The functional renormalisation group

and

applications to the phase structure of QCD

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Universität Heidelberg & ExtreMe Matter Institute

Trento, June 11th - 15th 2018

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

ISOQUANT SFB1225

EMMI

Material

Lecture notes

Non-perturbative methods in gauge theories
hand-written

Critical phenomena
hand-written

QCD
tex

Topical reviews

Collection of reviews & lecture notes on the FRG & DSE

Structure of the FRG: Aspects of the FRG **JMP '05, Annals Phys.322:2831-2915,2007**

talks

The FRG approach to gauge theories & applications to QCD **JMP, ERG 2012 Aussois**

Aspects of the QCD phase diagram and the EoS **B.-J. Schaefer, CompStar 2012 School Zadar**

Schladming 2011: Physics at all scales: The Renormalization Group

Schladming 2013: The phase diagram of QCD: Thermodynamics, order parameters & dynamics

JMP, Schladming 2013

Outline

- (I) Introduction to the phase diagram of QCD
- (II) Functional Renormalisation group for QCD
- (II) Phase structure of QCD & dynamics

(I) Introduction to the phase diagram of QCD

- **Heavy ion collisions**
  - Phases of a heavy ion collision
- **Phase structure of QCD**
  - Perturbative QCD & asymptotic freedom
  - chiral symmetry breaking
  - confinement

(II) Functional Renormalisation group for QCD

- **Introduction to the functional renormalisation group**
  - Derivation of the flow equation
  - Expansion schemes
  - Optimisation and error control*

- **FRG for QCD**
  - FRG for QCD and T=0 Yang-Mills theories
  - Dynamical hadronisation
  - QCD correlation functions at T=0

(III) Phase structure of QCD and dynamics

- **Yang-Mills theory at finite temperature**
  - Order parameter potential for confinement
  - Correlation functions at finite temperature
  - Polyakov loop from functional methods

- **Application to the phase structure of QCD and dynamics***
  - QCD-assisted hydrodynamics*
  - QCD-assisted transport*
  - QCD at imaginary chemical potential*

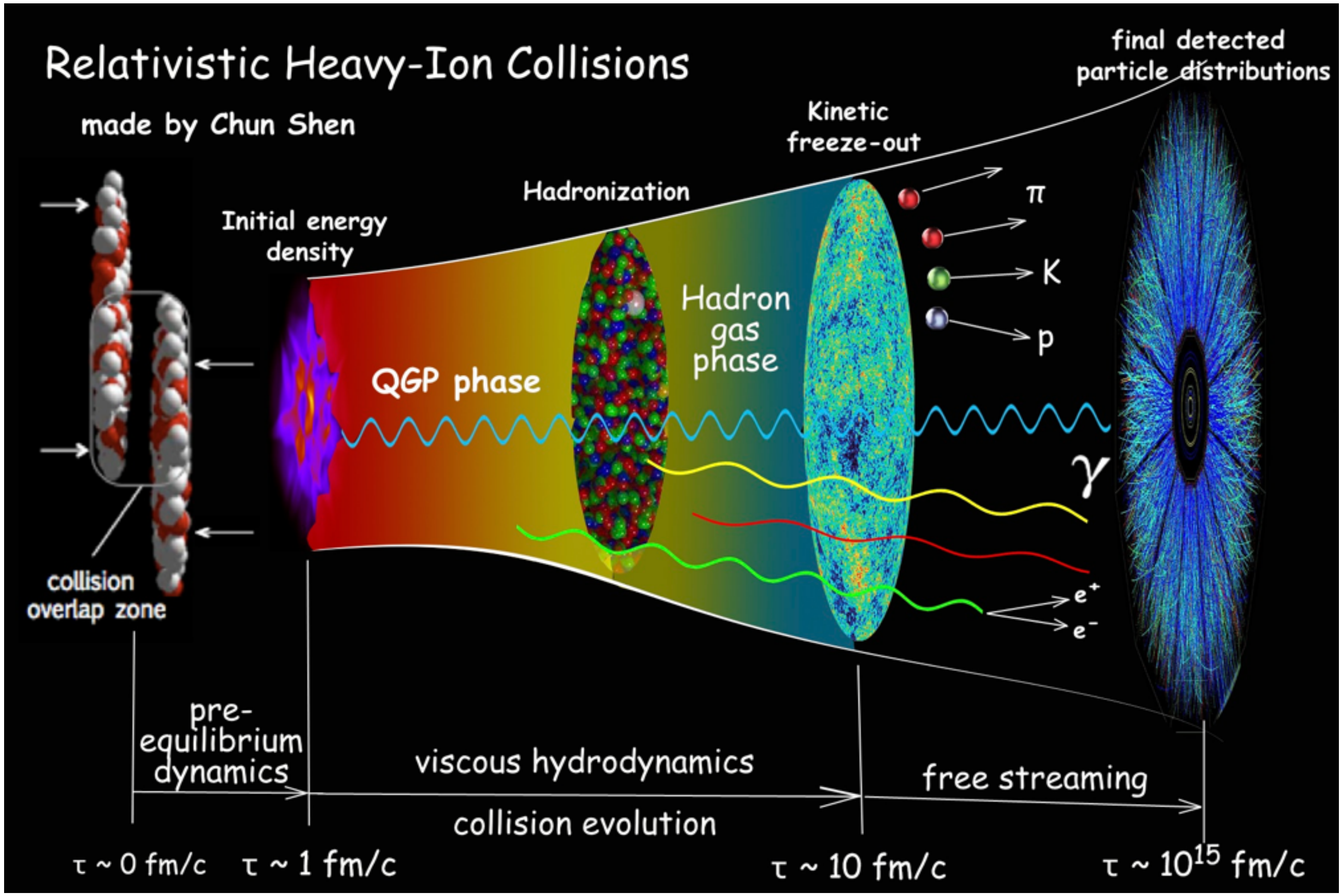
(I) Introduction to the phase diagram of QCD

- **Heavy ion collisions**
  - Phases of a heavy ion collision

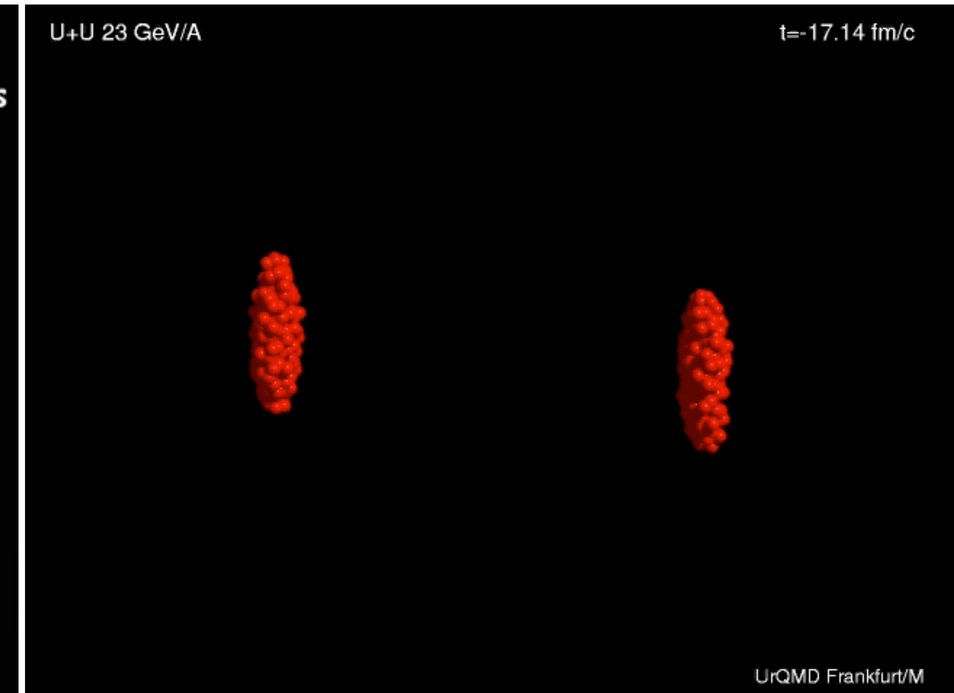
- **Phase structure of QCD**
  - Perturbative QCD & asymptotic freedom
  - chiral symmetry breaking
  - confinement

Heavy ion collisions

'Phases/Epochs' of a heavy ion collision & time scales



Simulation of a heavy ion collision



UrQMD Fr

$$1\mathrm{fm/c} \sim 3 \times 10^{-24}\mathrm{seconds}$$

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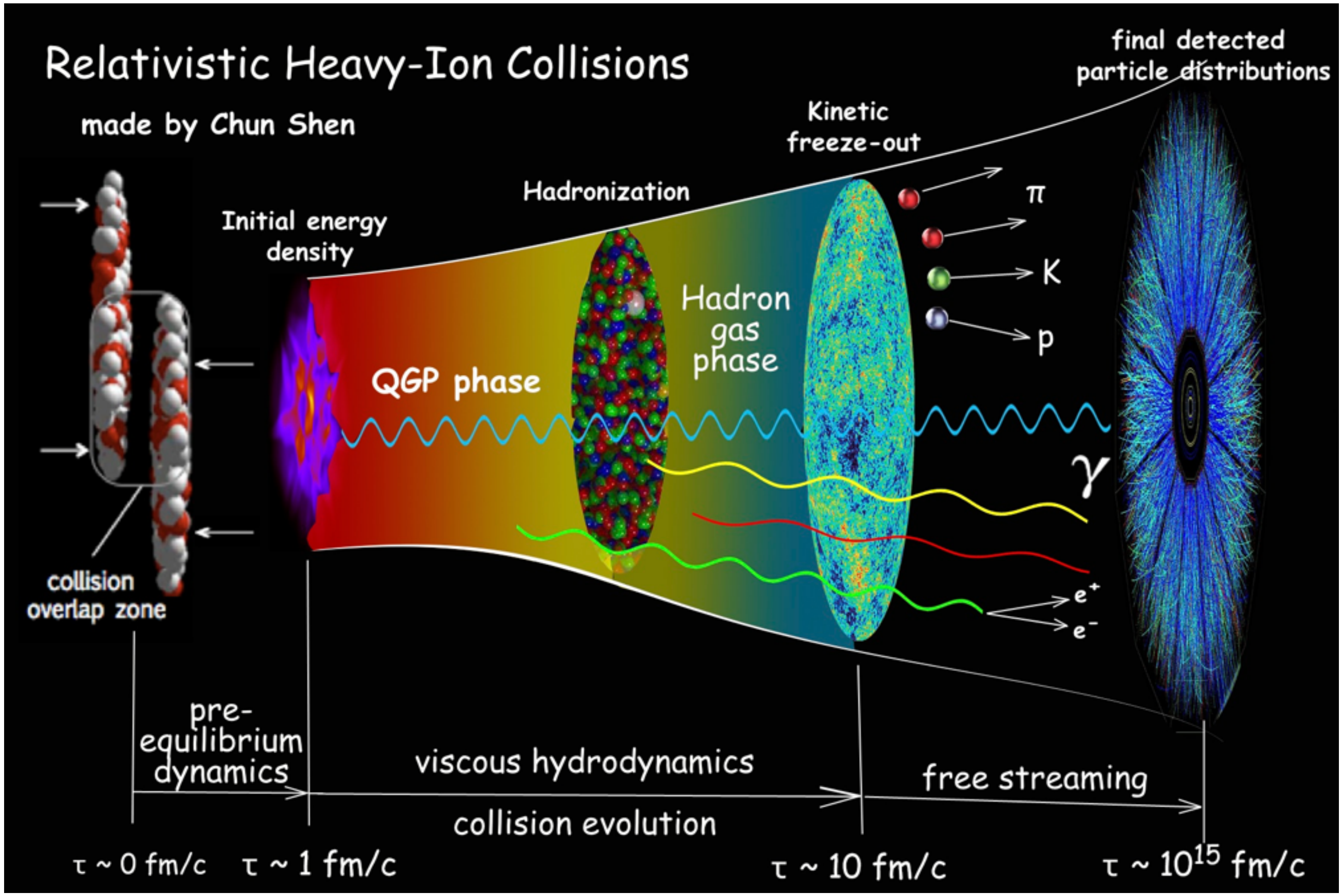
'Phases/Epochs' of a heavy ion collision & time scales

Simulation of a heavy ion collision



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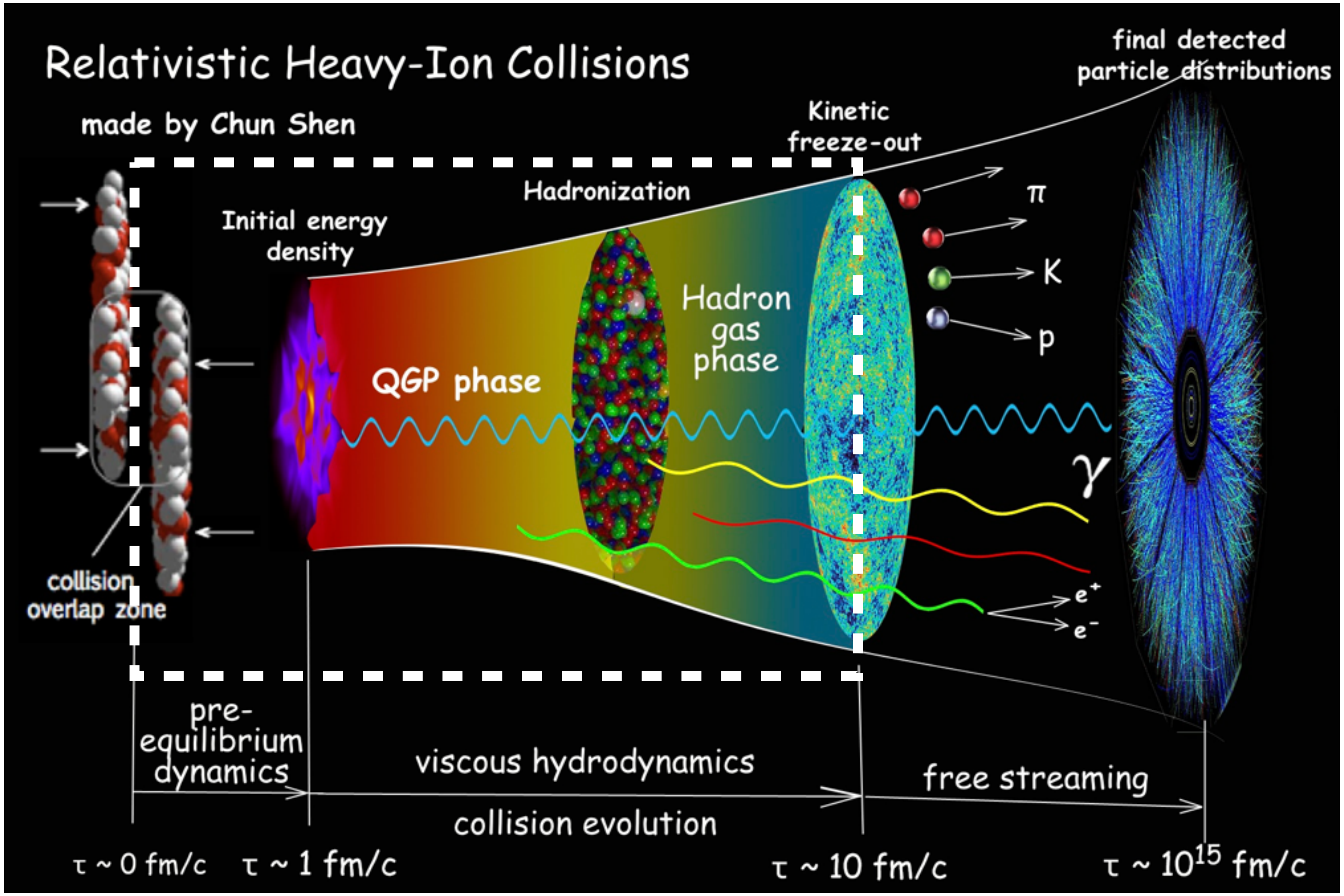

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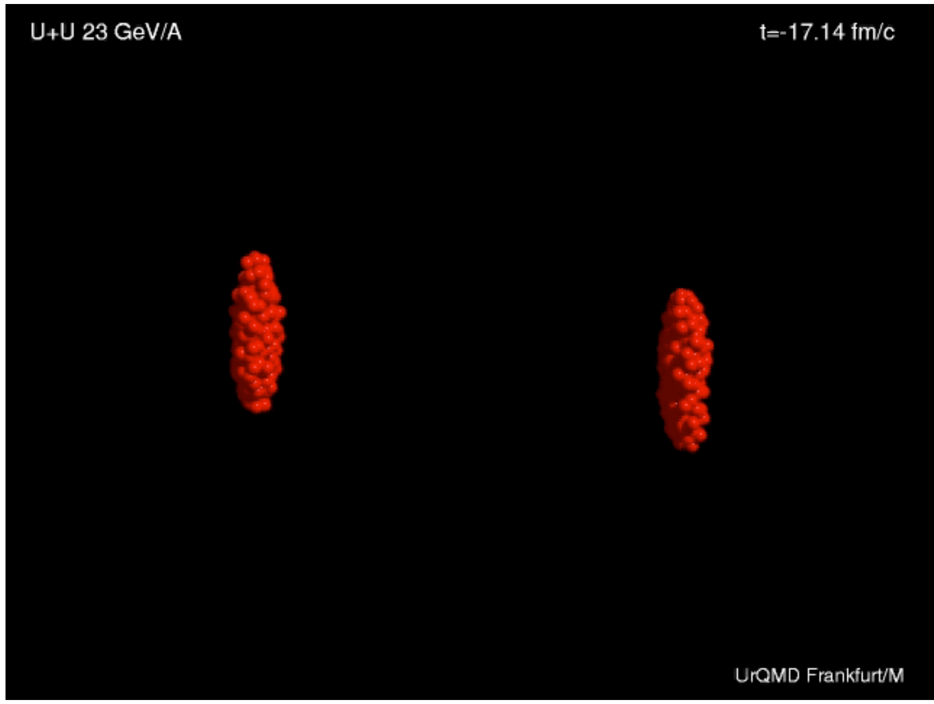
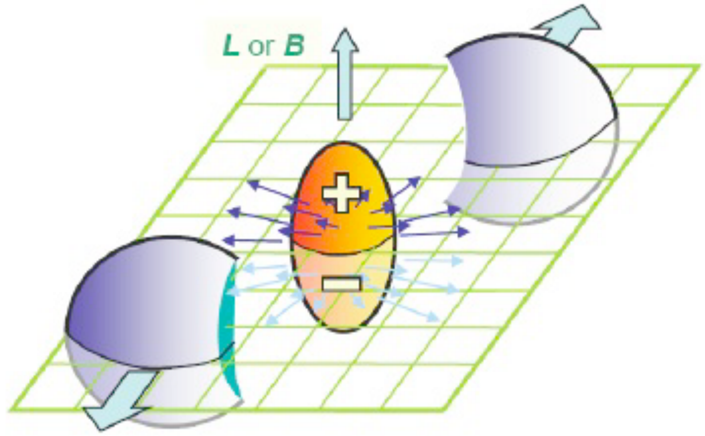


UrQMD Fr

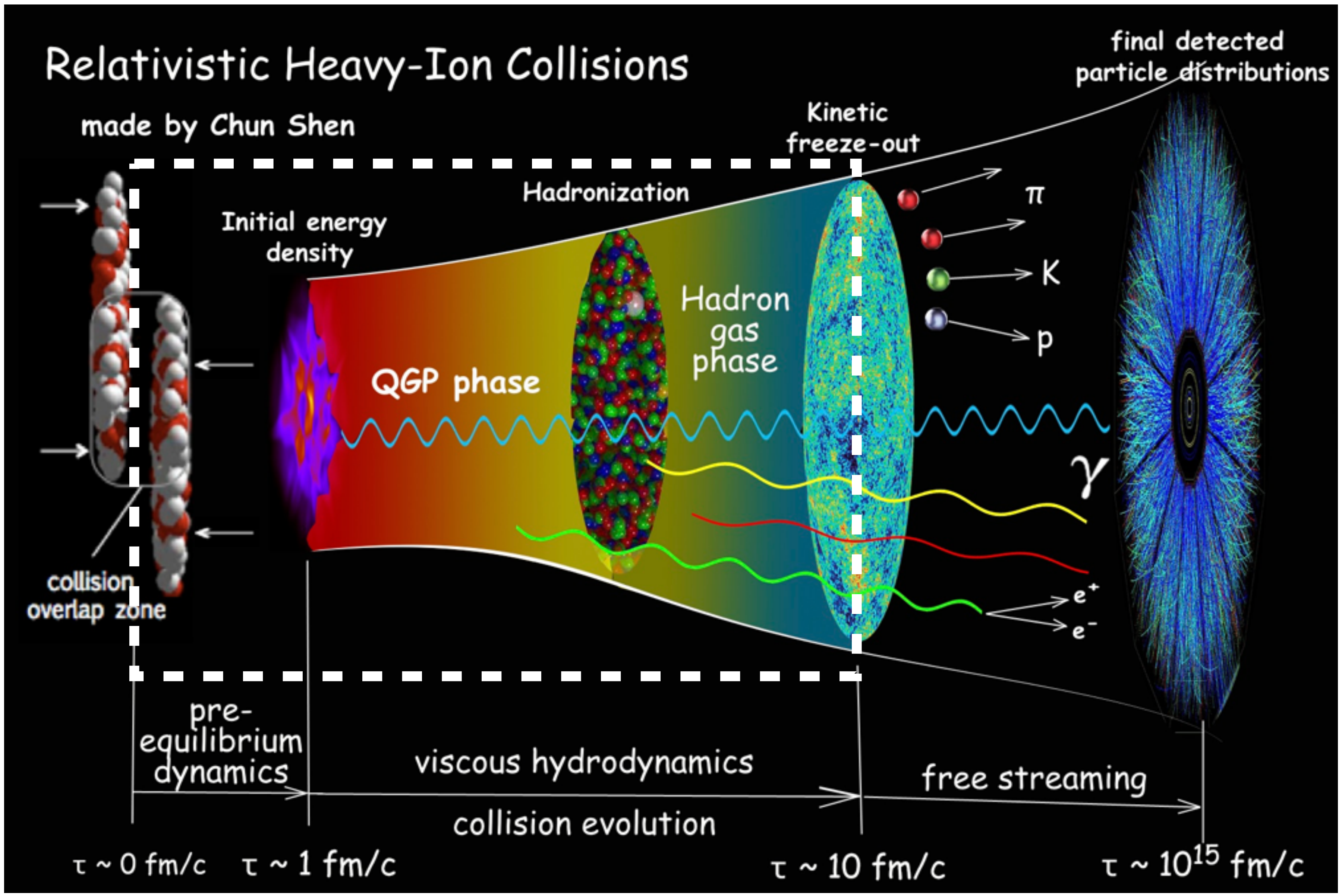
Flow



$$\frac{dN}{d(\varphi - \Psi_R)} = \frac{N_0}{2\pi}\left(1 + 2\sum_n v_n \cos[n\,(\varphi - \Psi_R)]\right)$$

Heavy ion collisions

‘Phases/Epochs’ of a heavy ion collision & time scales



Simulation of a heavy ion collision

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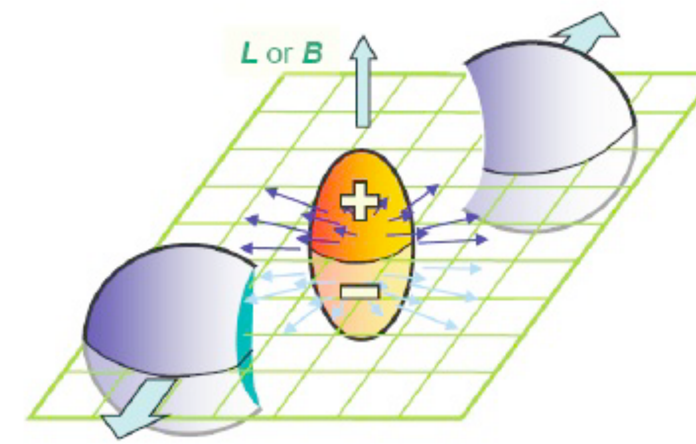
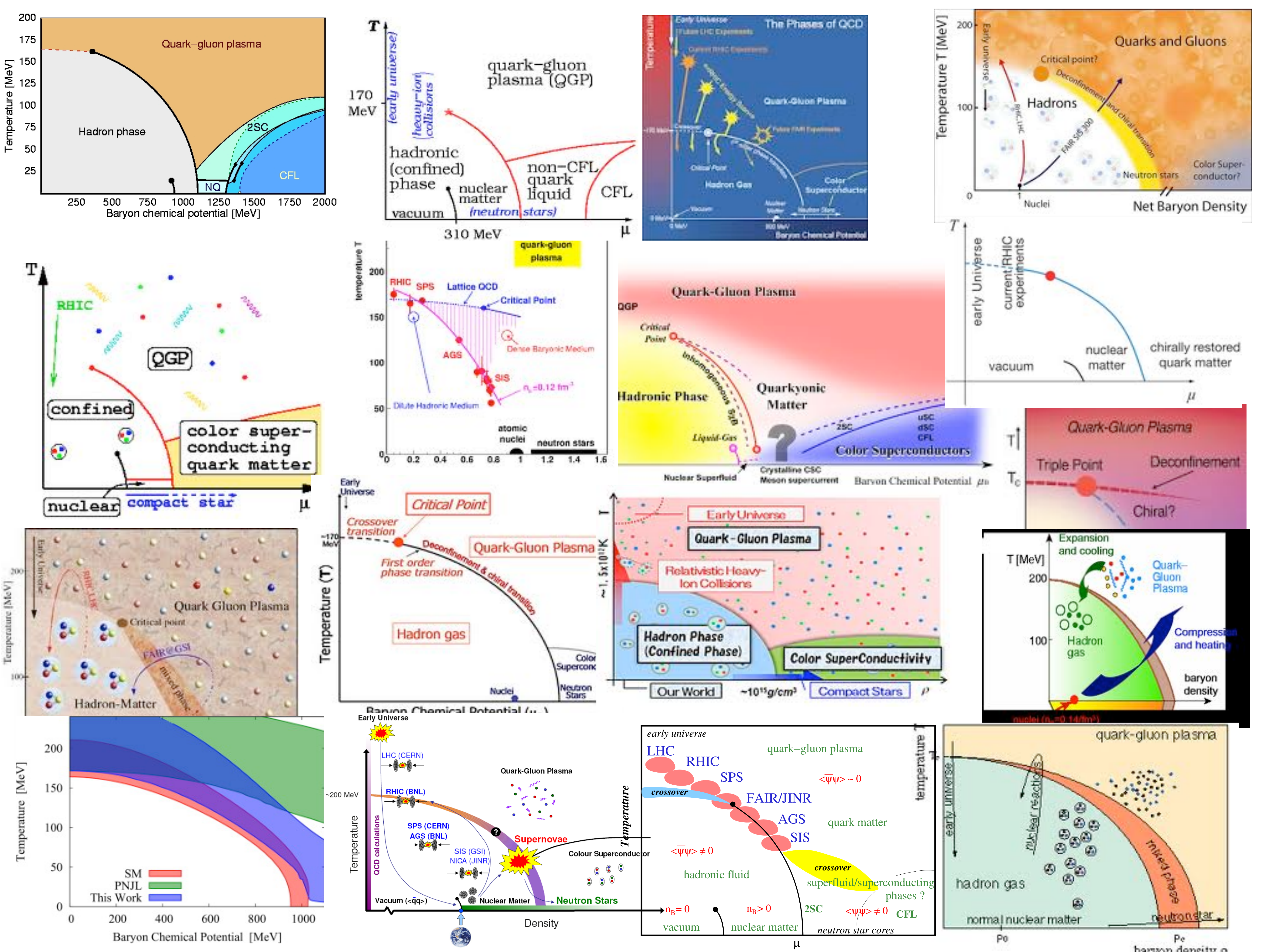
Flow



QCD: $\boxed{v_n}$

$$\frac{dN}{d(\varphi - \Psi_R)} = \frac{N_0}{2\pi}\left(1 + 2\sum_n \boxed{v_n} \cos[n\,(\varphi - \Psi_R)]\right)$$



Quark–gluon plasma
Hadron phase
2SC
CFL
NQ
Temperature [MeV]
Baryon chemical potential [MeV]
quark–gluon plasma (QGP)
hadronic (confined) phase
nuclear matter
non–CFL quark liquid
CFL
vacuum
(neutron stars)
170 MeV
310 MeV
early universe
heavy–ion collisions
The Phases of QCD
Quark-Gluon Plasma
Hadron Gas
Color Superconductor
Baryon Chemical Potential
Quarks and Gluons
Critical point?
Hadrons
Deconfinement and chiral transition
Neutron stars
Color Super-conductor?
Nuclei
Net Baryon Density
Temperature T [MeV]
RHIC
QGP
confined
color super-conducting quark matter
nuclear
compact star
quark-gluon plasma
RHIC
SPS
Lattice QCD
Critical Point
AGS
SIS
Dense Baryonic Medium
Dilute Hadronic Medium
atomic nuclei
neutron stars
temperature T
Quark-Gluon Plasma
QGP
Critical Point
Hadronic Phase
Quarkyonic Matter
Liquid-Gas
Color Superconductors
2SC
uSC
dSC
CFL
Crystalline CSC
Meson supercurrent
Nuclear Superfluid
Baryon Chemical Potential
early Universe
current RHIC experiments
vacuum
nuclear matter
chirally restored quark matter
Quark-Gluon Plasma
Triple Point
Deconfinement
Chiral?
Early Universe
Quark Gluon Plasma
Critical point
FAIR@GSI
mixed phase
Hadron-Matter
Temperature [MeV]
Early Universe
Critical Point
Crossover transition
First order phase transition
Deconfinement & chiral transition
Quark-Gluon Plasma
Hadron gas
Color Supercond
Nuclei
Neutron Stars
Temperature (T)
Baryon Chemical Potential
Early Universe
Quark-Gluon Plasma
Relativistic Heavy-Ion Collisions
Hadron Phase (Confined Phase)
Color SuperConductivity
Our World
~10^{15}g/cm^3
Compact Stars
Expansion and cooling
T [MeV]
Quark-Gluon Plasma
Hadron gas
Compression and heating
baryon density
SM
PNJL
This Work
Temperature [MeV]
Baryon Chemical Potential [MeV]
Early Universe
LHC (CERN)
RHIC (BNL)
SPS (CERN)
AGS (BNL)
SIS (GSI)
NICA (JINR)
Quark-Gluon Plasma
Supernovae
Colour Superconductor
Nuclear Matter
Neutron Stars
Vacuum
QCD calculations
~200 MeV
Temperature
Density
early universe
LHC
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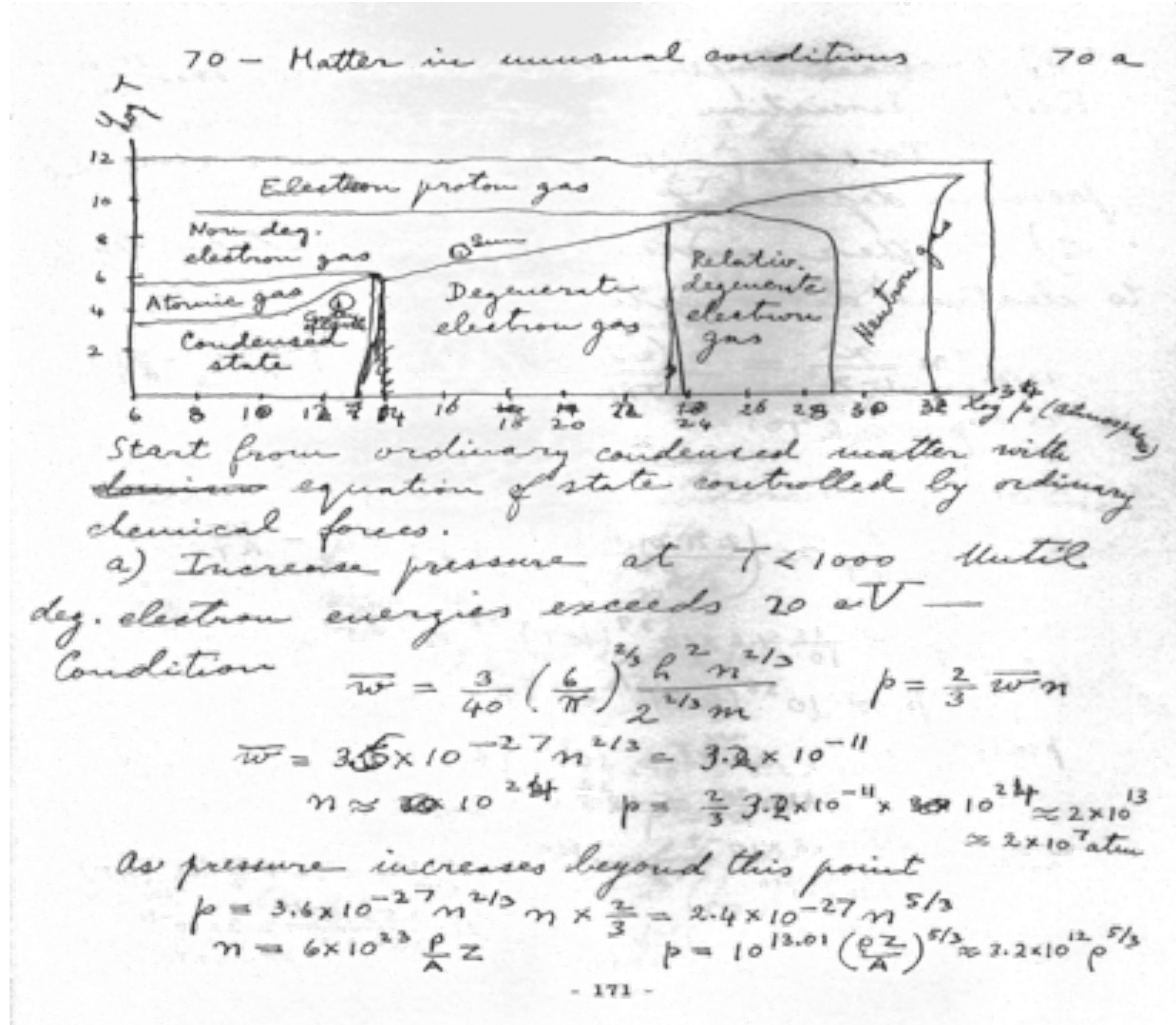
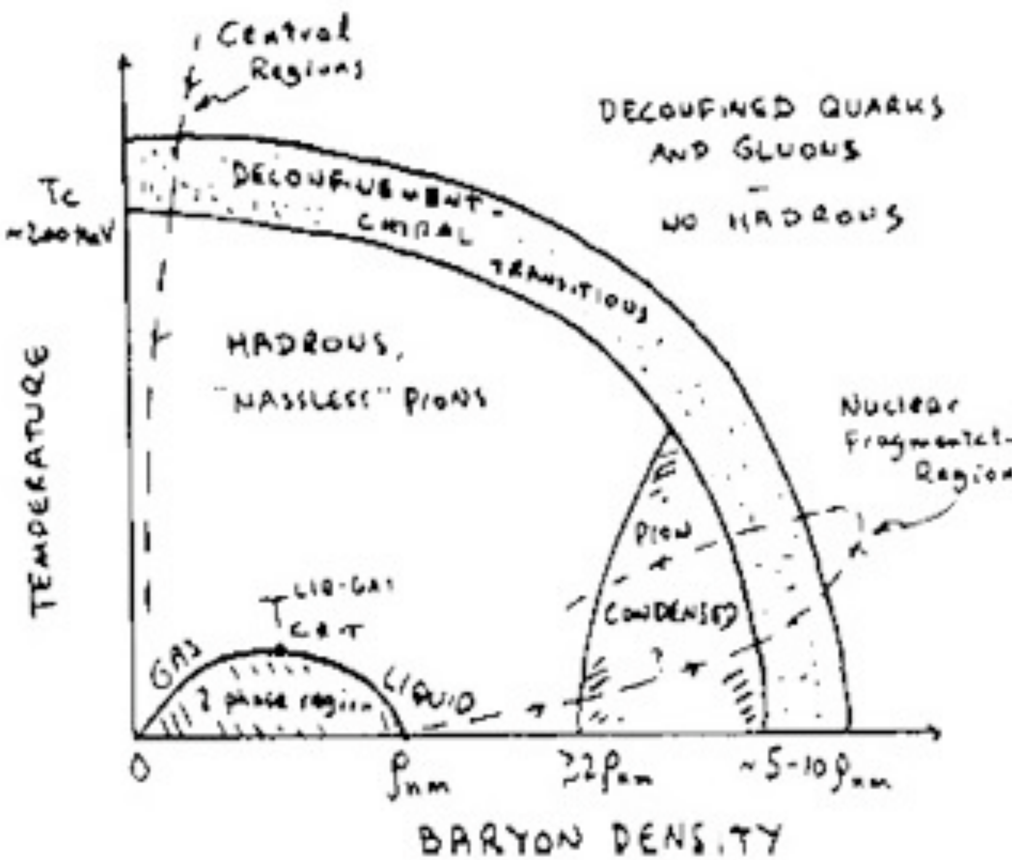
SPS
FAIR/JINR
AGS
SIS
quark–gluon plasma
crossover
quark matter
hadronic fluid
superfluid/superconducting phases ?
vacuum
nuclear matter
neutron star cores
2SC
CFL
temperature
quark-gluon plasma
early universe
nuclear reactions
hadron gas
mixed phase
normal nuclear matter
neutron star

