$

- pure gauge theory: string $\to$ linear rising potential
- with fundamental matter: string between static sources breaks
- dynamical $q\bar{q}$ screen individual static charges
- $\langle P\rangle \neq 0 \Longrightarrow P$ no (strict) order parameter

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- centre needed for confinement scenarios
- dual condensates: decisive role of center for confinement/CSB

- gauge group without center $\rightarrow$
- P not order parameter $\rightarrow$ string breaking $\rightarrow$ screening
- is there confinement? if yes: what is confinement mechanism?

Centers $\mathcal{Z}$ and generators $\mu^\vee_{(z)}$ of the centers: $z = \exp(2\pi i \mu^\vee_{(z)})$

group	A_r	B_r	C_r	$D_r,\ r$ even	$D_r,\ r$ odd
$\mathcal{Z}$	$\mathbb{Z}_{r+1}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_4$
$\mu^\vee_{(z)}$	$\mu^\vee_{(1)}$	$\mu^\vee_{(1)}$	$\mu^\vee_{(r)}$	$\mu^\vee_{(1)}, \mu^\vee_{(r)}$	$\mu^\vee_{(r)}$

E_6	E_7	E_8	F_4	G_2
$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$
$\mu^\vee_{(1)}$	$\mu^\vee_{(7)}$			

Deconfinement in G_2 Gauge Theory: Holland, Minkowski, Pepe, Wiese

- smallest simply connected gauge group with trivial center
- rank = 2, dimension = 14, subgroup of $SO(7)$

$$T_{abc} = T_{def}\, g_{da}\, g_{eb}\, g_{fc}.$$

T total antisymmetric

$$T_{127} = T_{154} = T_{163} = T_{235} = T_{264} = T_{374} = T_{576} = 1$$

- fundamental representations $\{7\}$, $\{14\}$ (= adjoint)
- evidence for first-order deconfinement PT at T_c (Bern group)
- chiral restoration at same T_c (Graz group)
- $SU(3)$ subgroup of G_2, QCD with dynamical scalar quarks

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Some (useful) facts

$$G_2/SU(3) \sim SO(7)/SO(6) \sim S^6$$

useful decomposition

$$\mathcal{U} = \mathcal{S} \cdot \mathcal{V} \quad \text{with} \quad \mathcal{V} \in SU(3) \quad \text{and} \quad \mathcal{S} \in G_2/SU(3)$$

fundamental weights $\mu_{(1)}$, $\mu_{(2)}$, highest weight $\mu = p\mu_{(1)} + q\mu_{(2)}$

$$\begin{aligned} \dim_{q,p} &= \frac{1}{120}(1+p)(1+q)(2+p+q)(3+p+2q) \\ &\quad \cdot (4+p+3q)(5+2p+3q) \end{aligned}$$

Casimir: $\mathcal{C}_{p,q} = 2p^2 + 6q^2 + 6pq + 10p + 18q$

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rep. $\mathcal{R}$	$[1,0]$	$[0,1]$	$[2,0]$	$[1,1]$	$[3,0]$	$[0,2]$	$[4,0]$	$[2,1]$
dim. $d_{\mathcal{R}}$	7	14	27	64	77	$77'$	182	189
Cas. $\mathcal{C}_{\mathcal{R}}$	12	24	28	42	48	60	72	64
rel. $\mathcal{C}'_{\mathcal{R}}$	1	2	$7/3$	3.5	4	5	6	$16/3$

- lowest representations:

V	$[1,0]$	$[0,1]$	$[2,0]$	$[1,1]$	$[0,2]$	$[3,0]$	$[4,0]$	$[2,1]$	$[0,3]$
dim	7	14	27	64	77	$77'$	182	189	273
C_2	12	24	28	42	60	48	72	64	108