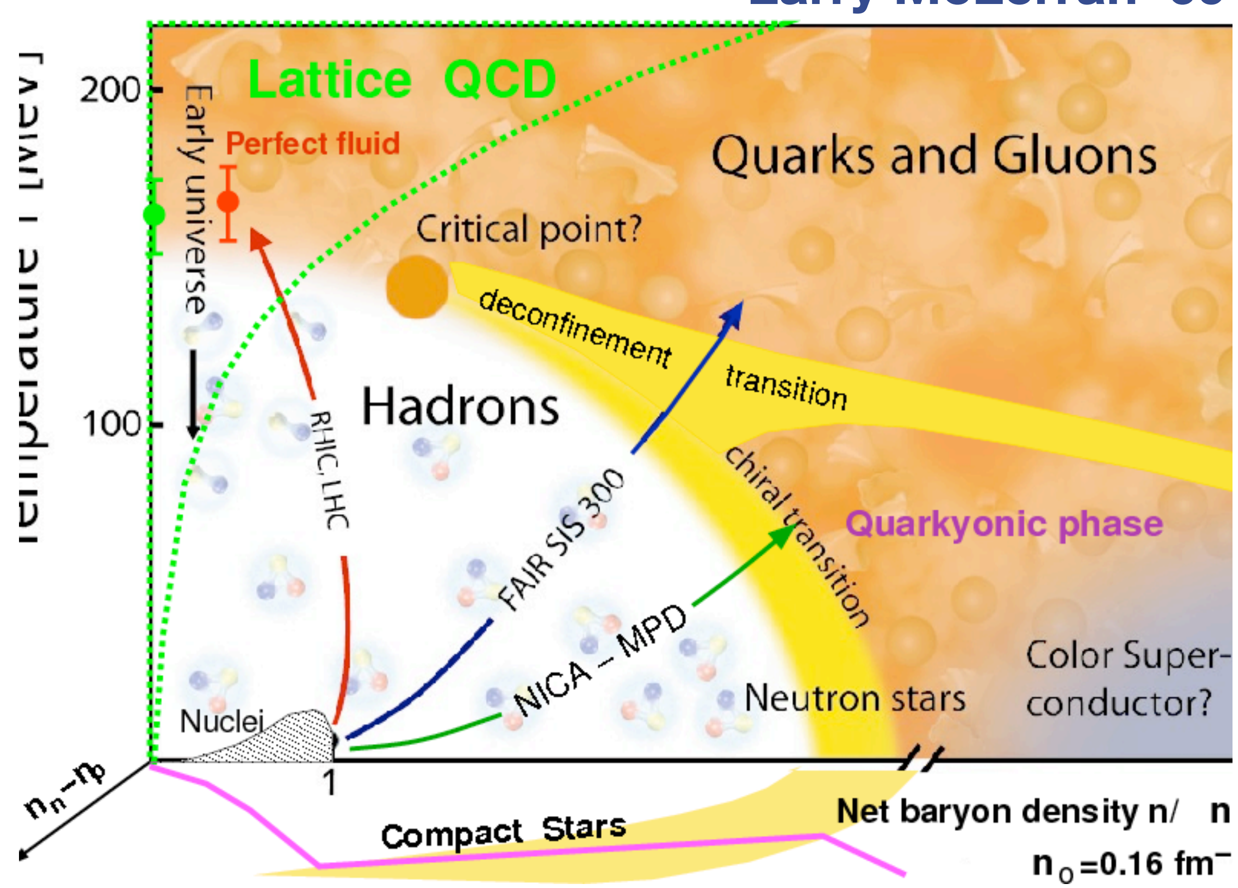


1953 Enrico Fermi

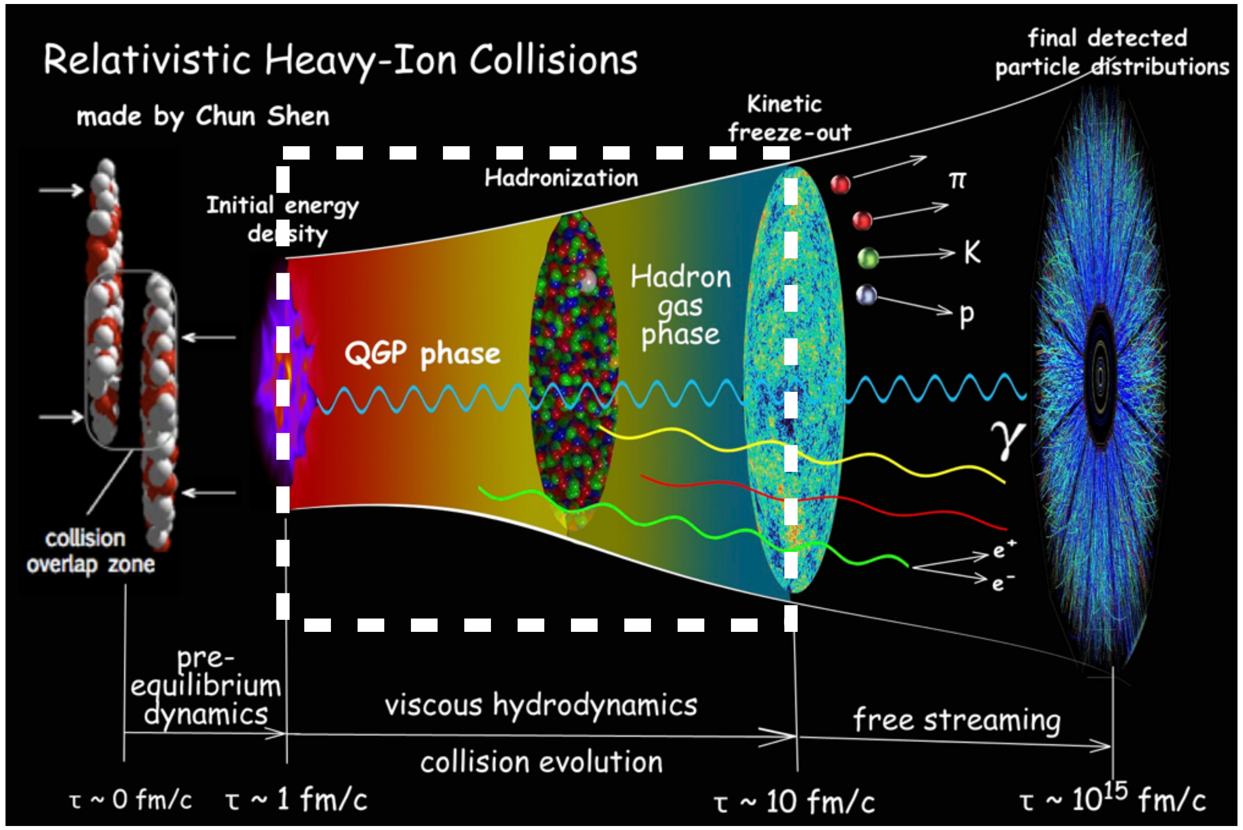
1983 US long range plan, Gordon Baym

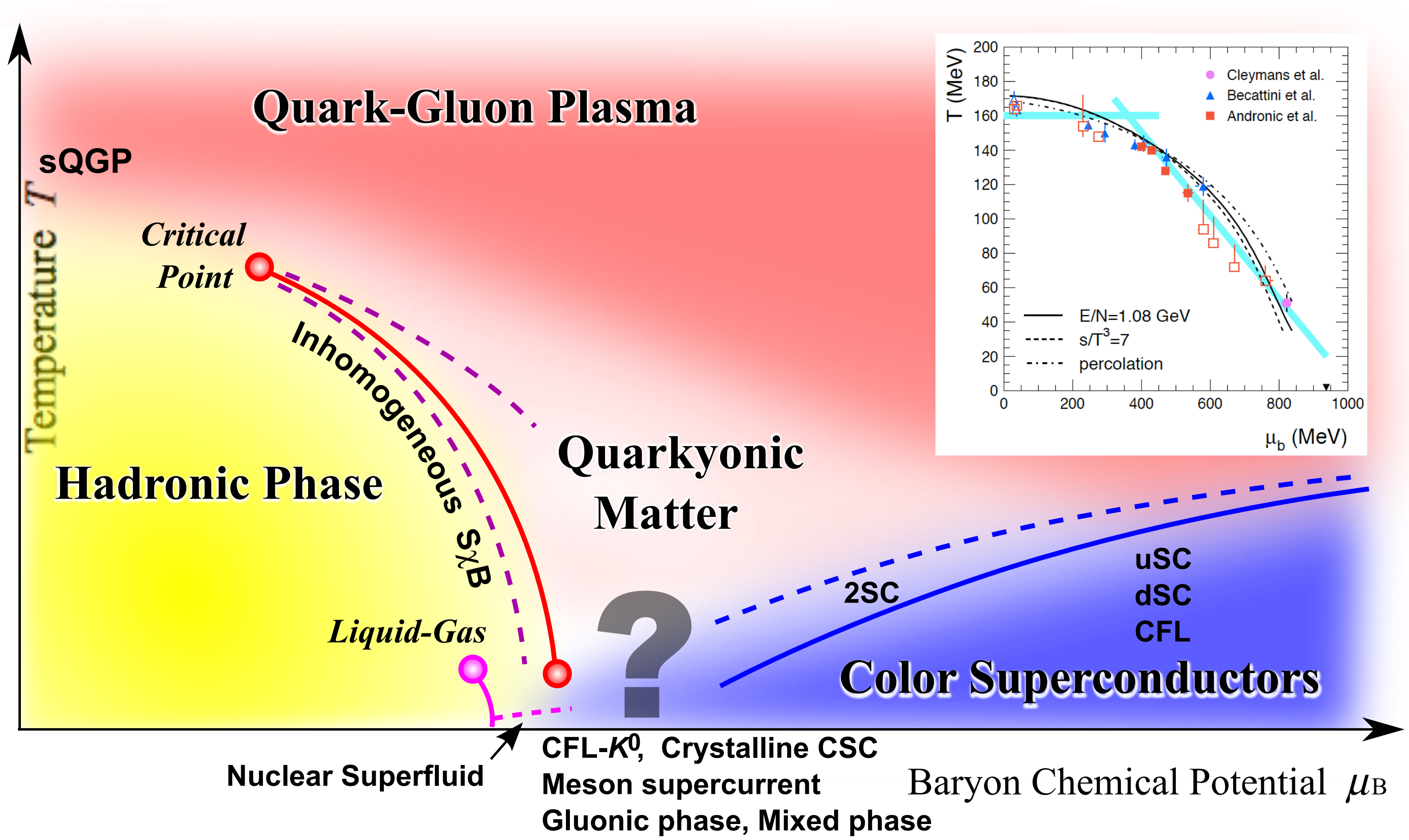


Larry McLerran '09

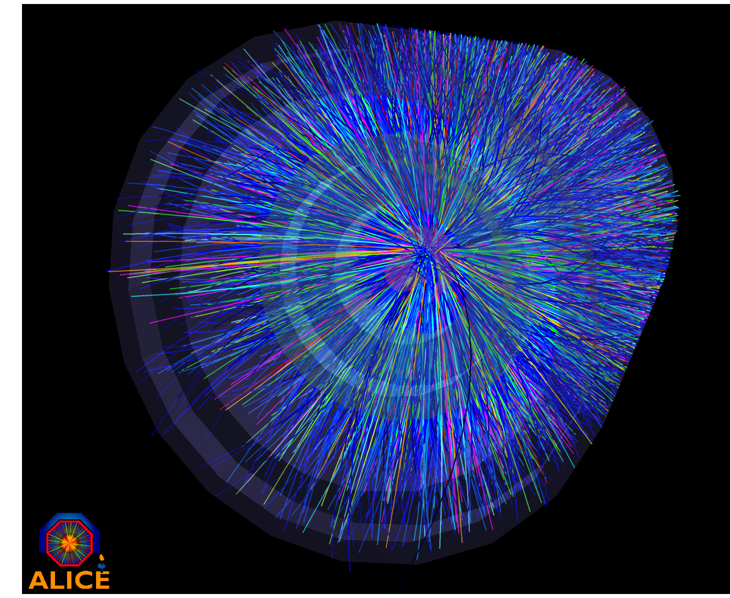


Heavy ion collisions

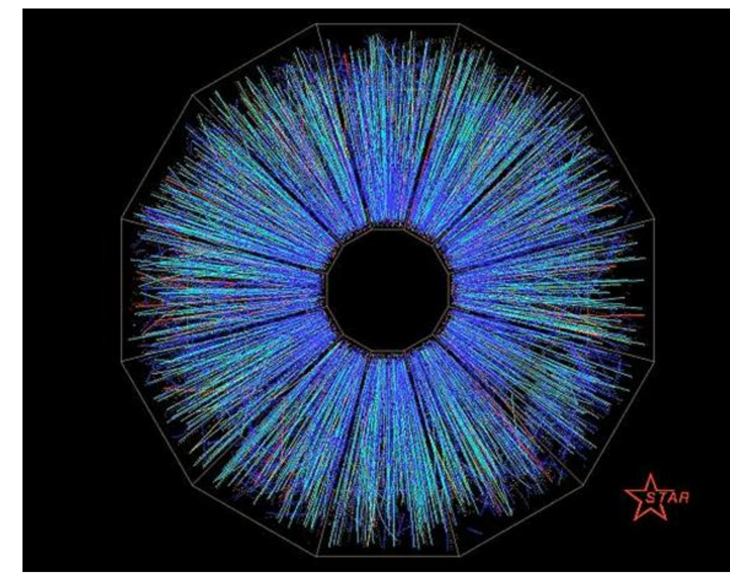




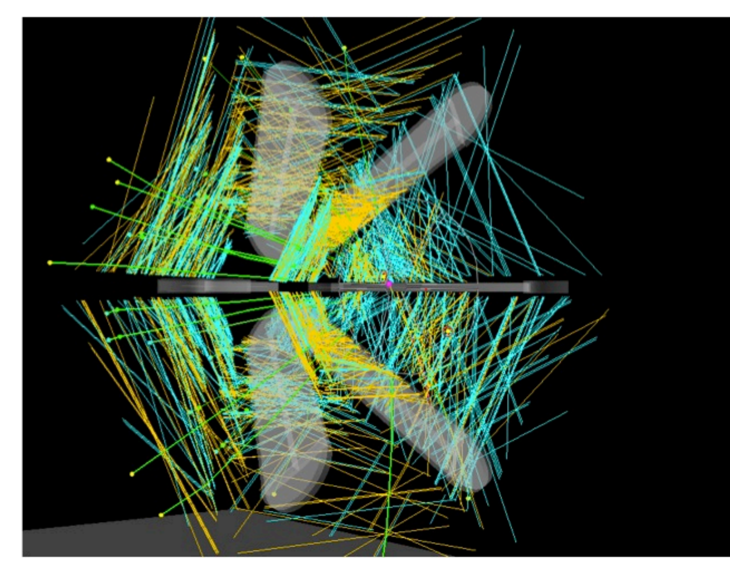
LHC



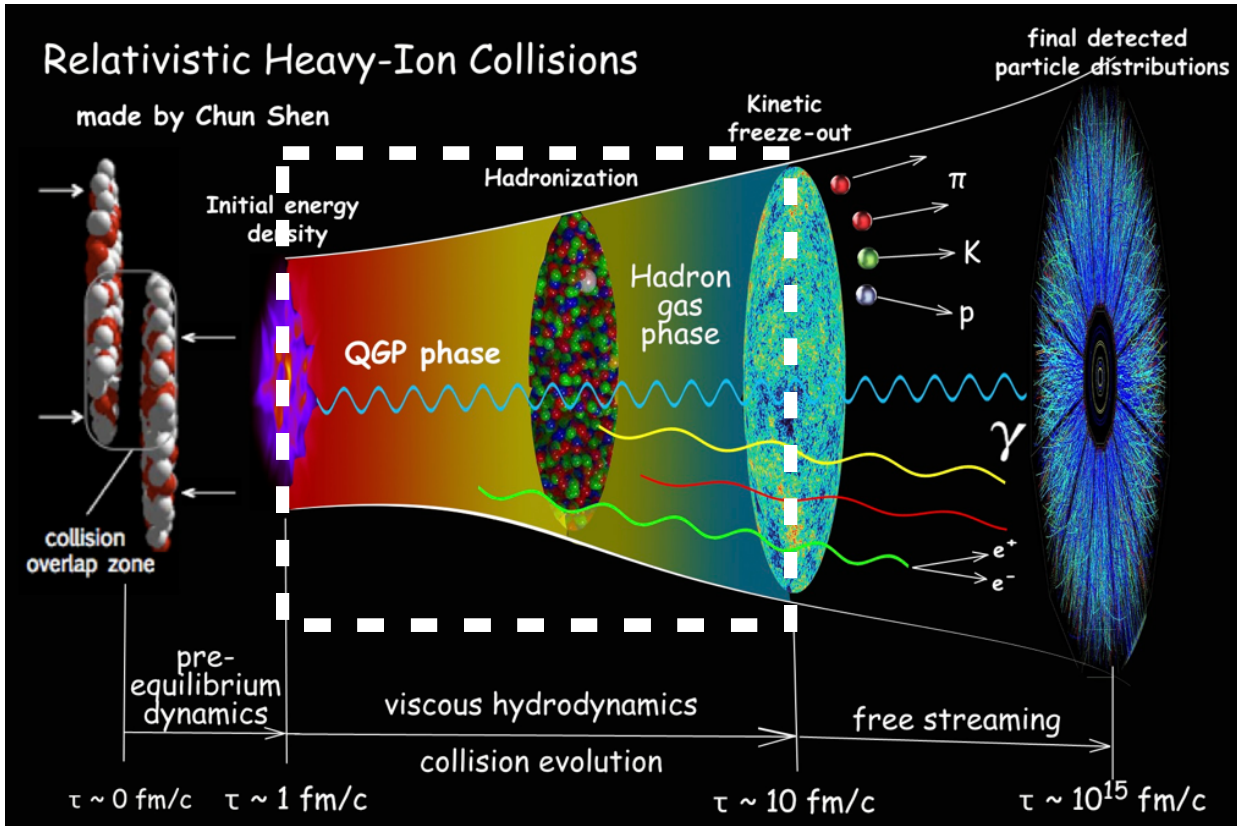
RHIC



HADES



Heavy ion collisions



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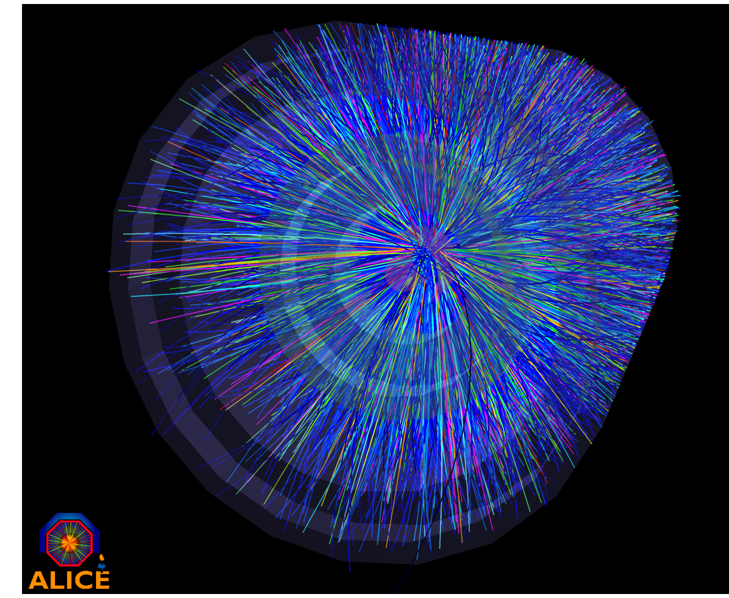
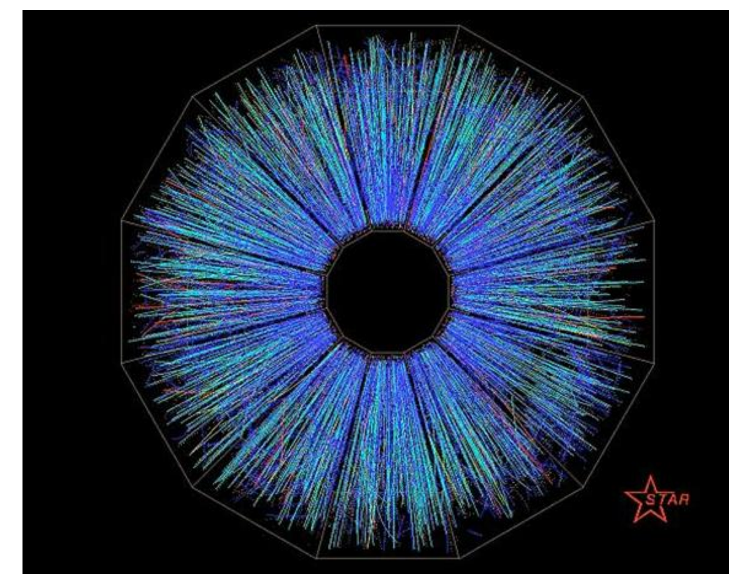
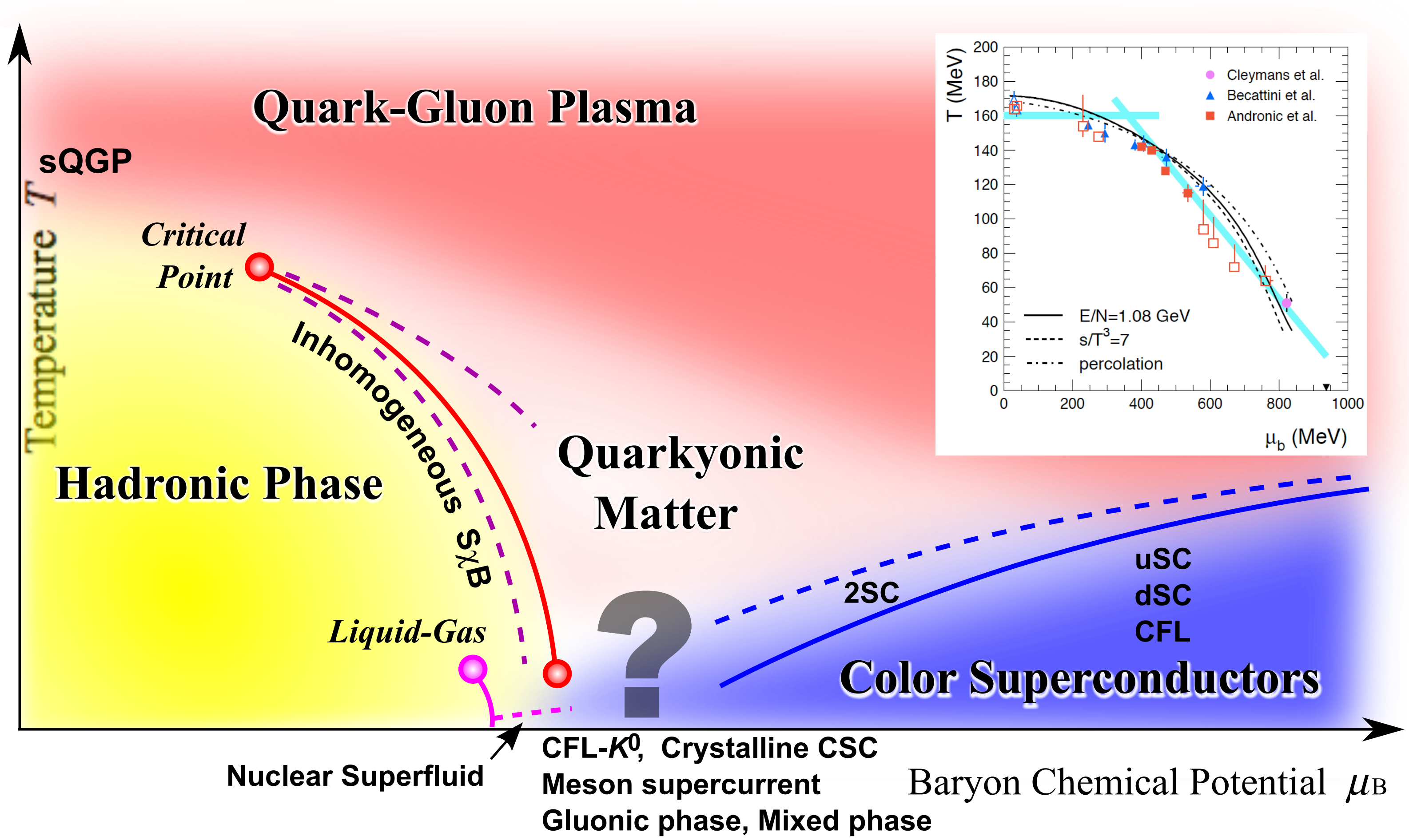
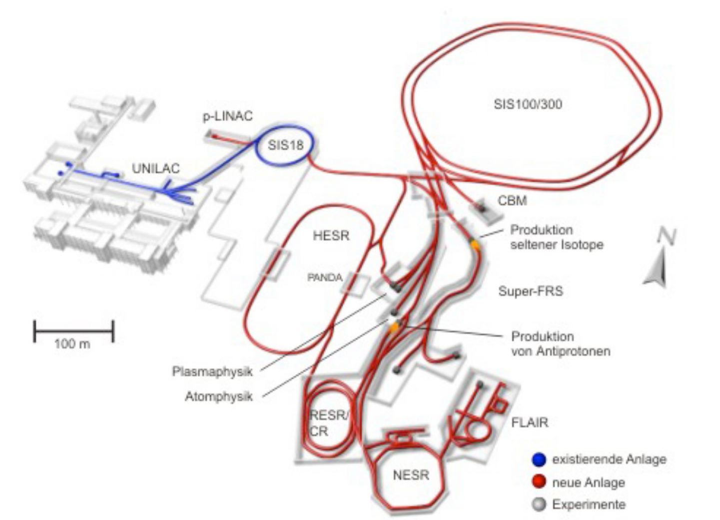


RHIC





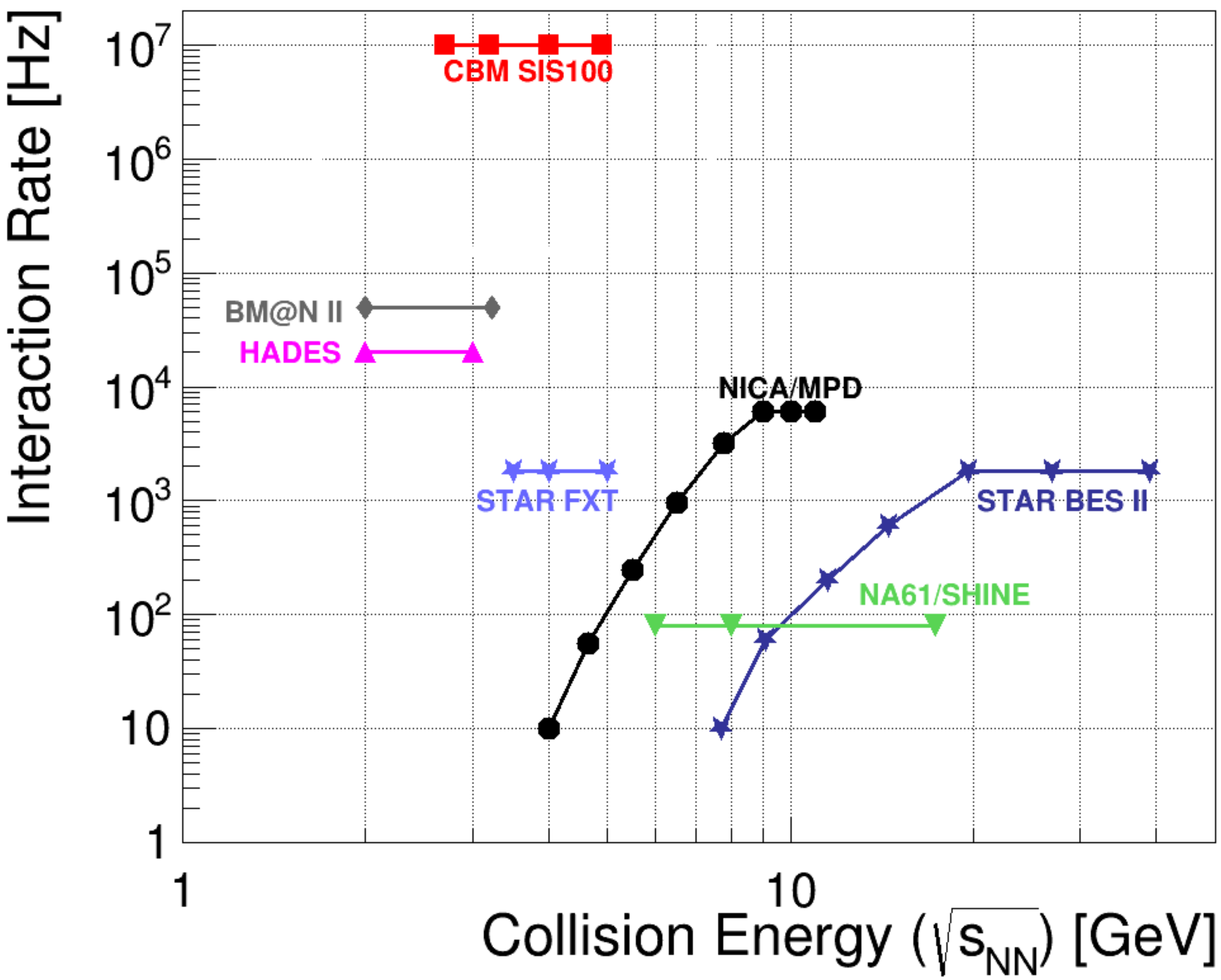
FAIR



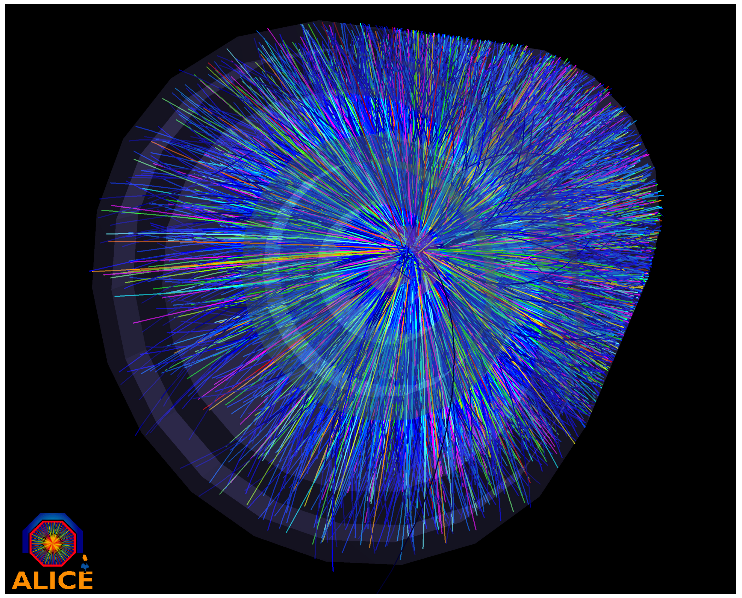
NICA

Heavy ion collisions

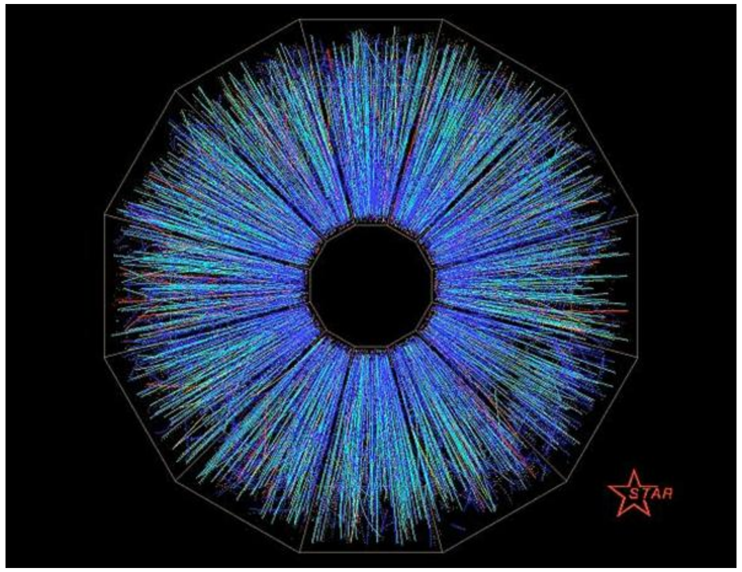
CBM collaboration, EPJ A53 (2017) 3, 60



LHC



RHIC



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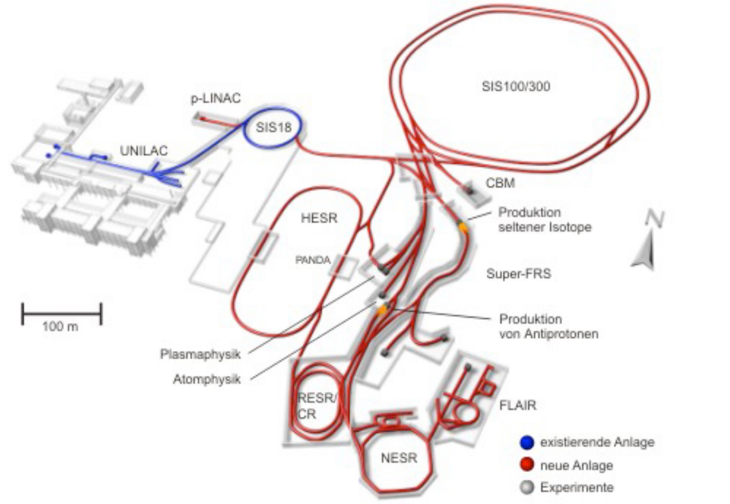
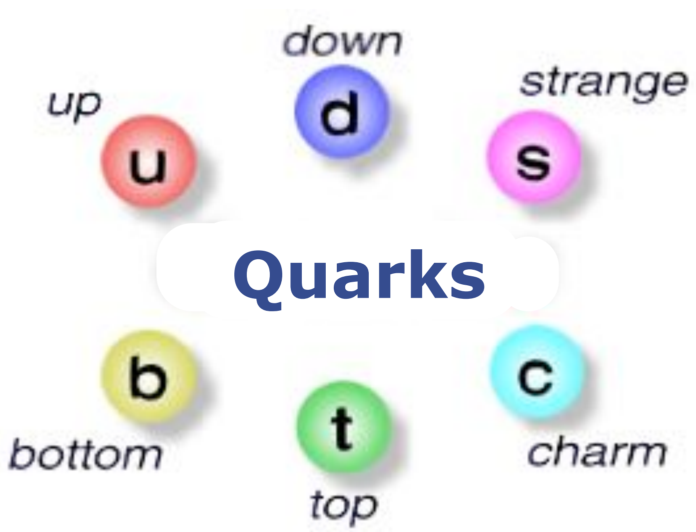


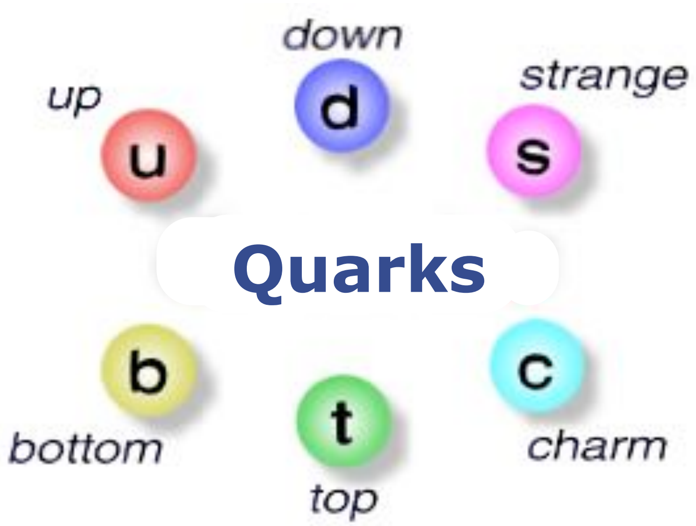
NICA

Phase structure of QCD



Gluons

Perturbative QCD & asymptotic freedom



Gluons

QCD, asymptotic freedom and all that

Action and interactions

QCD action S_{QCD}

Yang-Mills **gauge fixing**

$$\frac{1}{4}\int_x F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}\int_x (\partial_\mu A^a_\mu)^2 + \int_x \bar{c}^a \partial_\mu D^{ab}_\mu c^b + \int_x \bar{q}\cdot(i\not{D} + i\, m_\psi + i\mu\gamma_0)\cdot q$$

gluon **ghost** **quarks**

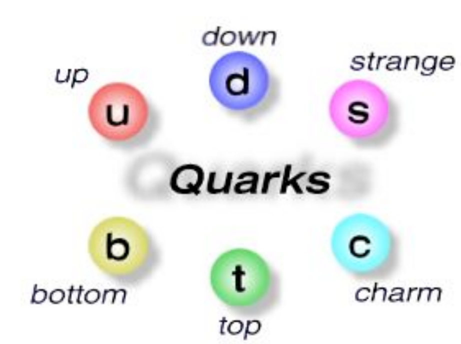
Pure gauge theory **matter sector**

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu\, A^c_\nu$$

$$\not{D} = \gamma_\mu D_\mu$$

$$\mathbf{a, b, c = 1, ..., N_c^2 - 1}$$

$$\mathbf{N_f = 6}$$



$$D_\mu(A) = \partial_\mu - i\, g\, A_\mu$$

QCD, asymptotic freedom and all that

Action and interactions

QCD action $S_{\rm QCD}$

Yang-Mills **gauge fixing**

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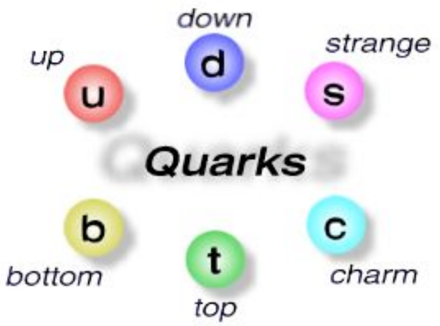


covariant derivative in adjoint representation

$$D^{ab}_\mu(A) = \partial_\mu\delta^{ab} - g\, f^{abc} A^c_\mu$$

QCD, asymptotic freedom and all that

Action and interactions

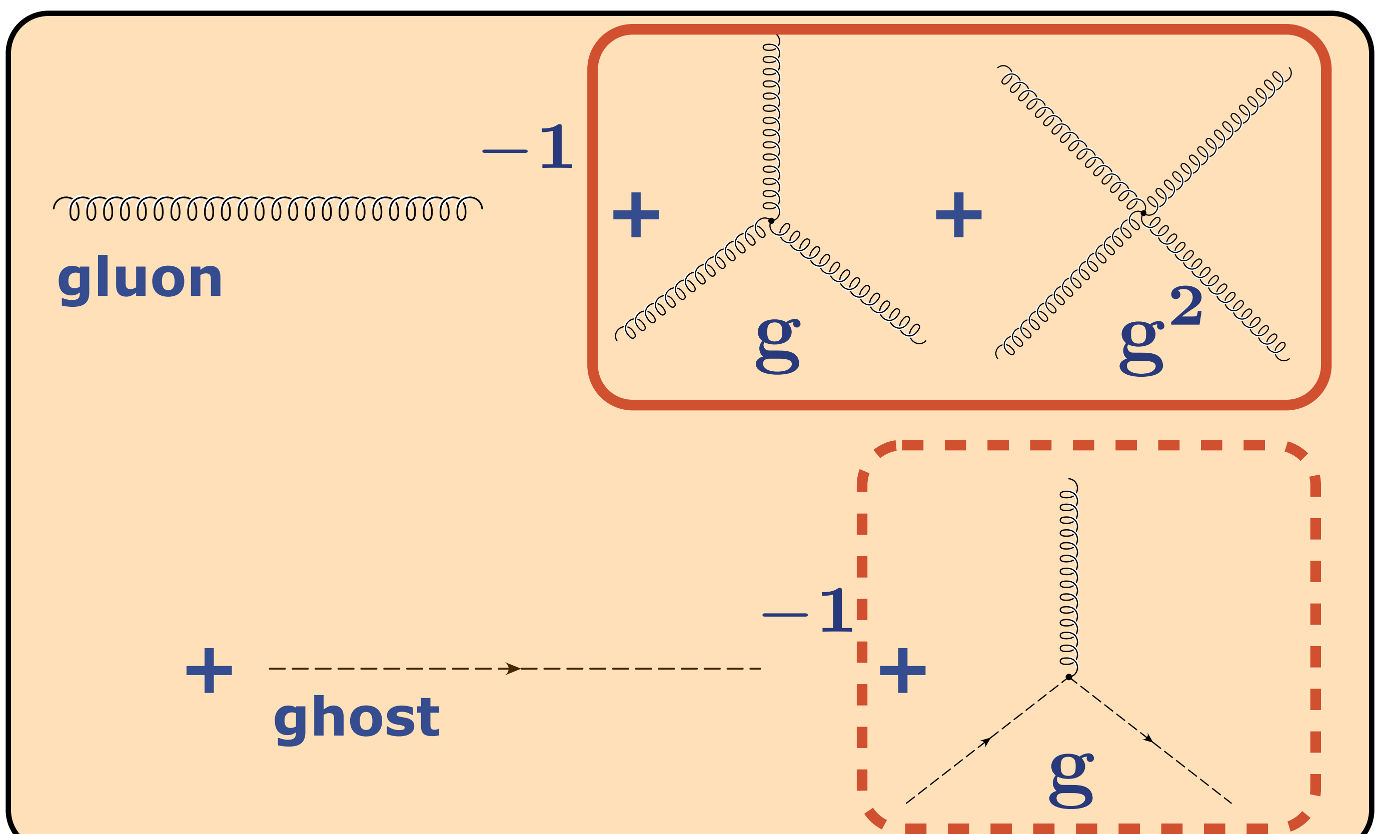
QCD action S_{QCD}

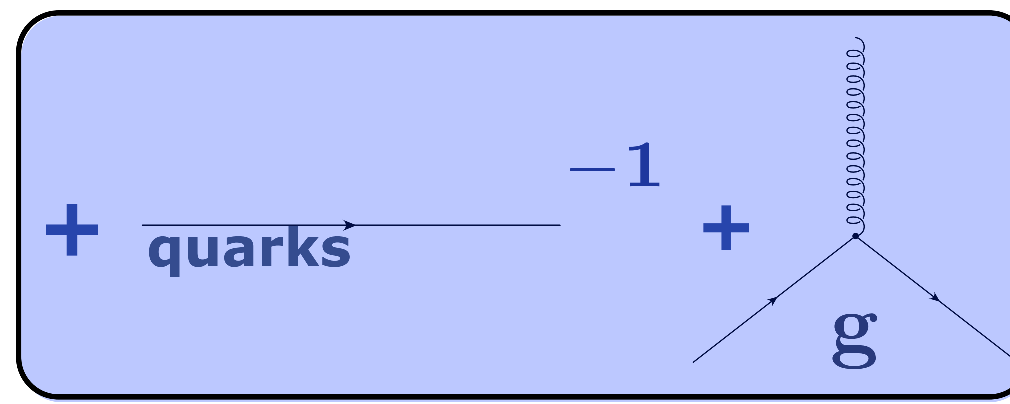
Yang-Mills **gauge fixing**

$$\frac{1}{4}\int_x F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}\int_x \left(\partial_\mu A^a_\mu\right)^2 + \int_x \bar{c}^a \partial_\mu D^{ab}_\mu c^b + \int_x \bar{q}\cdot\left(i\not{D} + i\, m_\psi + i\mu\gamma_0\right)\cdot q$$

gluon **ghost** **quarks**

Pure gauge theory **matter sector**



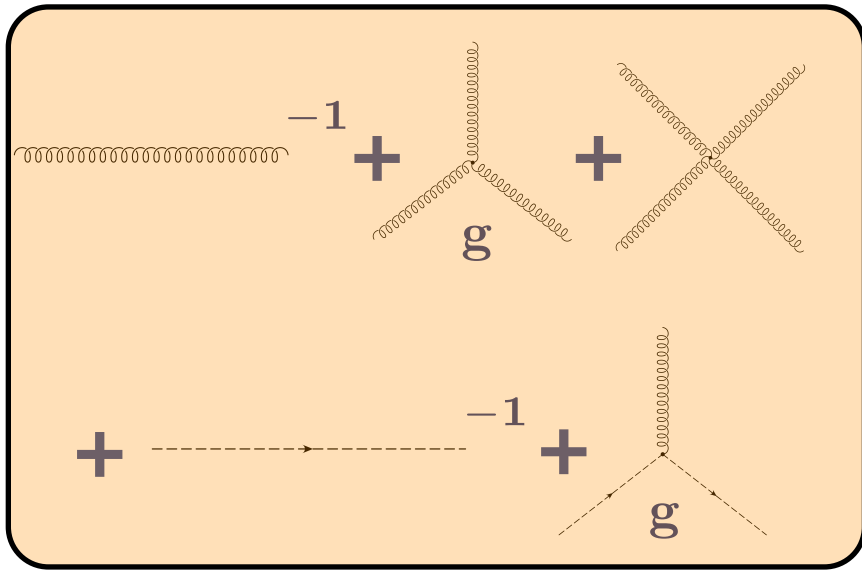


parameters

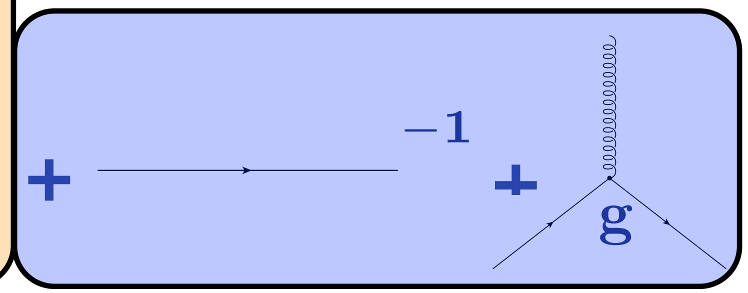
- **1 coupling** g
- **mass matrix** m_ψ $\mathrm{N_f \times N_f}$

QCD, asymptotic freedom and all that

Running coupling at low and high energies



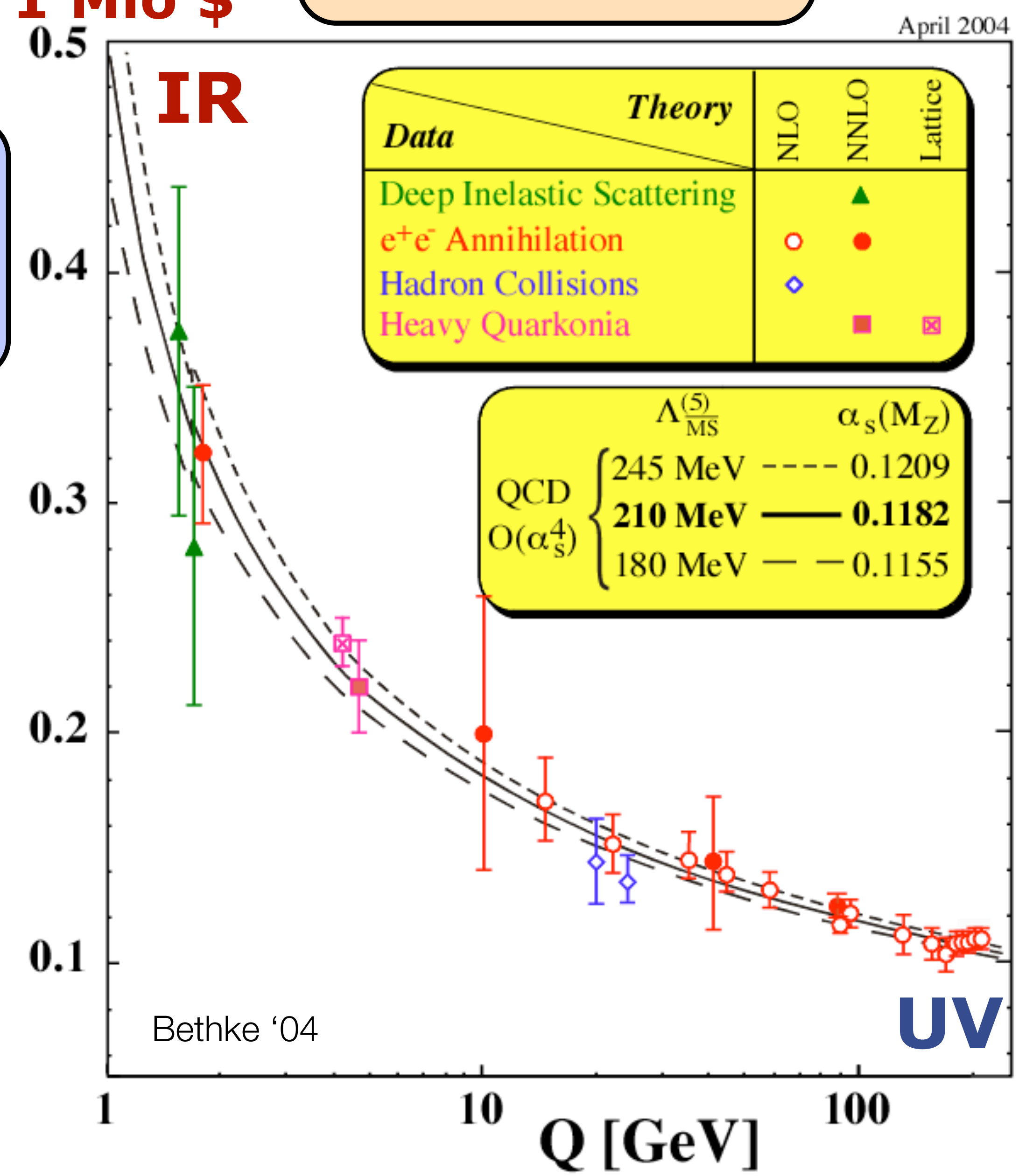
Pure gauge theory



matter sector

Millenium Prize 1 Mio $

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$

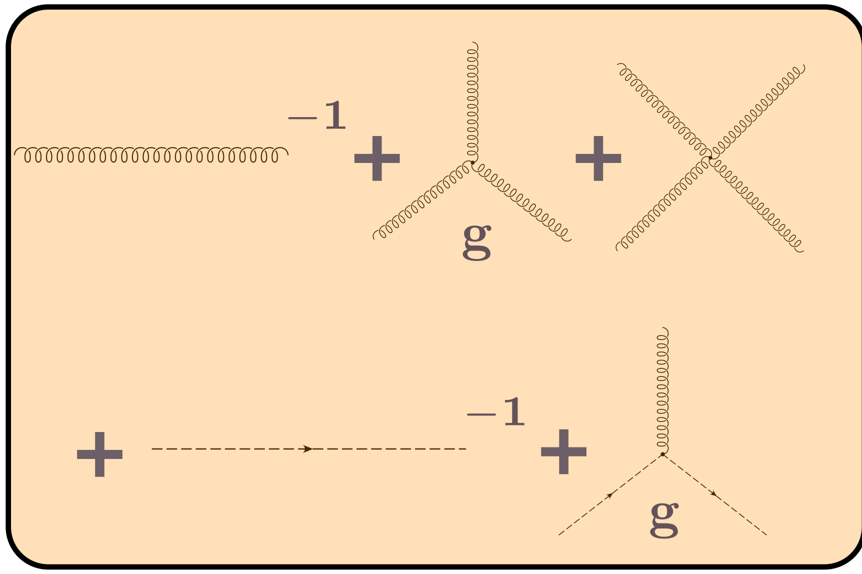


Nobel Prize '04

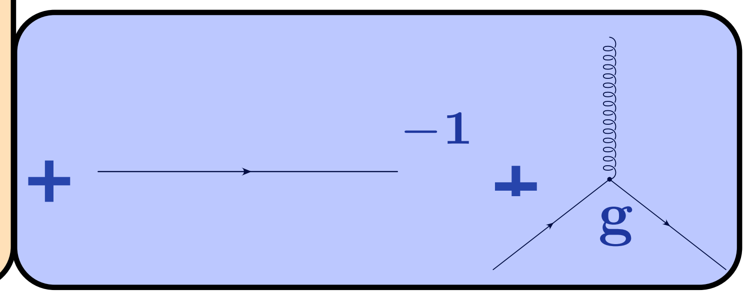
Gross, Politzer, Wilczek

QCD, asymptotic freedom and all that

Running coupling at low and high energies



Pure gauge theory



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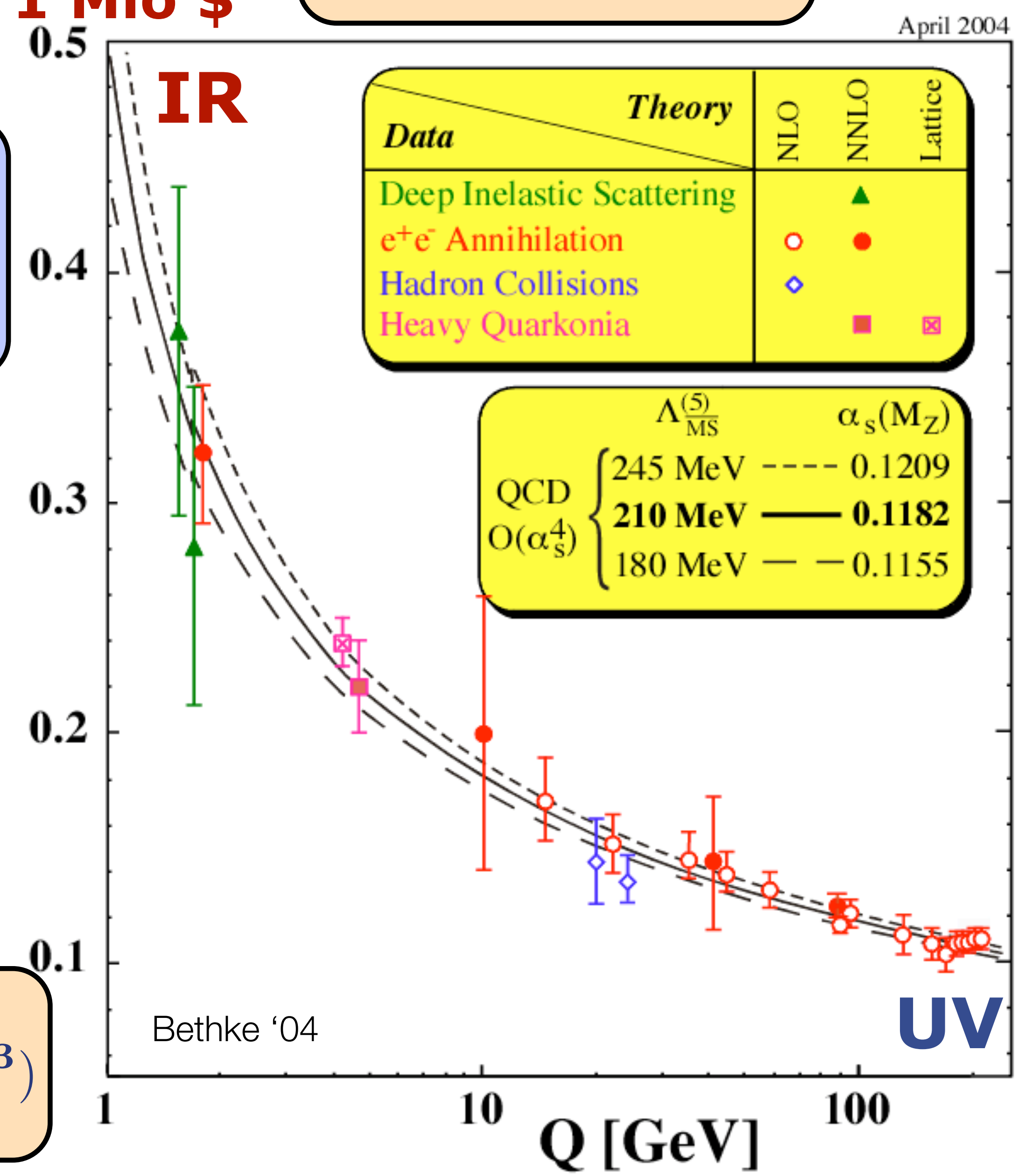


- **running coupling (1-loop)**

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 - \alpha_s(\mu)\beta_0 \log(Q^2/\mu^2)}$$

- **beta function**

$$\beta = Q^2 \frac{\partial \alpha_s(Q)}{\partial Q^2} = \beta_0 \alpha_s(\mu)^2 + O(\alpha_s(\mu)^3)$$

$$\beta = -\frac{1}{12\pi}(33 - 2N_f)\alpha_s^2 + O(\alpha_s^3)$$

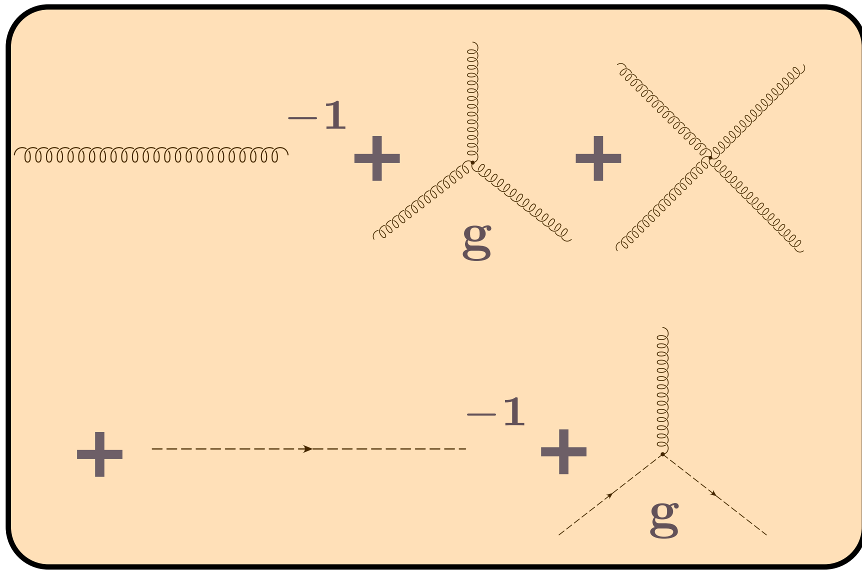
Nobel Prize '04

Gross, Politzer, Wilczek

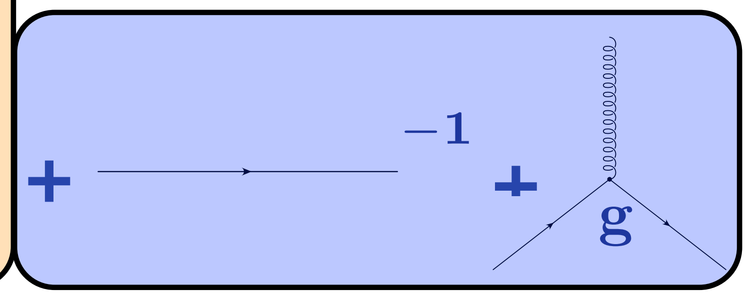
QCD, asymptotic freedom and all that

Running coupling at low and high energies

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$

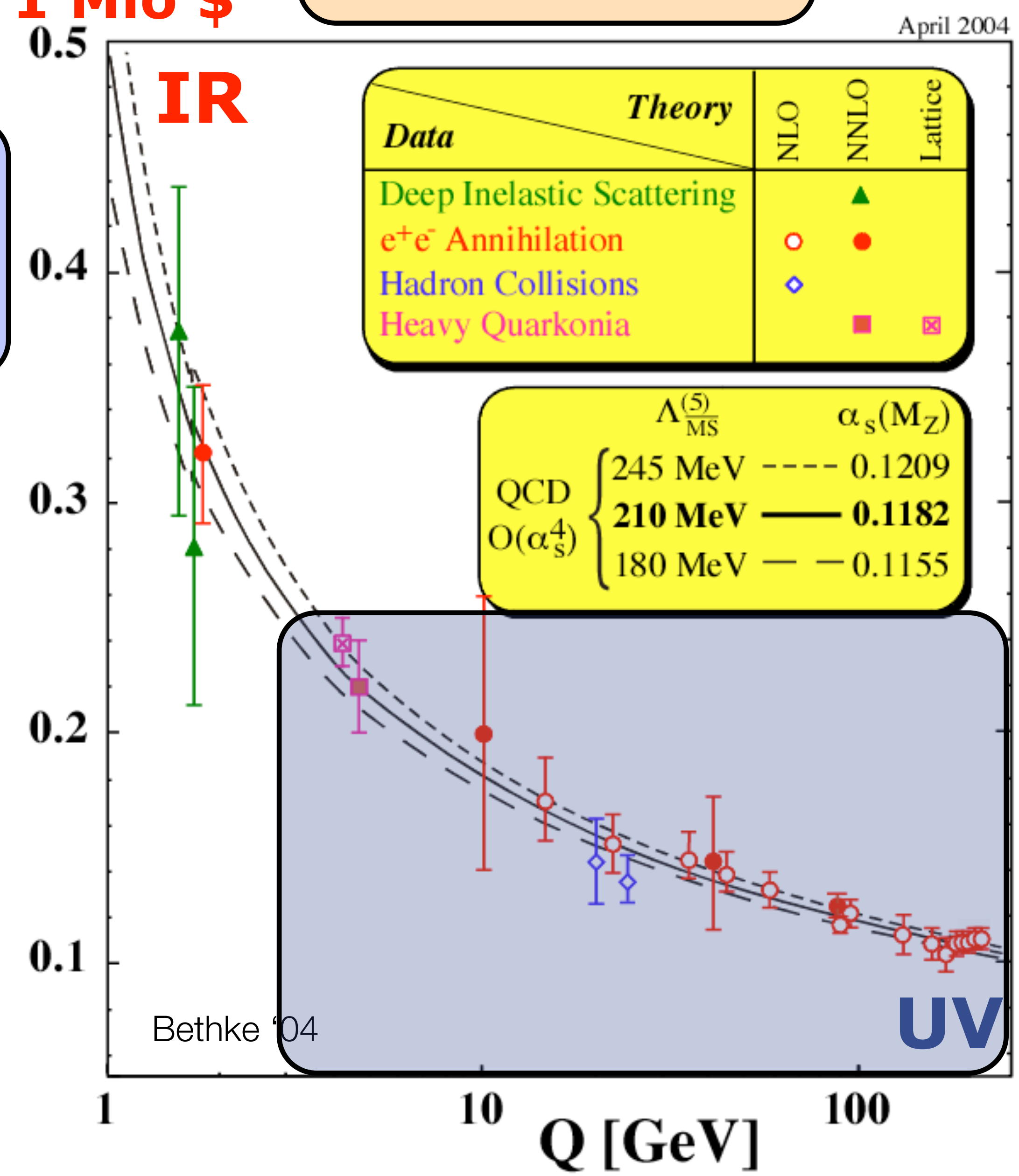


Pure gauge theory



matter sector

Millenium Prize 1 Mio $



- **running coupling (1-loop)**

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 - \alpha_s(\mu)\beta_0 \log(Q^2/\mu^2)}$$

- **UV: asymptotic freedom**

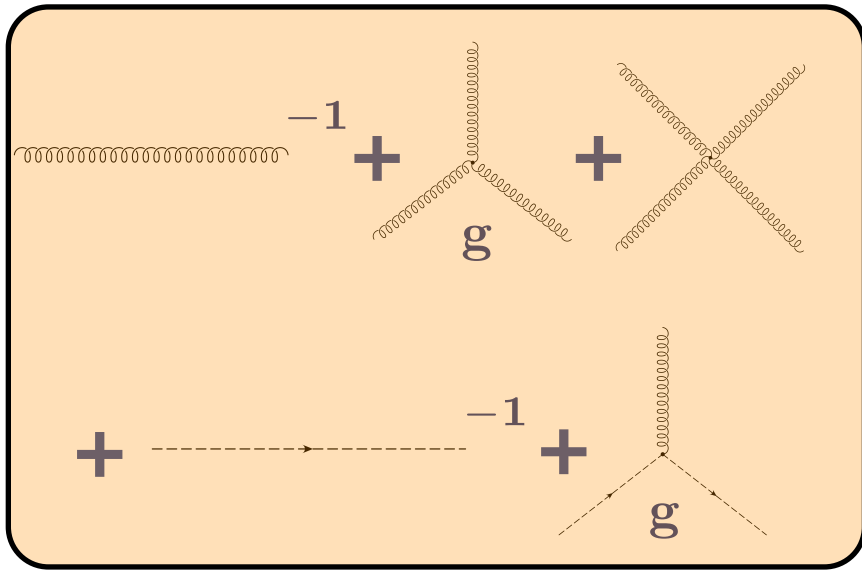
$$\alpha_s(Q \to \infty) = 0$$

Nobel Prize '04

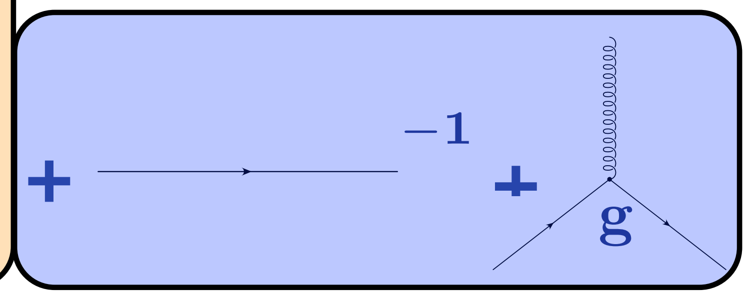
Gross, Politzer, Wilczek

QCD, asymptotic freedom and all that

Running coupling at low and high energies



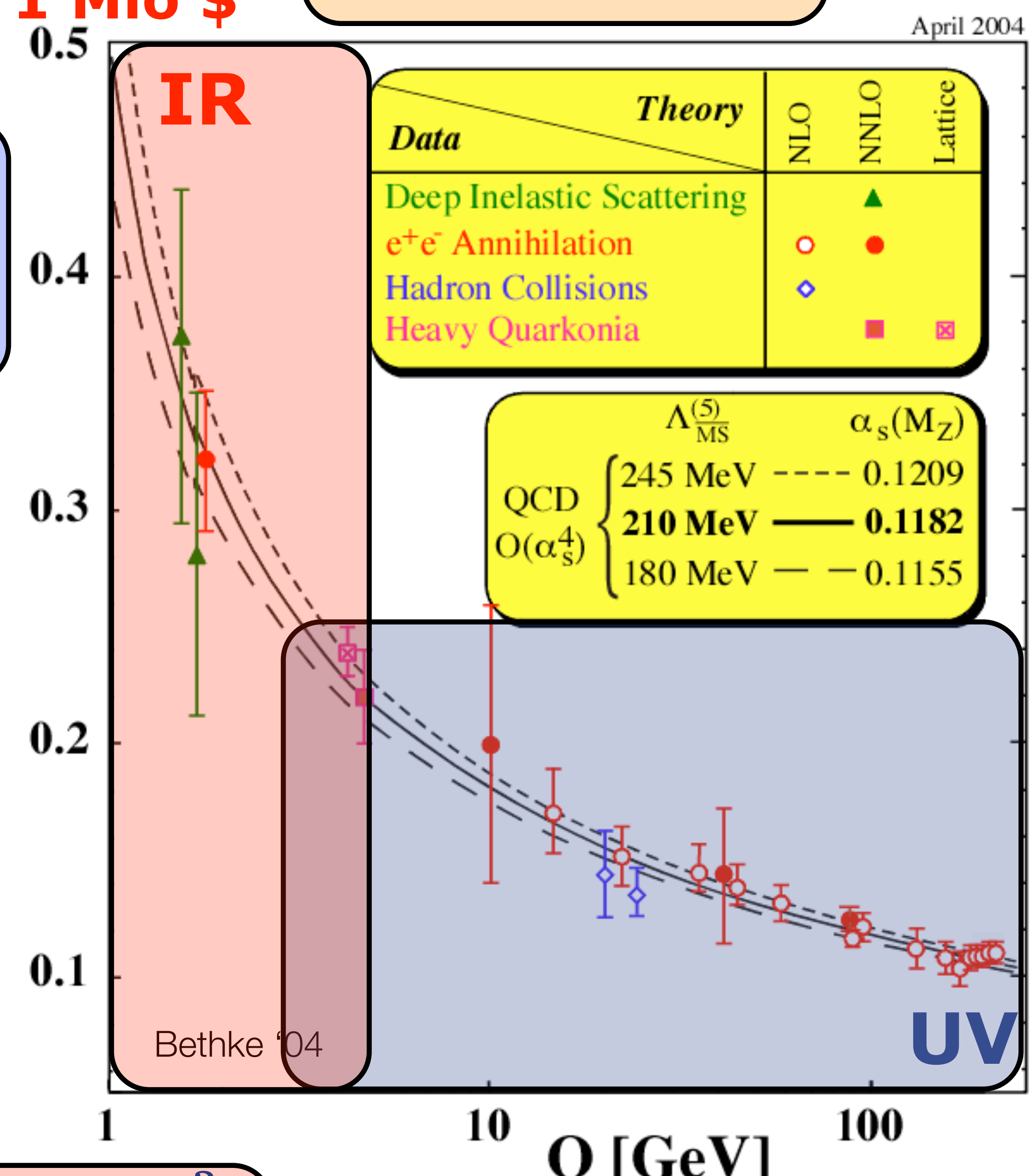
Pure gauge theory



matter sector

Millenium Prize 1 Mio $

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$



- **running coupling (1-loop)**

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 - \alpha_s(\mu)\beta_0 \log(Q^2/\mu^2)}$$

- **IR: failure of perturbation theory**

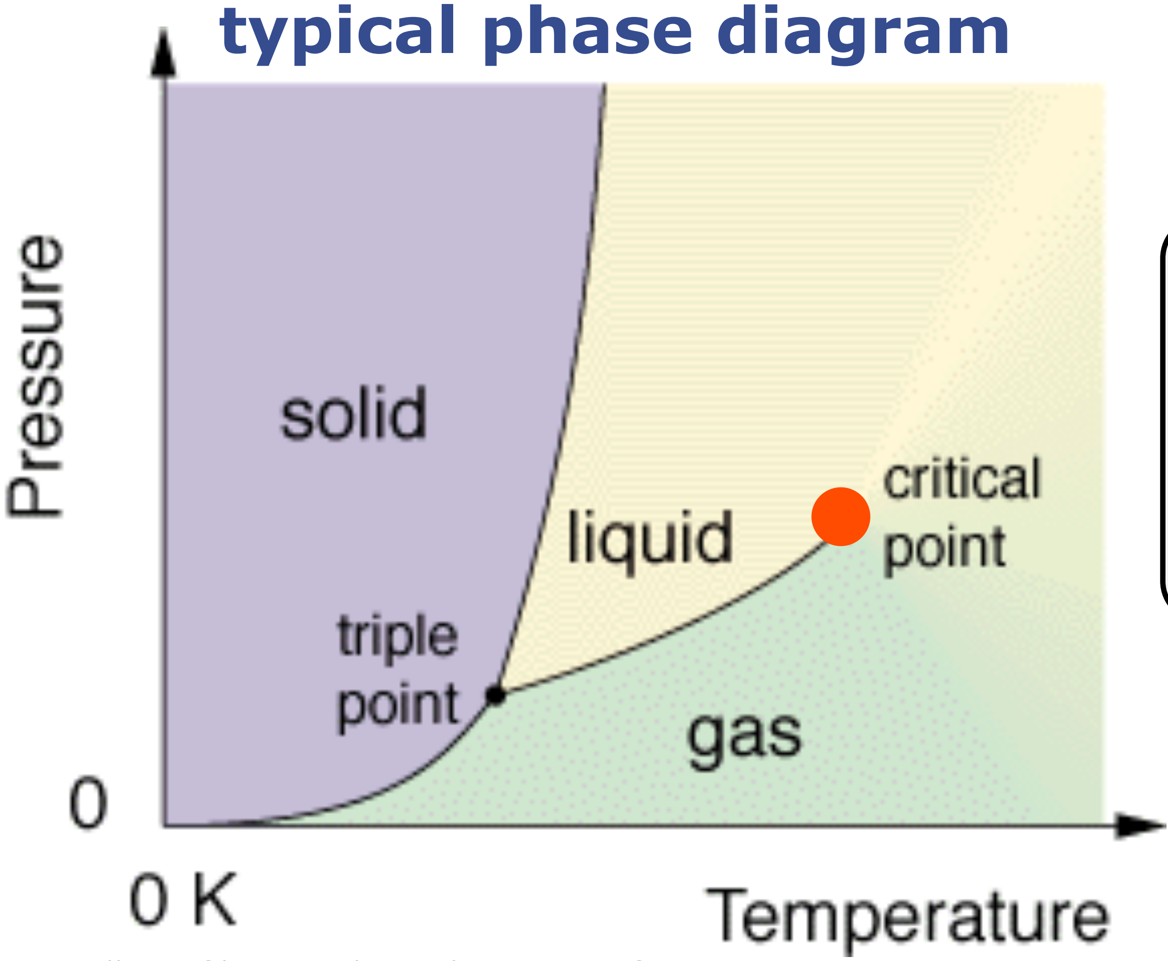
$$\alpha_s(\Lambda^2_{QCD}) = \infty \quad \text{at} \quad \Lambda^2_{QCD} = \mu^2 e^{-\beta_0/\alpha_s(\mu^2)}$$

$$\Lambda_{QCD} = 217^{+25}_{-23}\ \text{MeV}$$

Nobel Prize '04

Gross, Politzer, Wilczek

typical phase diagram



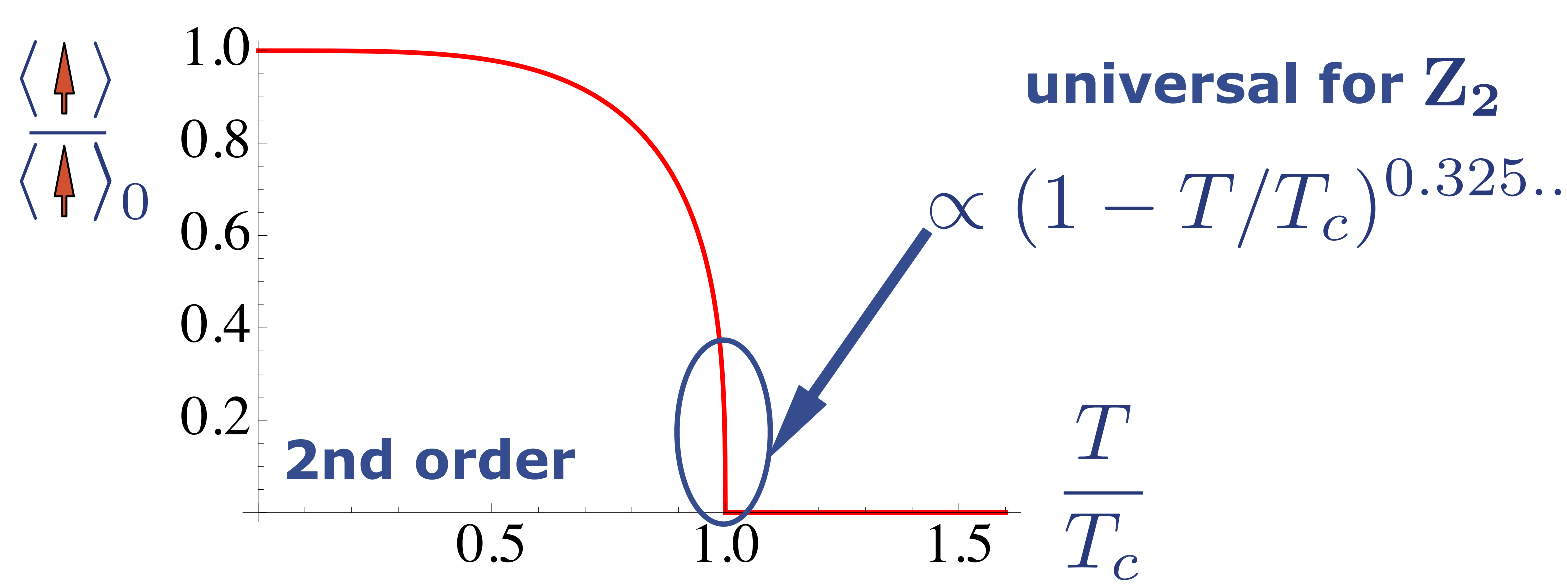
http://ltl.tkk.fi/research/theory/TypicalPD.gif

Order parameter: density n

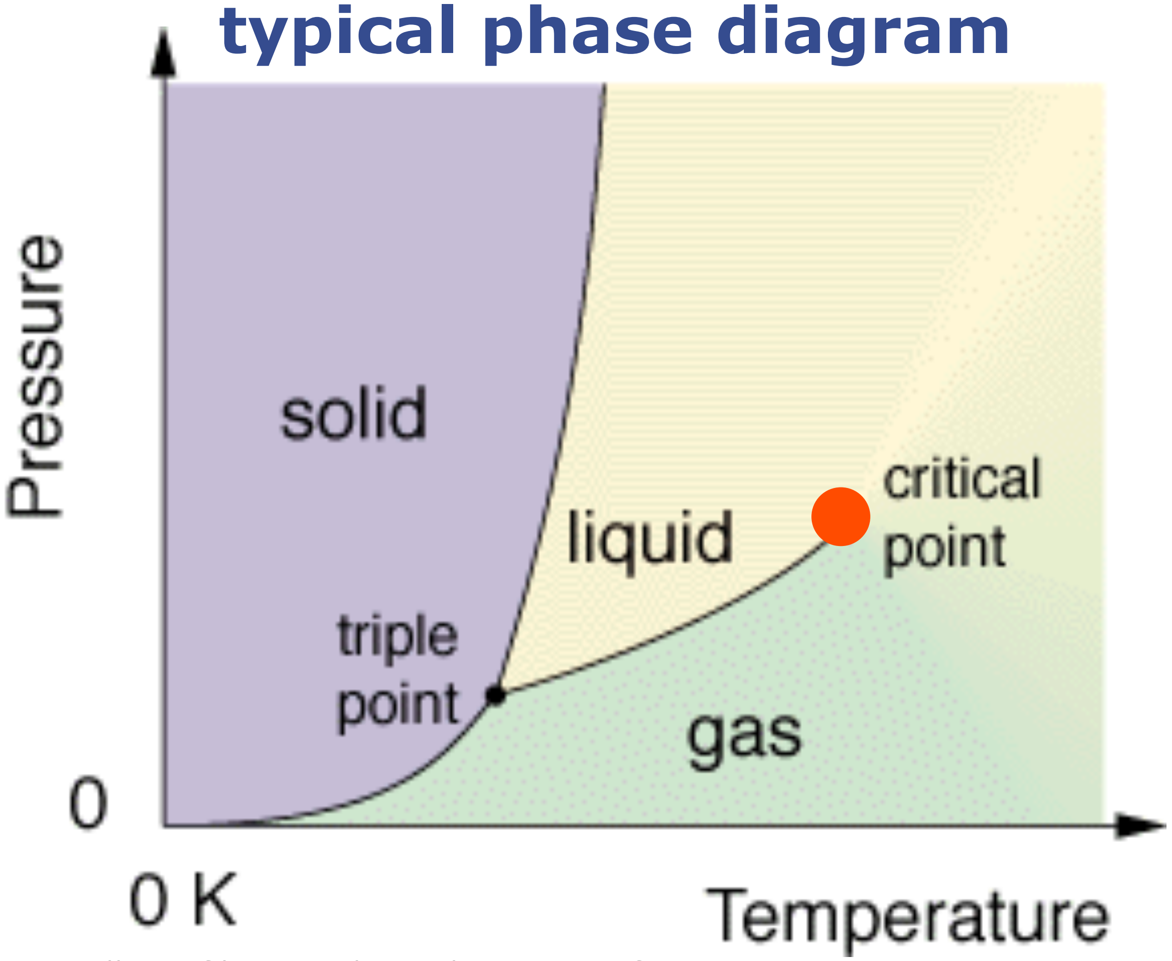
density jumps	1st order phase transition
derivative of density jumps	2nd order phase transition
density smooth	cross-over

Ising model in 3d: (↓ ↑) -spin system

Order parameter: $\langle \uparrow \rangle$

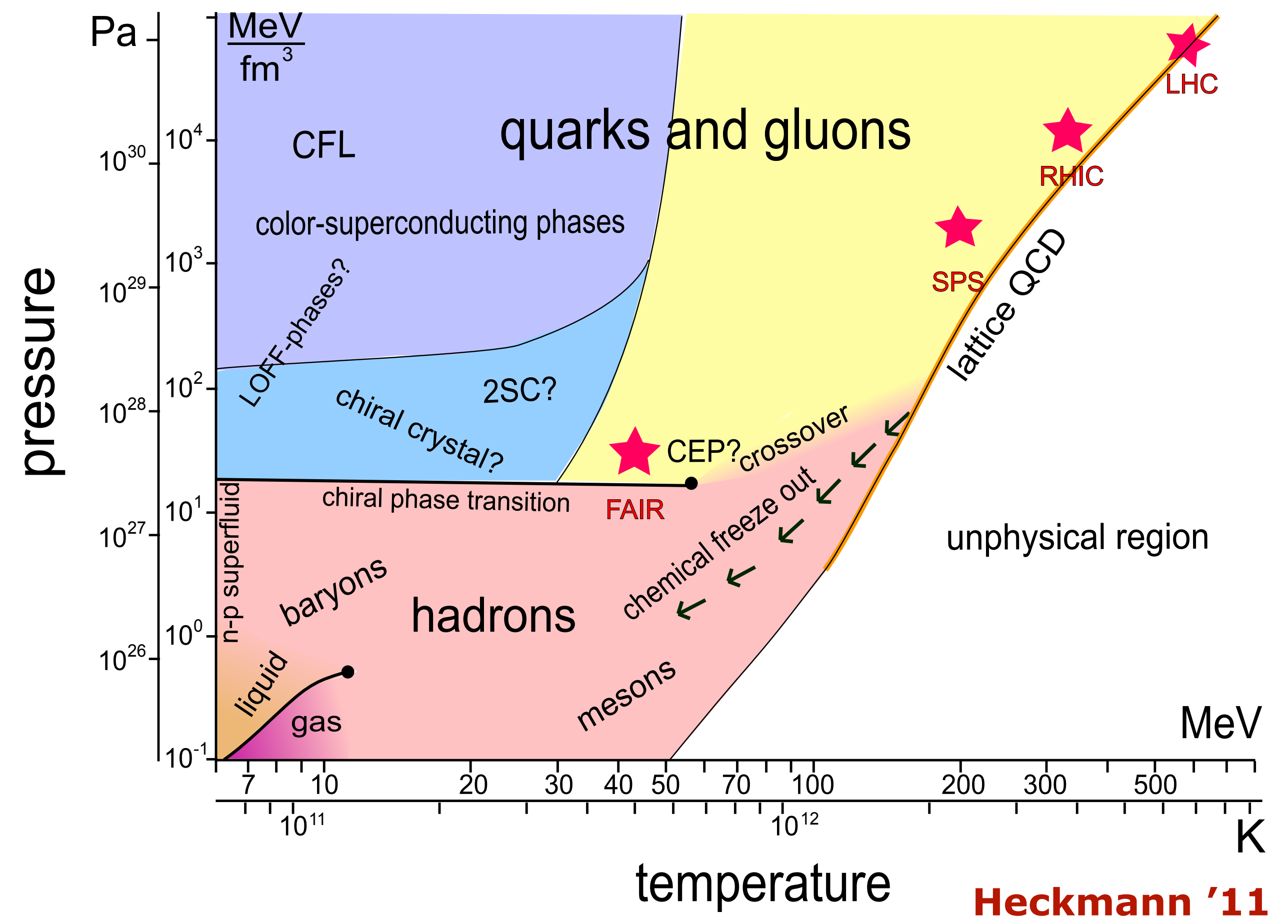


typical phase diagram



http://ltl.tkk.fi/research/theory/TypicalPD.gif

phase diagram of QCD



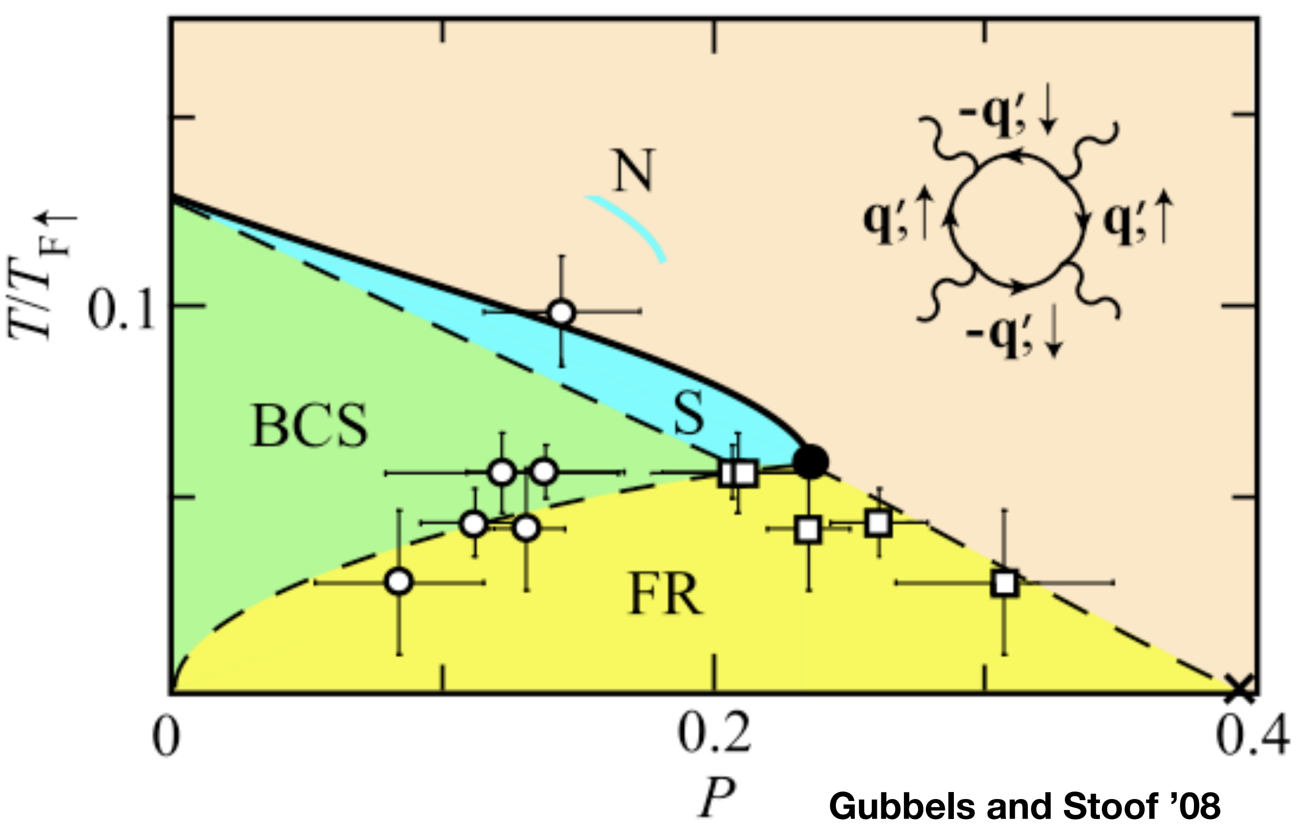
Heckmann '11

Phases in QCD

quarks massless - massive

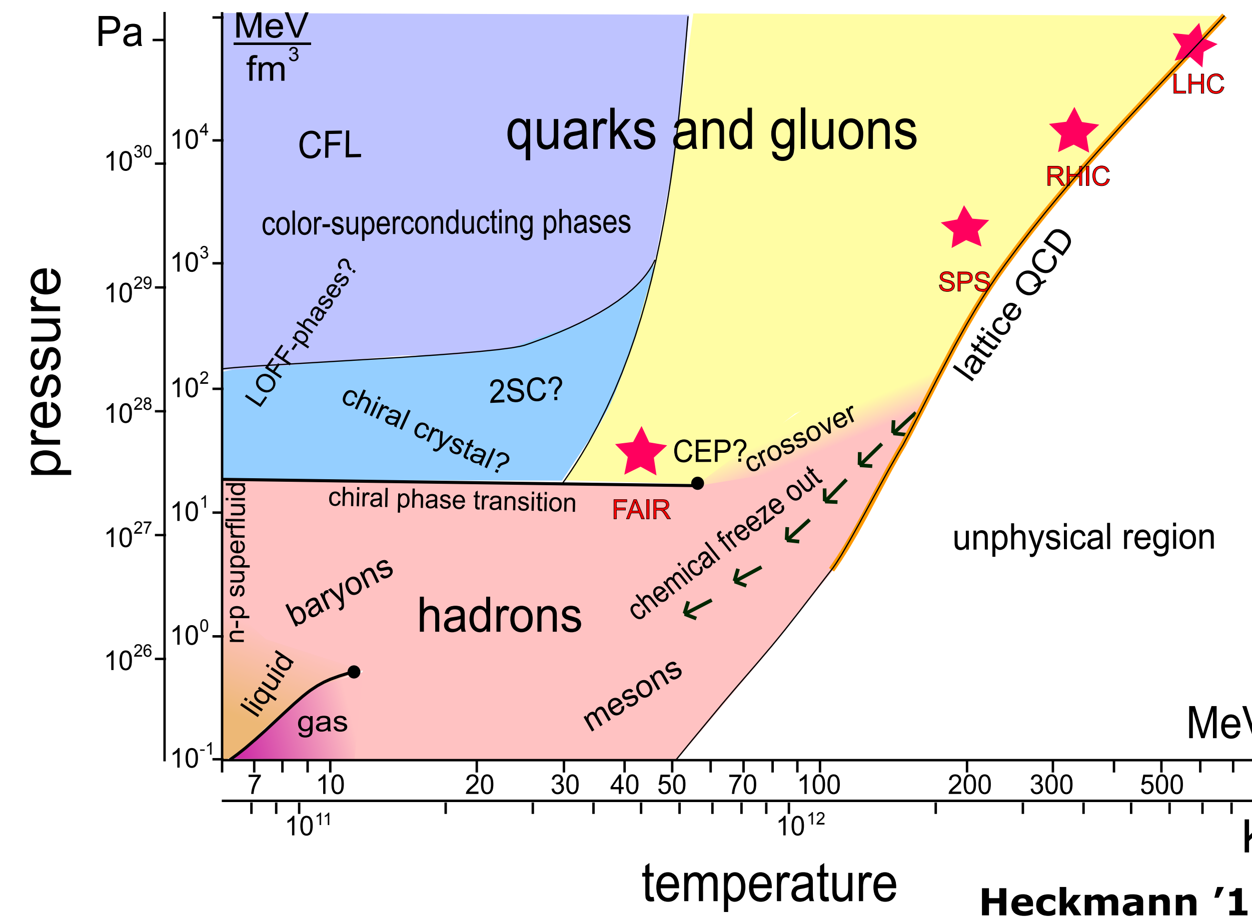
quarks confined - deconfined

Phase diagram of cold atoms



Gubbels and Stoof '08

phase diagram of QCD



Heckmann '11

Phases in QCD

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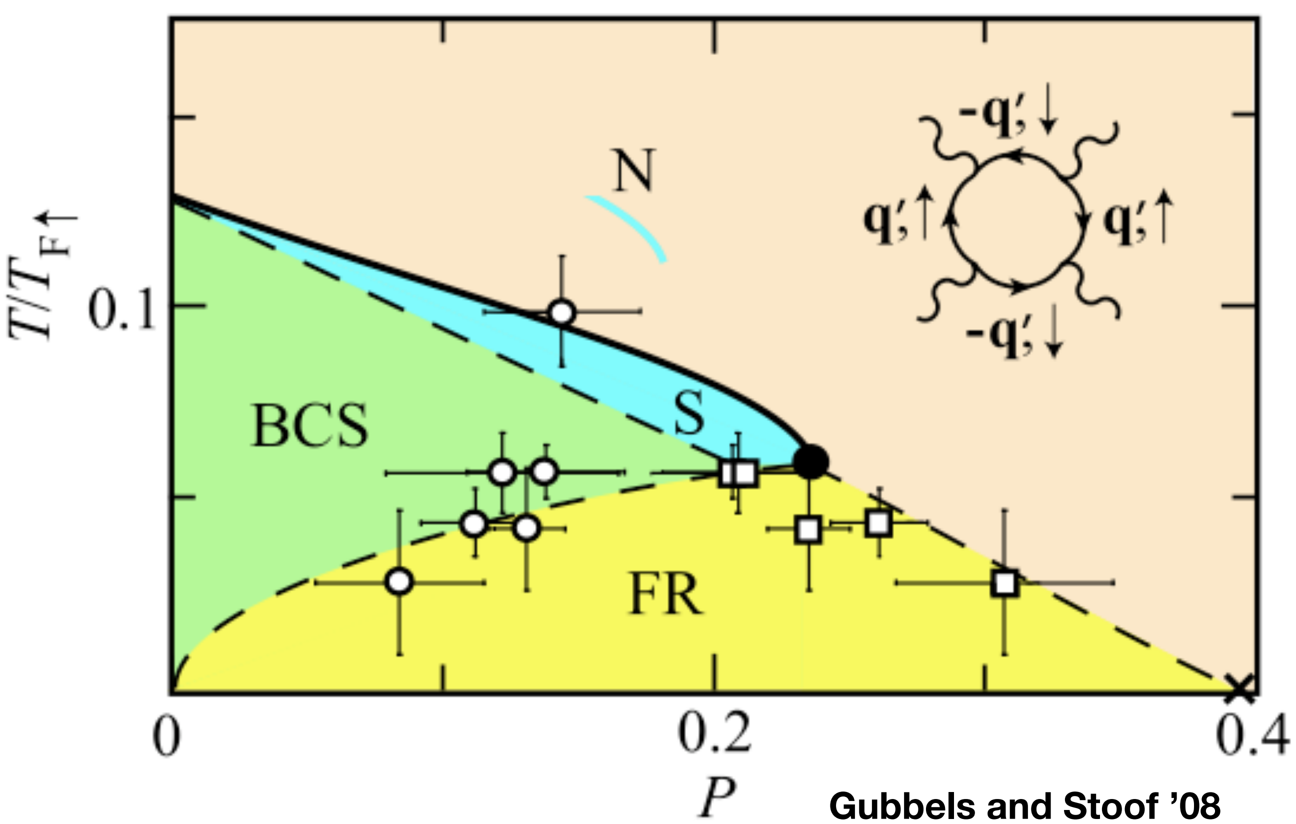
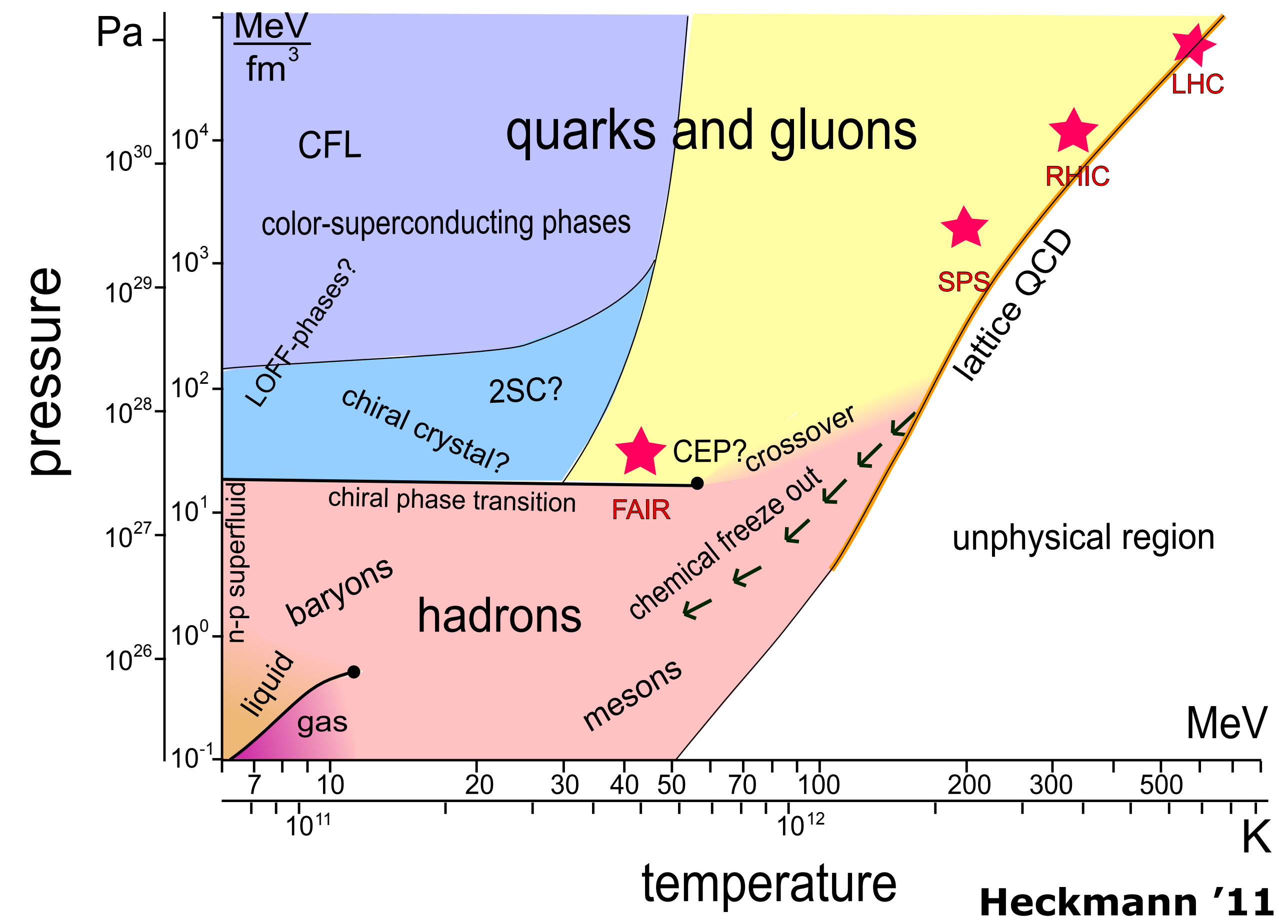
Phase diagram of cold atoms



Gubbels and Stoof '08

phase diagram of QCD



Heckmann '11

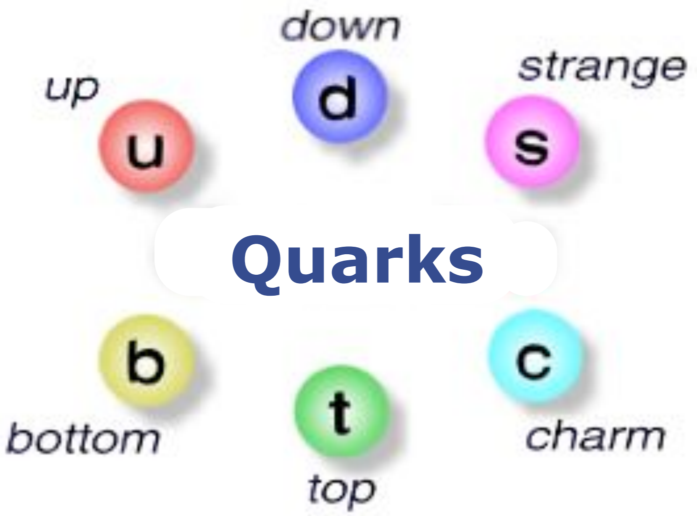
Phases in QCD

quarks massless - massive

quarks confined - deconfined

Strong chiral symmetry breaking makes up for 99% of the mass of the visible part of matter in the universe

Chiral symmetry breaking



Chiral symmetry breaking

strong chiral symmetry breaking $\Delta m_{\chi SB} \approx 400\,\mathrm{MeV}$

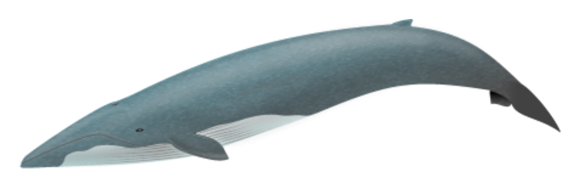
Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
Quark	u	c	t	$\frac{2}{3}$
Quark	d	s	b	$-\frac{1}{3}$
Mass [MeV]	4-8	80-130	$(4.1\text{-}4.4)\times10^3$	

two light flavours and one heavy flavour: 2+1

up

charm

top



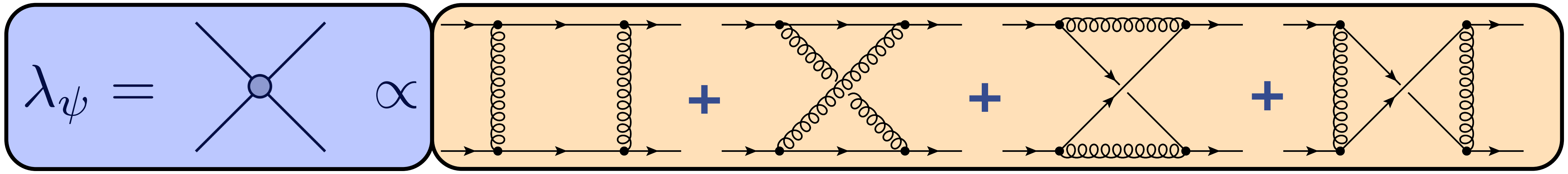
down

strange

bottom

Chiral symmetry breaking

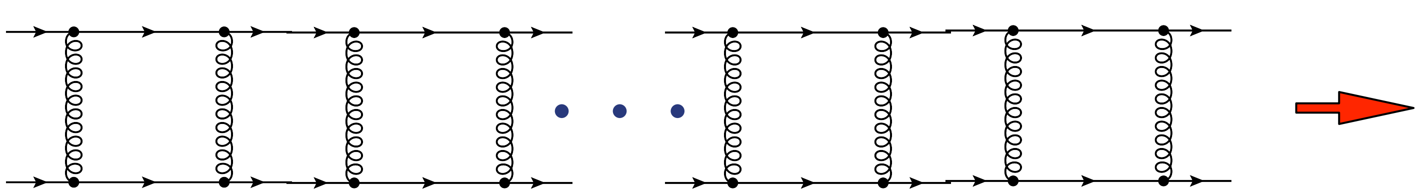
- **Perturbative four-fermi coupling** $\frac{\lambda_\psi}{2}\int\left[(\bar q q)^2 + (i\bar q\gamma_5\vec\tau q)^2\right]$



$\lambda_\psi \propto \alpha_s^2$

$\mathbf{N_f = 2}:\ \ \vec\tau = (\sigma_1, \sigma_2, \sigma_3)$

- **Fermionic mass term for** $\langle \bar q q\rangle \neq 0$

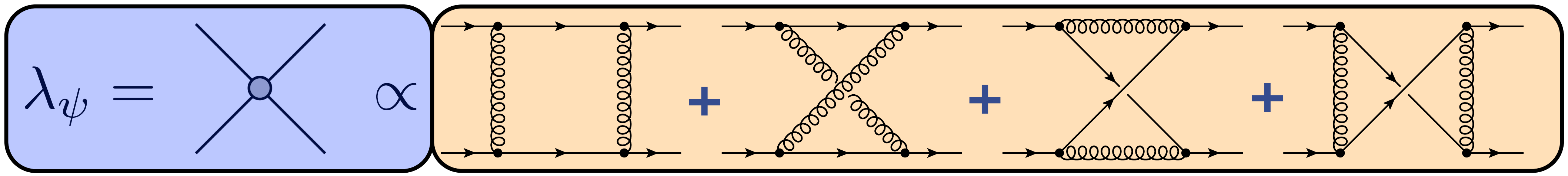


$$\frac{\lambda_\psi}{2}\int(\bar q q)^2 \longrightarrow \frac{\lambda_\psi}{2}\int\langle\bar q q\rangle \bar q q$$

mean field

Chiral symmetry breaking

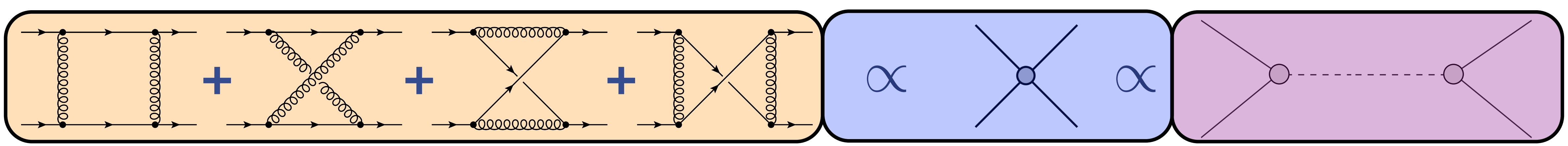
- **Perturbative four-fermi coupling** $\frac{\lambda_\psi}{2}\int\left[(\bar{q}q)^2+(i\bar{q}\gamma_5\vec{\tau}q)^2\right]$



$\lambda_\psi \propto \alpha_s^2$

$\mathbf{N_f = 2}:\ \ \vec{\tau} = (\sigma_1, \sigma_2, \sigma_3)$

- **Bosonisation (Hubbard-Stratonovich)** $\langle\sigma\rangle \neq 0$



$$\frac{\lambda_\psi}{2}\int\left[(\bar{\psi}\psi)^2+(i\bar{\psi}\gamma_5\vec{\tau}\psi)^2\right] = \frac{m_\sigma^2}{2}\int_x\left(\sigma^2+\vec{\pi}^2\right)+i\,h\int_x\bar{\psi}(\sigma+i\gamma_5\vec{\tau}\vec{\pi})\psi$$

$\mathbf{EOM}(\sigma)$

Chiral symmetry breaking

- **Chirality for massless particles**

Right-handed: *Left-handed:*

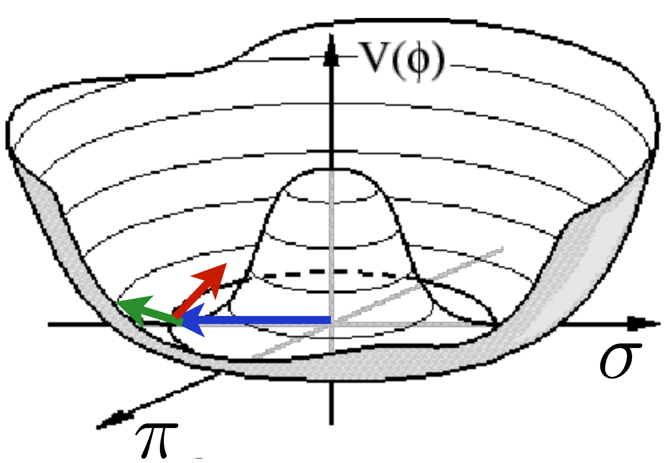
- **Order parameter**

$$\sigma \simeq \langle \bar{q}q \rangle$$ **chiral condensate**

$$\bar{q}q = q_R^\dagger q_L + q_L^\dagger q_R$$

  - **Chiral symmetry** $\sigma = 0$
  - **Symmetry broken** $\sigma \neq 0$

- **Meson potential**



Chiral symmetry breaking

- **Chirality for massless particles**

Right-handed: *Left-handed:*

- **Order parameter**

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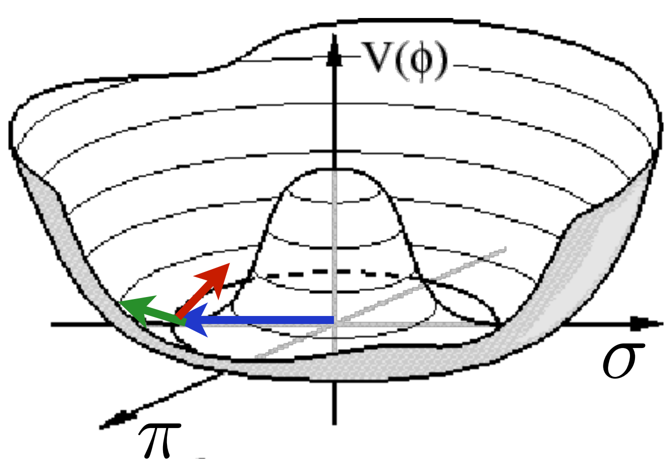
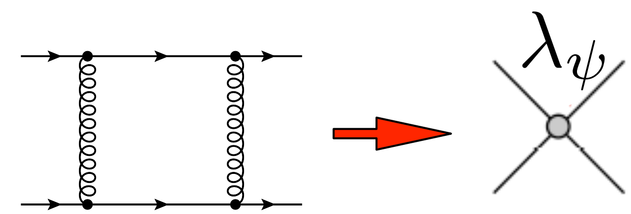