G_2-representations

- algebra of confinement and string breaking

$$
\begin{aligned}
\{7\}\otimes\{7\} &= \{1\}\oplus\{7\}\dots \quad &\text{mesons}\\
\{7\}\otimes\{7\}\otimes\{7\} &= \{1\}\oplus 4\cdot\{7\}+\dots \quad &\text{baryons}\\
\{14\}\otimes\{14\} &= \{1\}\oplus\{14\}\dots &\\
\{14\}\otimes\{14\}\otimes\{14\} &= \{1\}\oplus\{7\}\dots \quad &\text{string breaking}\\
\{7\}\otimes\{14\}\otimes\{14\}\otimes\{14\} &= \{1\}\oplus\dots \quad &\text{string breaking}
\end{aligned}
$$

character of any irreducible representation: polynomial of χ_7 and χ_{14}

$$
\begin{aligned}
\chi_7\cdot\chi_7 &= \chi_1+\chi_7+\chi_{14}+\chi_{27}\\
\chi_7\cdot\chi_{14} &= \chi_7+\chi_{27}+\chi_{64}
\end{aligned}
$$

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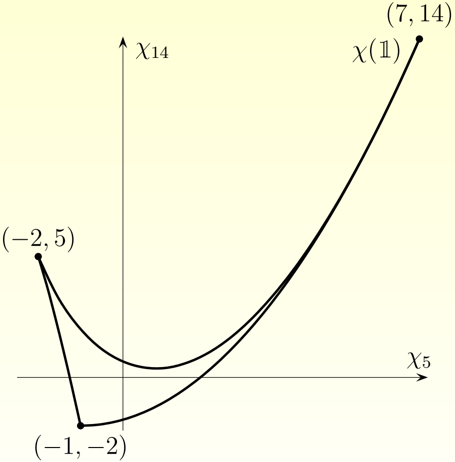
domain of characters χ_7 and χ_{14}



local hybrid Monte-Carlo for G_2

- geometric Lagrangian: $\mathcal{U}_{x,\mu}(t) \in G_2$:

$$L = \frac{1}{2}\mathrm{tr}\sum_{x,\mu}\left(i\dot{\mathcal{U}}_{x,\mu}\mathcal{U}_{x,\mu}^{-1}\right)^2 - S_{\mathrm{YM}}[\mathcal{U}]$$

- conjugated momentum in Lie algebra:

$$\mathfrak{P}_{x,\mu} = i\frac{\partial L}{\partial(\dot{\mathcal{U}}_{x,\mu}\mathcal{U}_{x,\mu}^{-1})} = i\mathcal{U}\frac{\partial L}{\partial\dot{\mathcal{U}}_{x,\mu}} = -i\dot{\mathcal{U}}_{x,\mu}\mathcal{U}_{x,\mu}^{-1}$$

- Hamiltonian

$$H = \frac{1}{2}\mathrm{tr}\,\mathfrak{P}_{x,\mu}^2 + \frac{\beta}{2N_c}\mathrm{tr}\sum_{x,\mu\nu}\left(2N_c - \mathcal{U}_{x,\mu\nu} - \mathcal{U}_{x,\mu\nu}^{\dagger}\right)$$

HMC equations

- variation $\Rightarrow$ staple $R_{x,\mu}$:

$$\begin{aligned}\delta H &= \operatorname{tr}\sum_{x,\mu}\left(\mathfrak{P}_{x,\mu}\delta\mathfrak{P}_{x,\mu}\right)\\ &- \frac{\beta}{2N_c}\operatorname{tr}\sum_{x,\mu}\left(\delta\mathcal{U}_{x,\mu}\mathcal{U}_{x,\mu}^{-1}\right)\left(\mathcal{U}_{x,\mu}R_{x,\mu}-R_{x,\mu}^{-1}\mathcal{U}_{x,\mu}^{-1}\right)\end{aligned}$$

- equation of motion

$$\dot{\mathfrak{P}}_{x,\mu}=\frac{i\beta}{2N_c}\left(\mathcal{U}_{x,\mu}R_{x,\mu}-R_{x,\mu}^{\dagger}U_{x,\mu}^{\dagger}\right)-G_{x,\mu}\equiv F_{x,\mu}-G_{x,\mu}$$