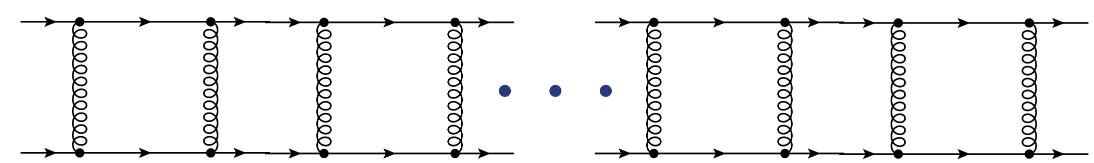
chiral symmetry



$\propto \alpha_s^2$

$$\int d^4x \, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2 \right]$$

strong correlations

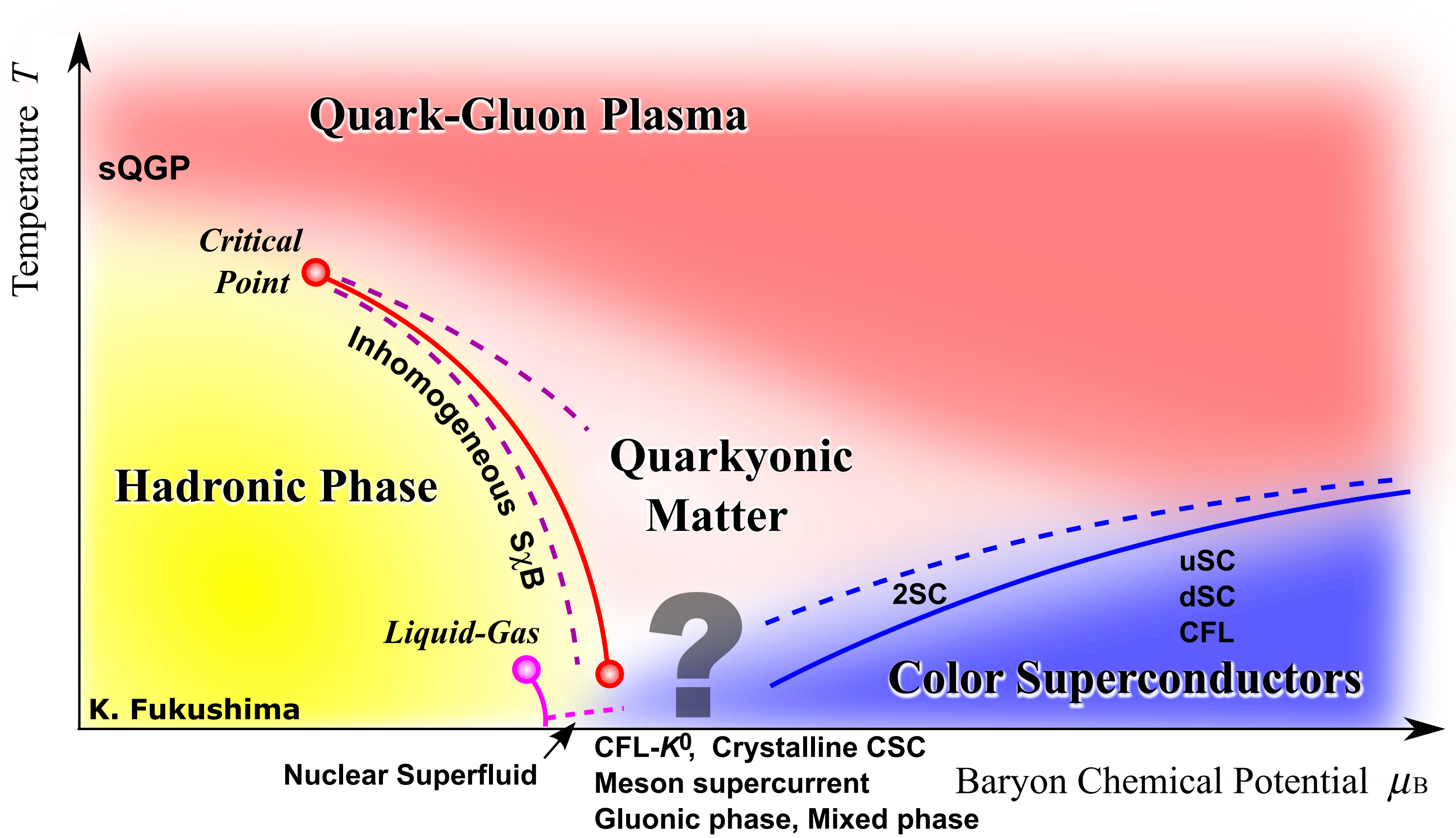


$$\langle \bar{q} q \rangle \neq 0$$

mass term: $\langle \bar{q} q \rangle \, \bar{q} q$

chiral symmetry broken

Phase diagram & order parameters



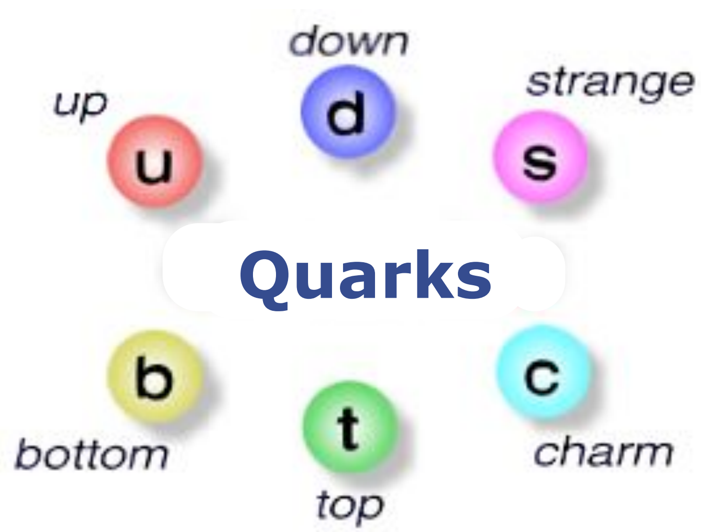
Phases in QCD

quarks massless - massive

quarks confined - deconfined

chiral condensate $\int_{\vec{x}} < \bar{q}(x)q(x) >$

Confinement

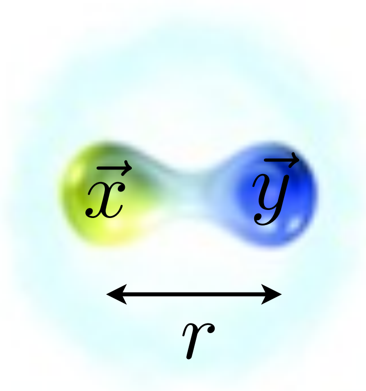


Quarks

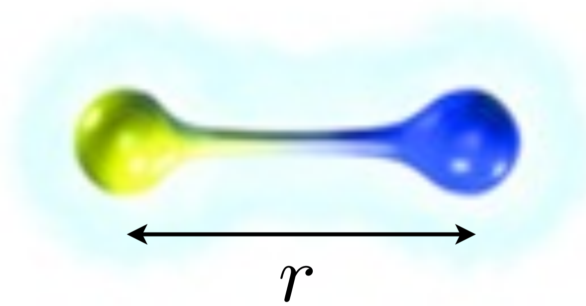
Gluons

Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

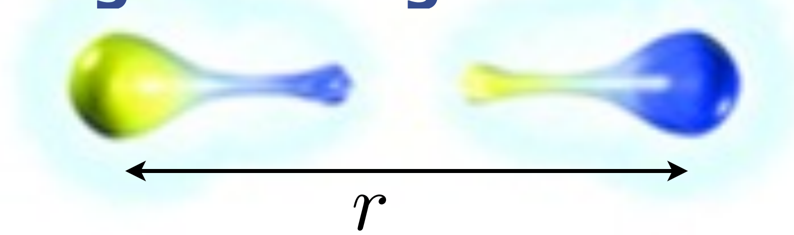


$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

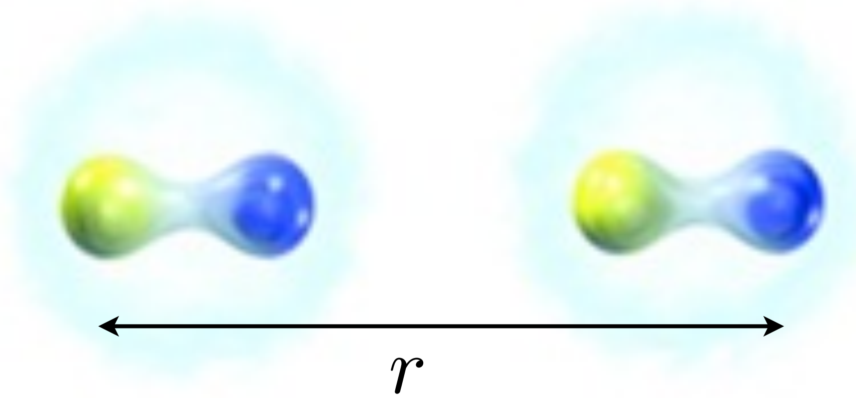


$$F_{q\bar{q}} \simeq \sigma r$$

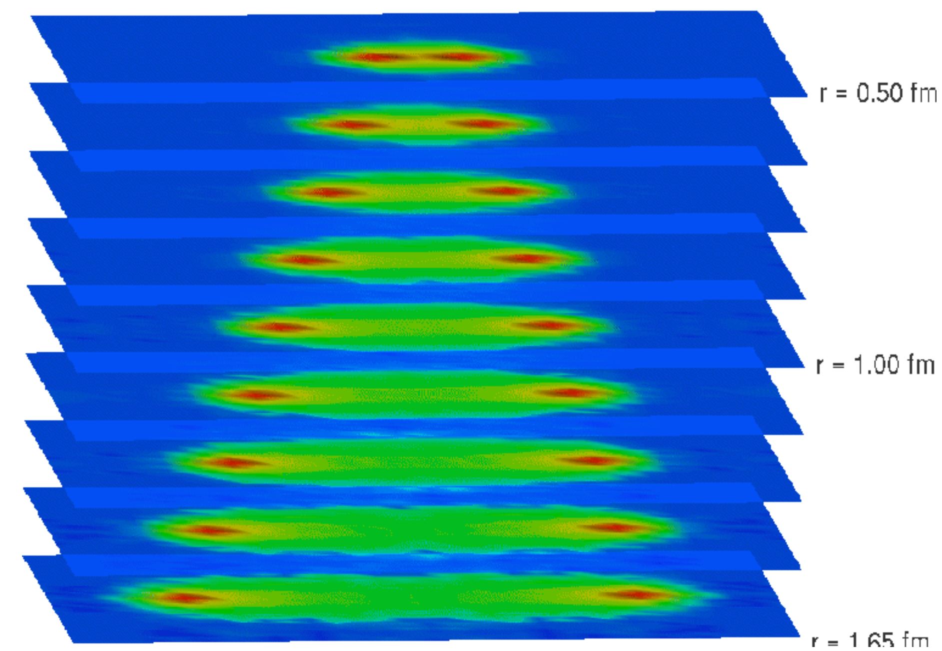
string breaking at $\mathrm{r} \approx 1\mathrm{fm}$



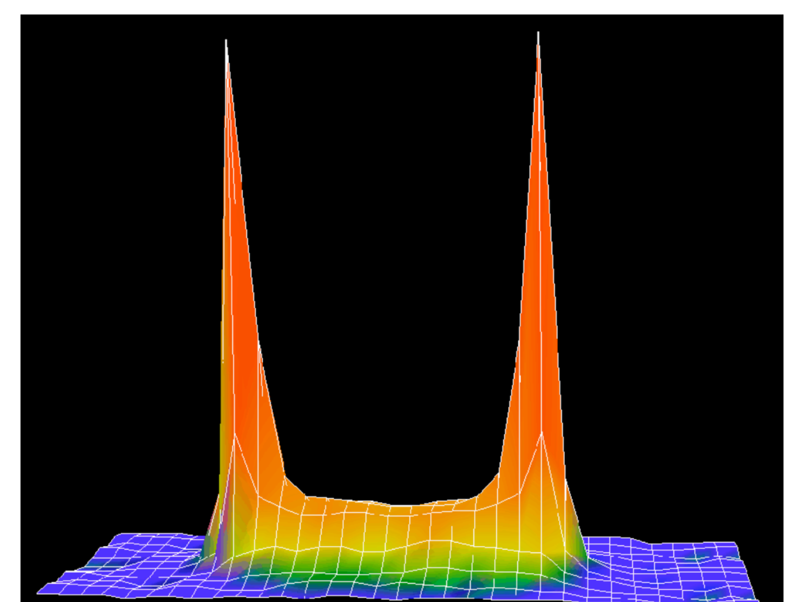
$$F_{q\bar{q}} \simeq \mathrm{const.}$$



pure gauge theory

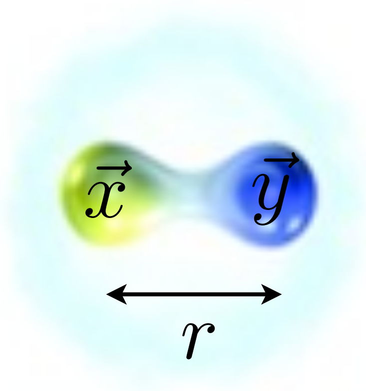


Energy density **Bali et al. '94**



Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair



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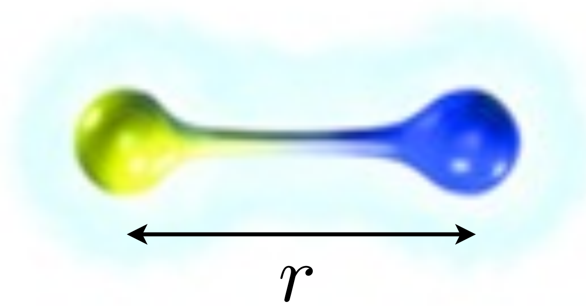

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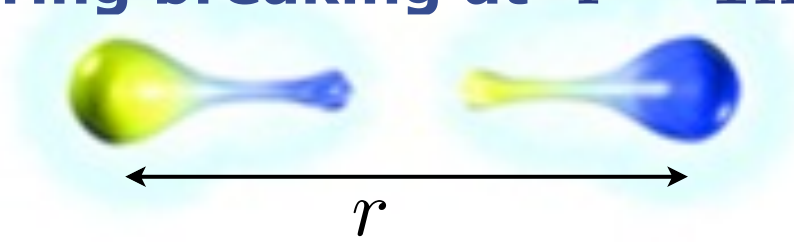
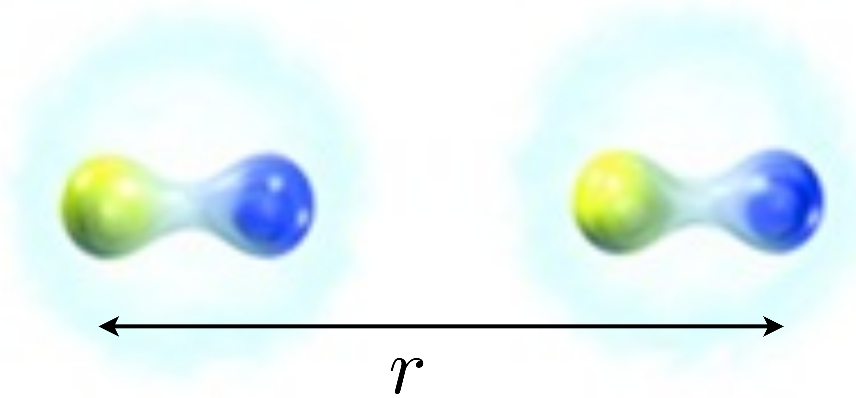
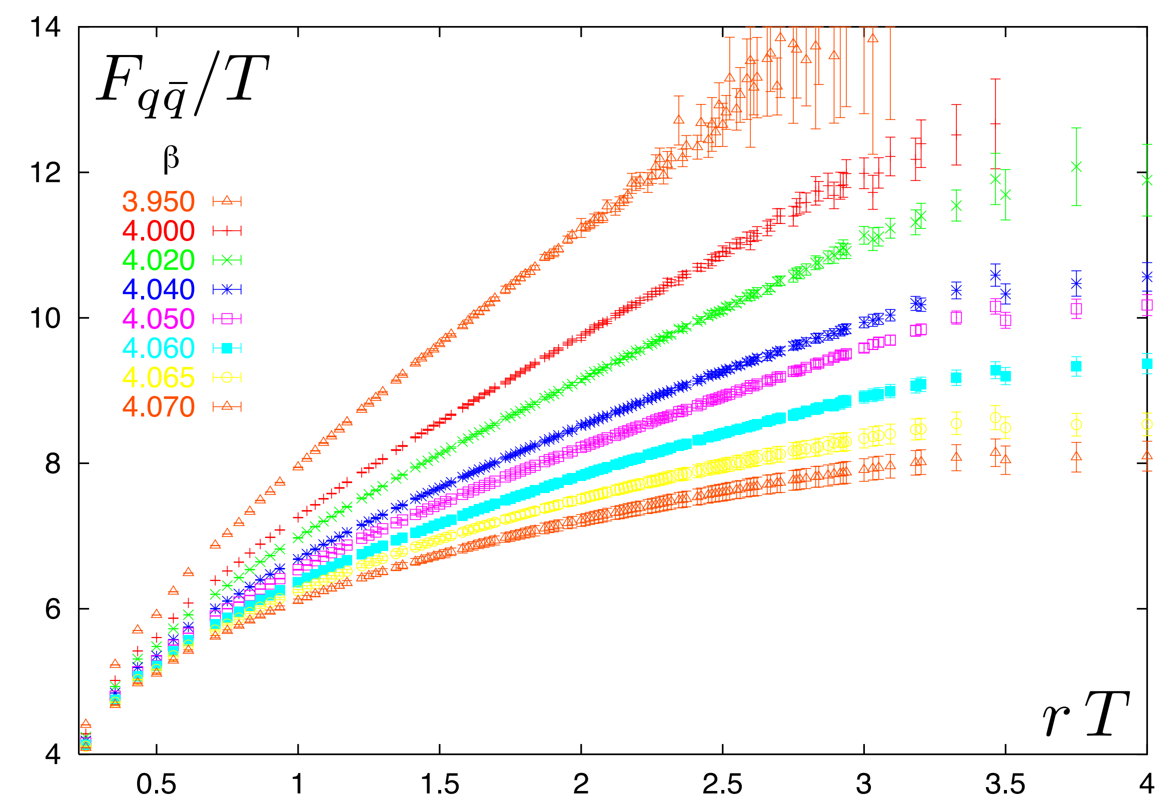


$$F_{q\bar{q}} \simeq \text{const.}$$



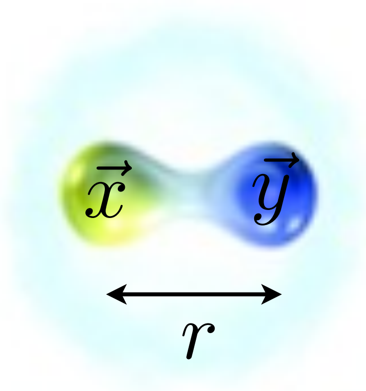
pure gauge theory



Kaczmarek et al '99

Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair



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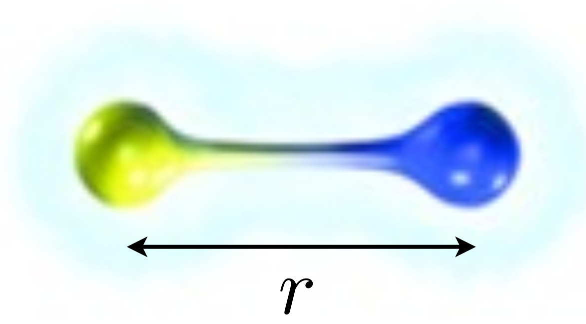

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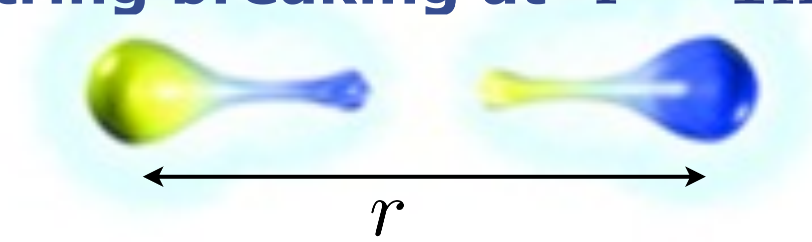
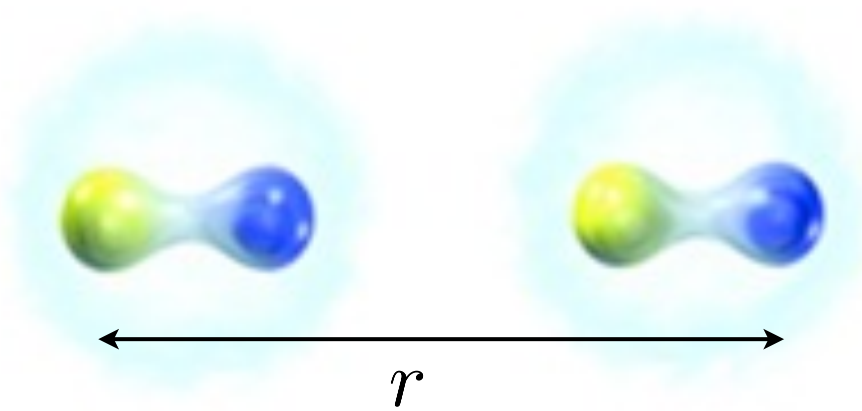




$$F_{q\bar{q}} \simeq \text{const.}$$

Order parameter $\sim \; '\langle q \rangle'$

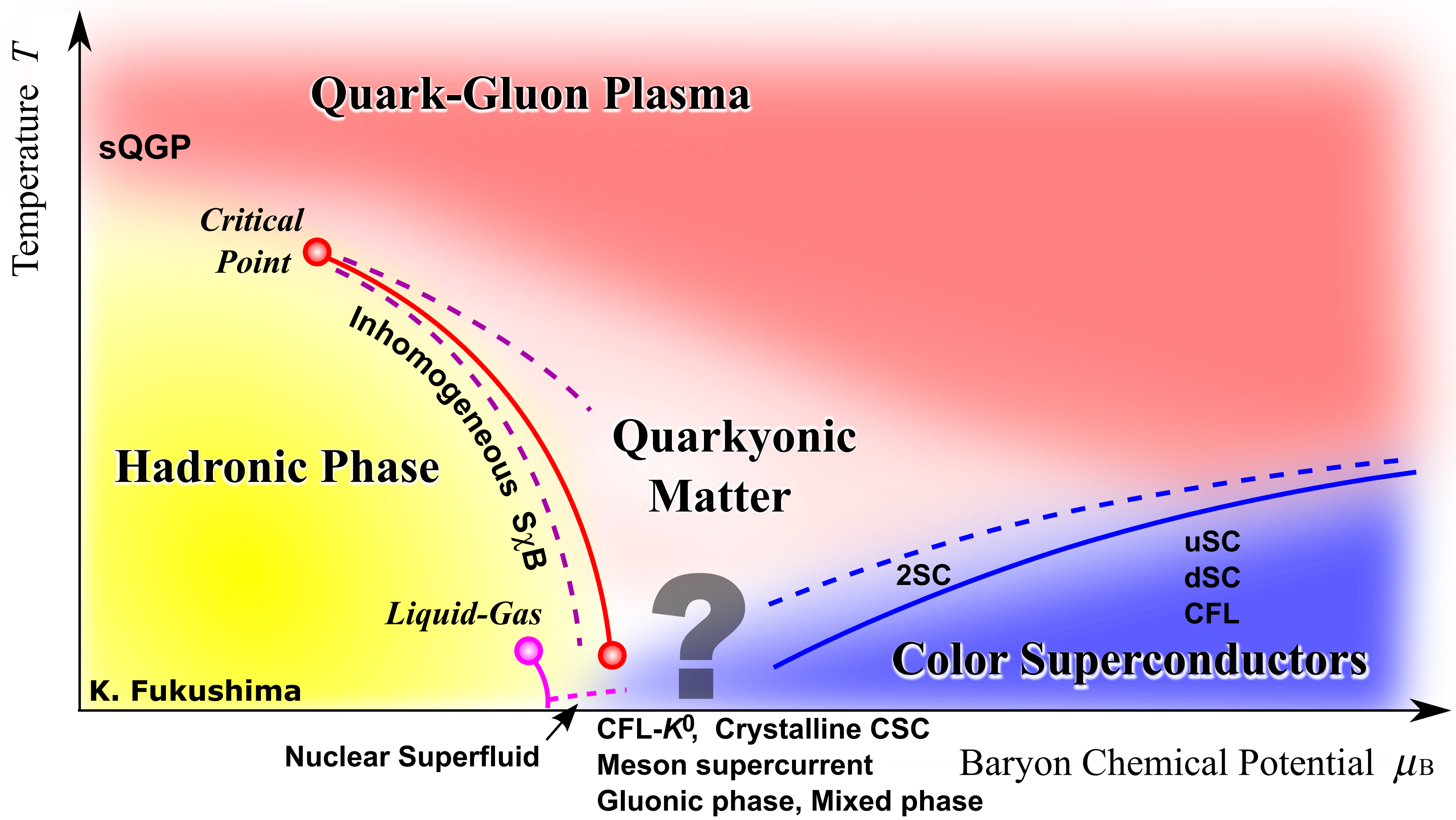
$$\Phi = e^{-\frac{1}{2T} F_{q\bar{q}}(\infty)}$$

- **Confinement** $\Phi = 0$
- **Deconfinement** $\Phi \neq 0$

Polyakov loop

$$\Phi = \frac{1}{3} \langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0\, A_0\} \rangle$$

Phase diagram & order parameters



Phases in QCD

quarks massless - massive

chiral condensate $\int_{\vec{\mathbf{x}}} < \bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{x}) >$

quarks confined - deconfined

Polyakov loop $\mathbf{\Phi} = \frac{\mathbf{1}}{\mathbf{N_c}} \langle \mathbf{tr}\, \mathcal{P} \mathbf{e}^{\mathbf{i\,g} \int_0^\beta \mathbf{A_0(x)}} \rangle$

(II) Functional Renormalisation group for QCD

- **Introduction to the functional renormalisation group**
  - Derivation of the flow equation
  - Expansion schemes
  - Optimisation and error control*

- **FRG for QCD**
  - FRG for QCD and T=0 Yang-Mills theories
  - Dynamical hadronisation
  - QCD correlation functions at T=0

Functional Renormalisation Group

Functional Renormalisation Group

Generating functional Z

$$Z[J] = \frac{1}{\mathcal{N}} \int d\varphi \, e^{-S[\varphi]+\int_x \, J\varphi}$$

partition function

$$\langle \varphi \rangle_J = \phi$$

$$S[\varphi] = \frac{1}{2} \int_x \left[\partial_\mu \varphi \partial_\mu \varphi + \mathrm{m}^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 \right]$$

classical action

zero-dimensional example: ‘Functional’ flows for integrals

Functional Renormalisation Group

Generating functional Z

$$Z[J] = \frac{1}{\mathcal{N}} \int d\varphi\, e^{-S[\varphi]+\int_x \, J\varphi}$$

partition function

$$\langle \varphi \rangle_J = \phi$$

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat\varphi\, e^{-S[\hat\varphi+\phi]+\int_x \, \hat\varphi \, \frac{\delta\Gamma[\phi]}{\delta\phi}}$$

free energy

$$\varphi = \hat\varphi + \phi$$

$$\langle \hat\varphi \rangle_{\frac{\delta\Gamma}{\delta\phi}} = 0$$

$$J = \frac{\delta\Gamma}{\delta\phi}$$

Functional Renormalisation Group

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$$\varphi = \hat{\varphi} + \phi$$

$$\langle \hat{\varphi} \rangle_{\frac{\delta\Gamma}{\delta\phi}} = 0$$

$$J = \frac{\delta \Gamma}{\delta \phi}$$

free energy

$$\Gamma[\phi] = \sup_J \left(\int_x J \cdot \phi - \log Z[J] \right)$$

Legendre transform

Functional Renormalisation Group

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$$\varphi = \hat{\varphi} + \phi$$

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Dyson-Schwinger equation

$$\frac{\delta \Gamma[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta \phi(x)} \right\rangle$$

quantum equation of motion

Functional Renormalisation Group

Dyson-Schwinger equation

$$\frac{\delta \Gamma[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi} + \phi]}{\delta \phi(x)} \right\rangle$$

Diagrammatics

$$S[\phi] = \frac{1}{2} \int_x \left[\partial_\mu \phi \partial_\mu \phi + \mathrm{m}^2 \phi^2 + \frac{\lambda}{4} \phi^4 \right]$$

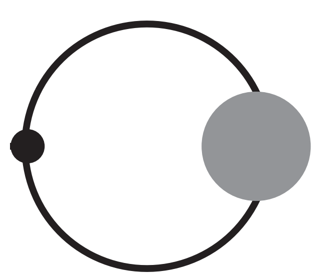
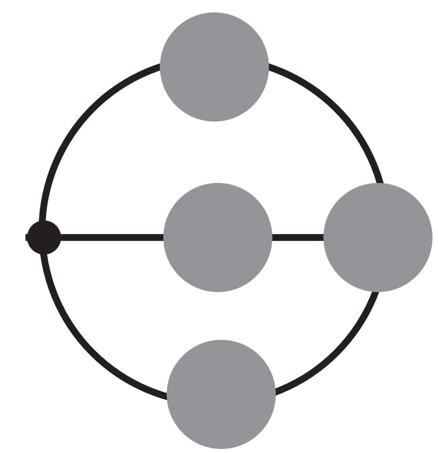
Functional Renormalisation Group

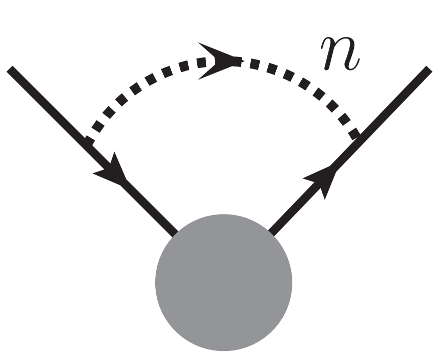
Dyson-Schwinger equation

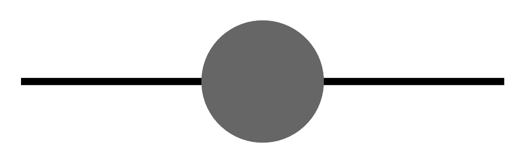
$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta\phi(x)} \right\rangle$$

Diagrammatics

$$S[\phi] = \frac{1}{2}\int_x \left[\partial_\mu\phi\partial_\mu\phi + \mathrm{m}^2\phi^2 + \frac{\lambda}{4}\phi^4\right]$$

$$\frac{\lambda}{2}\langle[\hat{\varphi}(x)+\phi(x)]^3\rangle = \frac{\lambda}{2}\phi^3(x) + \frac{3\lambda}{2}\phi(x)$$  $$- \quad \frac{\lambda}{2}$$ 

$$\mathbf{\Gamma}^{(\mathbf{n})} =$$ 

G =  $$= \langle\hat{\varphi}(x)\hat{\varphi}(y)\rangle$$

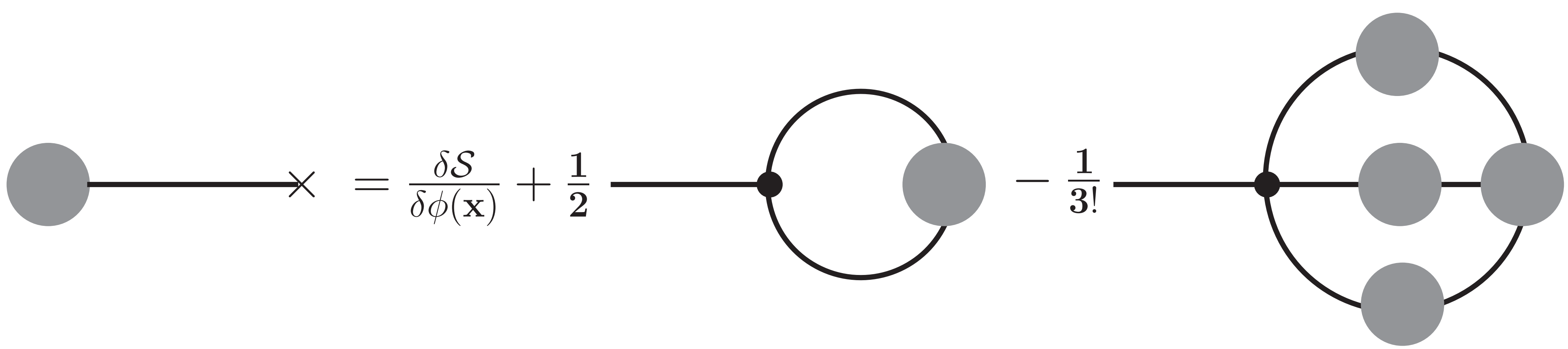
Functional Renormalisation Group

Dyson-Schwinger equation

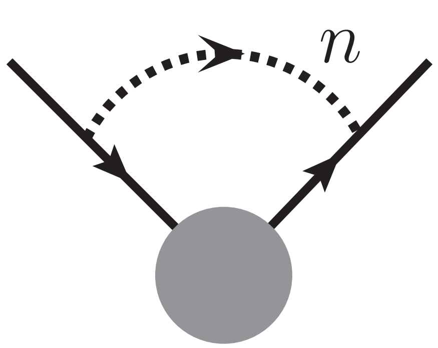
$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta\phi(x)} \right\rangle$$

Diagrammatics

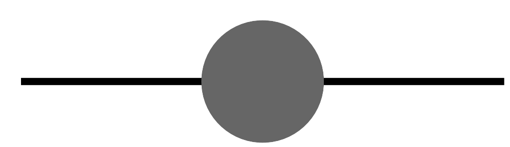
$$S[\phi] = \frac{1}{2}\int_x \left[\partial_\mu\phi\partial_\mu\phi + \mathrm{m}^2\phi^2 + \frac{\lambda}{4}\phi^4\right]$$



$\Gamma^{(\mathbf{n})} =$



G =



$= \langle\hat{\varphi}(x)\hat{\varphi}(y)\rangle$

Functional Renormalisation Group

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi]+\int_x \hat{\varphi}\, \frac{\delta\Gamma[\phi]}{\delta\phi}}$$

No quantum fluctuations

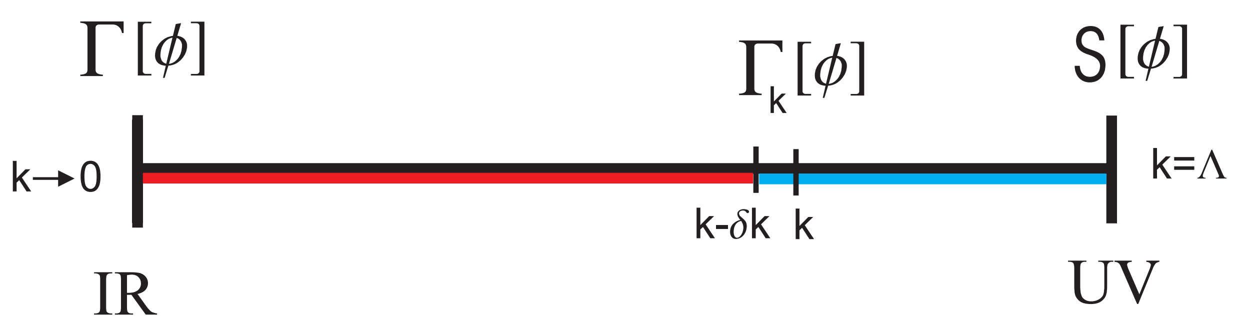
$$\Gamma[\phi] = -\log e^{-S[\phi]} = S[\phi]$$

Functional Renormalisation Group

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi] + \int_x \hat{\varphi}\, \frac{\delta\Gamma[\phi]}{\delta\phi}}$$

UV quantum fluctuations up to $p^2 = k^2$



Functional Renormalisation Group

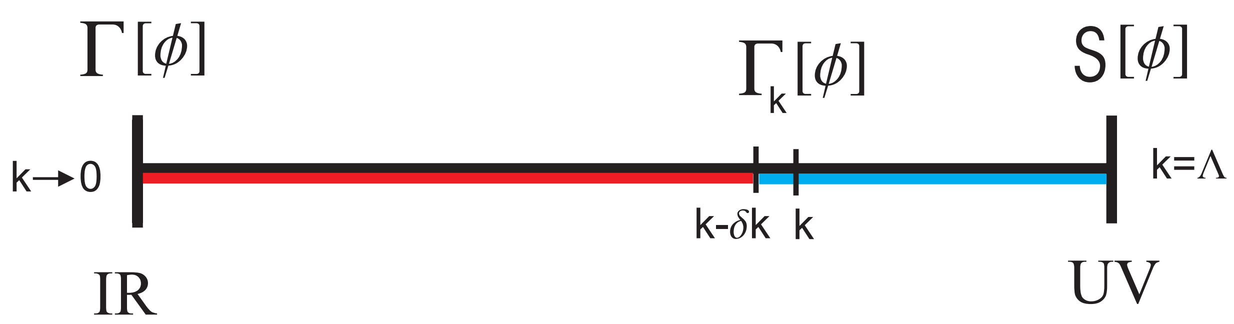
Effective action Γ_k

$$\Gamma_k[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi] + \frac{1}{2}\int_p \hat{\varphi}(p) R_k(p^2)\hat{\varphi}(-p) + \int_x \hat{\varphi}\,\frac{\delta \Gamma_k[\phi]}{\delta \phi}}$$

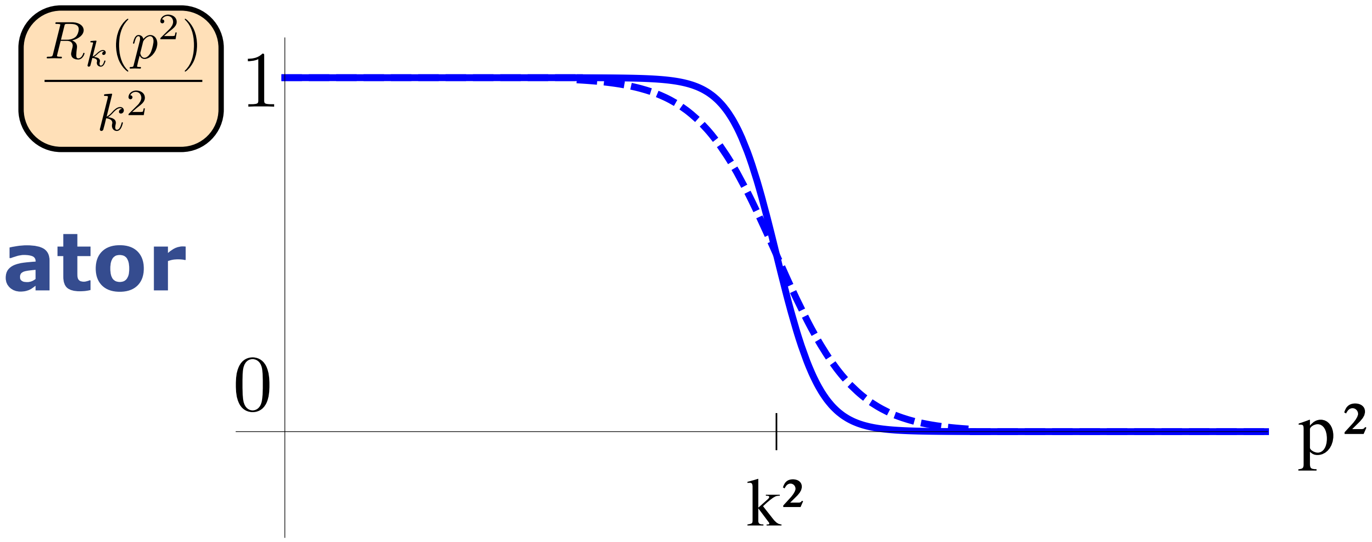
$$\frac{\delta \Gamma_k[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta \phi(x)} \right\rangle$$

DSE

UV quantum fluctuations up to $p^2 = k^2$



Regulator

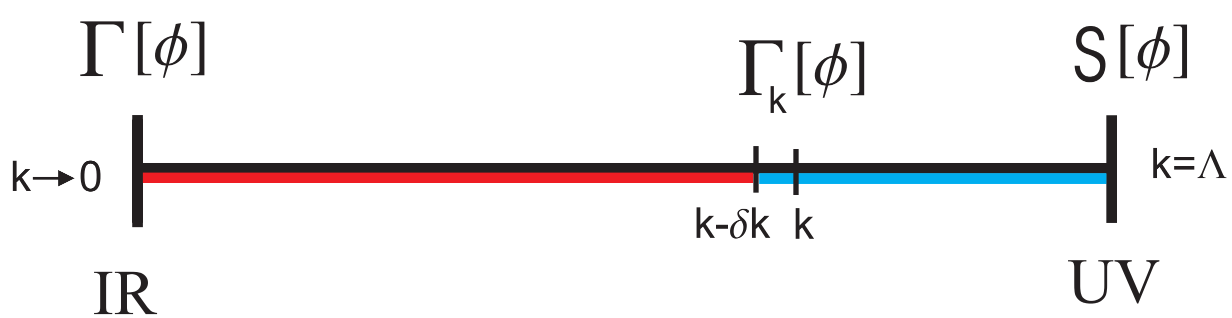


Functional Renormalisation Group

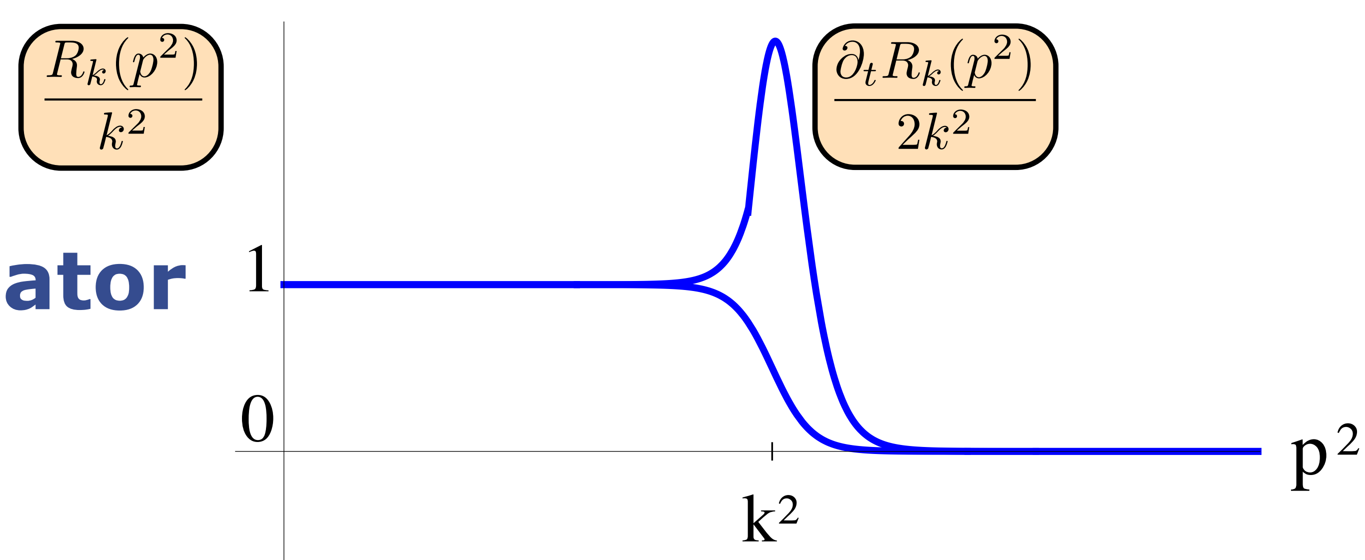
Effective action Γ_k

$$\Gamma_k[\phi] = -\log \int d\hat\varphi\, e^{-S[\hat\varphi+\phi]+\frac{1}{2}\int_p \hat\varphi(p) R_k(p^2)\hat\varphi(-p)+\int_x \hat\varphi \frac{\delta\Gamma_k[\phi]}{\delta\phi}}$$

UV quantum fluctuations up to $p^2 = k^2$



Regulator



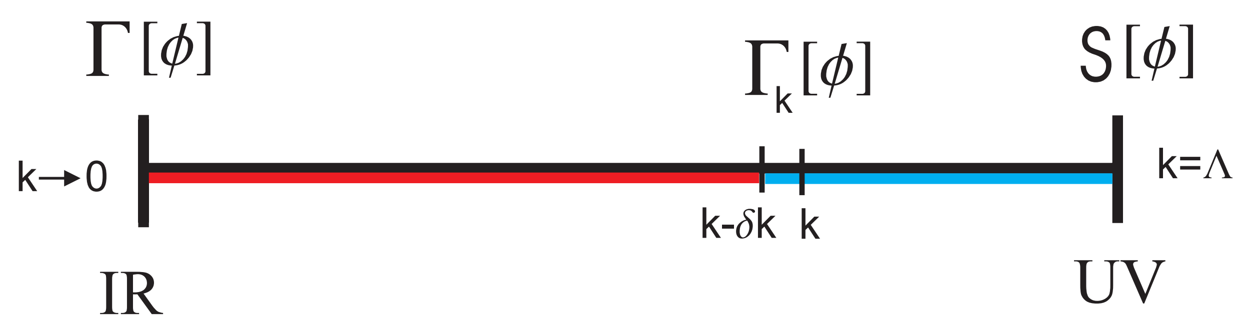
$$t = \log \frac{k}{\Lambda}$$

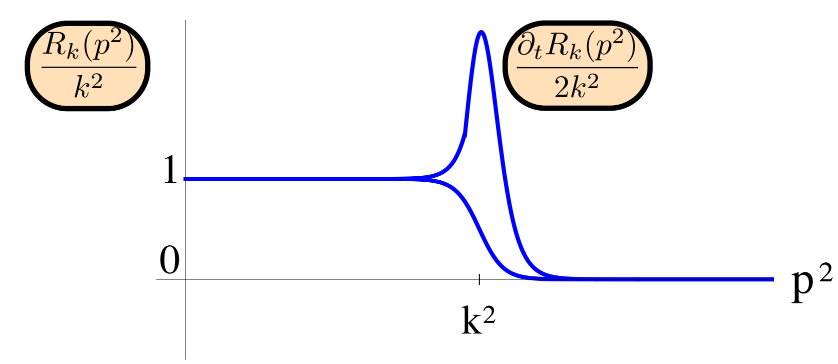
Functional Renormalisation Group

Effective action Γ_k

$$\Gamma_k[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi] + \frac{1}{2}\int_p \hat{\varphi}(p) R_k(p^2)\hat{\varphi}(-p) + \int_x \hat{\varphi}\, \frac{\delta \Gamma_k[\phi]}{\delta \phi}}$$

UV quantum fluctuations up to $p^2 = k^2$





Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \langle \hat{\varphi}(p)\hat{\varphi}(-p)\rangle\, \partial_t R_k(p^2)$$

$$t = \log \frac{k}{\Lambda}$$

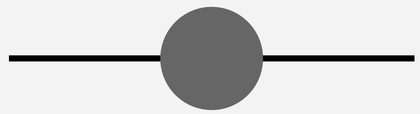
Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \, \langle \hat{\varphi}(p) \hat{\varphi}(-p) \rangle \, \partial_t R_k(p^2)$$

$$t = \log \frac{k}{\Lambda}$$

Propagator

G =  $= \langle \hat{\varphi}(x) \hat{\varphi}(y) \rangle$

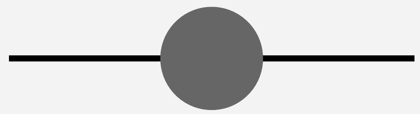
Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\,\langle \hat\varphi(p)\hat\varphi(-p)\rangle\,\partial_t R_k(p^2)$$

$$t = \log\frac{k}{\Lambda}$$

Propagator

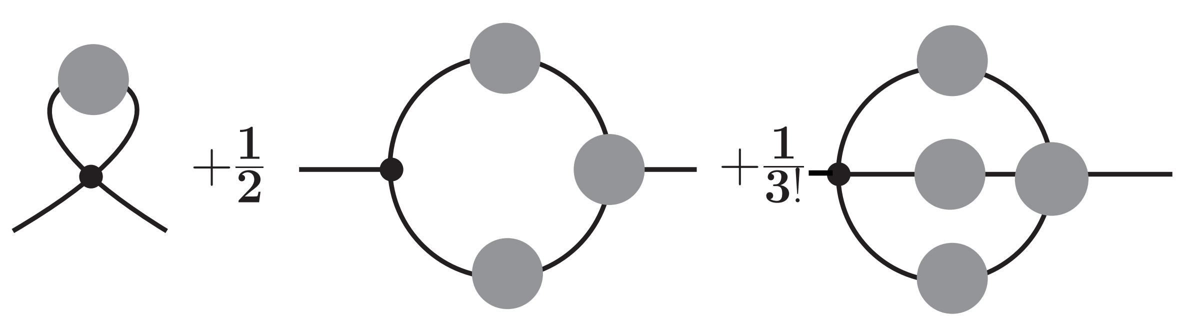
G =  $= \langle \hat\varphi(x)\hat\varphi(y)\rangle$

$$\frac{\delta\Gamma_k[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat\varphi+\phi]}{\delta\phi(x)}\right\rangle$$

$$\Gamma_k[\phi] = -\log\int d\hat\varphi\, e^{-S[\hat\varphi+\phi]+\frac{1}{2}\int_p \hat\varphi(p)R_k(p^2)\hat\varphi(-p)+\int_x \hat\varphi\,\frac{\delta\Gamma_k[\phi]}{\delta\phi}}$$

DSE

$$\Gamma_k^{(2)}[\phi] - S^{(2)}[\phi] = \frac{1}{2}$$



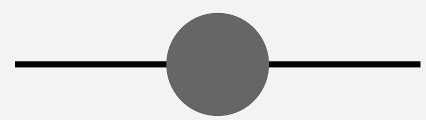
Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \, \langle \hat{\varphi}(p) \hat{\varphi}(-p) \rangle \, \partial_t R_k(p^2)$$

$$t = \log \frac{k}{\Lambda}$$

Propagator

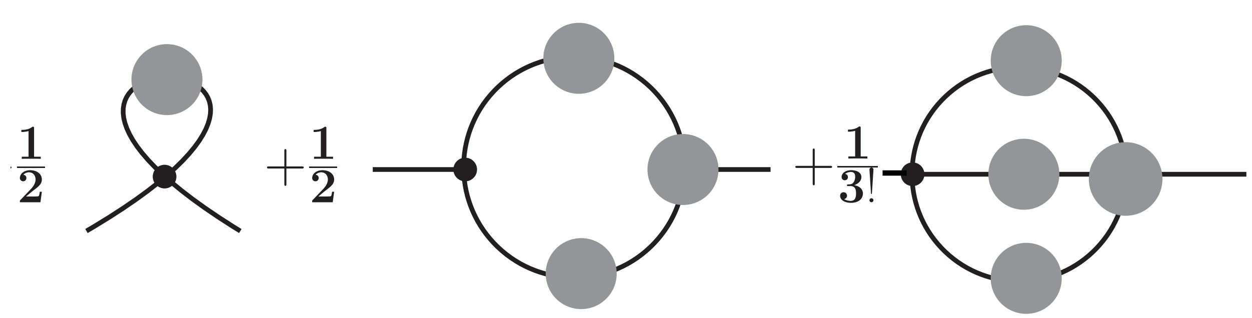
G =  $= \langle \hat{\varphi}(x) \hat{\varphi}(y) \rangle$

$$G^{-1}[\phi] = \Gamma_k^{(2)}[\phi] + R_k$$

$$\frac{\delta \Gamma_k[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi} + \phi]}{\delta \phi(x)} \right\rangle$$

$$\Gamma_k[\phi] = -\log \int d\hat{\varphi} \, e^{-S[\hat{\varphi}+\phi] + \frac{1}{2} \int_p \hat{\varphi}(p) R_k(p^2) \hat{\varphi}(-p) + \int_x \hat{\varphi} \frac{\delta \Gamma_k[\phi]}{\delta \phi}}$$

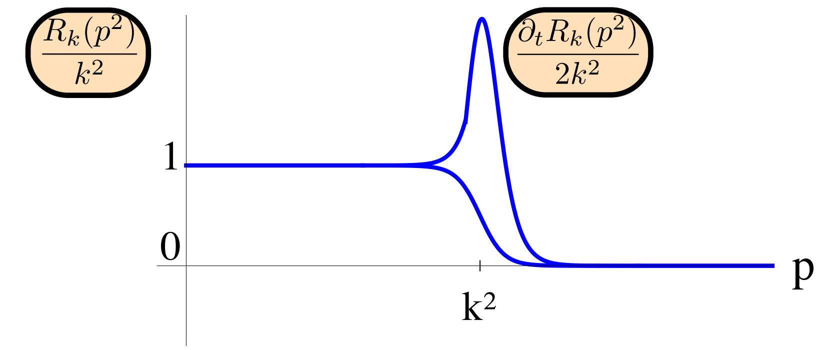
DSE

$\Gamma_k^{(2)}[\phi] - S^{(2)}[\phi] =$ 

Functional Renormalisation Group

Flow

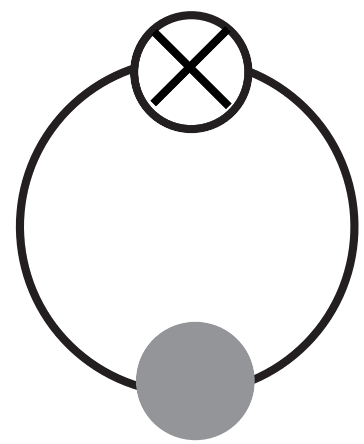
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



regulator

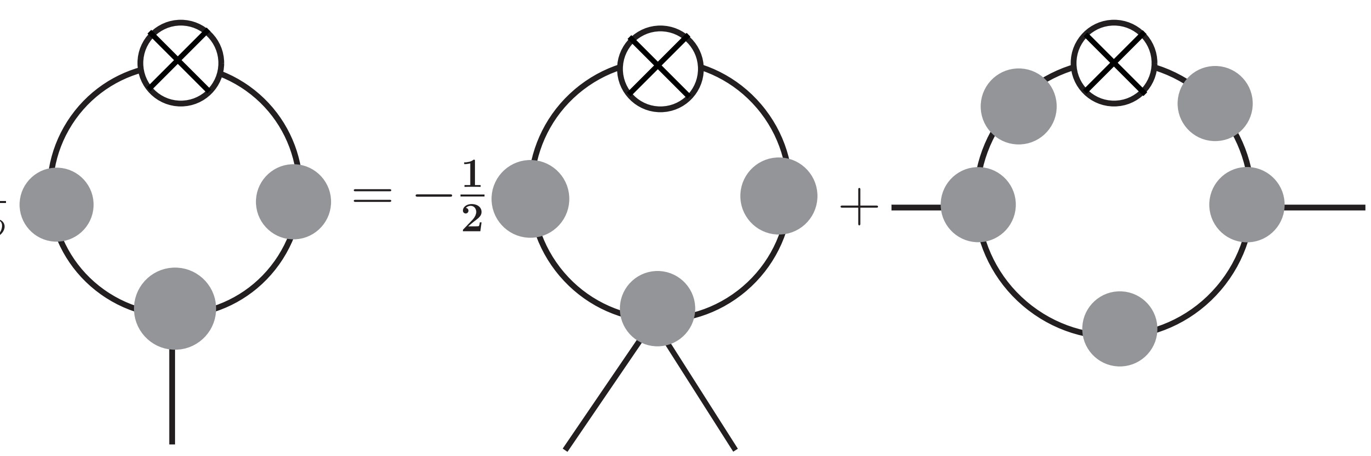
Diagrammatics

$$\partial_t \mathbf{\Gamma_k}[\phi] = \frac{1}{2}$$



Propagator

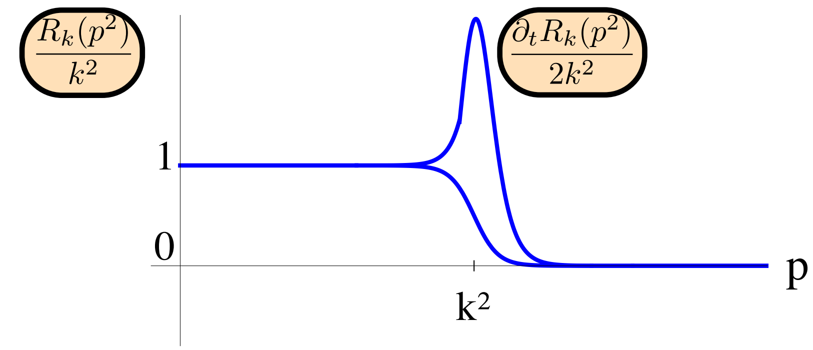
$$\partial_t \mathbf{\Gamma_k^{(2)}}[\phi] = -\frac{1}{2}\frac{\delta}{\delta\phi} \quad = -\frac{1}{2} \quad +$$