- final equation of motion:

$$\dot{\mathcal{U}}_{x,\mu}=i\,\mathfrak{P}_{x,\mu}\mathcal{U}_{x,\mu}\quad\text{and}\quad\dot{\mathfrak{P}}_{x,\mu}=\sum_a\operatorname{tr}(F_{x,\mu}T_a)T_a$$

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Implementing HMC

- SU(3) subgroup of G_2 : $\mathcal{U} = \mathcal{S} \cdot \mathcal{V}$ with $\mathcal{V} \in \mathsf{SU}(3)$

$$\mathcal{U} = \mathrm{e}^{\delta\tau \boldsymbol{u}} = \mathrm{e}^{\delta\tau \boldsymbol{s}} \cdot \mathrm{e}^{\delta\tau \boldsymbol{v}} = \mathcal{S} \cdot \mathcal{V}$$

- $[\boldsymbol{v}, \boldsymbol{v}'] = \boldsymbol{v}'', \;\; [\boldsymbol{v}, \boldsymbol{s}] = \boldsymbol{s}', \;\; [\boldsymbol{s}, \boldsymbol{s}'] = \boldsymbol{u} + \boldsymbol{s}''$
- $\boldsymbol{s} \to \mathcal{S}$ and $\boldsymbol{v} \to \mathcal{V}$ simple to calculate
- depending on order of symplectic integrator:
  keep corresponding order in $\delta\tau$ in

$$\delta\tau \boldsymbol{u} = \delta\tau(\boldsymbol{s} + \boldsymbol{v}) + \tfrac{1}{2}\delta\tau^2[\boldsymbol{s}, \boldsymbol{v}] + \dots$$

- use $\boldsymbol{s}$ and $\boldsymbol{v}$ in calculations: this is time reversible

static potential

- string tension σ for areas $LT = 0$ up to $LT > 100$
- W covers range of 40 orders of magnitude
- brute force approach by increasing statistics fails miserably
- Lüscher and Weisz method reduces absolute error exponentially
- split lattice in time slices, calculate mean values with fixed boundary conditions on each slice
- full result: integral over boundary conditions
- iteration $\rightarrow$ *multilevel algorithm*
- here: lattice also split in the spatial direction of Wilson line

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Casimir scaling $\qquad \dfrac{\sigma_{\mathcal{R}}}{c_{\mathcal{R}}} = \dfrac{\sigma_{\mathcal{R}'}}{c_{\mathcal{R}'}}$

rectangular Wilson loops in $\mathcal{R}$ for large T

$$\langle W_{\mathcal{R}}(R,T)\rangle = \exp\big(\kappa_{\mathcal{R}}(R) - V_{\mathcal{R}}(R)T\big), \quad V_{\mathcal{R}}(R) = \gamma_{\mathcal{R}} - \frac{\alpha_{\mathcal{R}}}{R} + \sigma_{\mathcal{R}} R.$$

static potential $\qquad V_{\mathcal{R}}(R) = \dfrac{1}{\tau}\ln\dfrac{\langle W_{\mathcal{R}}(R,T)\rangle}{\langle W_{\mathcal{R}}(R,T+\tau)\rangle}.$

string tension $\sigma_{\mathcal{R}}$ from the Creutz ratio

$$\begin{aligned}\sigma_{\mathcal{R}}(R+\rho/2) &= \frac{\alpha_{\mathcal{R}}}{R(R+\rho)} + \sigma_{\mathcal{R}} \\ &= -\frac{1}{\tau\rho}\ln\frac{\langle W_{\mathcal{R}}(R+\rho,T+\tau)\rangle\,\langle W_{\mathcal{R}}(R,T)\rangle}{\langle W_{\mathcal{R}}(R+\rho,T)\rangle\,\langle W_{\mathcal{R}}(R,T+\tau)\rangle}.\end{aligned}$$