Functional Renormalisation Group

Flow

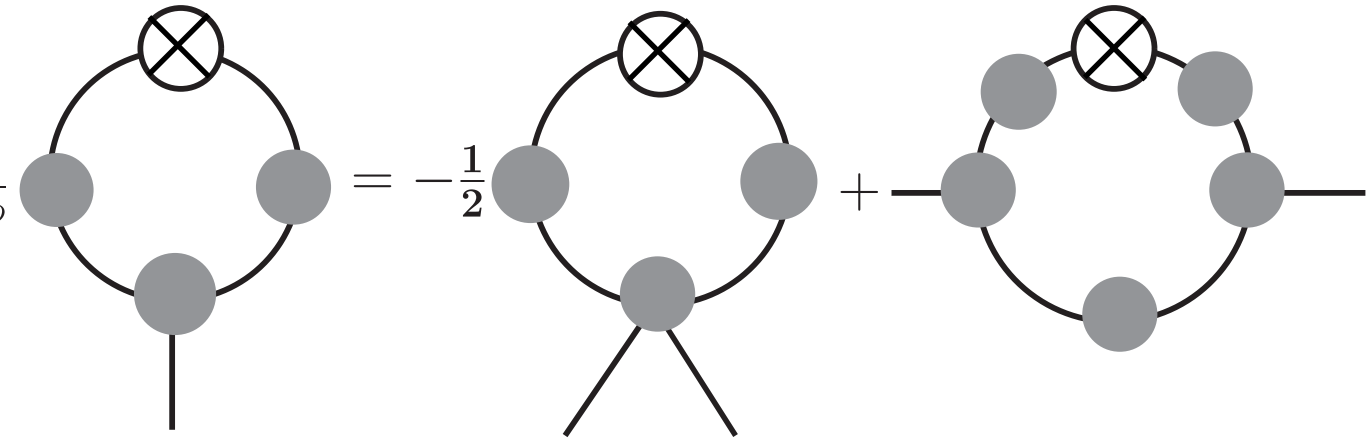
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



Propagator

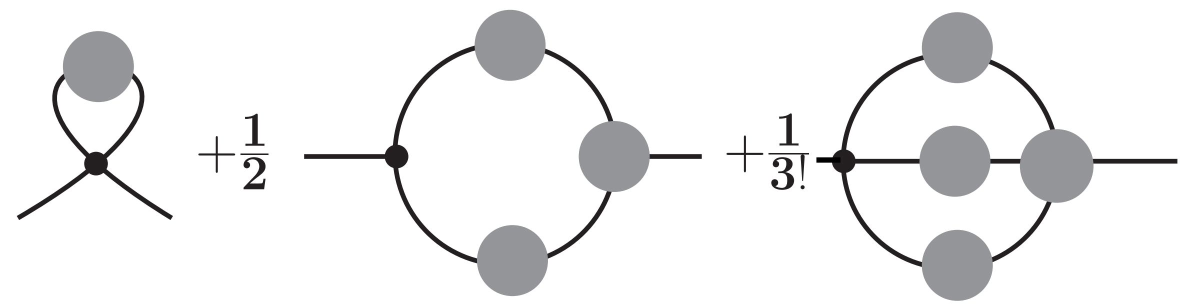
FRG

$$\partial_t \mathbf{\Gamma_k^{(2)}}[\phi] = -\frac{1}{2}\frac{\delta}{\delta\phi}$$



DSE

$$\Gamma^{(2)}[\phi] - S^{(2)}[\phi] = \frac{1}{2}$$



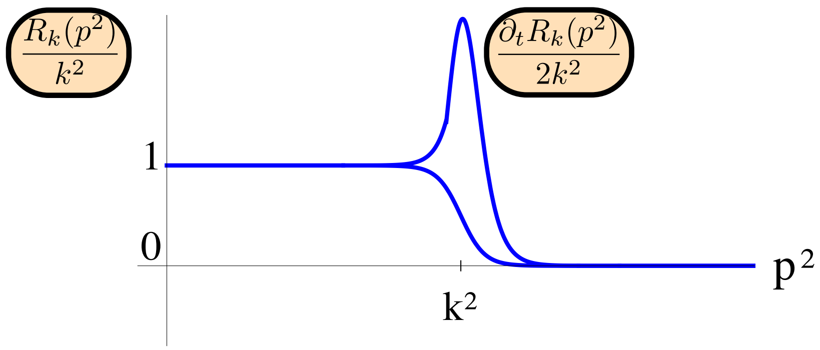
$$\partial_t \Gamma^{(n)} = \mathrm{Flow}_n[\Gamma^{(m)}; m = 2, ..., n+2]$$

$$\Gamma^{(n)} = \mathrm{DSE}_n[S^{(m)}, \Gamma^{(m)}; m = 2, ..., n+2]$$

Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



Properties

	FRG	DSE	2PI	3PI	4PI
• 1-loop exact	✓	–			
• closed	✓	✓			
• RG-scaling	✓	–	–	–	✓
• Energy/particle-number conserv.	–	–	✓	✓	✓

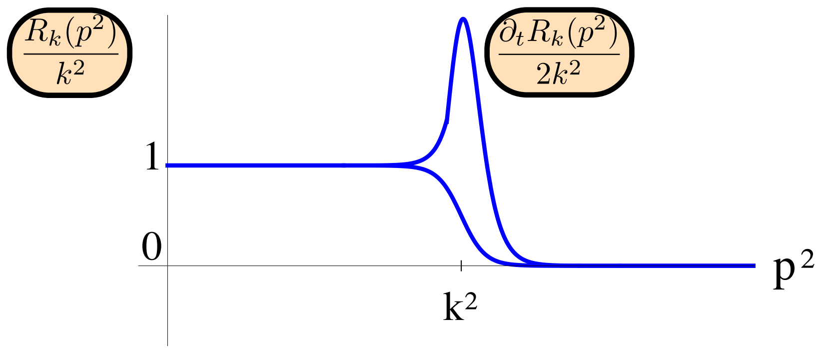
✓ automatic

– only in specific approximation schemes

Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



Properties

FunMethods

- **1-loop exact** ✓
- **closed** ✓
- **RG-scaling** ✓
- **Energy/particle-number conserv.** ✓

✓ **automatic**

– **only in specific approximation schemes**

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

$$\partial_t \Gamma^{(n)} = \mathrm{Flow}_n[\Gamma^{(m)}; m = 2, ..., n+2]$$

Derivative expansion

- Expansion in powers of momenta
- controlled in the presence of a mass gap m_{gap}
- Expansion parameter $\dfrac{p^2}{\max(k^2, m_{\mathrm{gap}}^2)}$

Vertex expansion

- Expansion in number n of external fields
- controlled in perturbation theory/presence of symmetries
- Expansion parameter n

Mixtures, exact resummation schemes, ...

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k}\partial_t R_k$$

Derivative expansion

- **Expansion in powers of momenta**
- **controlled in the presence of a mass gap** m_{gap}
- **Expansion parameter** $\dfrac{p^2}{\max(k^2, m_{\mathrm{gap}}^2)}$

Lowest order: 0th order

$$\Gamma_k[\phi] = \frac{1}{2}\int_p \phi\, p^2 \phi + \int_x V_k(\phi) + O(p^2)$$

$$\Gamma_k^{(2)}[\phi](p,q) = \left(p^2 + V_k''(\phi)\right)(2\pi)^d \delta(p-q)$$

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

Lowest order: 0th order

$$\Gamma_k[\phi] = \frac{1}{2} \int_p \phi \, p^2 \phi + \int_x V_k(\phi) + O(p^2)$$

$$\Gamma_k^{(2)}[\phi](p,q) = \left(p^2 + V_k''(\phi)\right) (2\pi)^d \delta(p-q)$$

$$R_{k,\mathrm{opt}}(p^2) = (k^2 - p^2)\theta(k^2 - p^2)$$

$$\partial_t R_{k,\mathrm{opt}}(p^2) = 2k^2 \theta(k^2 - p^2)$$

Flow

$$\partial_t V_k[\phi] = \frac{1}{2d} \frac{\Omega_d}{(2\pi)^d} k^d \frac{k^2}{k^2 + V''(\phi)}$$

$$\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

Lowest order: 0th order

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$$\Gamma_k^{(2)}[\phi](p,q) = \left(p^2 + V_k''(\phi)\right) (2\pi)^d \delta(p-q)$$

$$\Gamma^{(2)}[\phi](p) + R_{k,\mathrm{opt}}(p^2) = \left[k^2 + V''(\phi)\right] \theta(k^2 - p^2) + (p^2 + V''(\phi))\theta(p^2 - k^2)$$

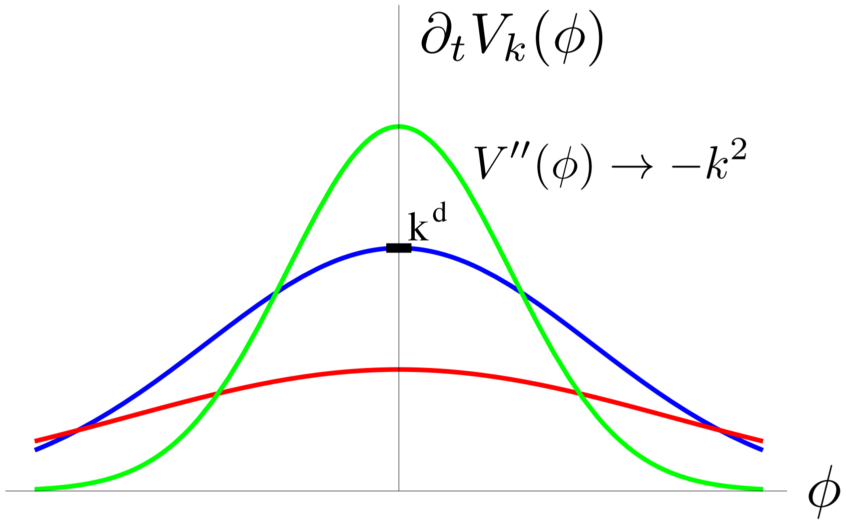
Flow

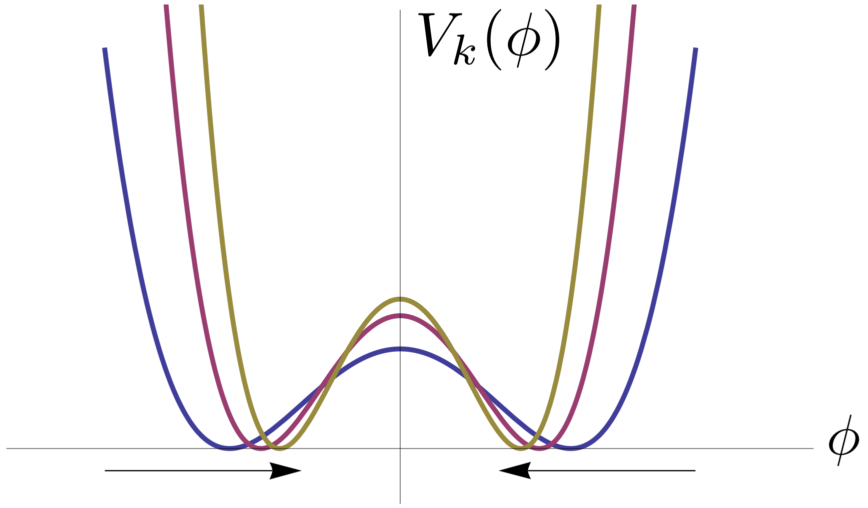
$$\partial_t V_k[\phi] = \frac{1}{2d} \frac{\Omega_d}{(2\pi)^d} k^d \frac{k^2}{k^2 + V''(\phi)}$$

$$\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

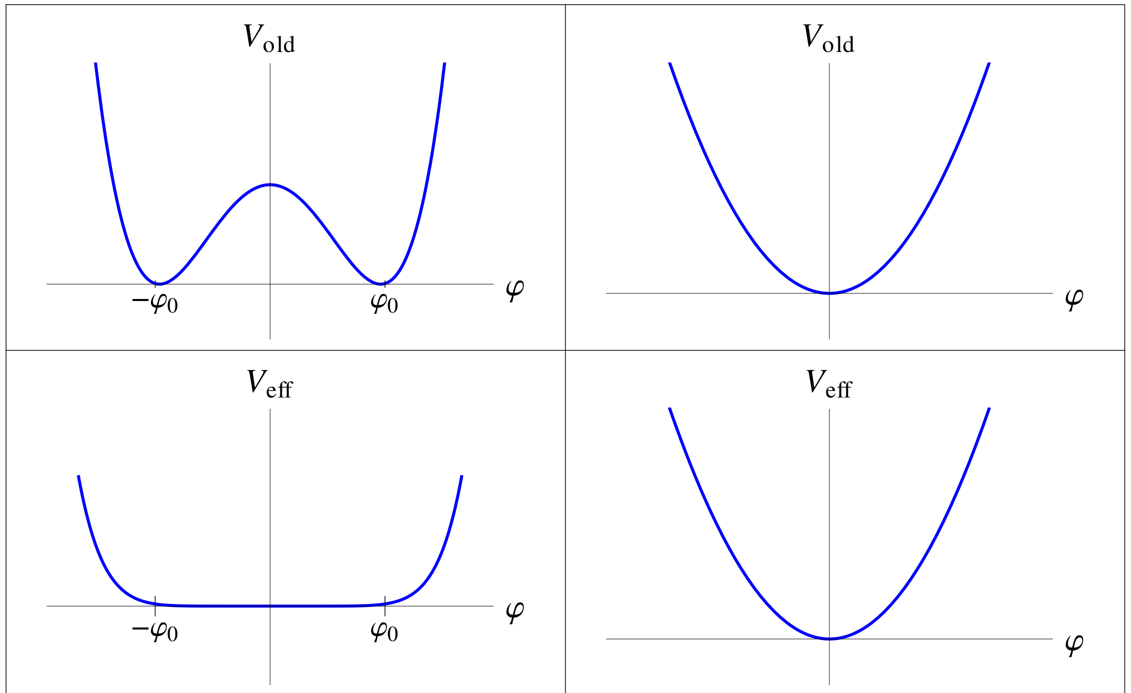
Approximation schemes & phase structure

$$\partial_t V_k[\phi] = \frac{1}{2d}\frac{\Omega_d}{(2\pi)^d}k^d\frac{k^2}{k^2+V''(\phi)}$$



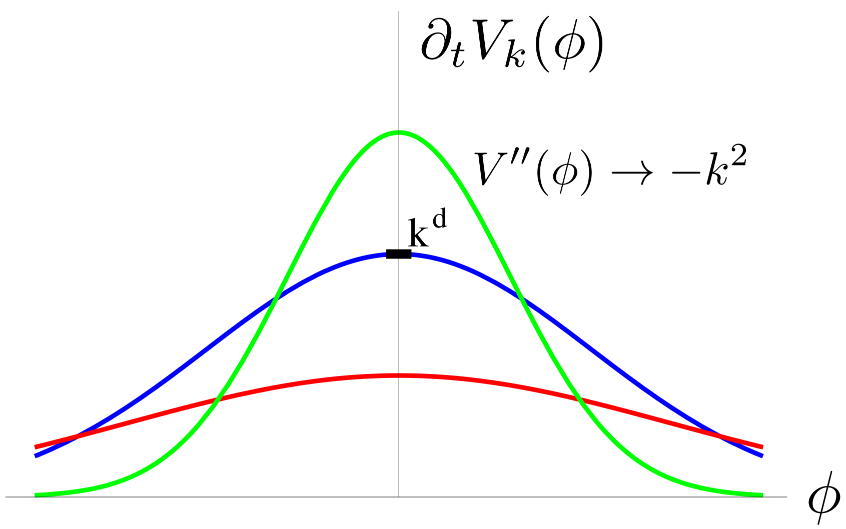


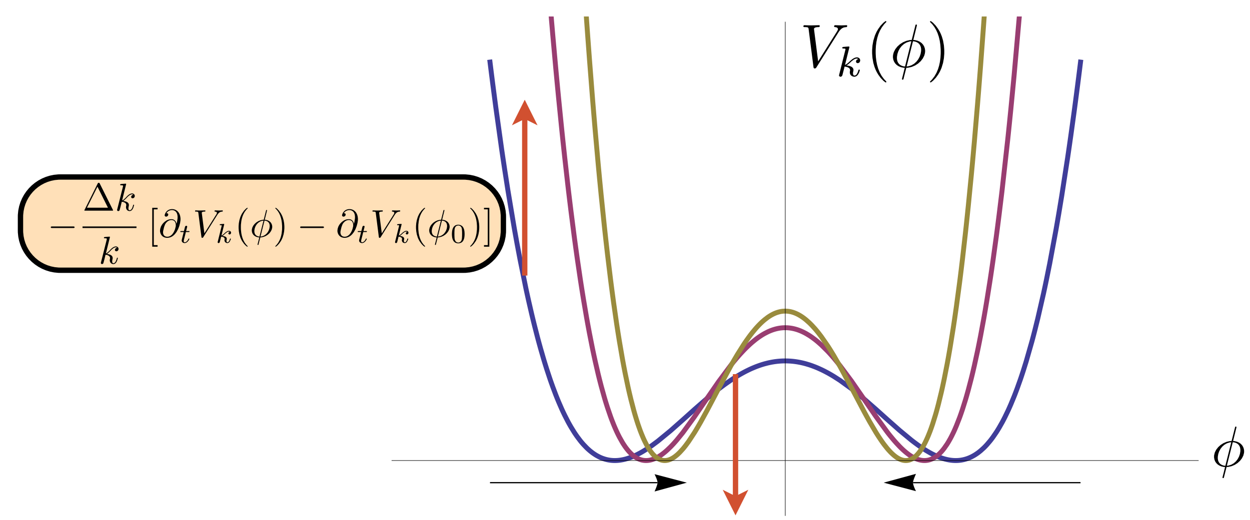
- **bosonic flow is symmetry-restoring**
- **flow guarantees convexity**



Approximation schemes & phase structure

$$\partial_t V_k[\phi] = \frac{1}{2d}\frac{\Omega_d}{(2\pi)^d}k^d\frac{k^2}{k^2+V''(\phi)}$$





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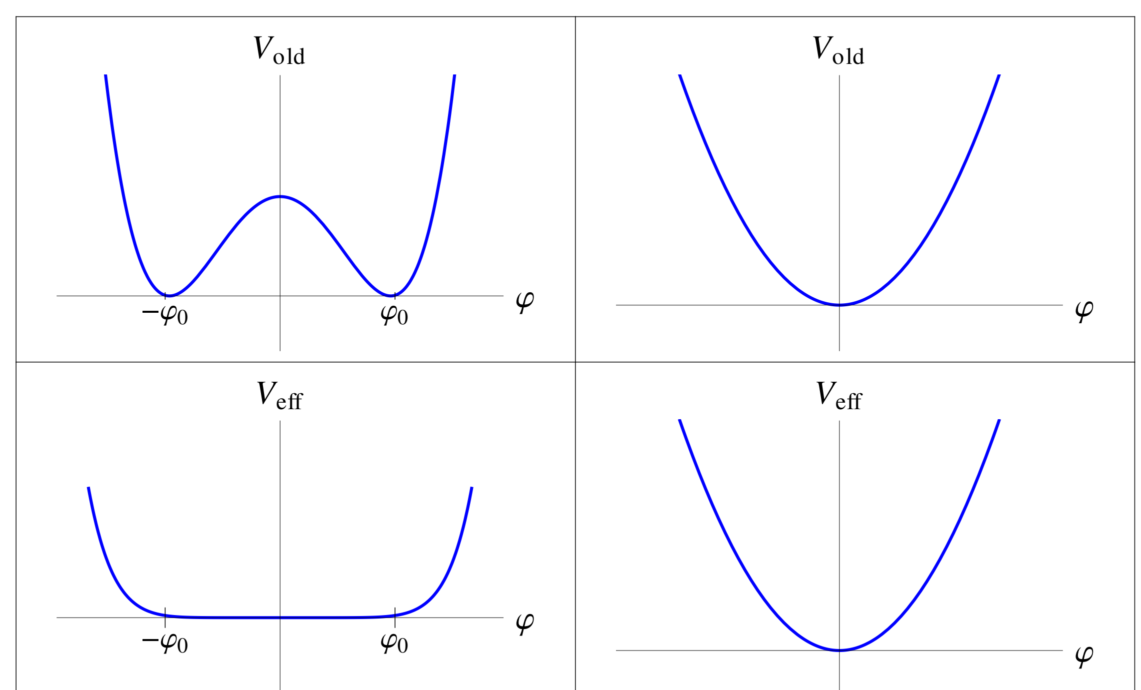
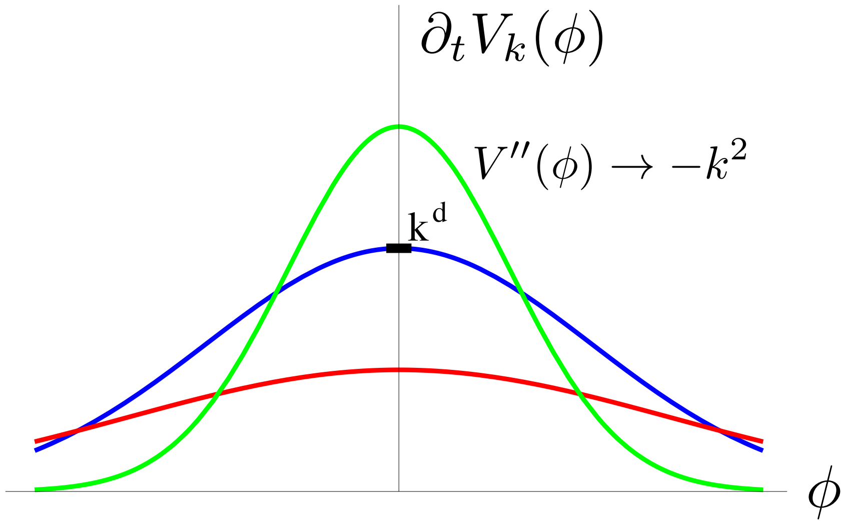
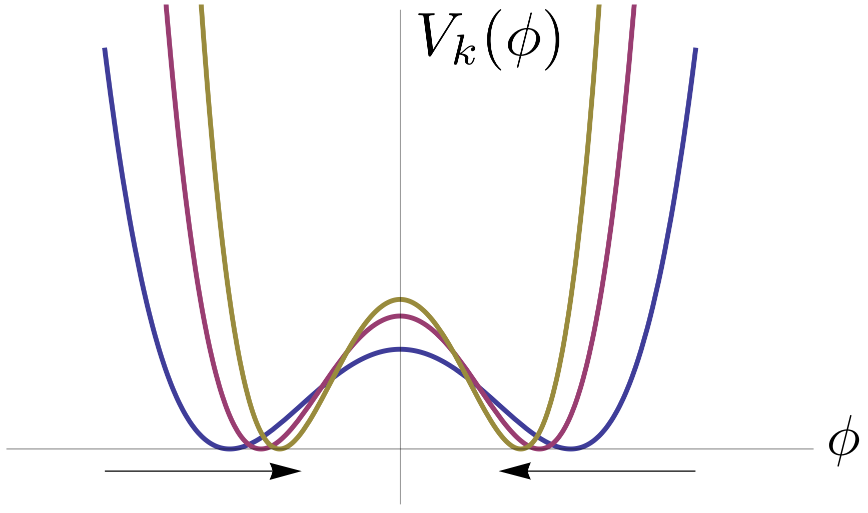


Approximation schemes & phase structure

$$\partial_t V_k[\phi] = \frac{1}{2d}\frac{\Omega_d}{(2\pi)^d} k^d \frac{k^2}{k^2 + V''(\phi)}$$





- **bosonic flow is symmetry-restoring**
- **flow guarantees convexity of effective action**

Litim, JMP, Vergara '06

Example: 3d critical exponents with FRG

$$\Gamma_k[\phi] = \frac{1}{2}\int_p Z_k\, \phi\, p^2 \phi + \int_x V_k(\phi)$$

$$V_k(\phi) = \sum_{n=1}^{N_{\max}} \frac{\lambda_n}{n!}(\phi^2 - \phi_{0,k}^2)^n$$

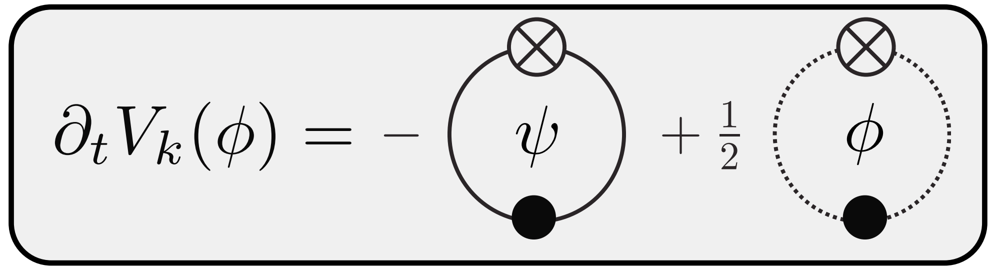
$N = 1: \; \nu_{\text{Ising}} = 0.630...$

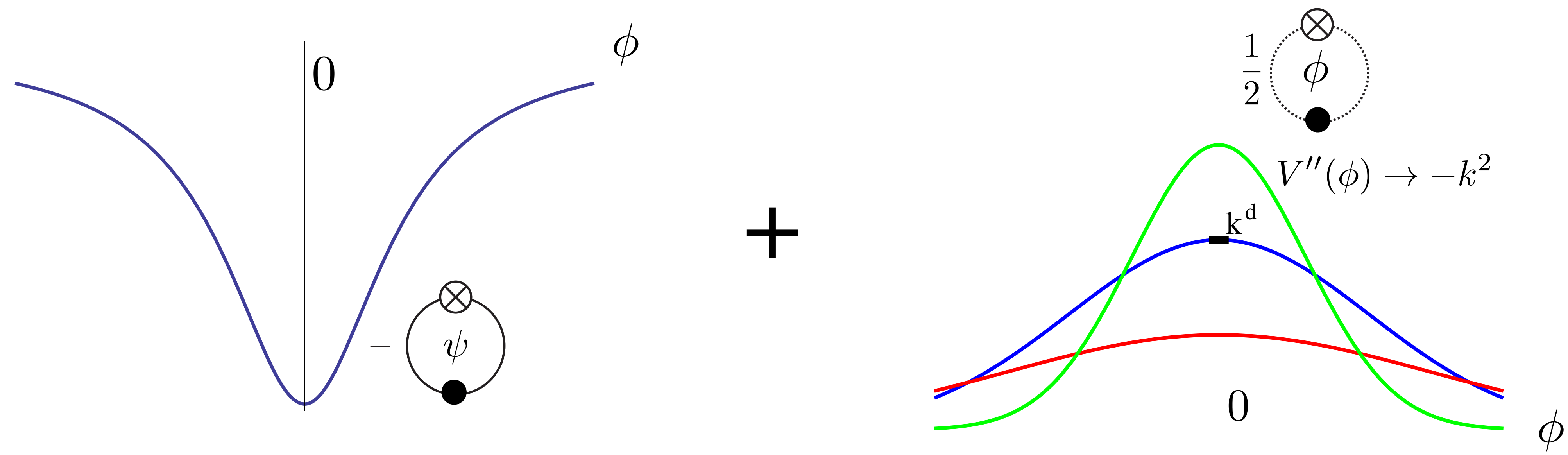
$N = 1: \; \nu_{\text{Ising}} = 0.637...$

A simple program to compute critical exponents in O(N)-models with the Wetterich equation

Michael Scherer

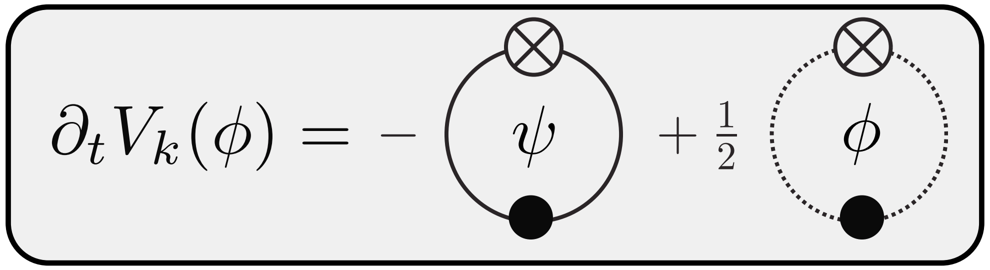
Approximation schemes & phase structure

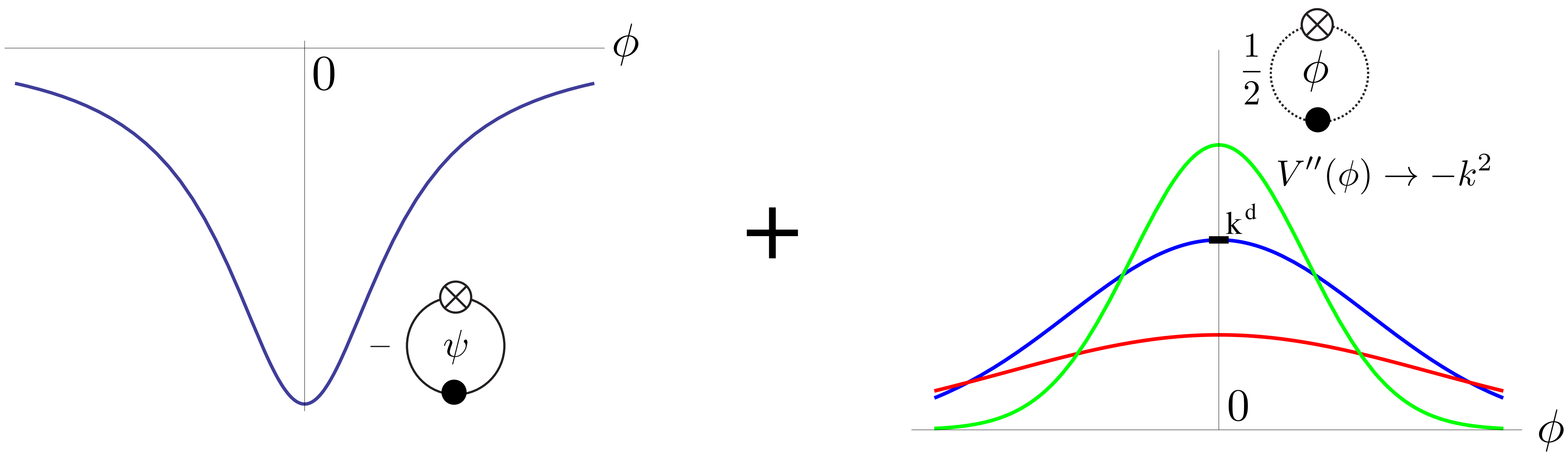




- **bosonic flow is symmetry-restoring**
- **fermionic flow is symmetry-breaking**
- **competing dynamics decides about fate of symmetries**
- **flow guarantees convexity**

Approximation schemes & phase structure





- bosonic flow is symmetry-restoring
- fermionic flow is symmetry-breaking
- competing dynamics decides about fate of symmetries
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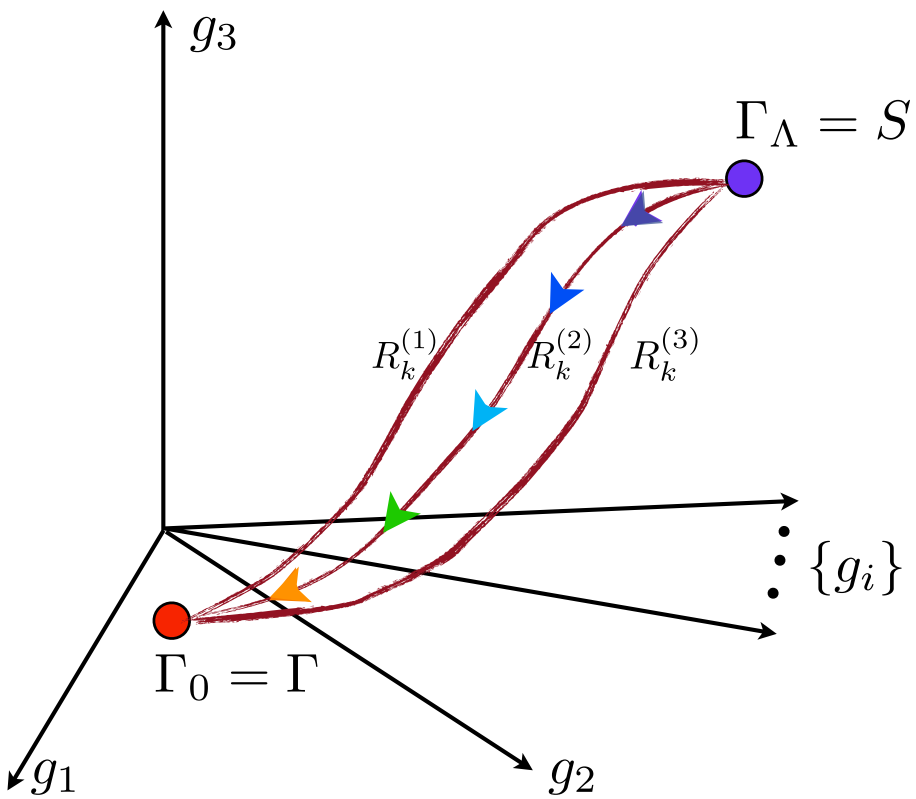
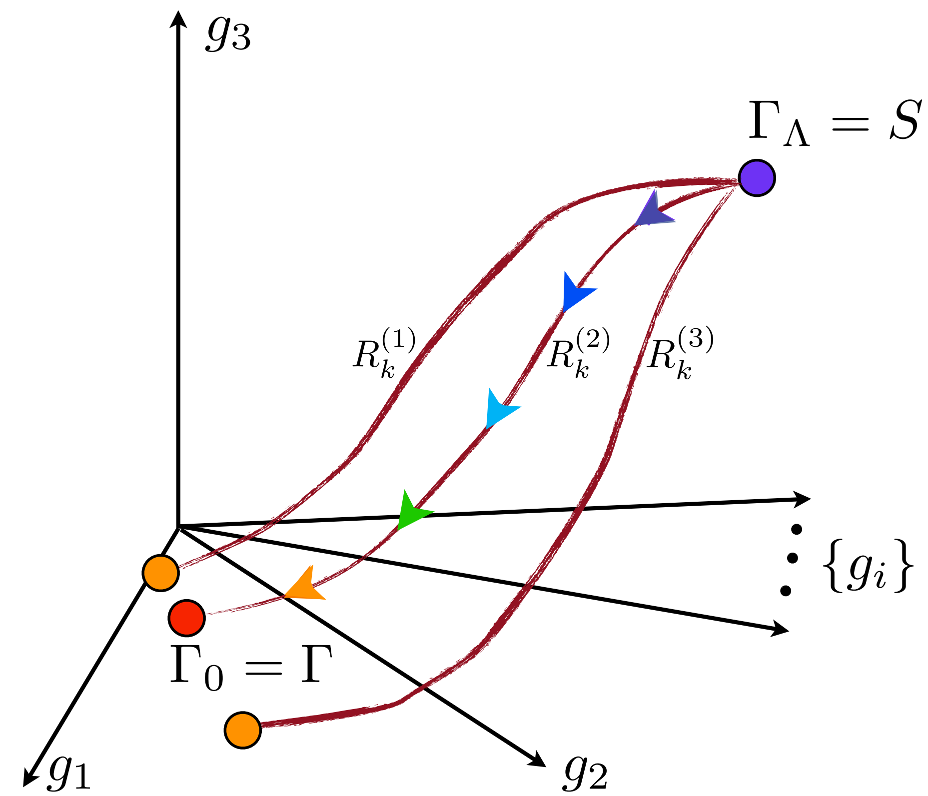
'governs general phase structures'

Approximation schemes & error control

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k}\partial_t R_k$$

Theory space





full flow

approximated flow

Optimisation: find $R_k^{(2)}$!

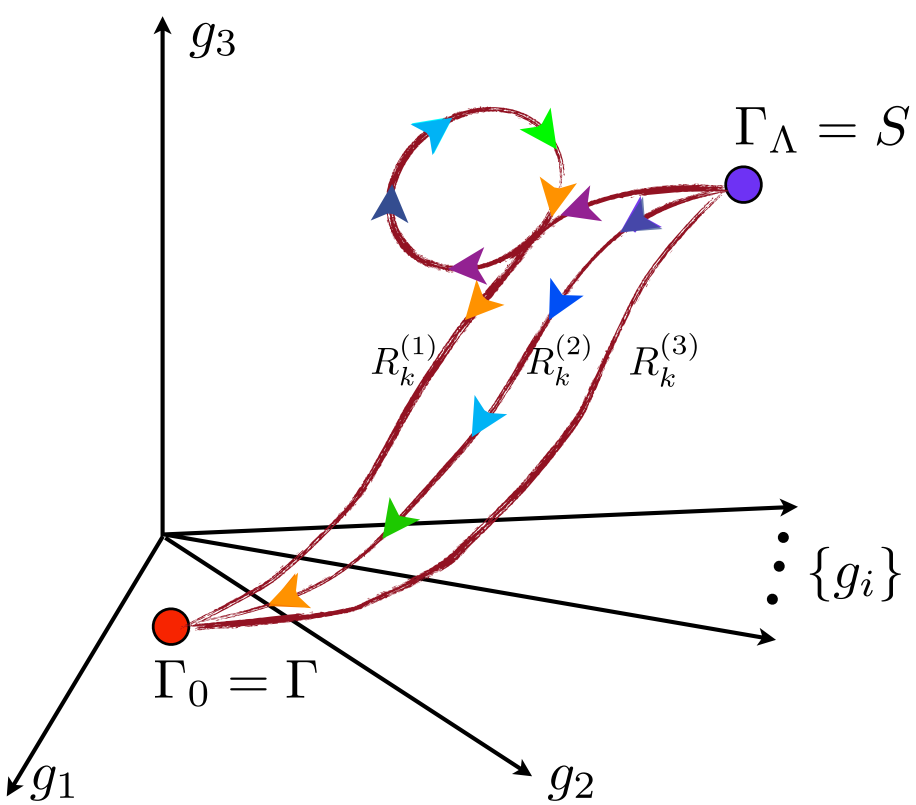
Litim '01: most rapid convergence

JMP '05: integrability

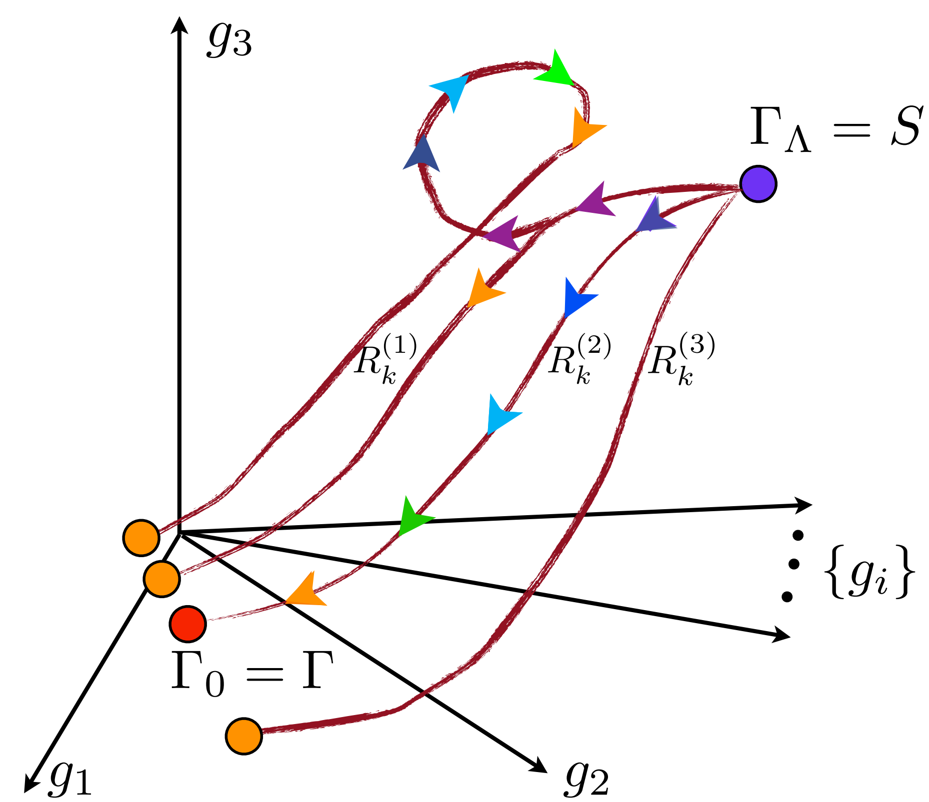
Approximation schemes & error control

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\,\partial_t R_k$$

Theory space



full flow



approximated flow

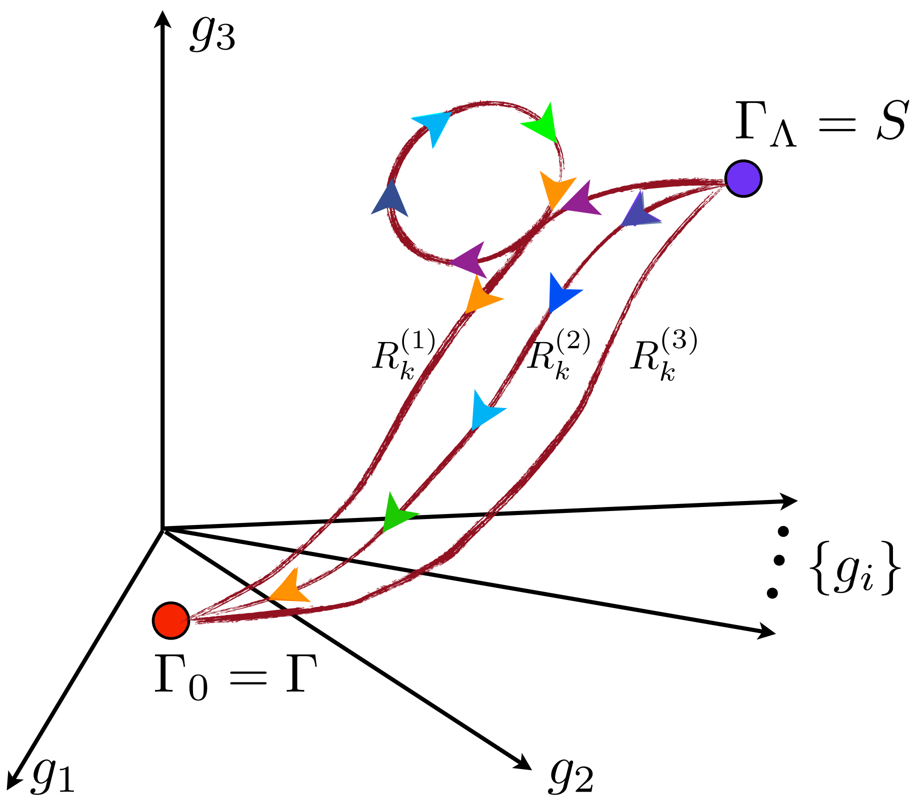
Optimisation: find $R_k^{(2)}$!

JMP '05: integrability

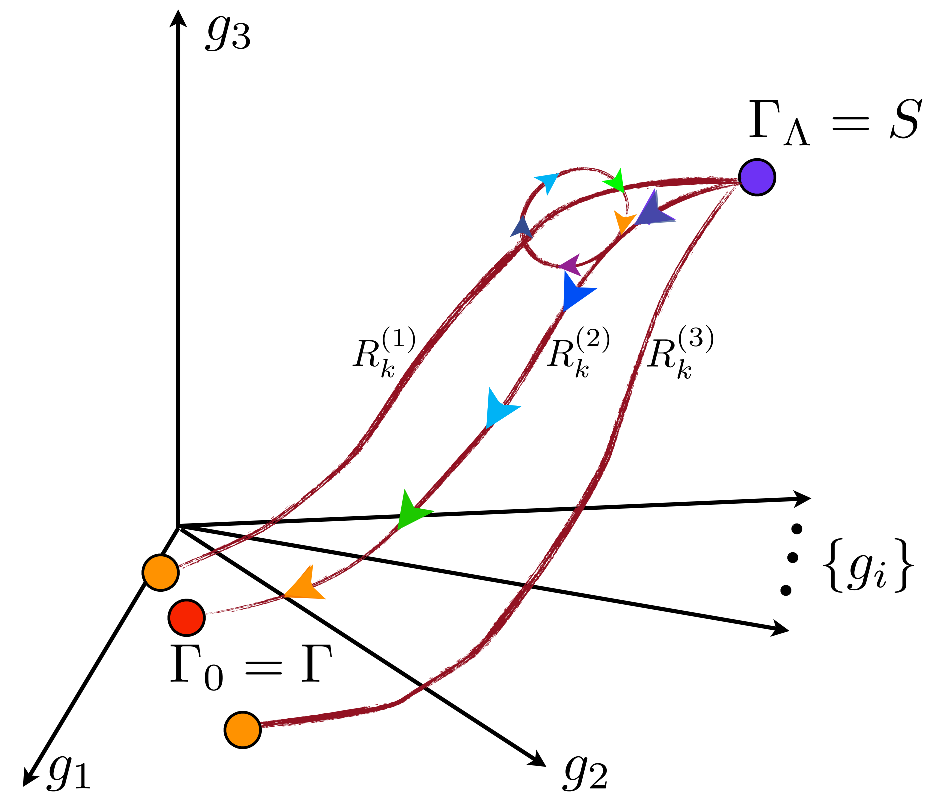
Approximation schemes & error control

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Theory space

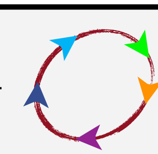


full flow



optimised flow

Optimisation: find $R_k^{(2)}$!