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Scaled potential

$\mathcal{R}$	$[1,0]$	$[0,1]$	$[2,0]$	$[1,1]$	$[3,0]$	$[0,2]$	$[4,0]$	$[2,1]$
$\alpha_{\mathcal{R}}/\alpha_{[1,0]}$	1	1.97	2.31	3.46	3.94	5.03	5.64	5.23
$\gamma_{\mathcal{R}}/\gamma_{[1,0]}$	1	2.04	2.25	3.38	3.80	5.07	5.21	5.07
$\sigma_{\mathcal{R}}/\sigma_{[1,0]}$	1	2.00	2.37	3.58	4.12	5.00	6.54	5.50

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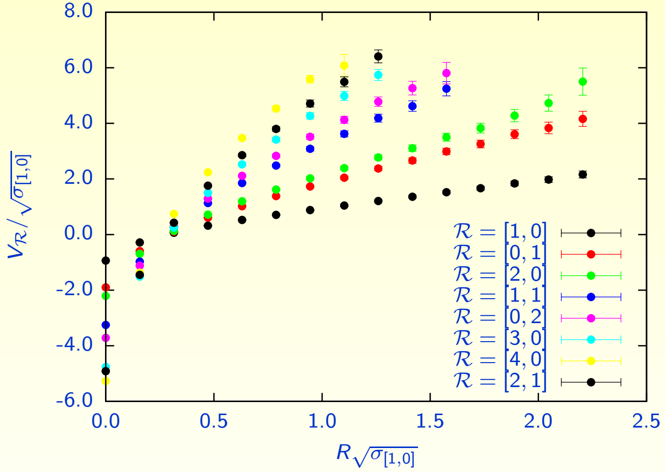

Figure: Potential with $\beta = 40$ unscaled on a 28^3 lattice

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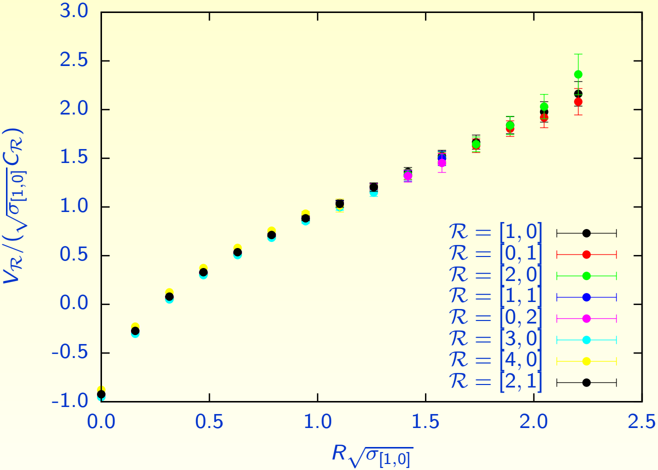