$\lim_{L\to 0} \frac{1}{L}$  $\to 0$

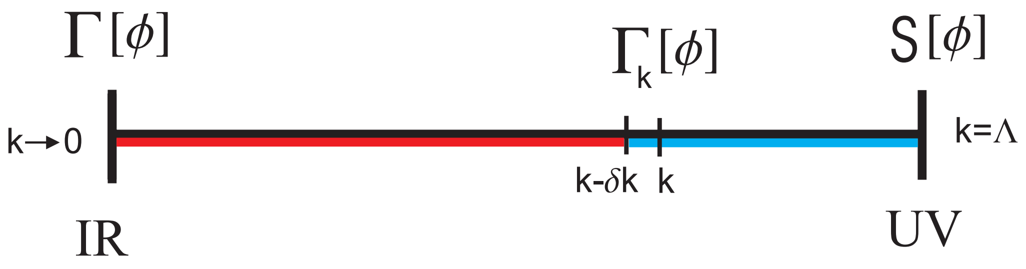
JMP '05: integrability

FRG for QCD

Functional RG for QCD

eg. JMP, AIP Conf.Proc. 1343 (2011)
NPA 931 (2014) 113

free energy at momentum scale k

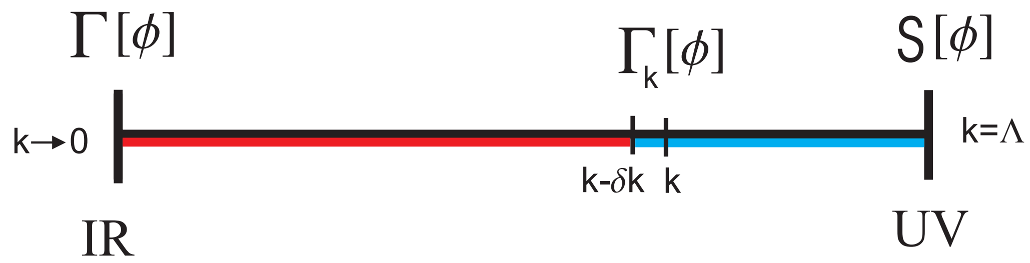


ab initio

Functional RG for QCD

eg. JMP, AIP Conf.Proc. 1343 (2011)
NPA 931 (2014) 113

free energy at momentum scale k

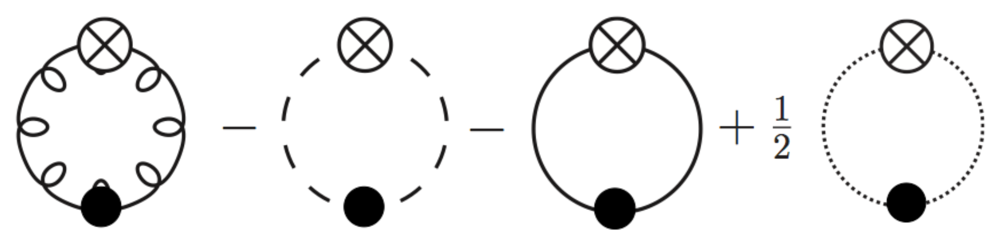


ab initio

glue quantum fluctuations

hadronic quantum fluctuations

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



free energy/ grand potential

quark quantum fluctuations

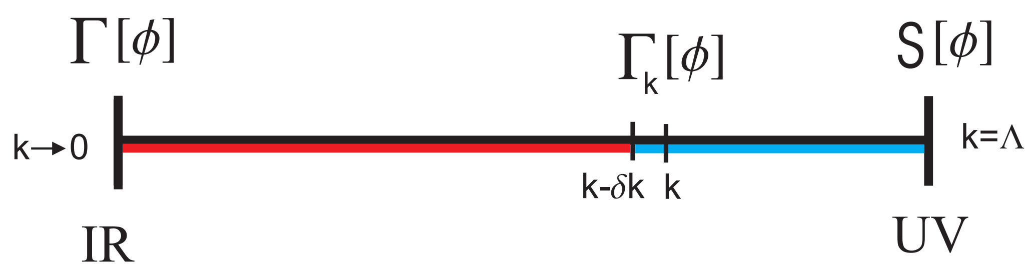
RG-scale k: $t = \ln k$

closed form

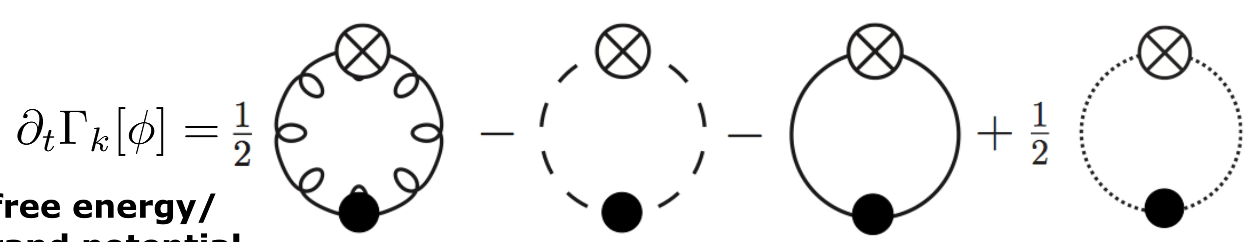
Functional RG for QCD

eg. JMP, AIP Conf.Proc. 1343 (2011)
NPA 931 (2014) 113

free energy at momentum scale k



ab initio



RG-scale k: $t = \ln k$

closed form

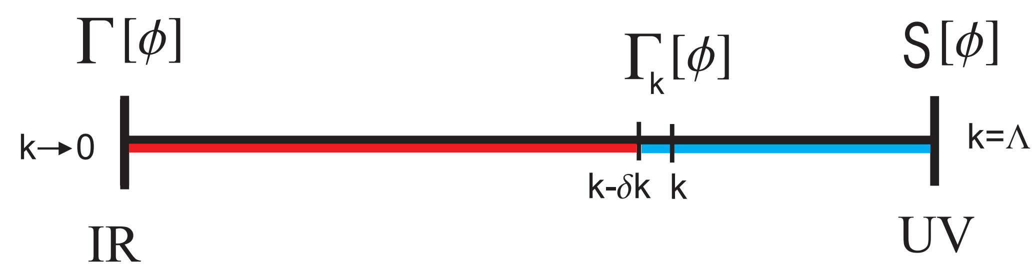
properties

- **access to physics**
- **numerically tractable, also at real time**
  no sign problem
  systematic error control via closed form
- **low energy models naturally encorporated**

Functional RG for QCD

eg. JMP, AIP Conf.Proc. 1343 (2011)
NPA 931 (2014) 113

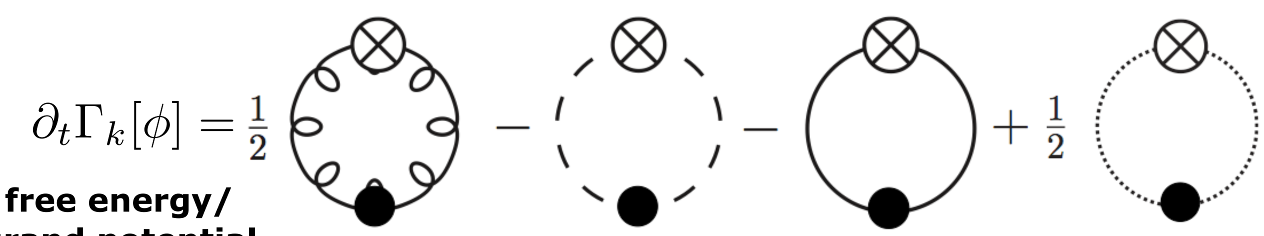
free energy at momentum scale k



ab initio

glue quantum fluctuations

hadronic quantum fluctuations



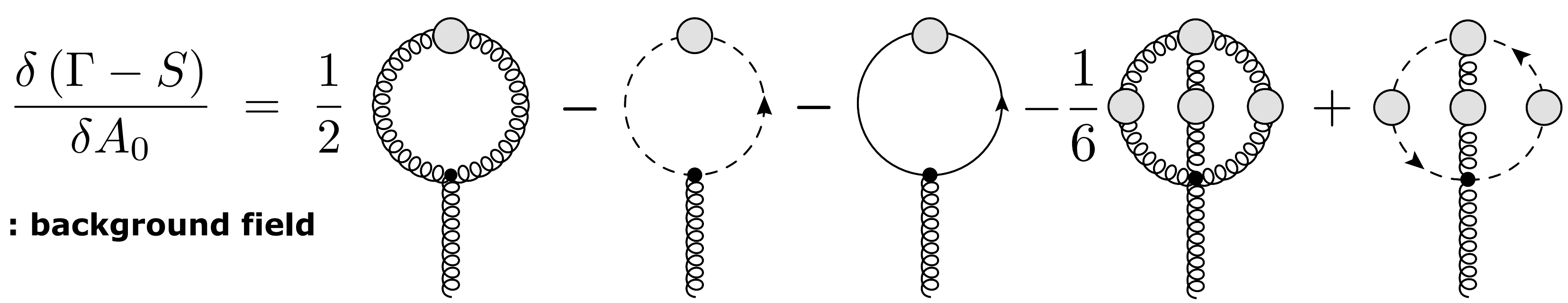
free energy/ grand potential

quark quantum fluctuations

RG-scale k: $t = \ln k$

closed form

functional DSE :



A_0 **: background field**

Functional RG for QCD

fQCD collaboration: J. Braun, L. Corell, A. Cyrol, W.-j. Fu, M. Leonhardt, M. Mitter, JMP, M. Pospiech, F. Rennecke, N. Wink

Heidelberg, Dalian, Darmstadt

Agenda

QCD at finite T & mu

- Phase structure
- Fluctuations
- Phenomenology

Real time correlation functions

- Hadron spectrum & decays
- Transport coefficients
- Dynamics

Functional RG for QCD

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- **Phase structure**
- **Fluctuations**
- **Phenomenology**

Real time correlation functions

- **Hadron spectrum & decays**
- **Transport coefficients**
- **Dynamics**

Selection of papers

quenched QCD: **Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005**

unquenched QCD: **Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016**

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

vector mesons: **Rennecke, PRD 92 (2015) 076012**

pure glue: **Mitter, JMP, Strodthoff, PRD 91 (2015) 054035**

finite T: **Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015**

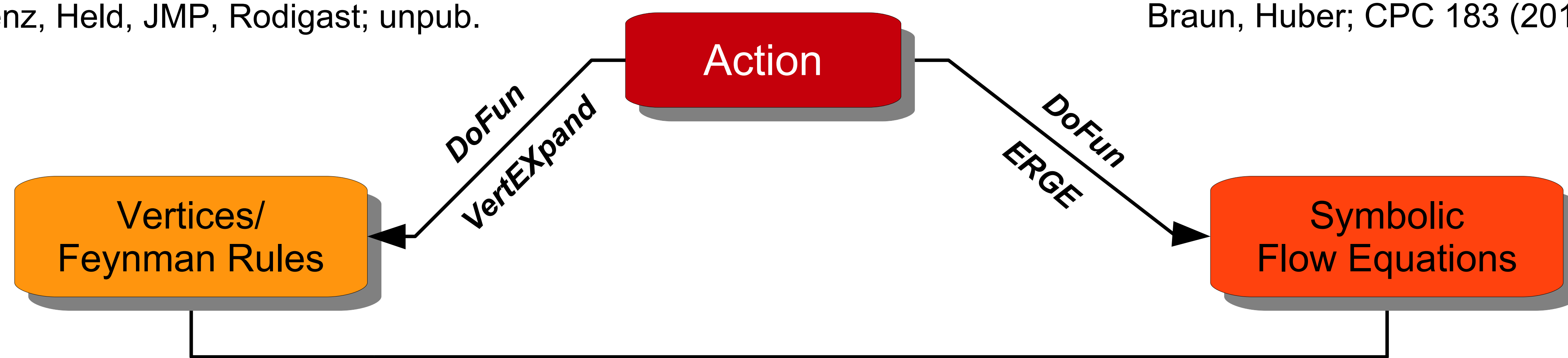
finite density: *fluctuations:* **Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 11, 116020**

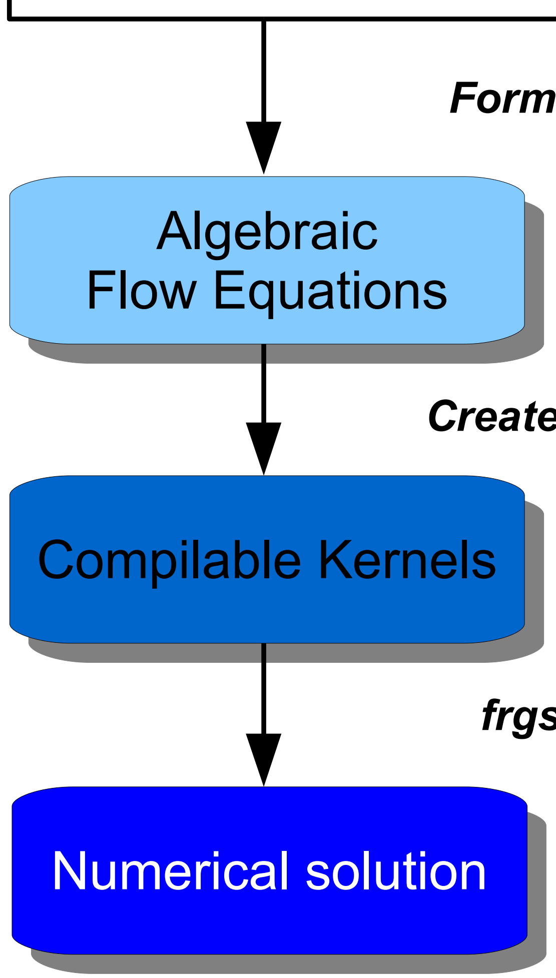
phase structure: **Braun, Leonhardt, Pospiech, PRD 96 (2017) 7, 076003**

fQCD: workflow

VertEXpand
Mathematica package for the derivation of vertices from a given action using FORM
Denz, Held, JMP, Rodigast; unpub.

DoFun
Mathematica package for the derivation of functional equations
Braun, Huber; CPC 183 (2012) 1290





FormTracer
High-performance, general purpose, easy-to-use Mathematica tracing tool using FORM
Cyrol, Mitter, JMP, Strodthoff;
Cyrol, Mitter, Strodthoff; CPC 219 (2017) 346

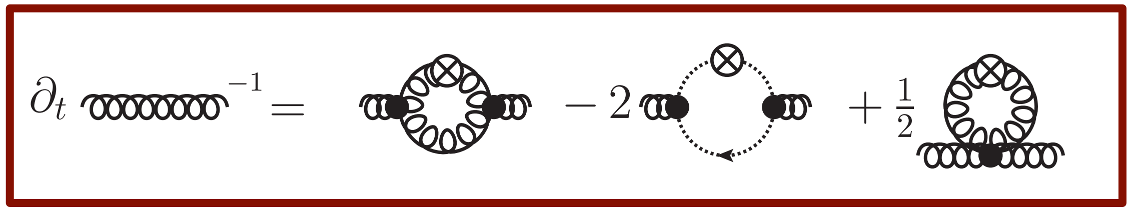
CreateKernels
Mathematica package for the automatic generation of compilable C++ kernels for use in connection with the *frgsolver*
Cyrol, Mitter, Pawlowski, Strodthoff; unpub.

frgsolver
Flexible, high-performance, parallelized C++ OOP framework for the numerical solution of functional equations
Cyrol, Mitter, Pawlowski, Strodthoff; unpub.

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung
FWF Der Wissenschaftsfonds.
European Research Council
Established by the European Commission

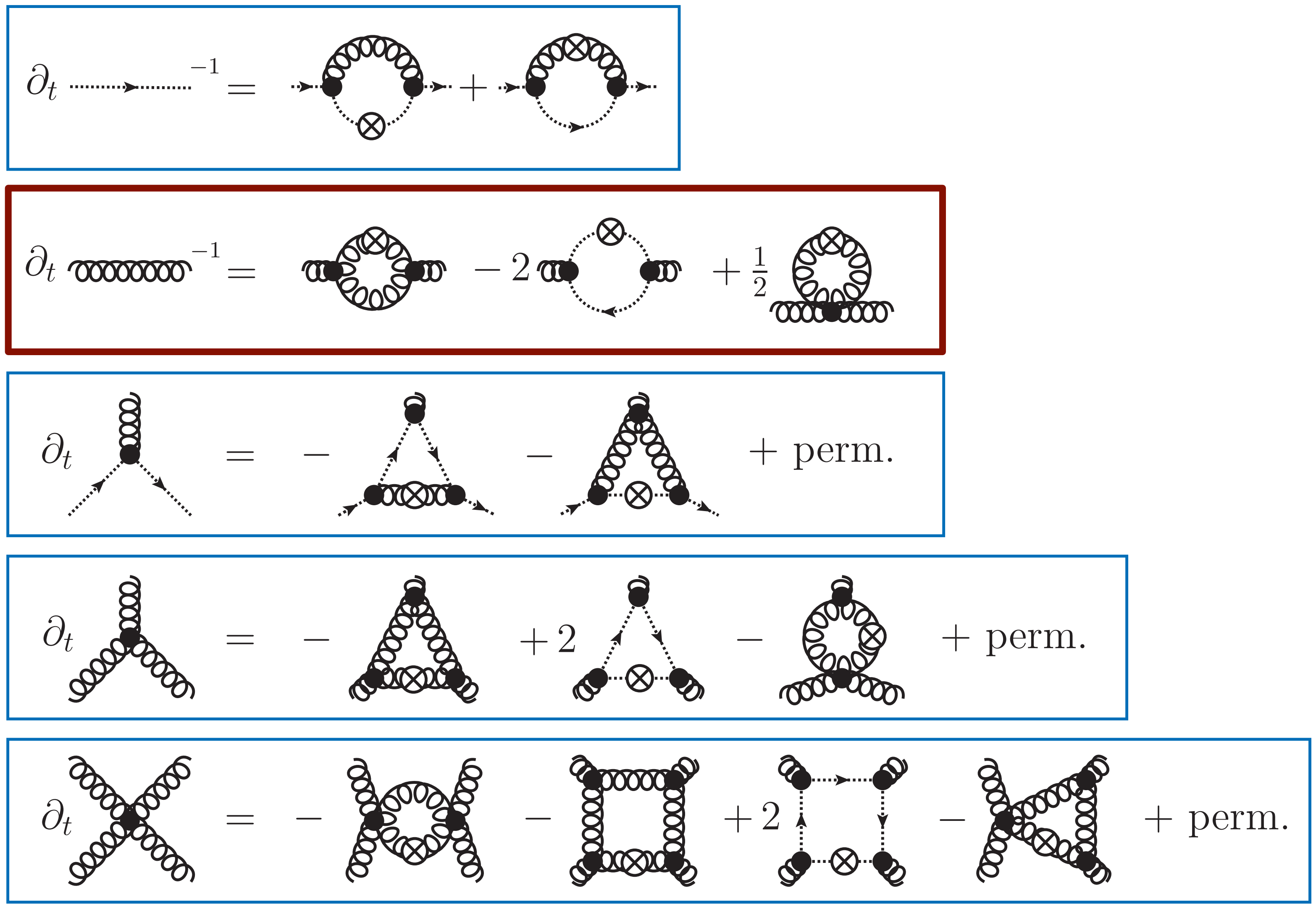
YM-theory: gluonic correlation functions

$\langle A\, A \rangle (p^2)$



Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: gluonic correlation functions



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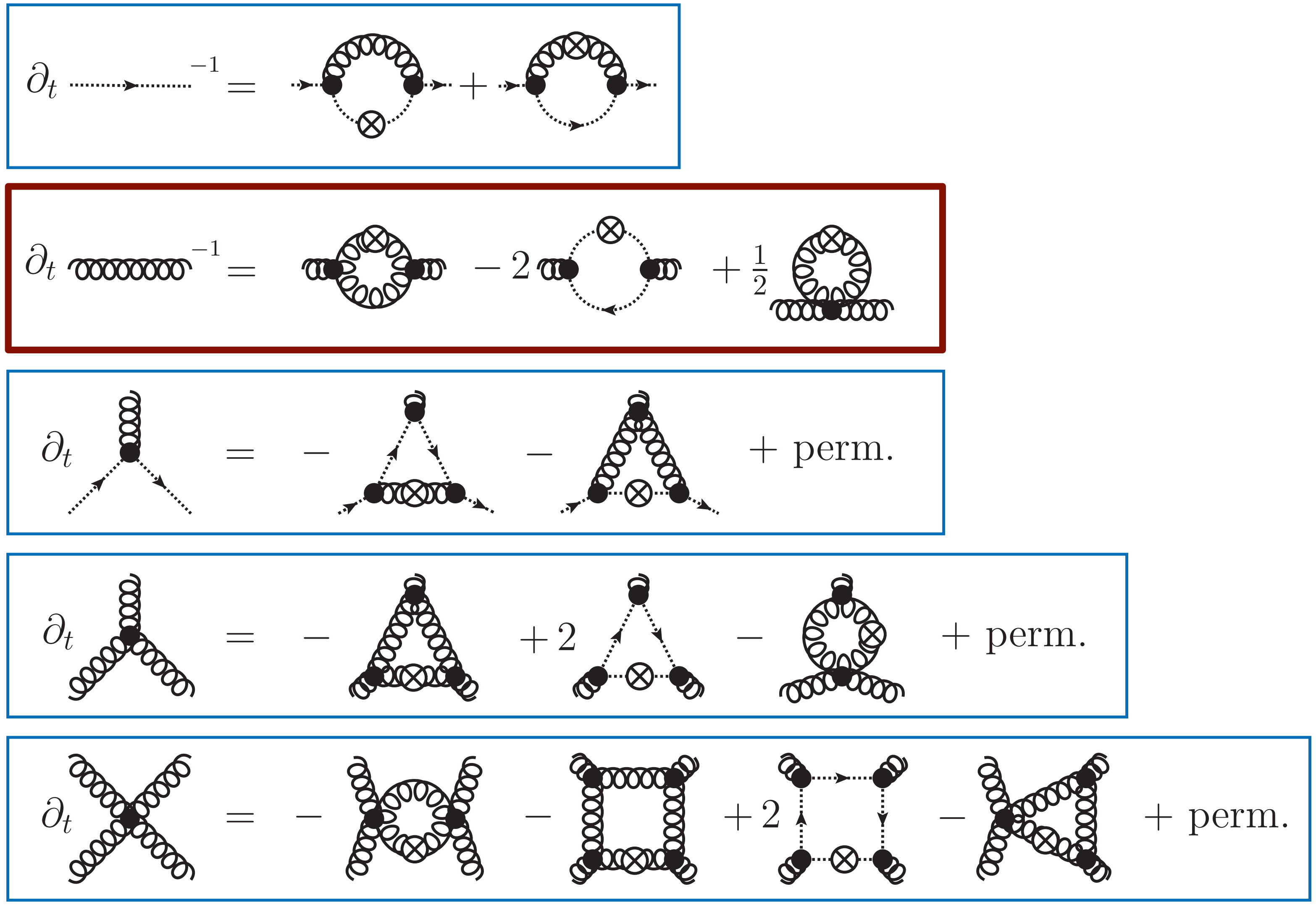
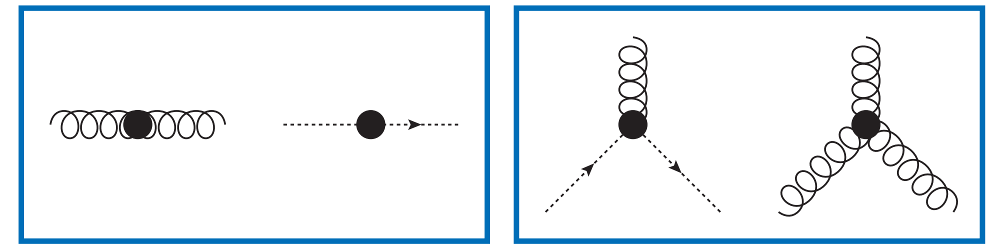
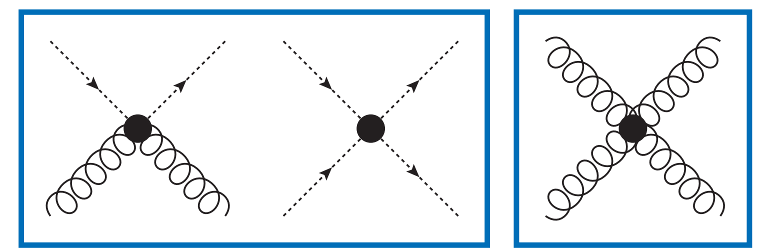




full. mom. dep.

full. mom. dep.
classical tensor structures

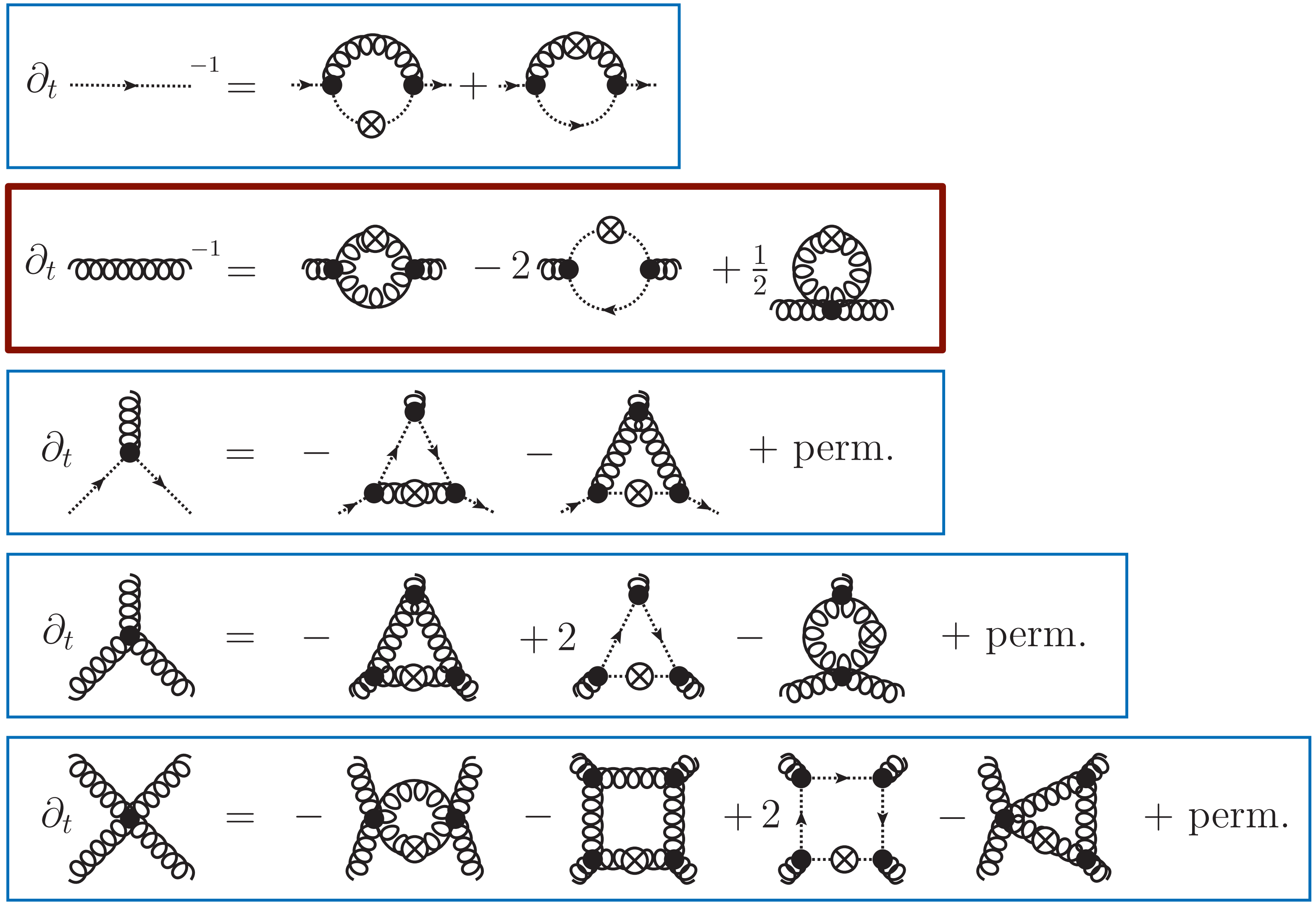


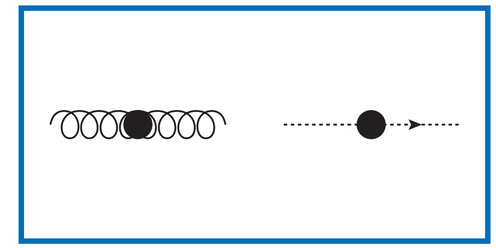
mom. dep. needed by tadpoles
full tensor basis

sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

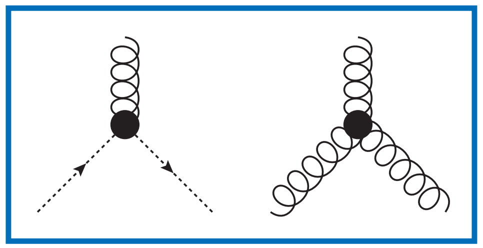
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: gluonic correlation functions





full. mom. dep.



full. mom. dep.
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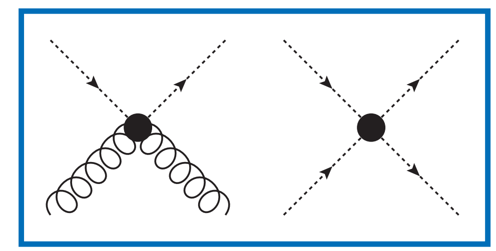
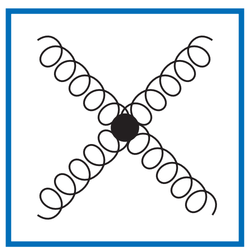


mom. dep. needed by tadpoles
full tensor basis



sym. point mom. dep. and
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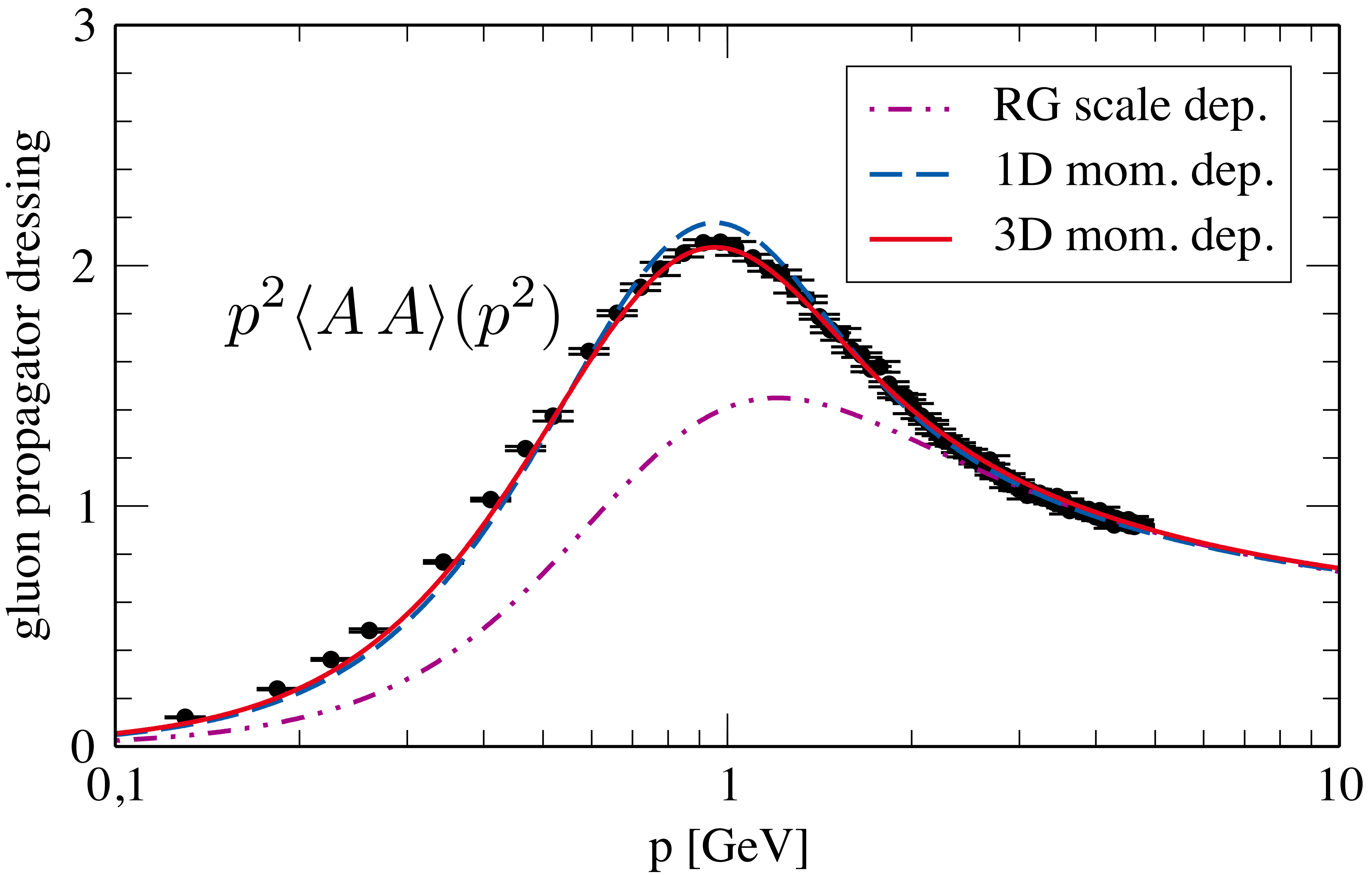
classical tensor structure

Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: Euclidean gluon propagator

Functional Renormalisation Group



Lattice: Sternbeck, Ilgenfritz, Müller-Preussker, Schiller, Bogolubsky, PoS LAT2006, 076

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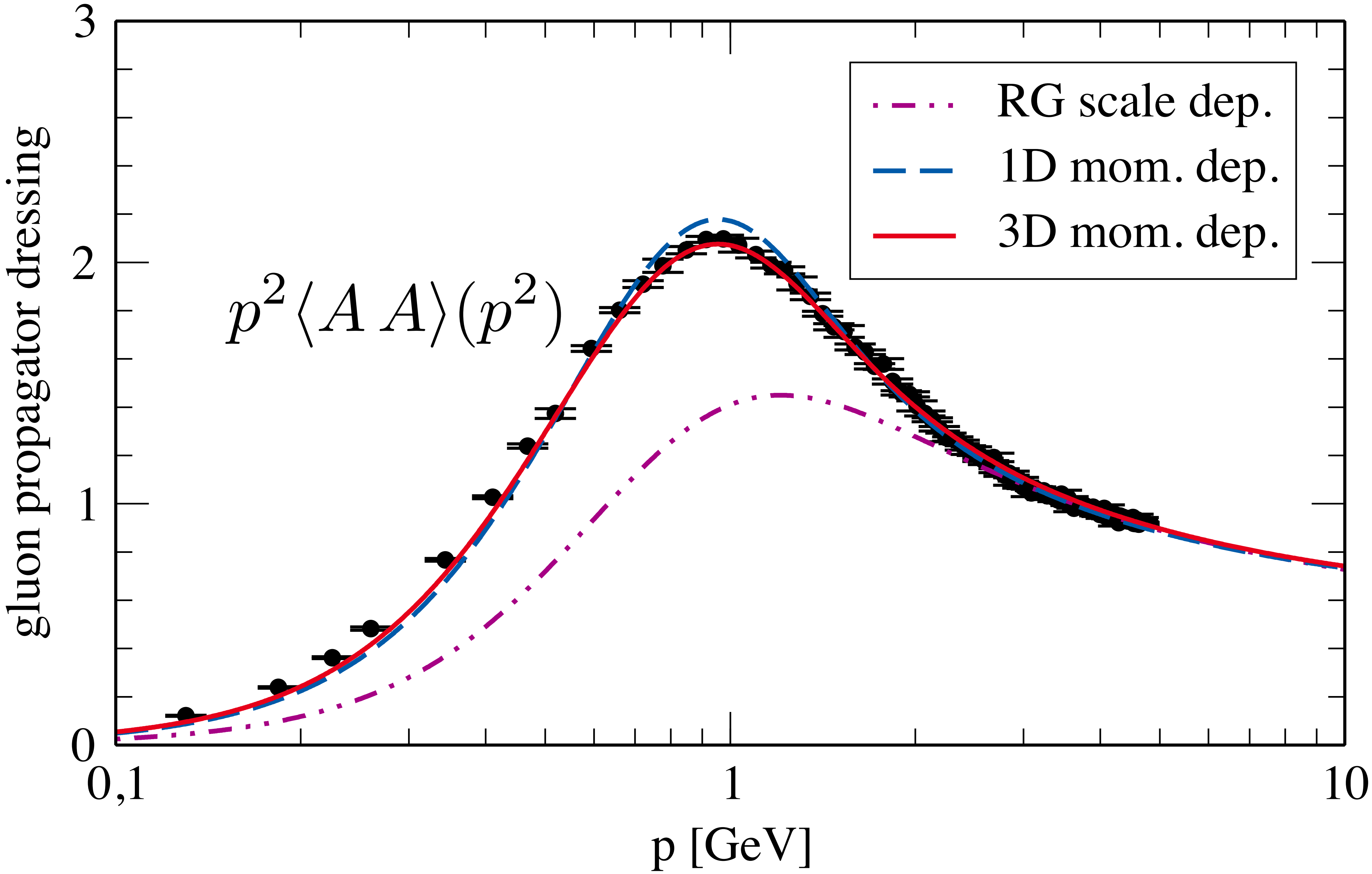
YM-theory: Euclidean gluon propagator

Functional Renormalisation Group



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Aiming at apparent convergence

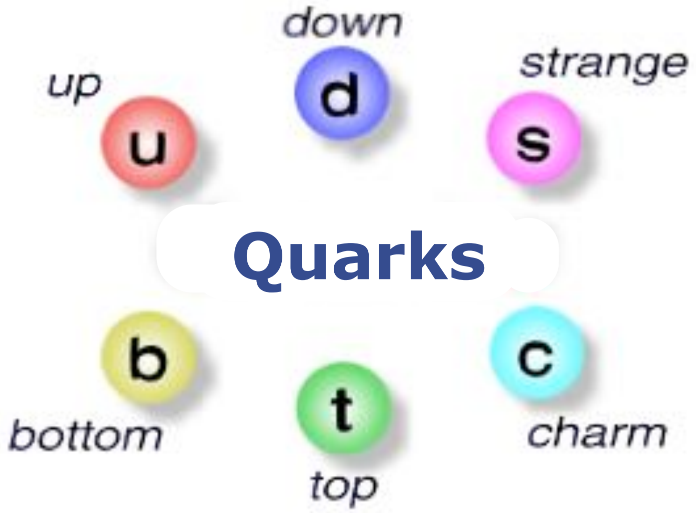
up to date pinch technique:
Aguilar, Binosi, Papavassiliou, PRD 89 (2014) 085032

up to date DSE:
Cyrol, Huber, Smekal, EPJ C75 (2015) 102

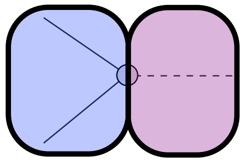
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Dynamical hadronisation

Gies, Wetterich ’01
JMP ’05
Flörchinger, Wetterich ’09

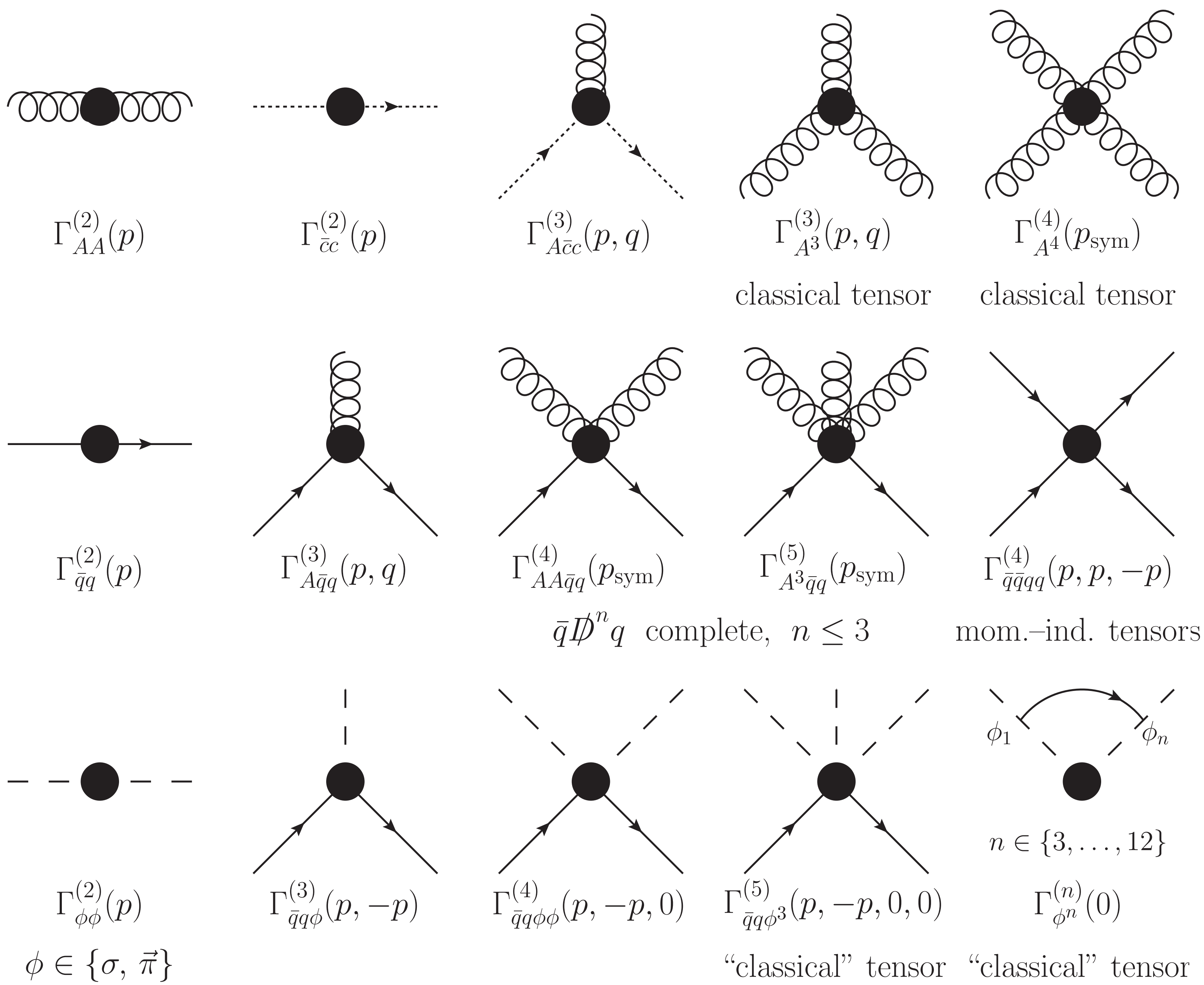


Gluons



Hadrons

QCD: current set of correlation functions



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

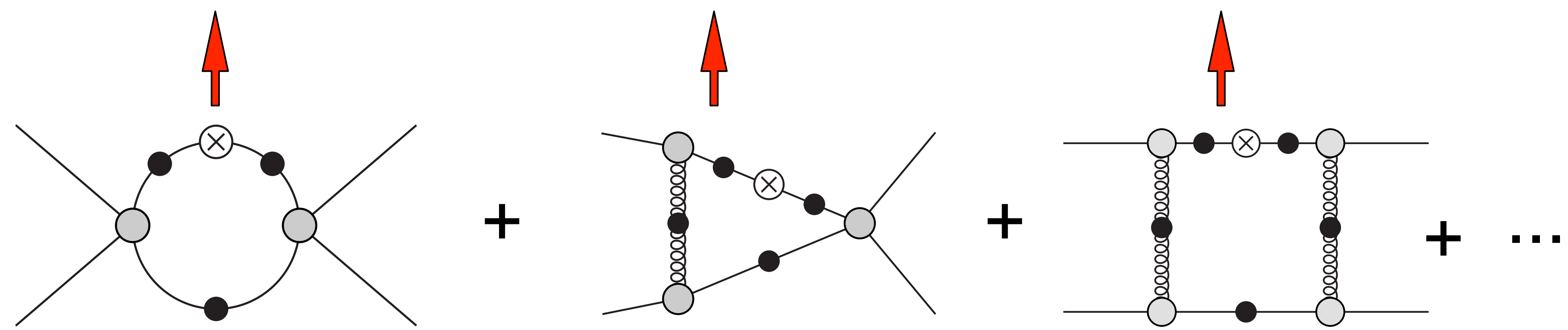
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

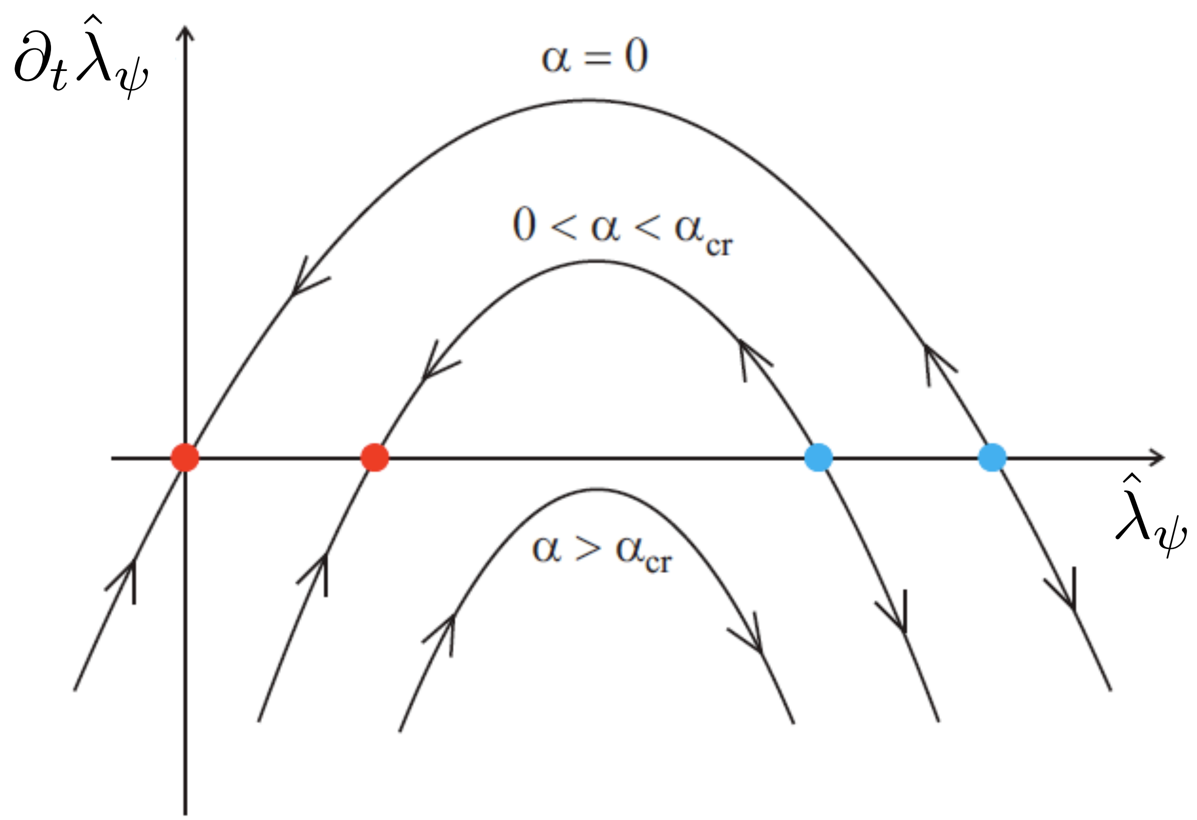
Chiral symmetry breaking

A glimpse at chiral symmetry breaking in QCD within the FRG

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ **with infrared scale** k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi + A\left(\frac{T}{k}\right)\hat{\lambda}_\psi^2 + B\left(\frac{T}{k}\right)\hat{\lambda}_\psi\,\alpha_s + C\left(\frac{T}{k}\right)\alpha_s^2 + \cdots$$





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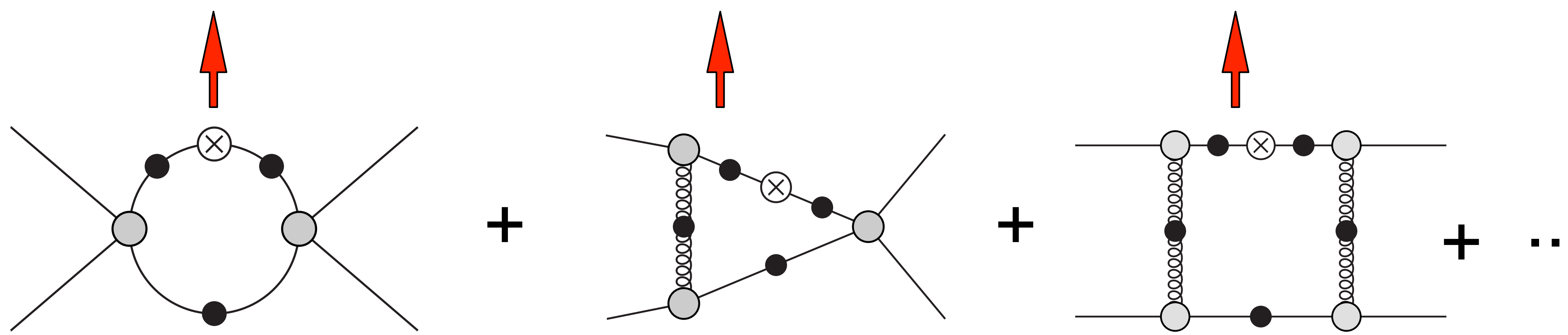

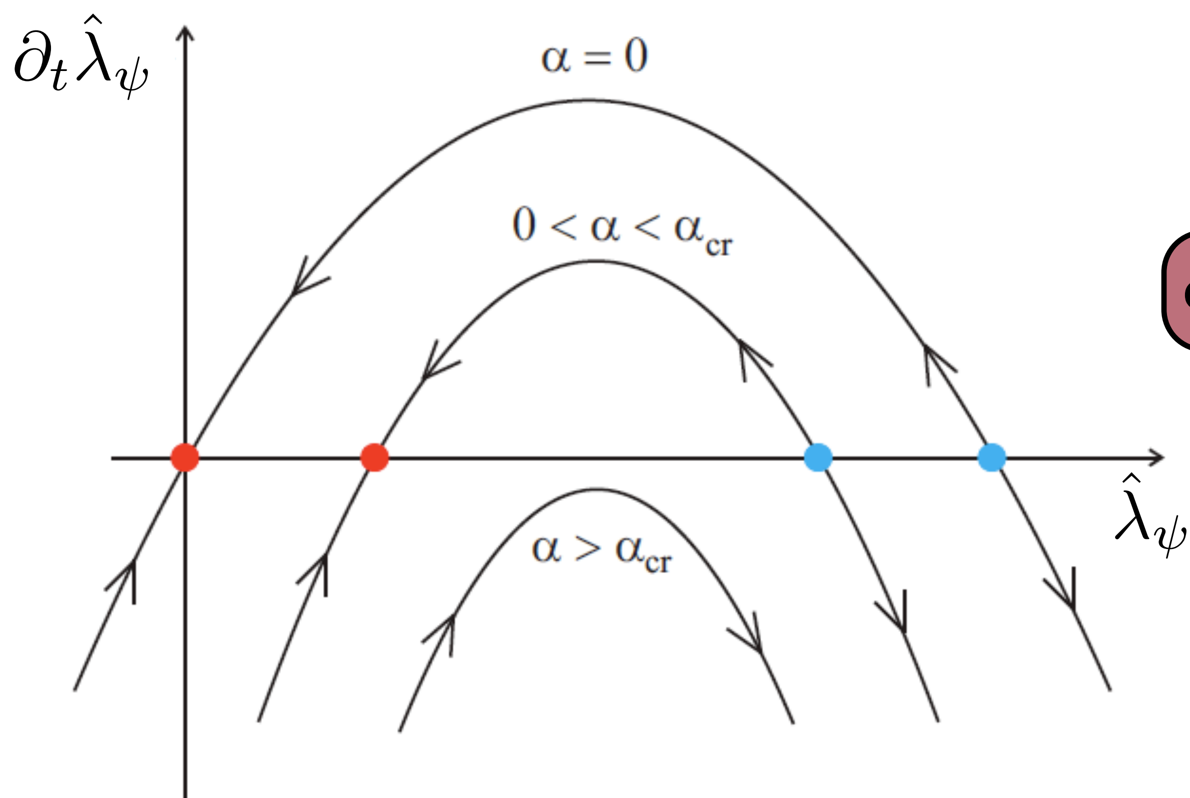

chiral symmetry breaking ⟷ $\alpha_s > \alpha_{s,cr}$

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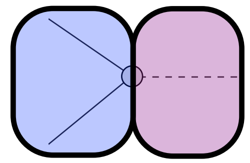
$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

$$\lambda_\psi=\frac{h^2}{m_\phi^2}$$

Hubbard-Stratonovich



$$\Phi=(\sigma,\vec\pi)$$

$$\tau\cdot\Phi=\sigma+i\gamma_5\vec\sigma\vec\pi$$

General dynamical hadronisation

hadronised Flow

$$\frac{\partial}{\partial t}\bigg|_\phi\Gamma_k[\phi]=\frac{1}{2}G_{k,\phi}\dot R_{k,\phi}+R_kG_{k,\phi}\frac{\delta\dot\phi}{\delta\phi}-\frac{\delta\Gamma}{\delta\phi}\dot\phi$$

JMP '05

$$\phi=(A_\mu,C,\bar C,q,\bar q,\Phi,...,n,\bar n,...)$$

mesons baryons

$$-\frac{1}{2}\int_p\phi_k^*\cdot R_k\cdot\phi_k+J\cdot\phi_k$$

guarantees 1-loop flow

Dynamical hadronisation

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JMP '05
Flörchinger, Wetterich '09

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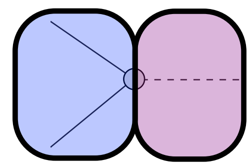

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JMP '05

$$\phi=(A_\mu,C,\bar C,q,\bar q,\Phi,...,n,\bar n,...)$$

mesons baryons

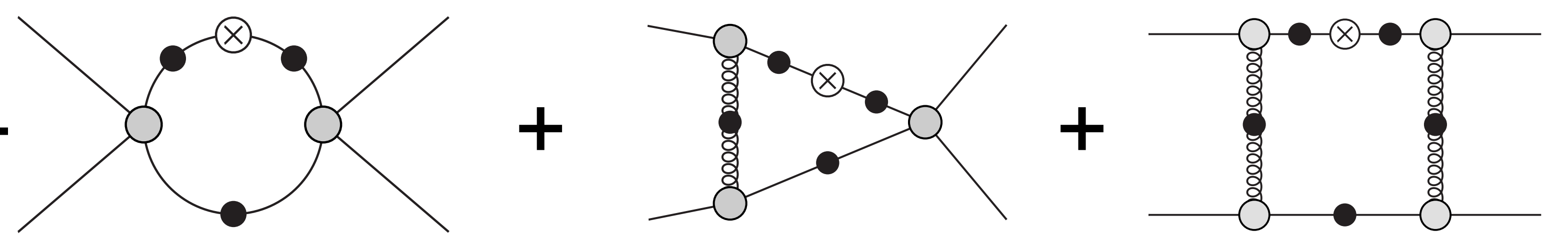
How to fix ϕ_k & $\dot\phi_k$?

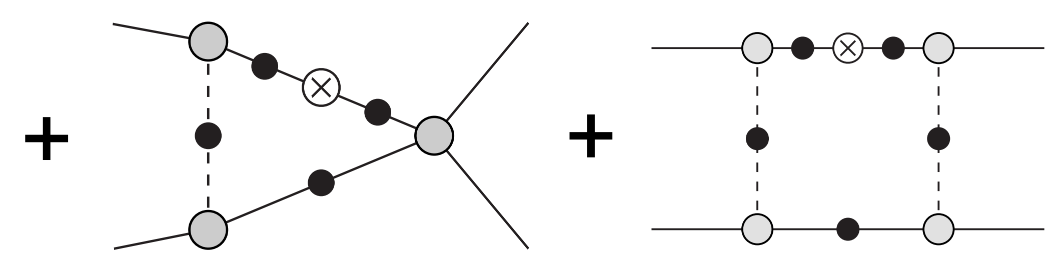
$$\dot\Phi_k\simeq\dot A_k\bar\psi\tau\psi+\dot B_k\Phi_k+\dot C_k$$

Dynamical hadronisation

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

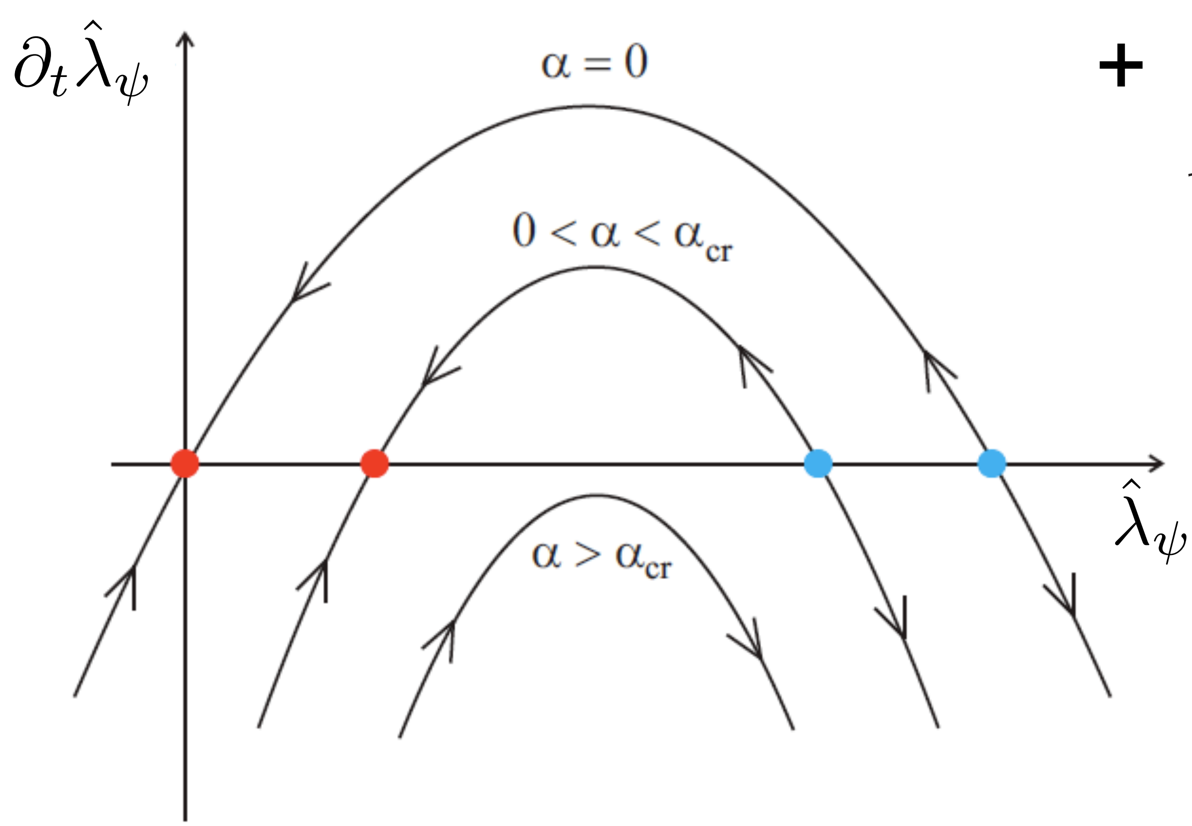
$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi$ +





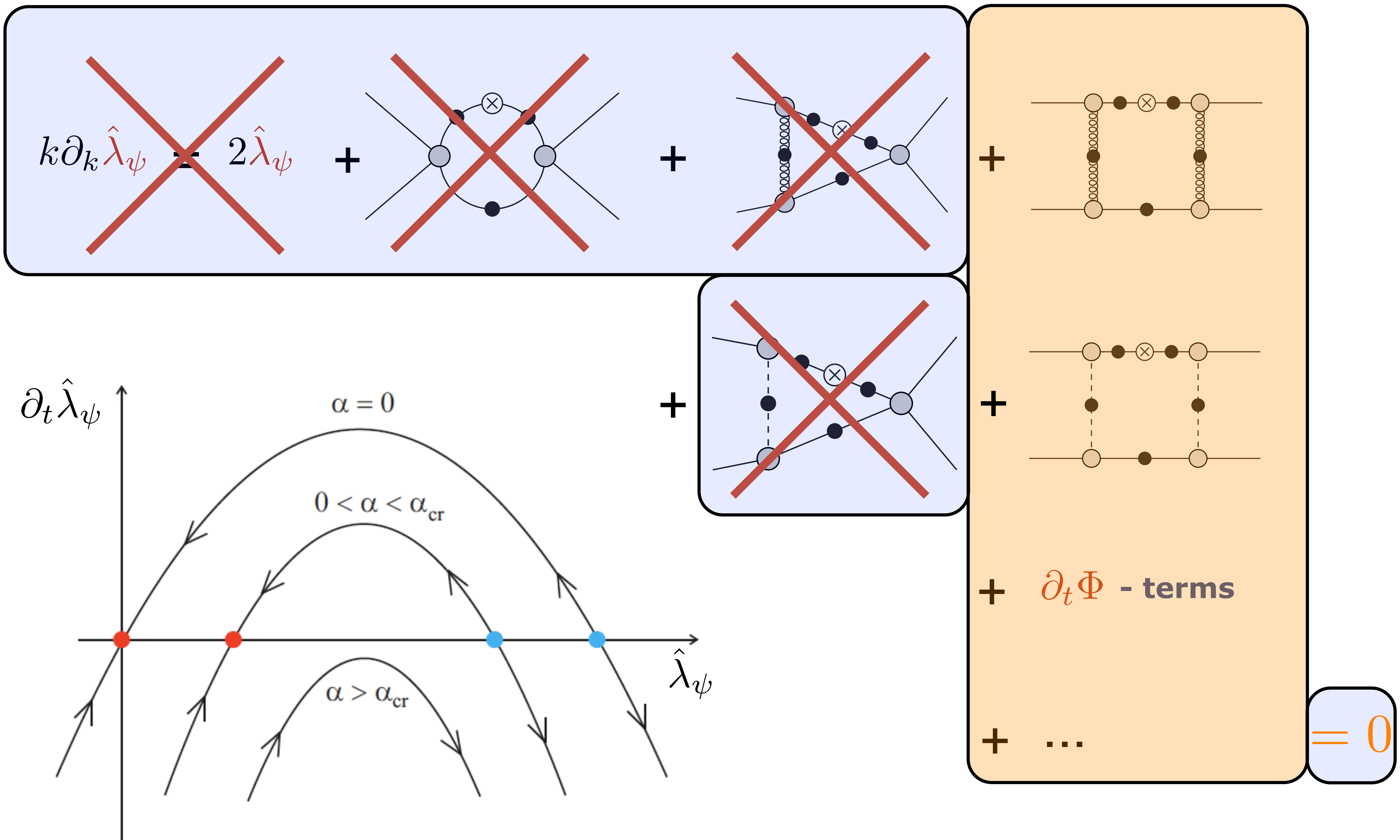
+ $\partial_t \Phi$ **- terms**

+ ...



Dynamical hadronisation

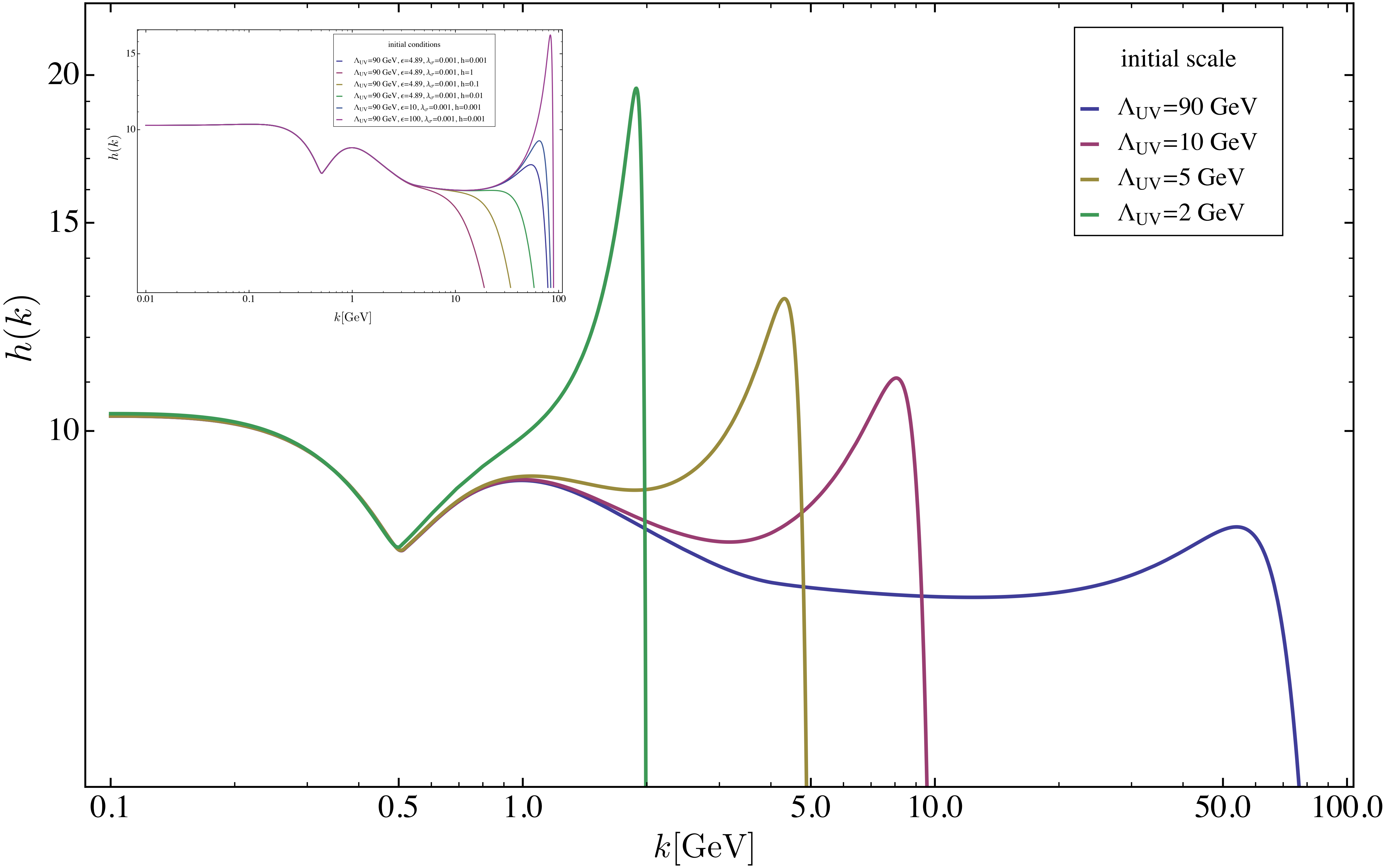
Full bosonisation $\hat{\lambda}_\psi = 0$



Dynamical hadronisation

Braun, Fister, Haas, JMP, Rennecke '14
Mitter, JMP, Strodthoff '14
Cyrol, Mitter, JMP, Strodthoff '17

Full bosonisation $\hat{\lambda}_\psi = 0$



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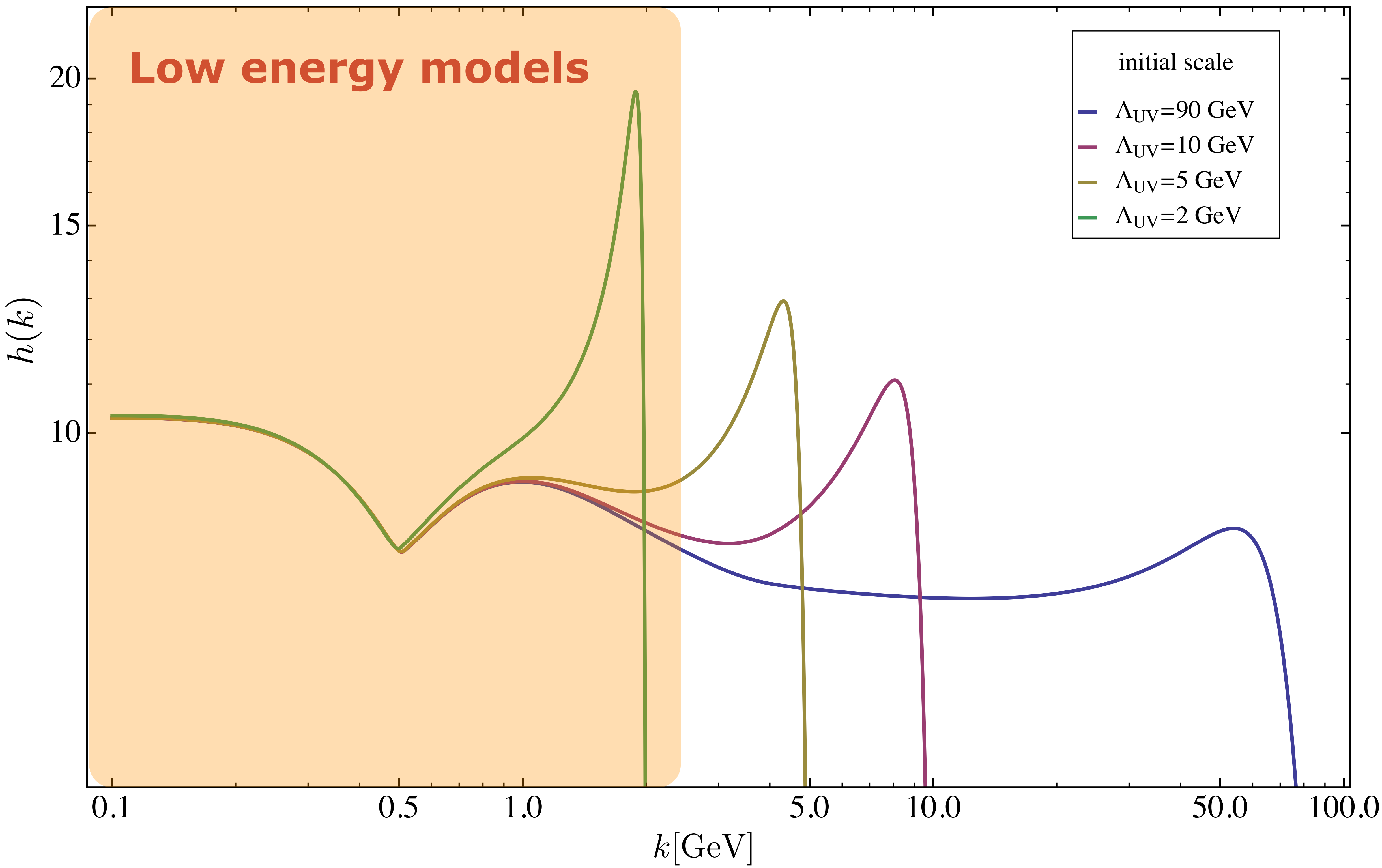

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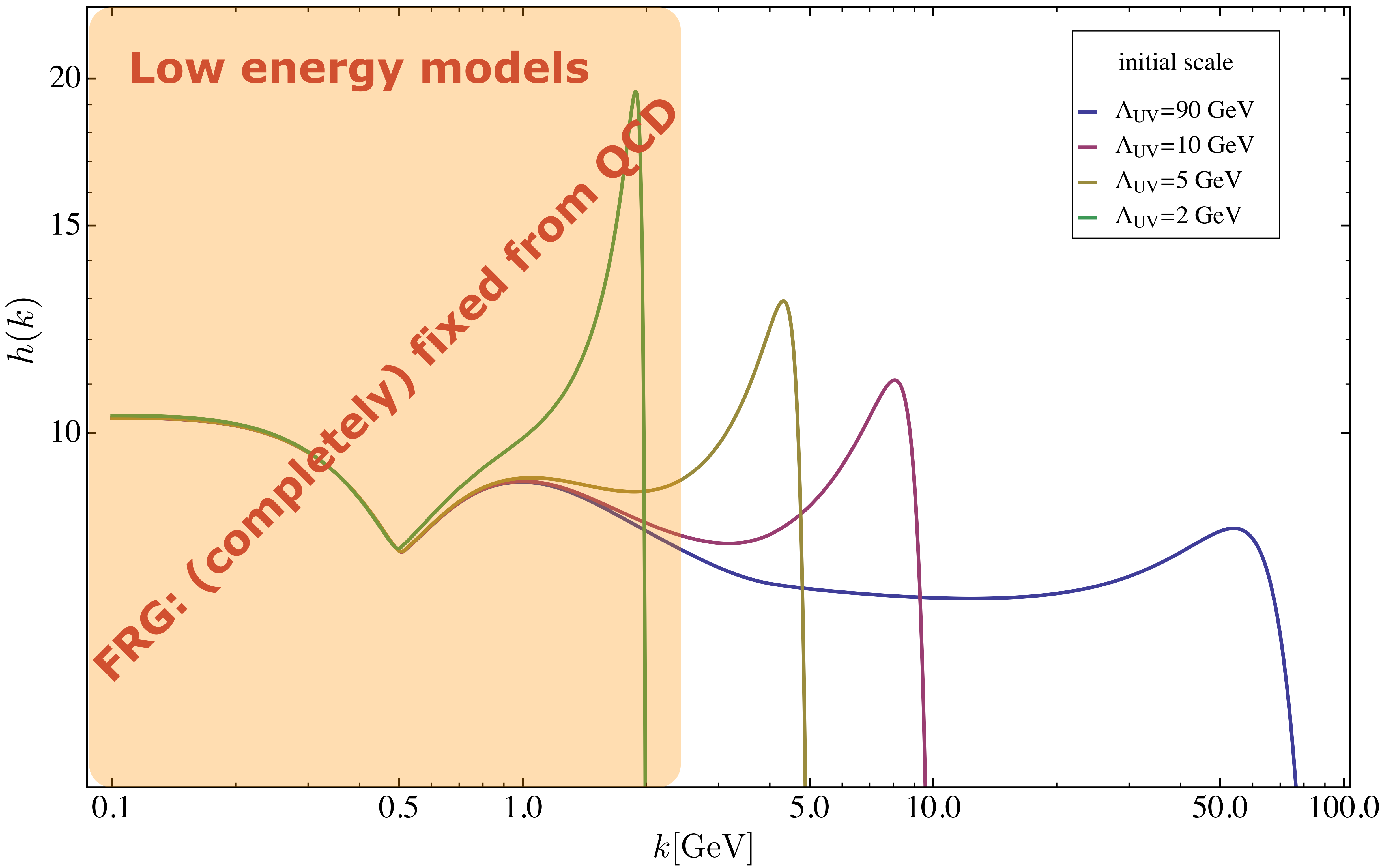
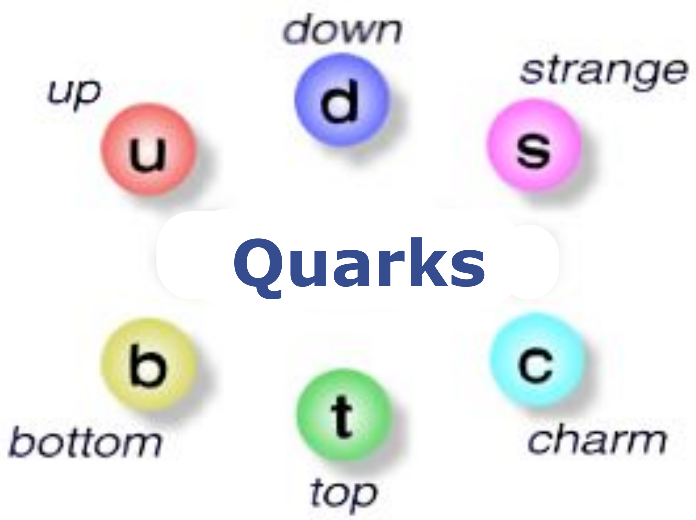
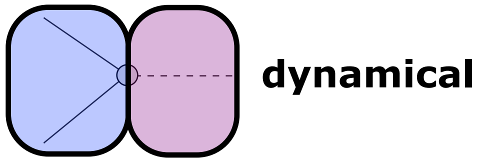


QCD results at T=0



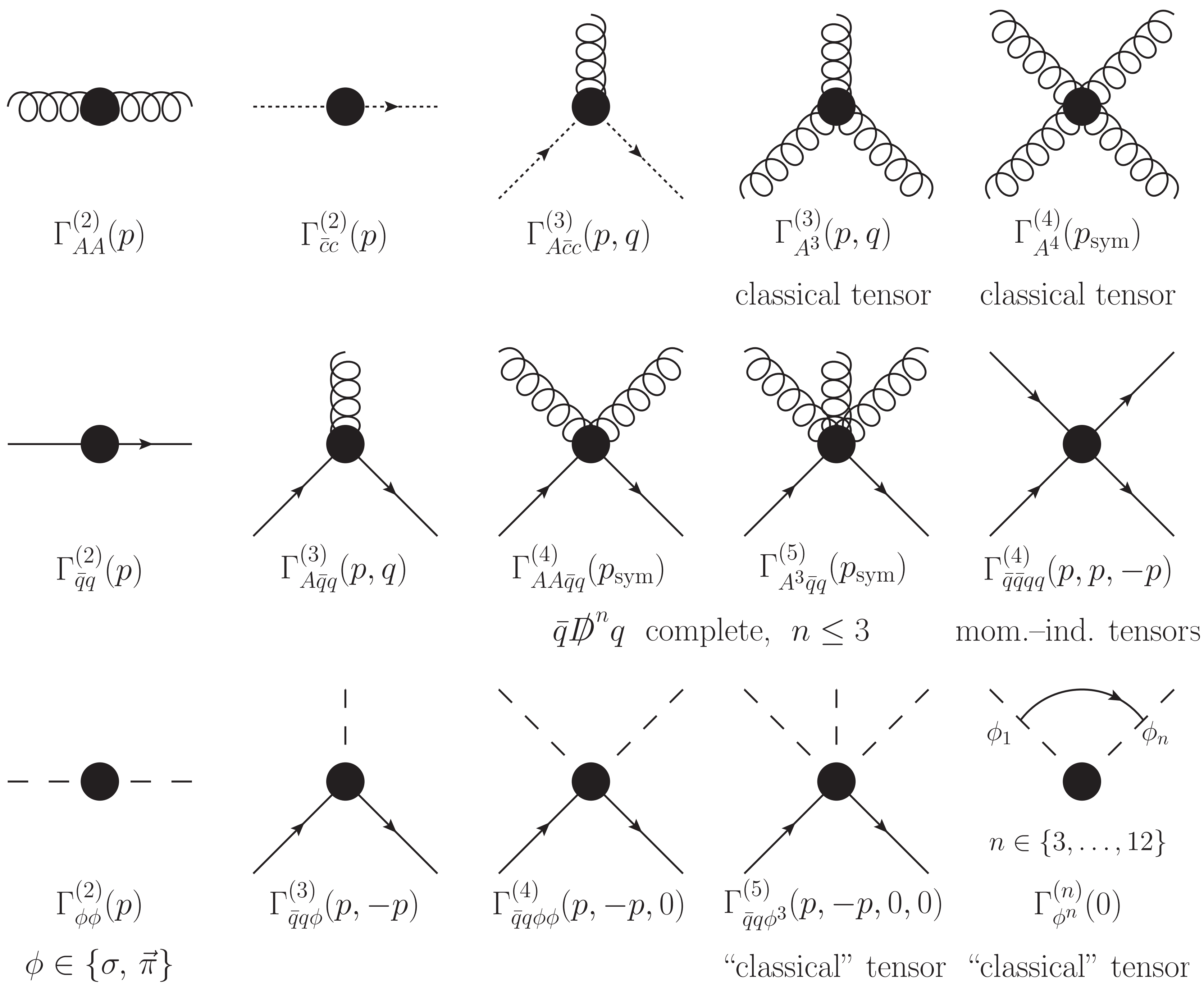
Quarks

Gluons



Hadrons

QCD: current set of correlation functions



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Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

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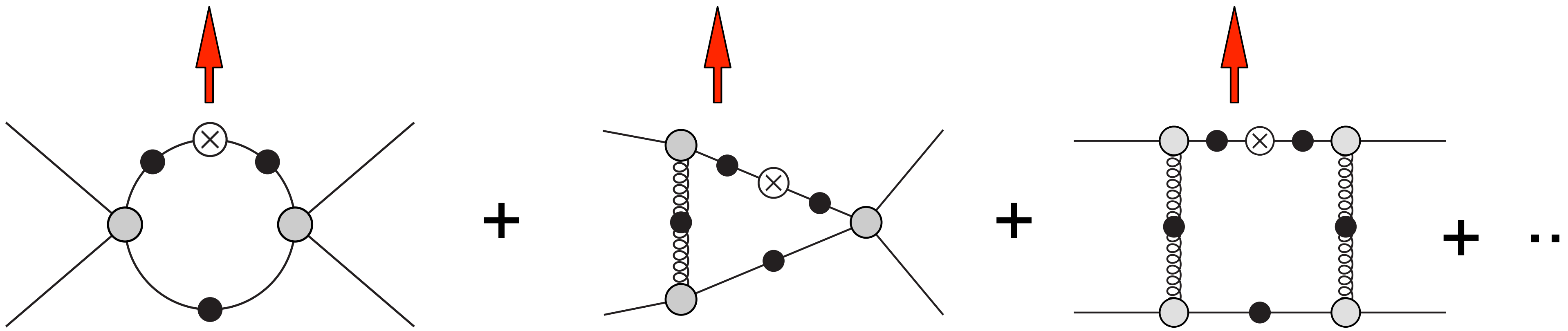

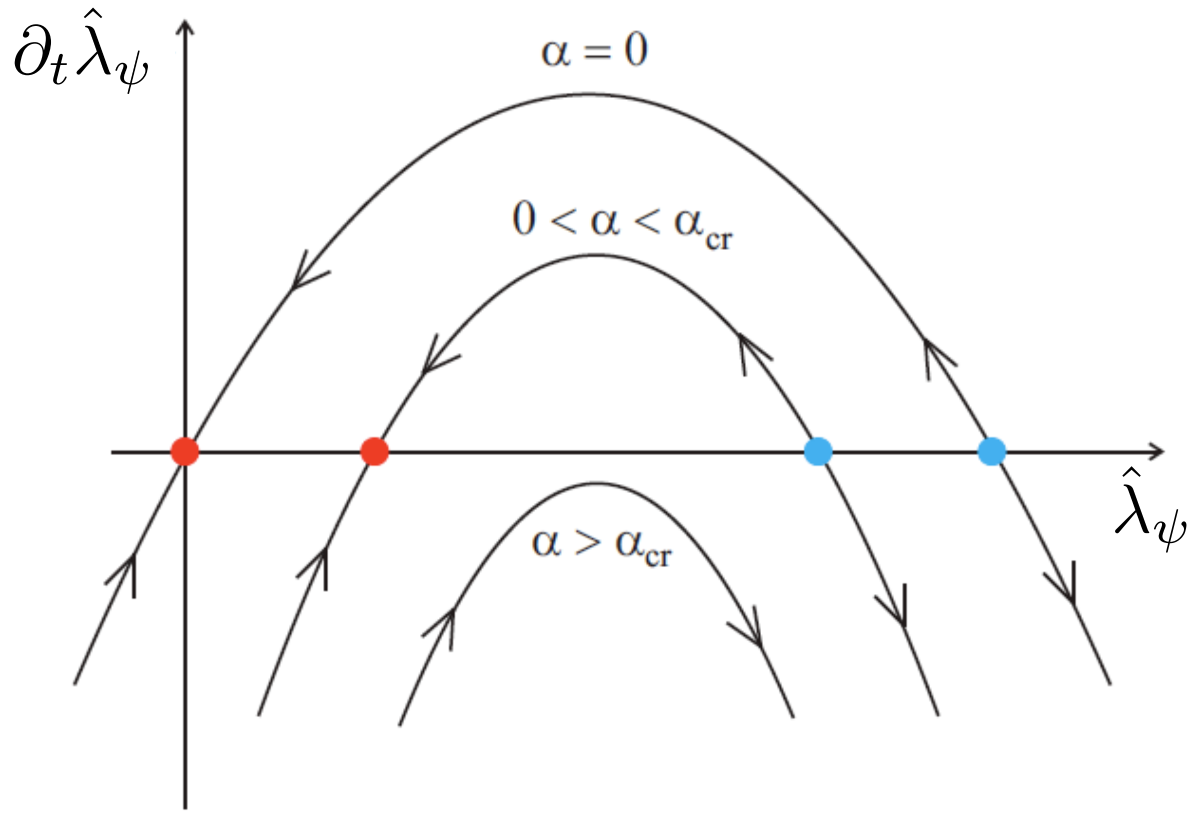

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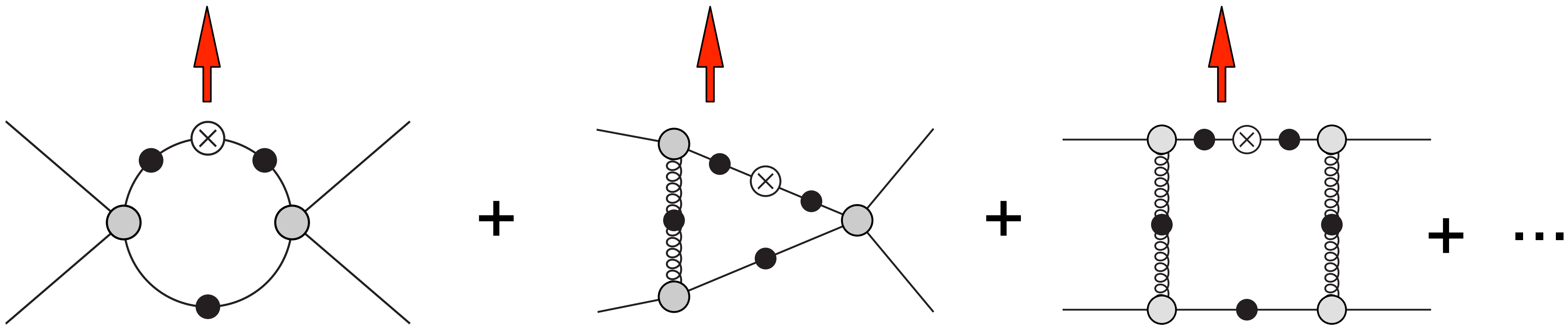

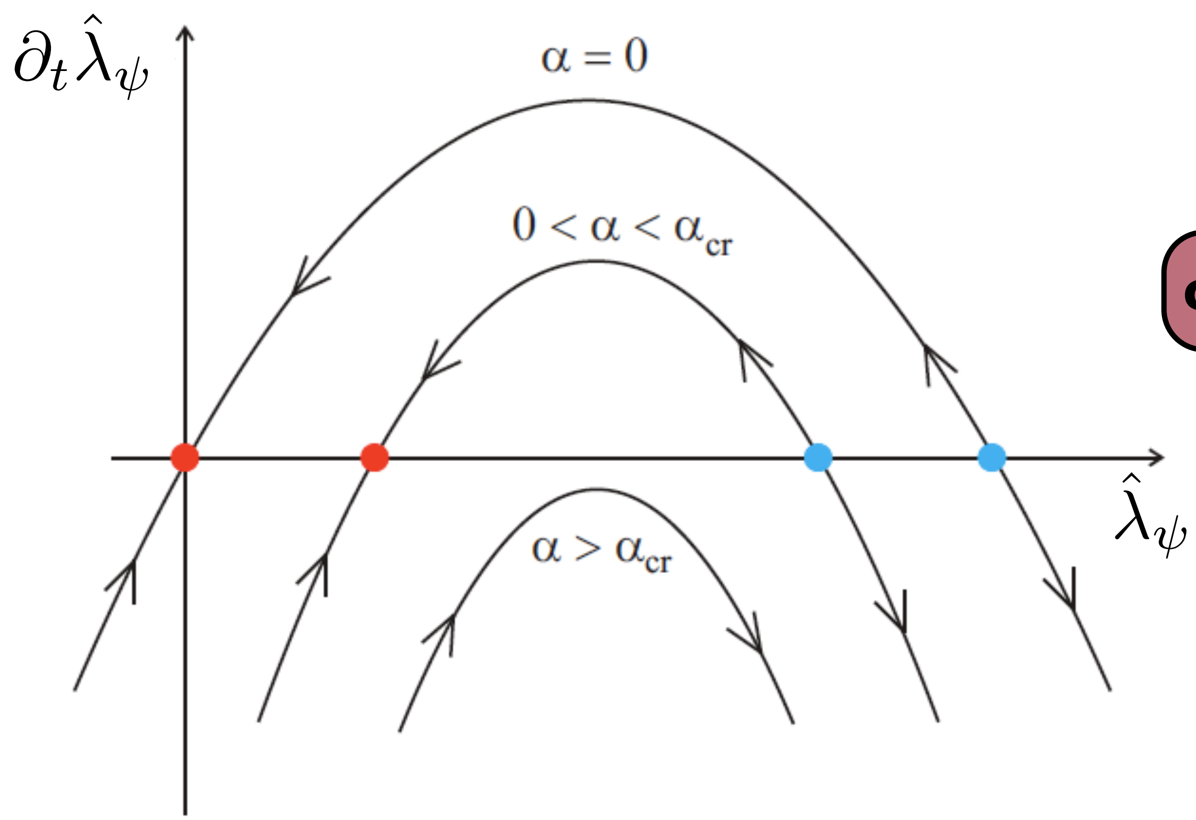

chiral symmetry breaking ⟷ $\alpha_s > \alpha_{s,cr}$

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JMP '05
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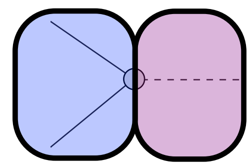

Hubbard-Stratonovich

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General dynamical hadronisation

hadronised Flow

$$\frac{\partial}{\partial t}\Big|_\phi\Gamma_k[\phi]=\frac{1}{2}G_{k,\phi}\dot R_{k,\phi}+R_kG_{k,\phi}\frac{\delta\dot\phi}{\delta\phi}-\frac{\delta\Gamma}{\delta\phi}\dot\phi$$

JMP '05

$$\phi=(A_\mu,C,\bar C,q,\bar q,\Phi,...,n,\bar n,...)$$

mesons baryons

$$-\frac{1}{2}\int_p\phi_k^*\cdot R_k\cdot\phi_k+J\cdot\phi_k$$

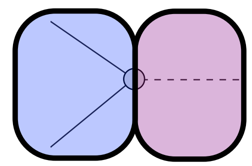
guarantees 1-loop flow

Dynamical hadronisation

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

$$\lambda_\psi=\frac{h^2}{m_\phi^2}$$



Hubbard-Stratonovich

$$\Phi=(\sigma,\vec\pi)$$

$$\tau\cdot\Phi=\sigma+i\gamma_5\vec\sigma\vec\pi$$

General dynamical hadronisation

hadronised Flow

$$\frac{\partial}{\partial t}\Big|_\phi\Gamma_k[\phi]=\frac{1}{2}G_{k,\phi}\dot R_{k,\phi}+R_kG_{k,\phi}\frac{\delta\dot\phi}{\delta\phi}-\frac{\delta\Gamma}{\delta\phi}\dot\phi$$

JMP '05

$$\phi=(A_\mu,C,\bar C,q,\bar q,\Phi,\ldots,n,\bar n,\ldots)$$

mesons baryons

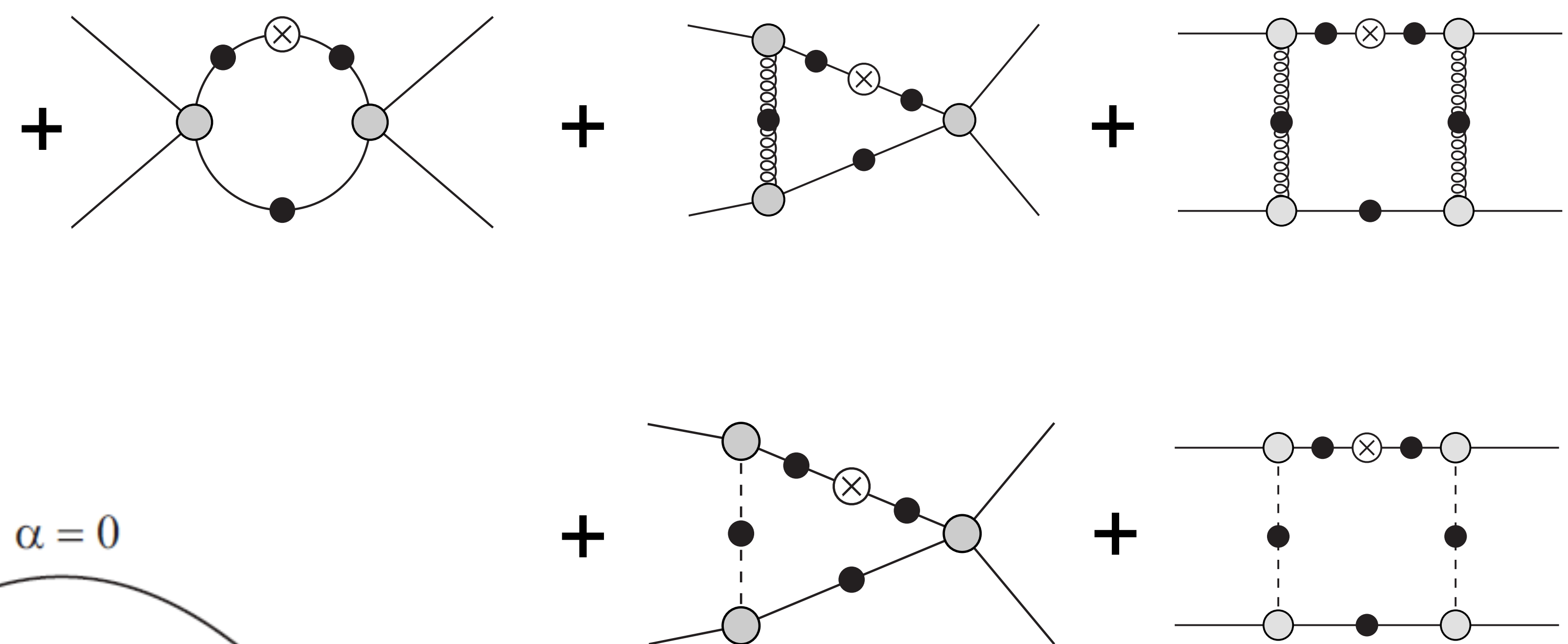
How to fix ϕ_k & $\dot\phi_k$?

$$\dot\Phi_k\simeq\dot A_k\bar\psi\tau\psi+\dot B_k\Phi_k+\dot C_k$$

Dynamical hadronisation

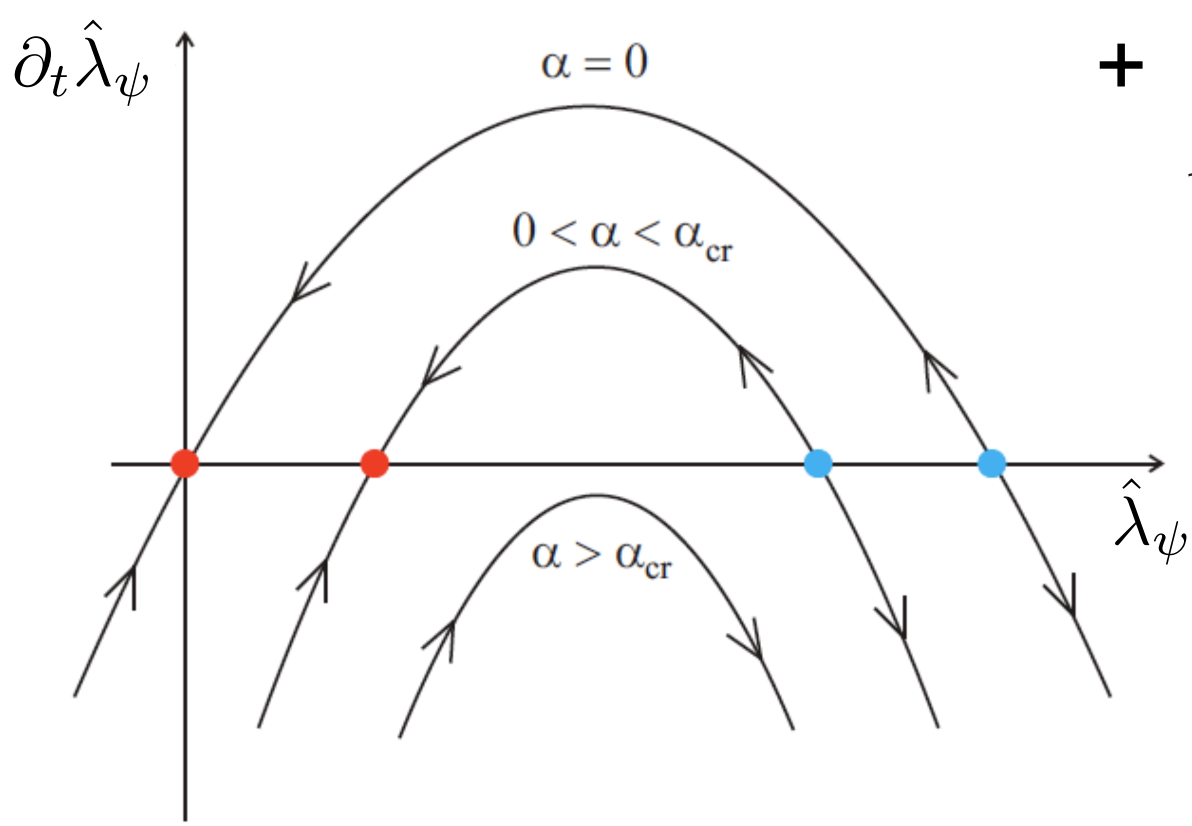
Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi + \ldots$$



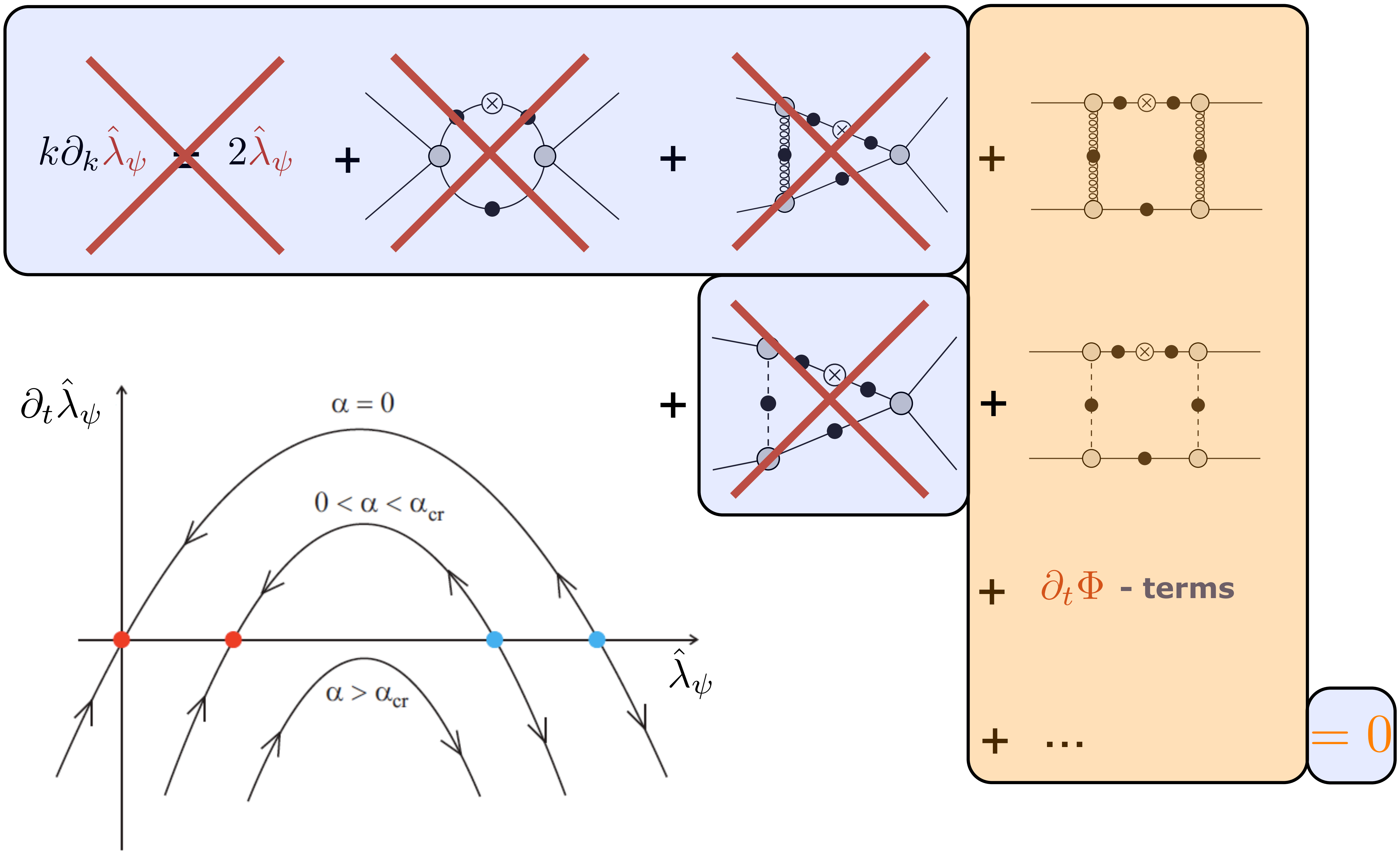
$+\ \partial_t \Phi$ **- terms**

$+\ \ldots$



Dynamical hadronisation

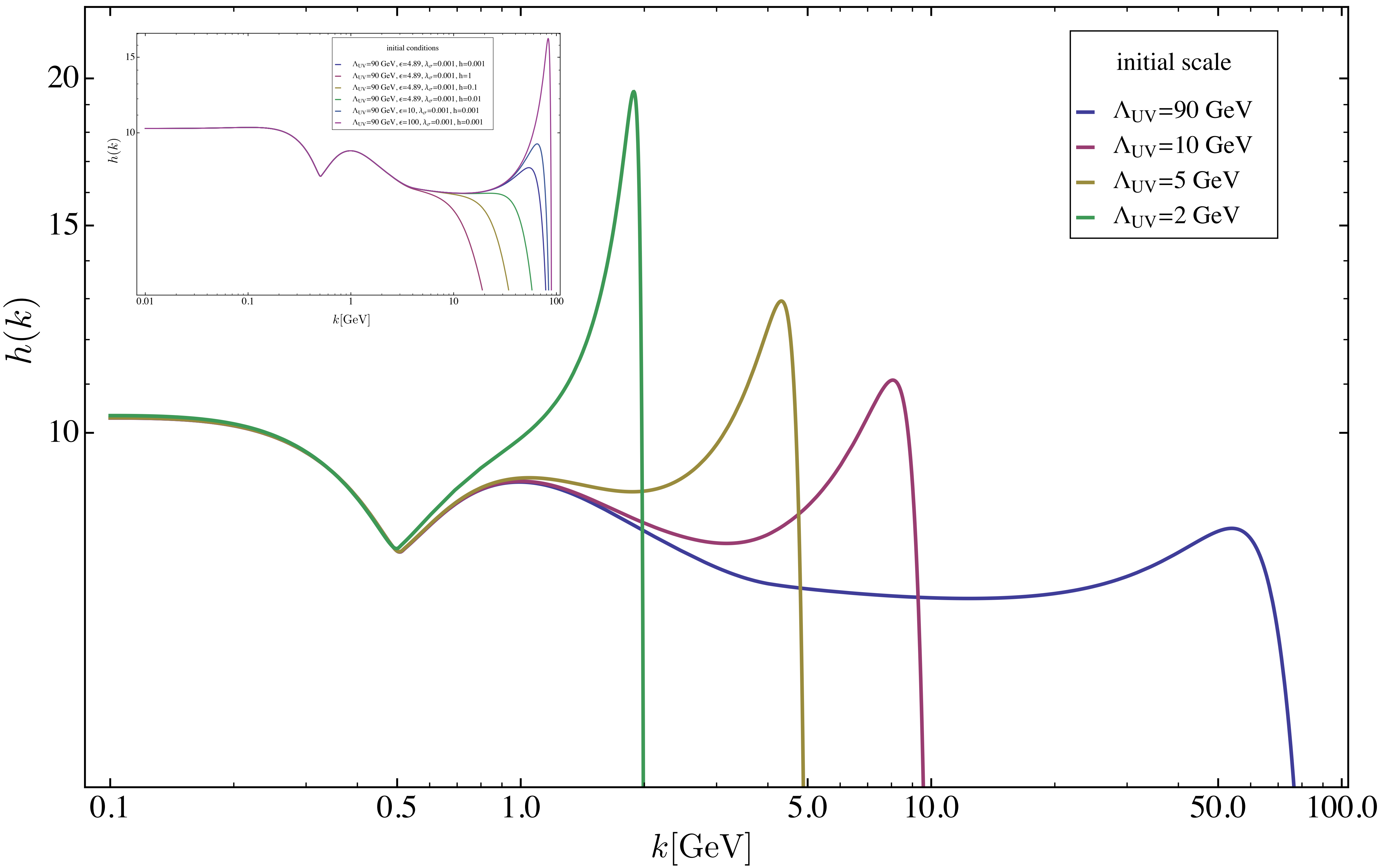
Full bosonisation $\hat{\lambda}_\psi = 0$