Figure: Potential with $\beta = 40$ scaled on a 28^3 lattice

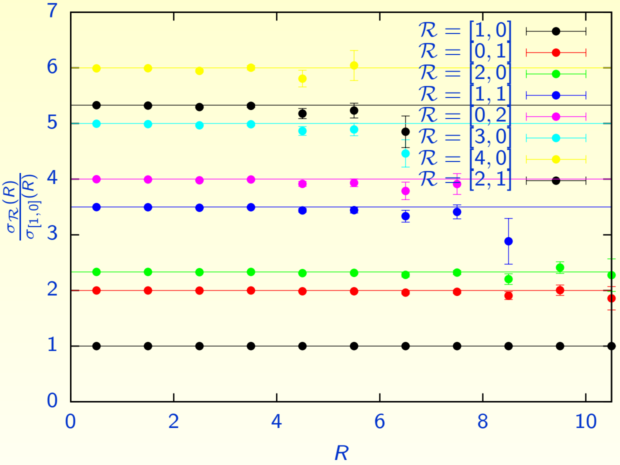

Figure: String Tension with $\beta = 40$ scaled on a 28^3 lattice

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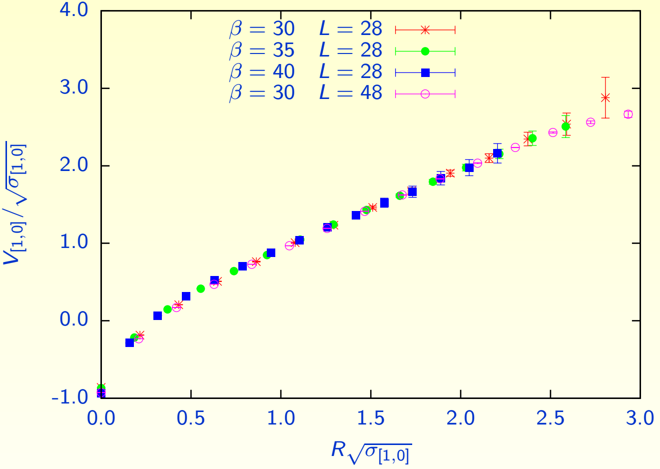

Figure: continuum scaling of the fundamental potential

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string breaking

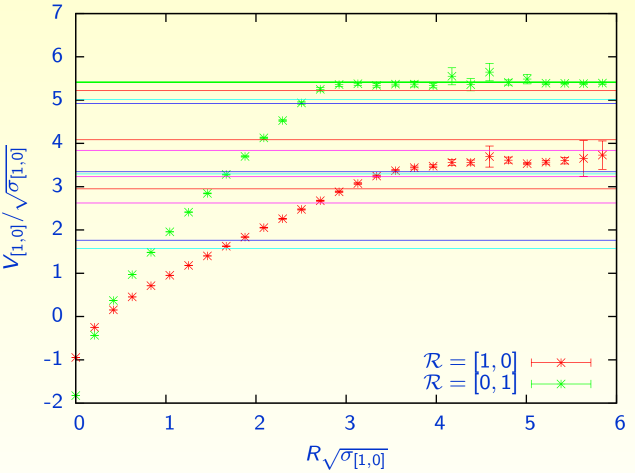

Figure: Potential (lattice 48^3, $\beta = 30$) and glue-lump mass

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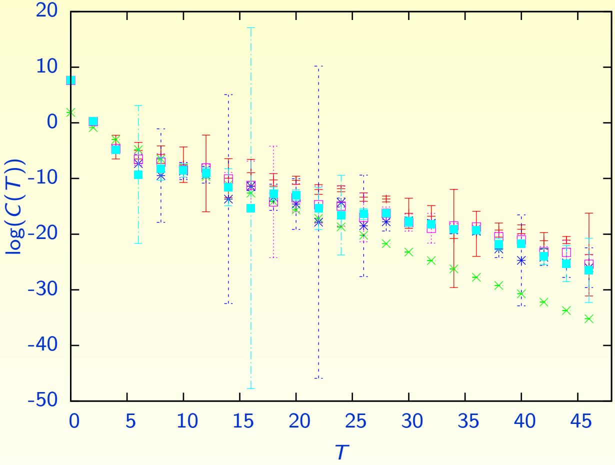

Figure: Glue-lump correlator (lattice 48^3, $\beta = 30$)

Adding a Higgs-Field

- G_2 gauge-Higgs model

$$\mathcal{L}_{\rm GH}[A,\varphi] = \mathcal{L}_{\rm YM}[A] + \tfrac{1}{2} D_\mu\varphi D_\mu\varphi + V(\varphi)$$

- $\varphi = (\varphi_1, \dots, \varphi)^T$ in $\{7\}$

$$V(\varphi) = \lambda(\varphi^2 - v^2)^2$$

- Higgs-mechanism for $v = \langle\varphi\rangle \neq 0$: $G_2 \longrightarrow SU(3)$
- $\{14\} \longrightarrow \{8\} \oplus \{3\} \oplus \{\bar{3}\}$
  $\{8\}$: SU(3) gluons
  $\{3\} + \{\bar{3}\}$: massive, play similar role as SU(3) quarks
- $\{7\} \longrightarrow \{1\} \oplus \{3\} \oplus \{\bar{3}\}$

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	$SO(7)$ unbroken	$SO(7)$ broken to $SO(6)$
massive scalars	7	1
massless vector bosons	14	8
massive vector bosons	0	6
massless gauge sector	G_2	$SU(3)$

Phases

- lattice action

$$S = \beta \sum_{\Box} \left(1 - \frac{1}{7} \mathrm{tr}\, \Re U_{\Box}\right) - \kappa \sum_{x,\mu} \Phi_{x+\hat{\mu}} U_{x,\mu} \Phi_x$$

- Φ_x as $\{7\}$, normalised $\Phi \cdot \Phi = 1$
- gauge coupling β, hopping parameter κ
- $\kappa = 0$: G_2 Yang-Mills theory
- $\kappa = \infty$: SU(3) gauge theory
- $\beta \to \infty$: SO(7) or SO(6) spin model

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- $\kappa = 0$: pure G_2 gauge theory:
  first order deconfinement transition
- $\kappa = \infty$: 6 gluons decouple, pure $SU(3)$
  first order deconfinement transition
- $\beta_c(G_2) \geq 7/6 \cdot \beta_c(SU_3)$
- expect first order transition line connecting two theories
- exploration on small lattice
  calculate $\langle P \rangle$ in large region ($\beta = 5 \dots 10$, $\kappa = 0 \dots 10^4$)
- jumps $\rightarrow$ deconfinement transition line from $\kappa = 0$ to $\kappa = 10^4 \sim \infty$

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Partial Phase diagram of G_2-YMH model

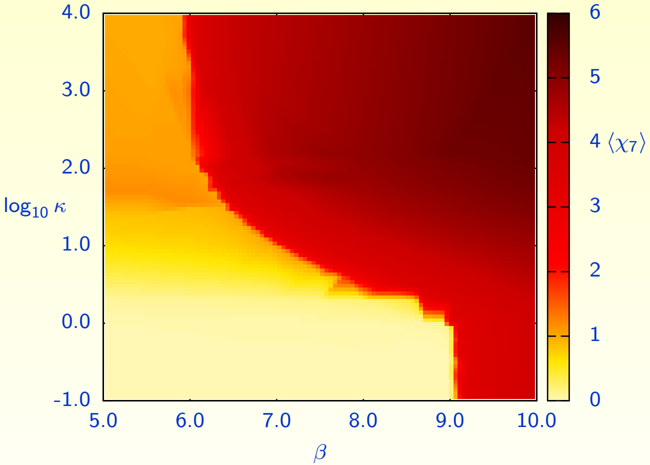

Figure: phase diagram on a $12^3 \times 2$ lattice

- check order on larger lattices.
- $\beta \to \infty$: $U_{x,\mu} = \mathbb{1}$ and $O(7)$ sigma model

$$S = -\kappa \sum_{x,\mu} \Phi_{x+\hat{\mu}} \Phi_x$$

- large κ: second order SB-transition $O(7) \to O(6)$
- triple point at

$$\beta_{\text{crit}} = 9.55(5) \quad , \quad \kappa_{\text{crit}} = 1.50(4)$$

- at single point confining phase meets two deconfining phases
- first order line meets second order line

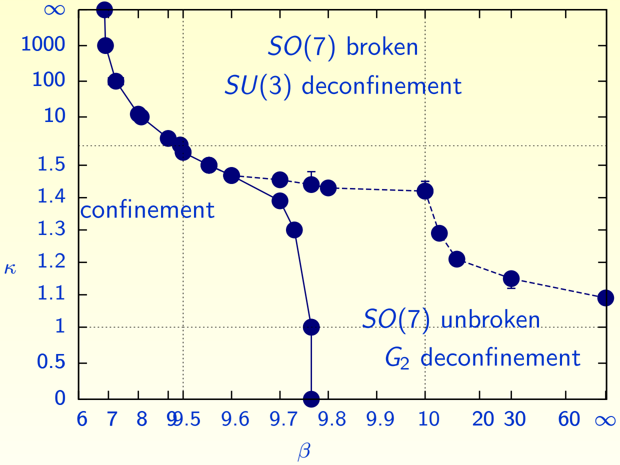

Figure: phase lines on a $16^3 \times 6$ lattice

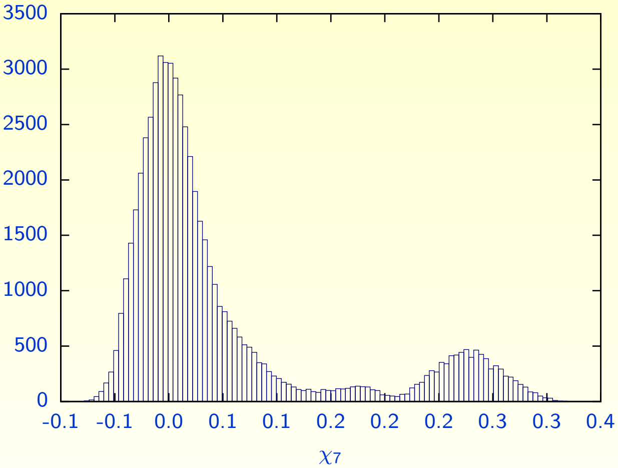

Figure: distribution of the polyakov loop at $(\beta = 9.76, \kappa = 1)$

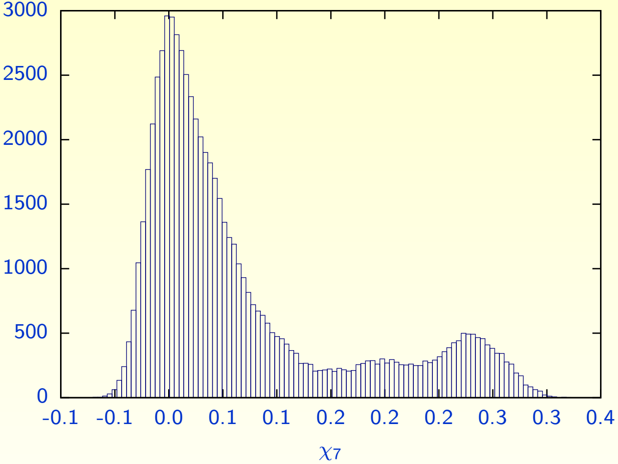

Figure: distribution of the polyakov loop at $(\beta = 9.725, \kappa = 1.3,)$

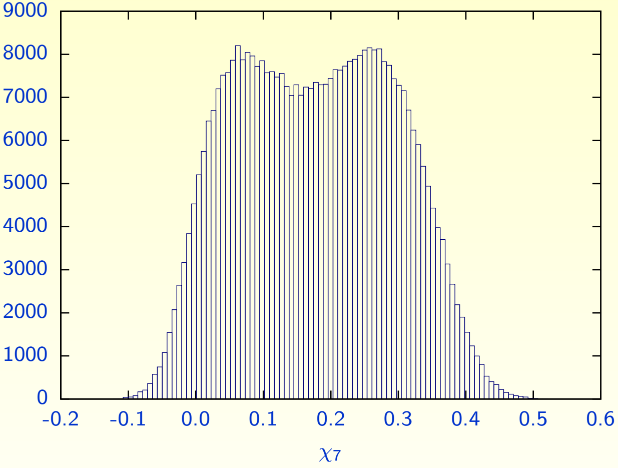

Figure: distribution of the polyakov loop at ($\beta = 9.5535, \kappa = 1.5$, $12^3 \times 6$)

- phase transitions by suszeptibility peaks of P and $S_{\rm Higgs}$
- histogram method for Polyakov loop
- order of the Higgs transition line: finite size scaling of

$$\partial_\kappa^n \langle V^{-1} \sum_{x,\mu} \Phi_{x+\hat{\mu}} U_{x,\mu} \Phi_x \rangle, \quad n = 1, 2$$

- lattices up to $20^3 \times 6$: point where second order $SO(7) \to SO(6)$ transition may turn into a crossover cannot be determined reliably.

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Conclusions

- Casimir scaling with Lüscher-Weiss (lowest reps)
- string breaking for static 7 and static 14 charges
- phase diagram of G_2 Yang-Mills-Higgs known

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