Dynamical hadronisation

Braun, Fister, Haas, JMP, Rennecke ’14
Mitter, JMP, Strodthoff ’14
Cyrol, Mitter, JMP, Strodthoff ‘17

Full bosonisation $\hat{\lambda}_\psi = 0$



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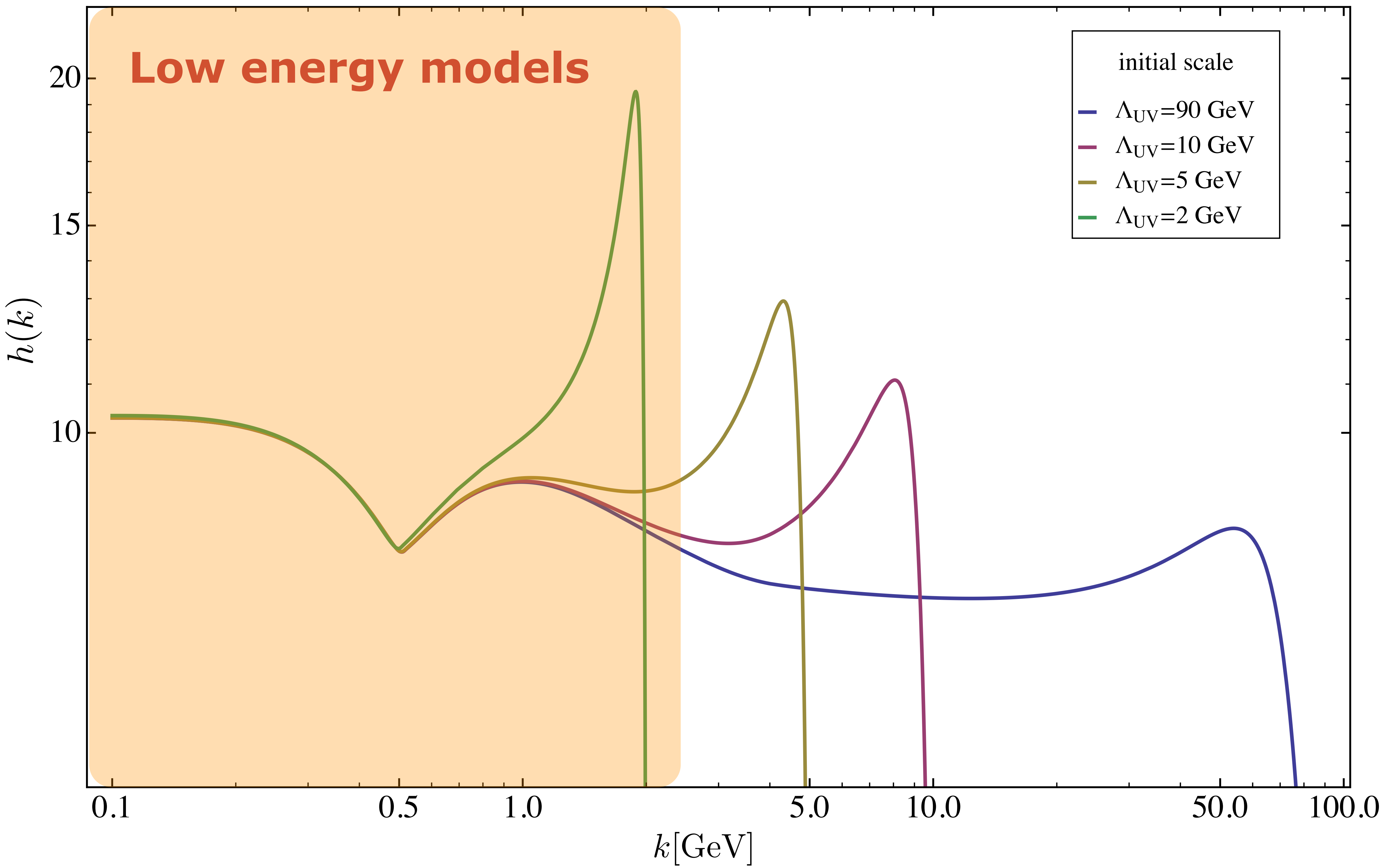

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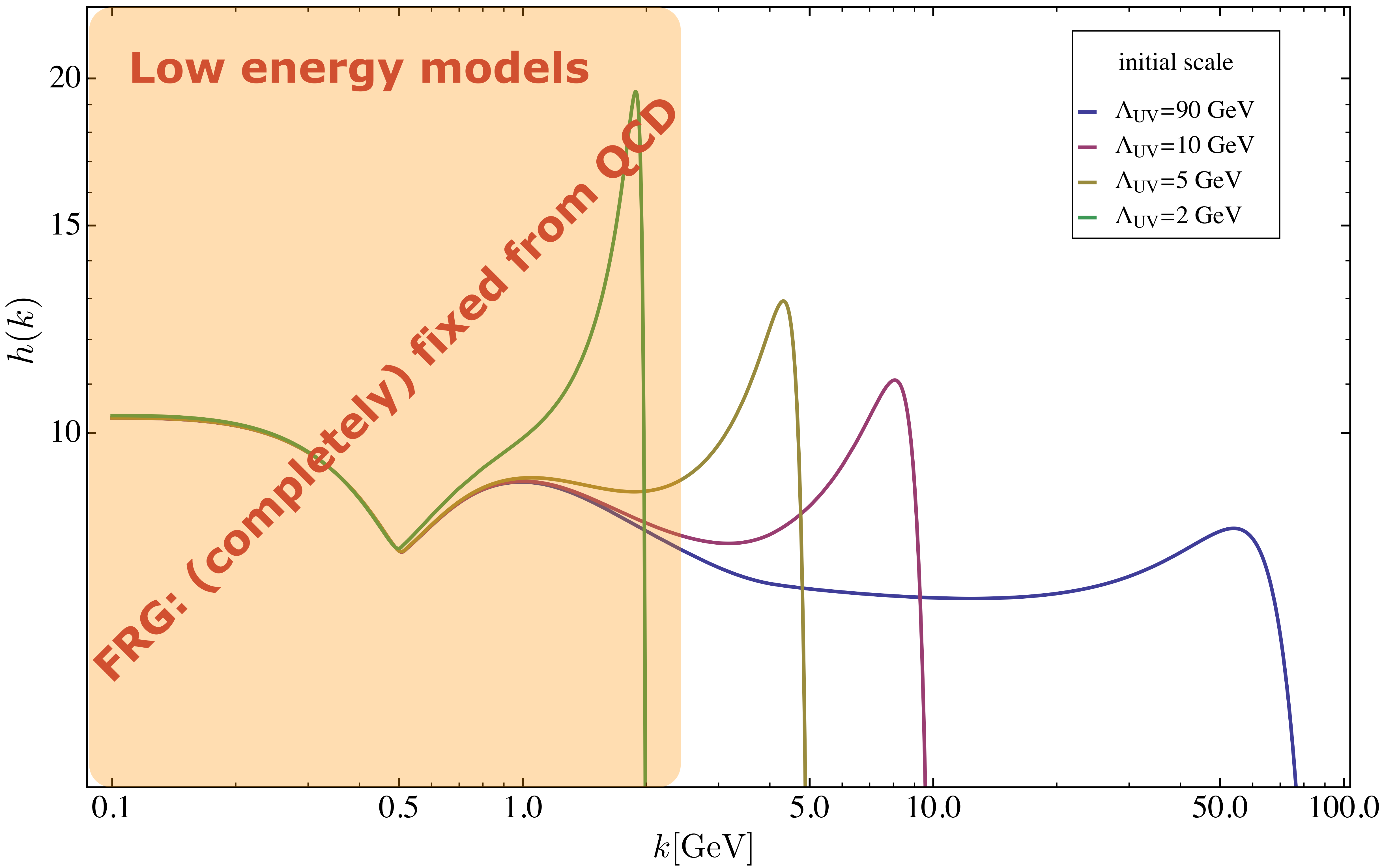
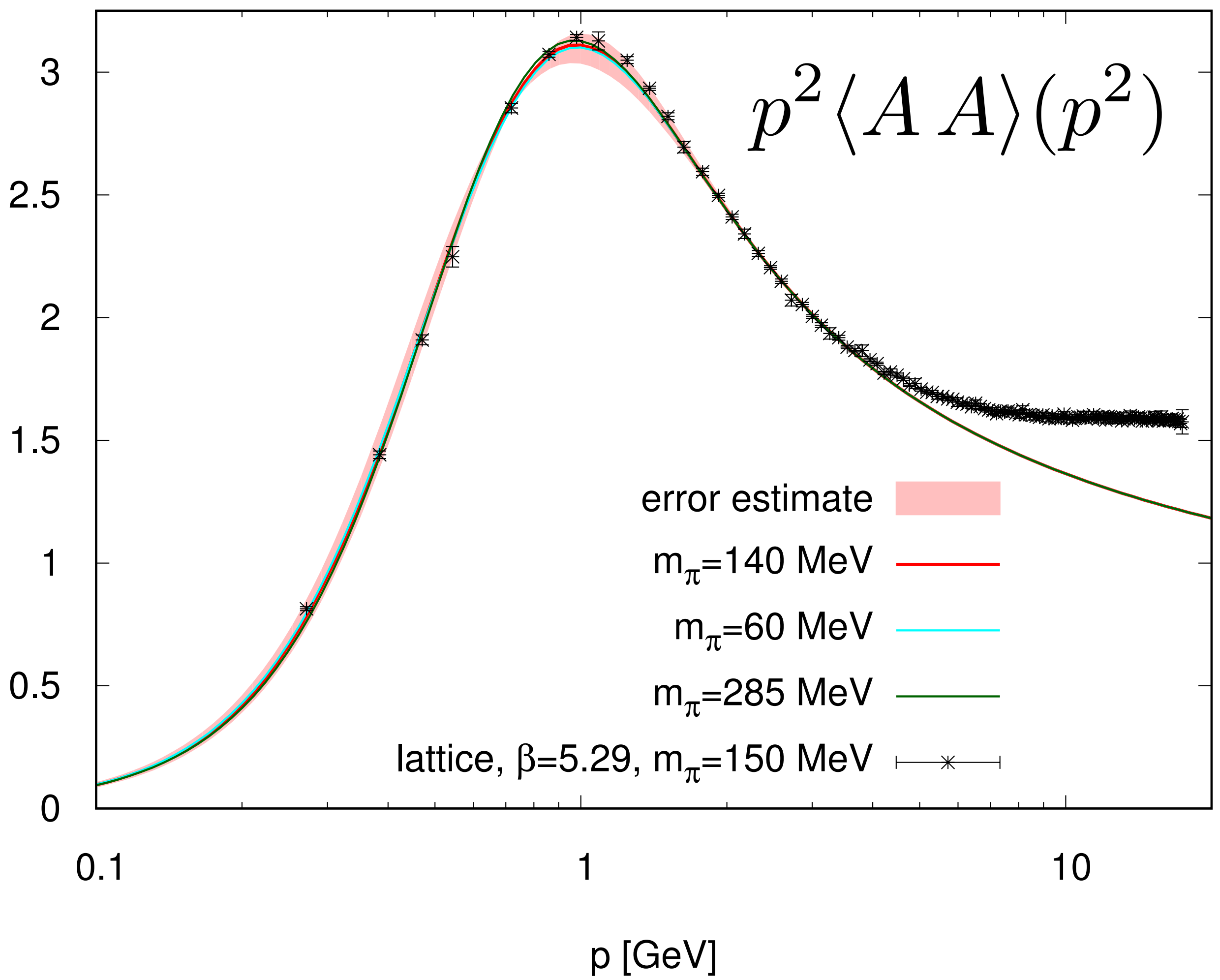
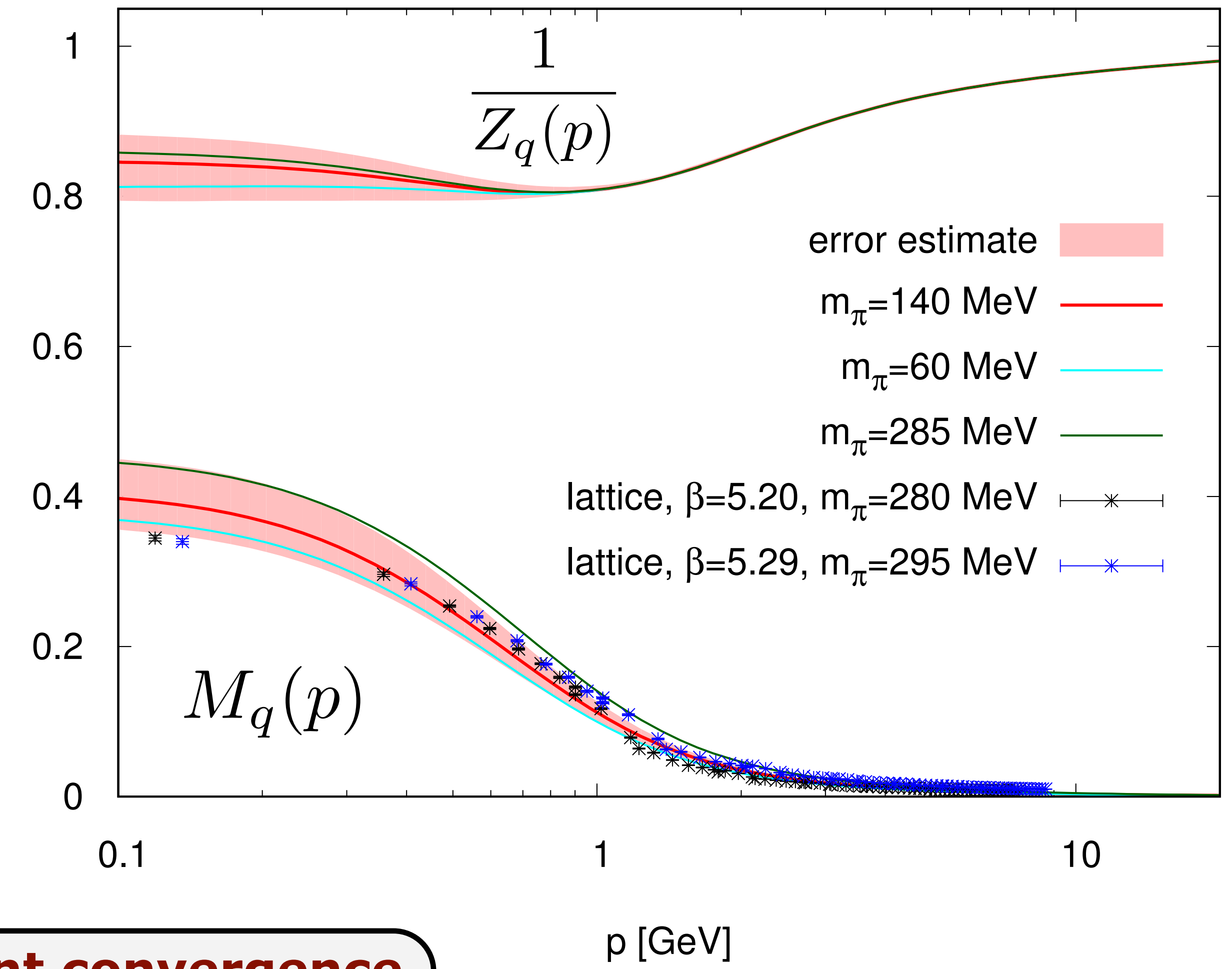


QCD: Euclidean propagators



lattice, e.g.: Oliviera et al, Acta Phys.Polon.Supp. 9 (2016) 363
Sternbeck et al, PoS LATTICE2016 (2017)
A. Athenodorou et al, PLB 761 (2016) 444

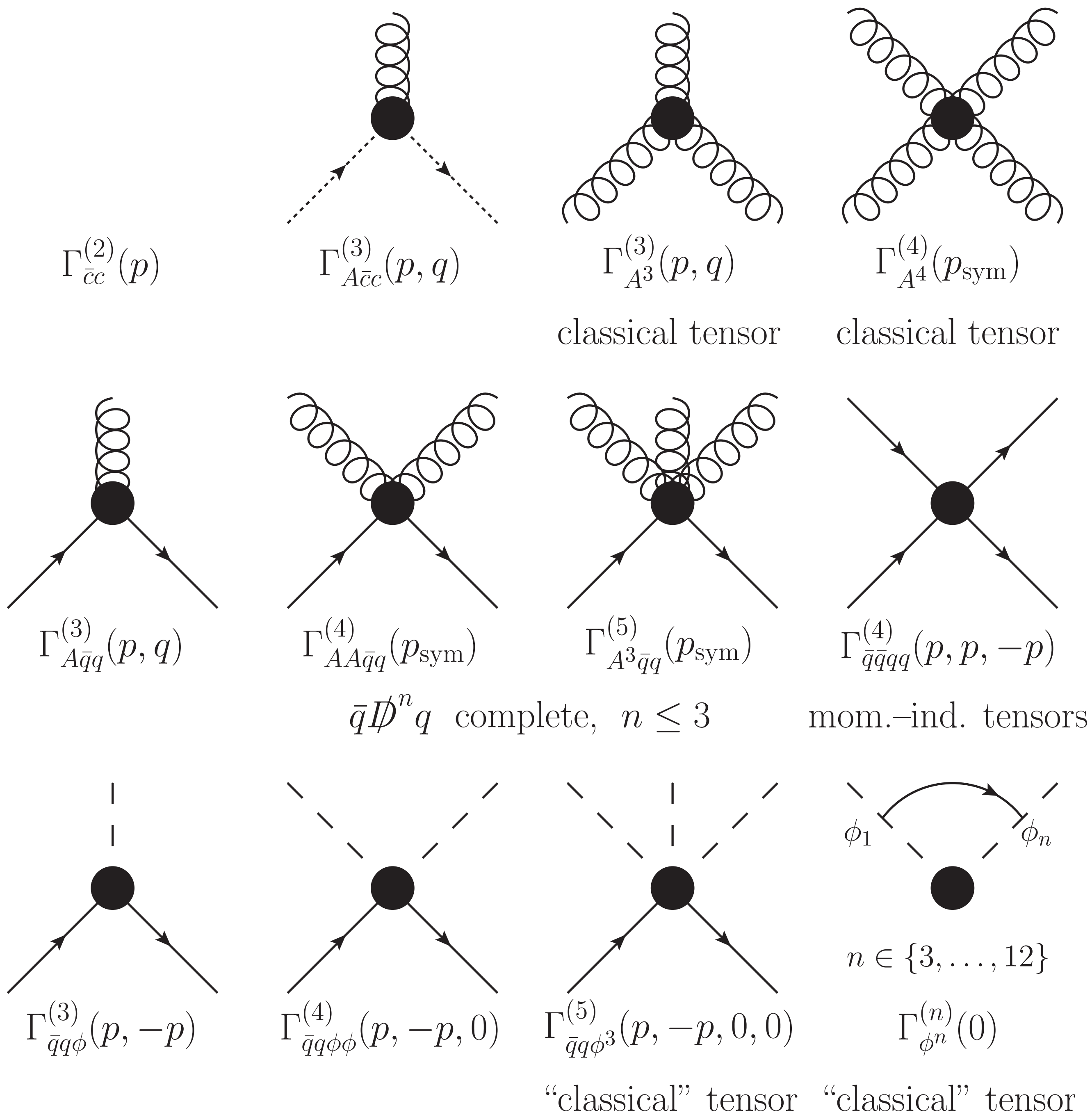
$$\frac{1}{Z_q(p)} \frac{1}{i\not{p} + M_q(p)}$$



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

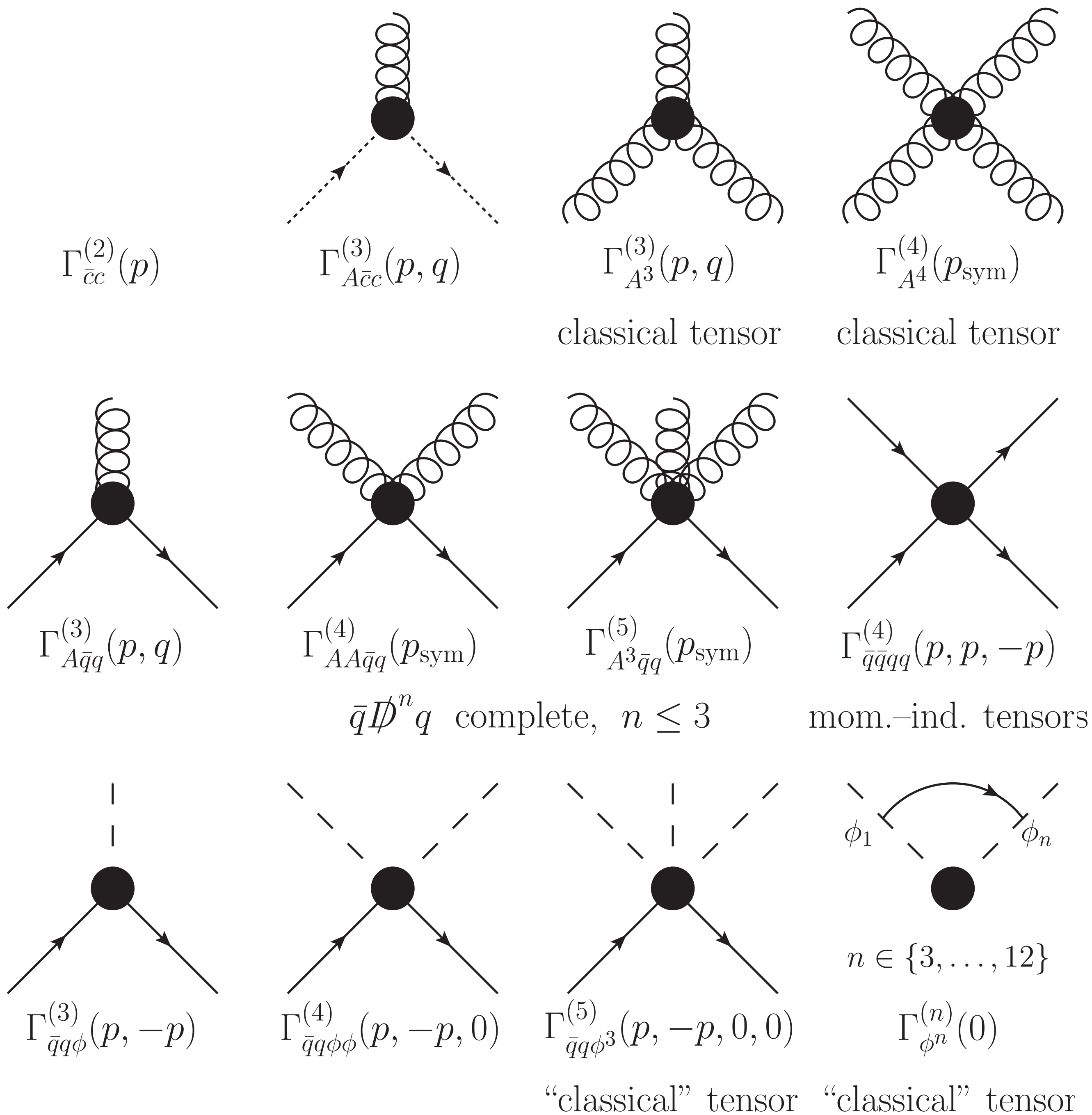
QCD: Vertices



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

QCD: Vertices

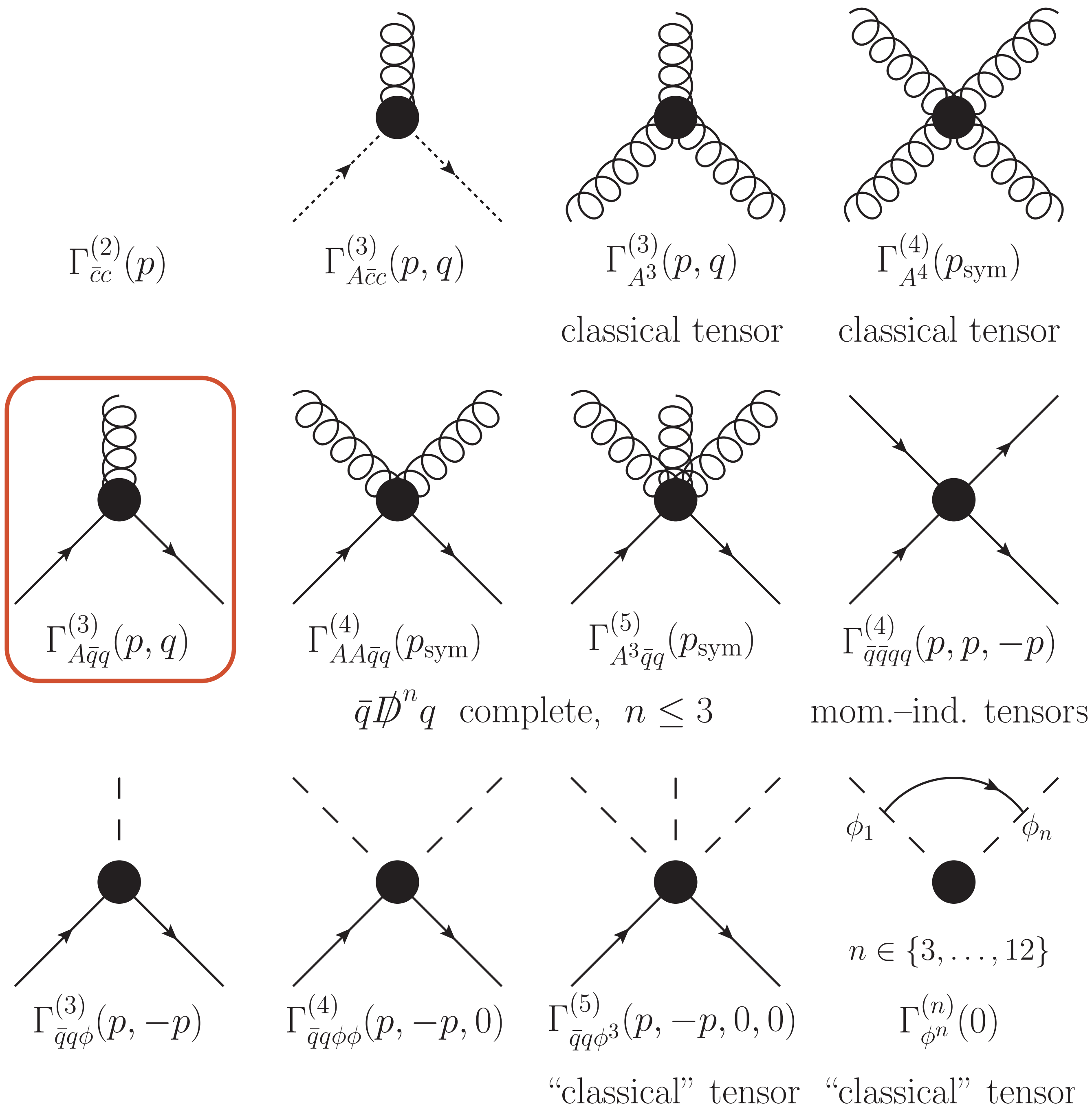


Welches Schweinderl hätten's denn gerne?

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

QCD: Vertices



Welches Schweinderl hätten's denn gerne?

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Quark-gluon vertex

$$\left[\Gamma^{(3)}_{\bar{q}qA}\right]^a_\mu (p,q) = \mathbb{1}^{\text{flav}}_{2\times 2}\, T^a \sum_{i=1}^{8} \lambda_i(p,q) \left[\mathcal{T}^{(i)}_{\bar{q}qA}\right]_\mu (p,q)$$

covariant expansion scheme

$\bar{q}\not{D}q :$ $$\left[\mathcal{T}^{(1)}_{\bar{q}qA}\right]_\mu (p,q) = -i\,\gamma_\mu$$

$\bar{q}\not{D}^2 q :$ $$\left[\mathcal{T}^{(2)}_{\bar{q}qA}\right]_\mu (p,q) = (p-q)_\mu\, \mathbb{1}_{4\times 4}$$

$$\left[\mathcal{T}^{(3)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}-\not{q})\gamma_\mu$$

$$\left[\mathcal{T}^{(4)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}+\not{q})\gamma_\mu$$

$\bar{q}\not{D}^3 q :$ $$\left[\mathcal{T}^{(5)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}+\not{q})(p-q)_\mu$$

$$\left[\mathcal{T}^{(6)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}-\not{q})(p-q)_\mu$$

$$\left[\mathcal{T}^{(7)}_{\bar{q}qA}\right]_\mu (p,q) = \frac{i}{2}[\not{p},\not{q}]\gamma_\mu$$

Aiming at apparent convergence

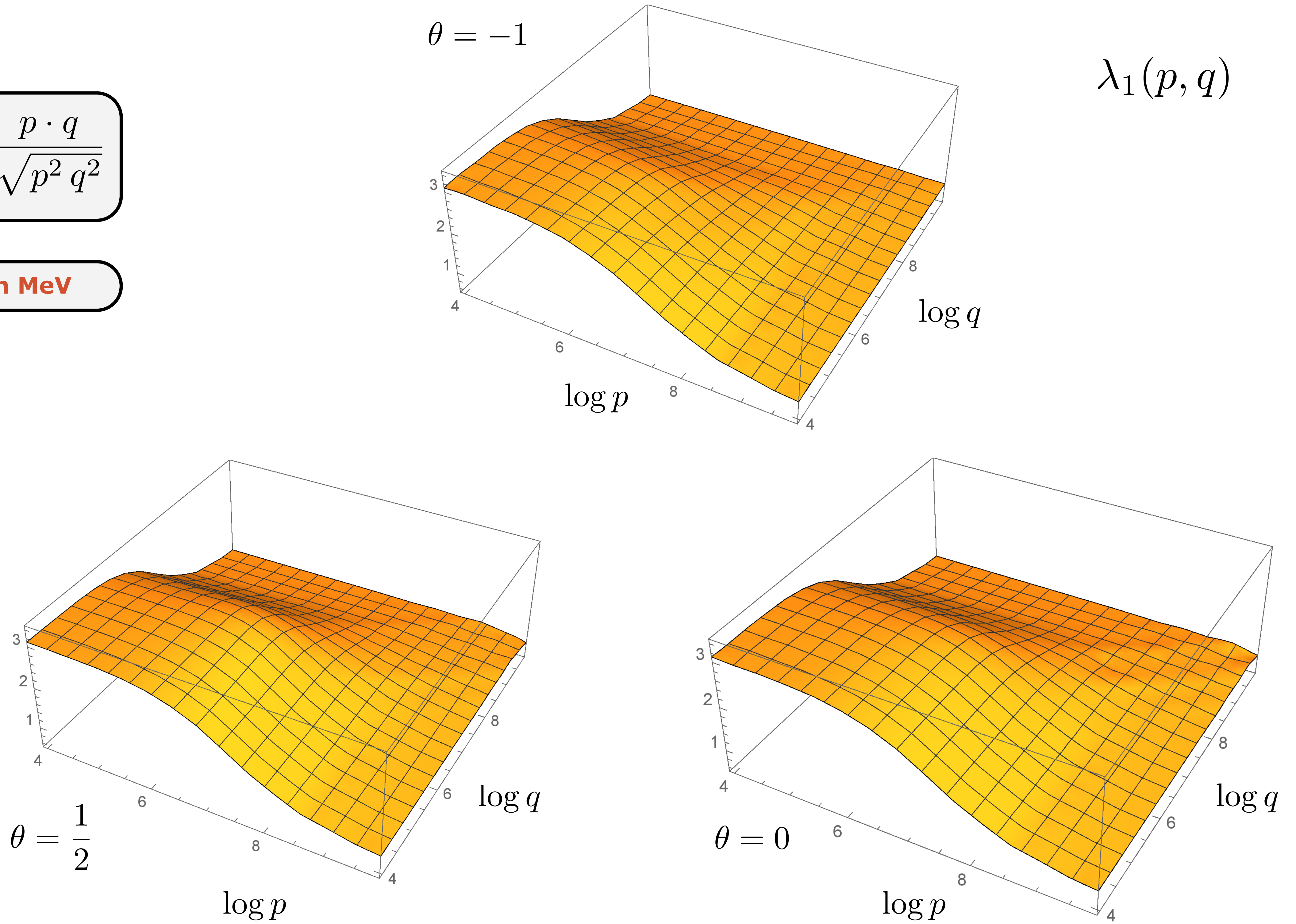
quenched: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Quark-gluon vertex

$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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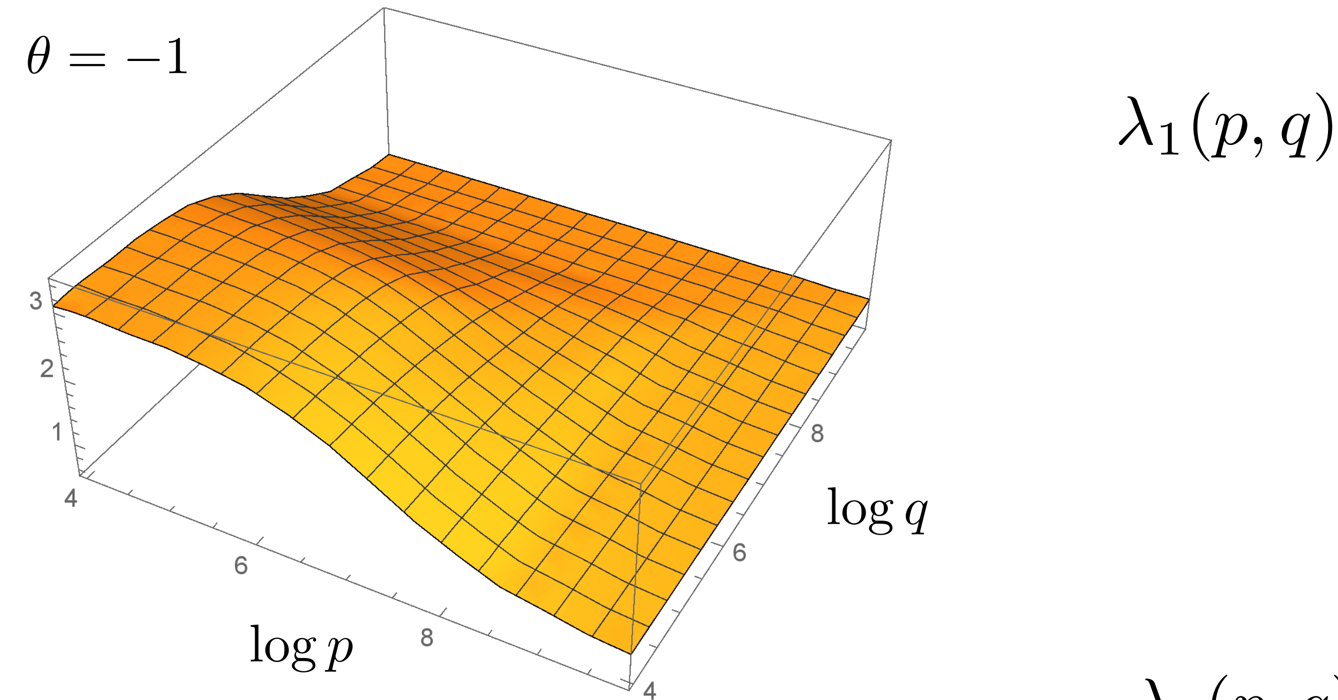
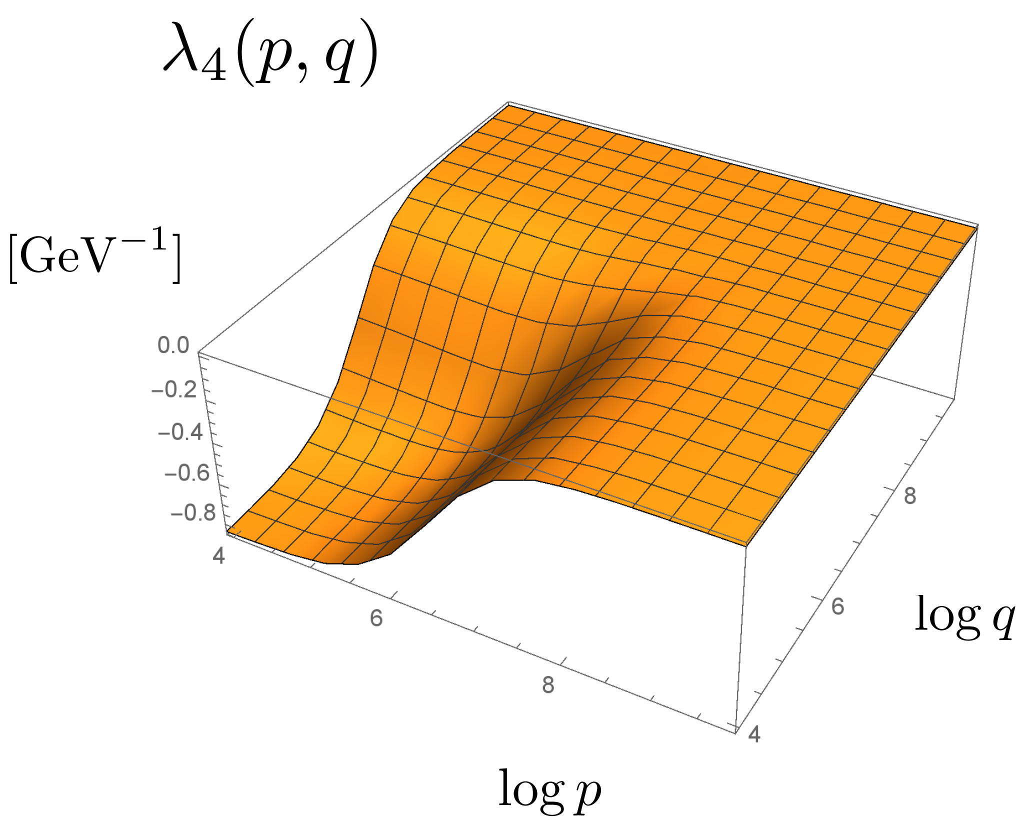
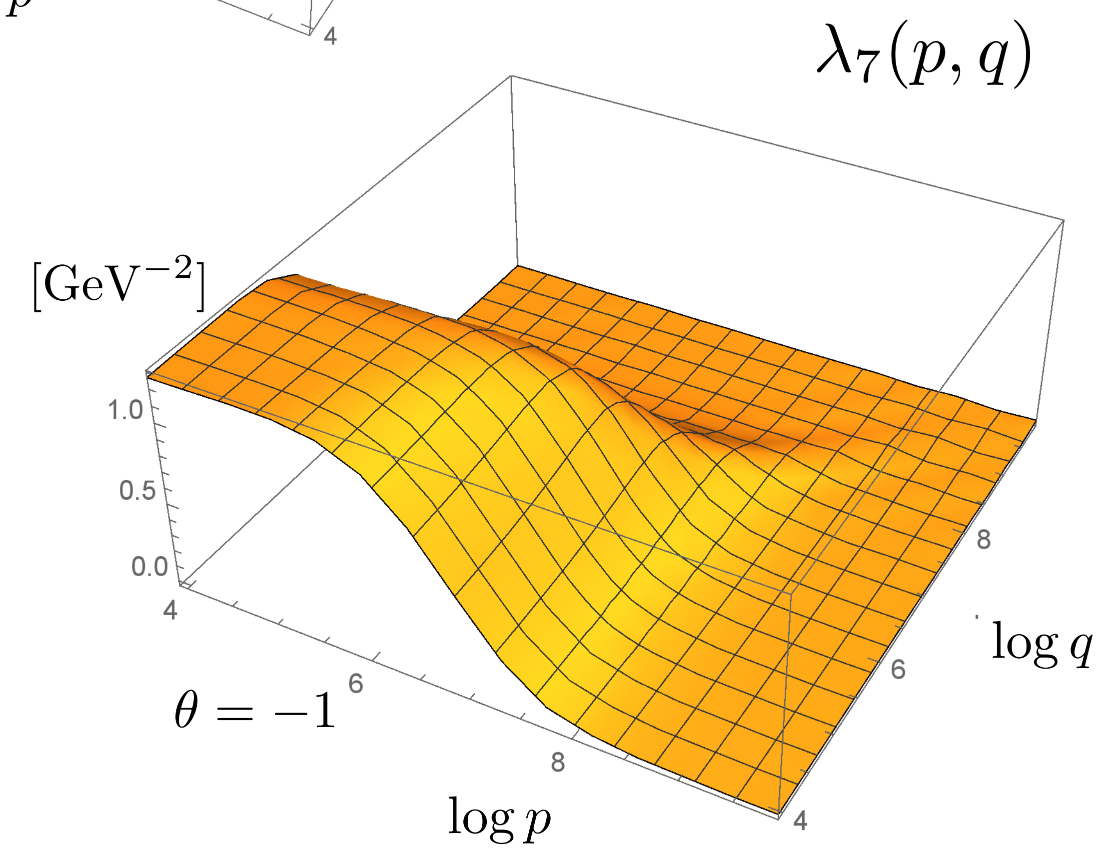
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV







Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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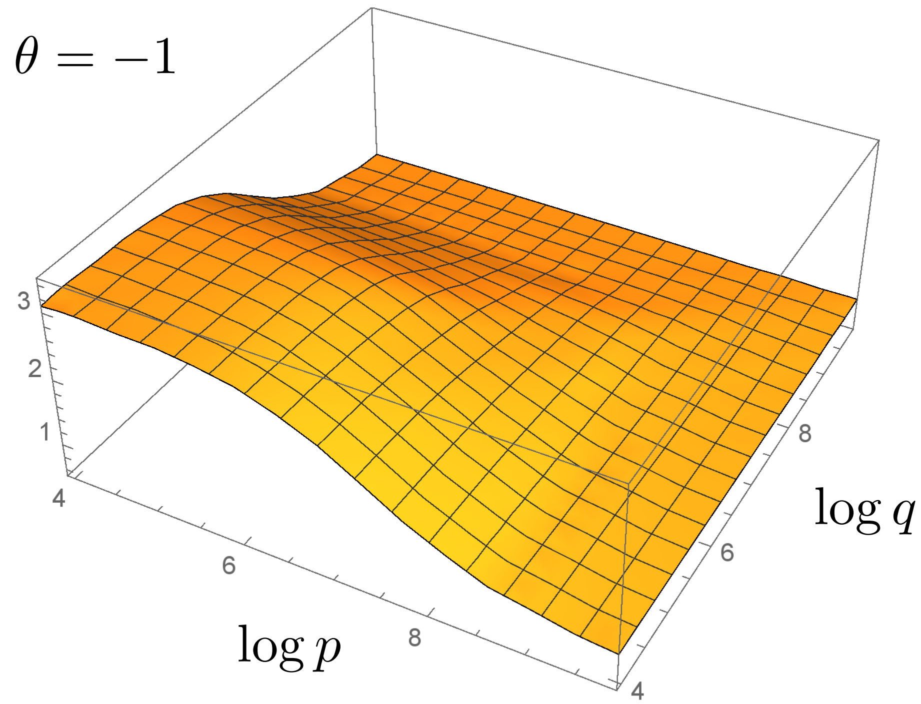


$\lambda_1(p, q)$

up-to-date 1st principles works:

FunMethods:

Williams, EPJ A51 (2015) 57
Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, Fischer, Heupel, PRD 93 (2016) 034026

Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027
Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 95 (2017) 031501
Aguilar, Cardona, Ferreira, Papavassiliou, arXiv:1610.06158

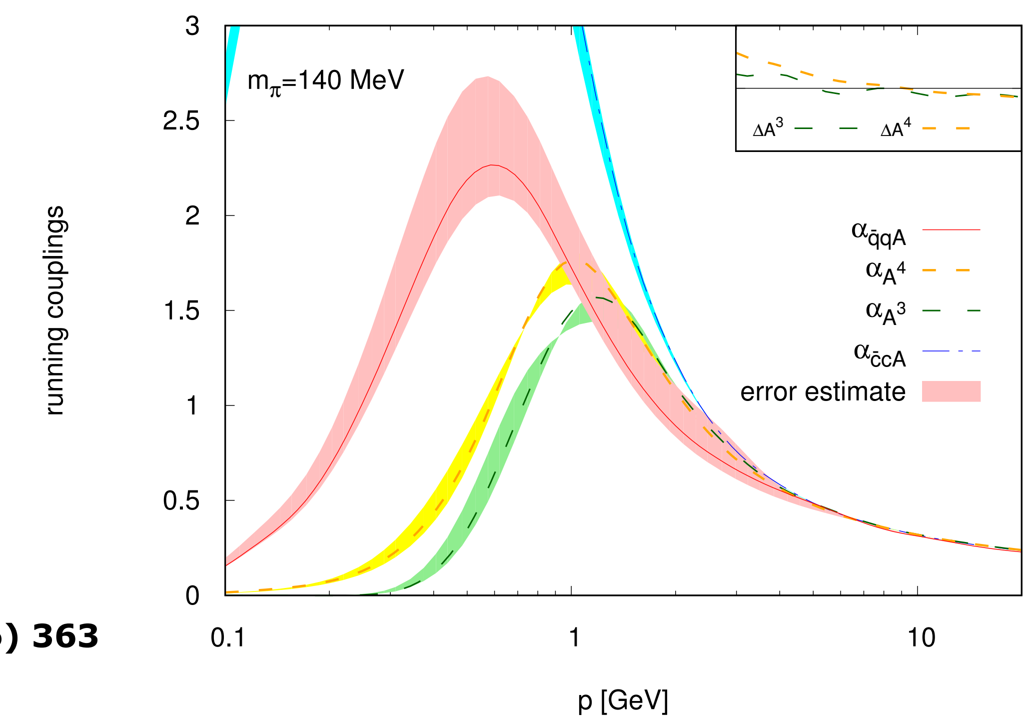
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012

Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

lattice: **Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363**

Beware of BRST



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

(III) Phase structure of QCD and dynamics

- **Yang-Mills theory at finite temperature**
  - Order parameter potential for confinement
  - Correlation functions at finite temperature
  - Polyakov loop from functional methods

- **Application to the phase structure of QCD and dynamics***
  - QCD-assisted hydrodynamics*
  - QCD-assisted transport*
  - QCD at imaginary chemical potential*

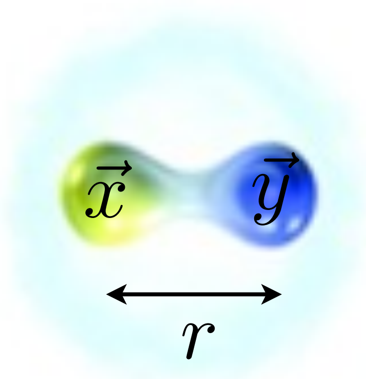
Yang-Mills theory at finite temperature

Order parameter potential for Confinement

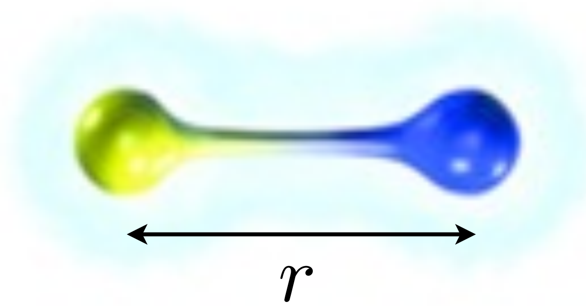
Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

Reminder

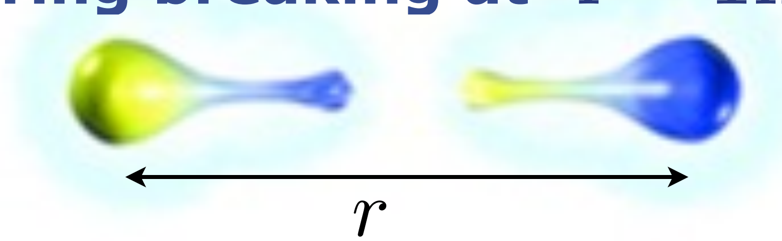


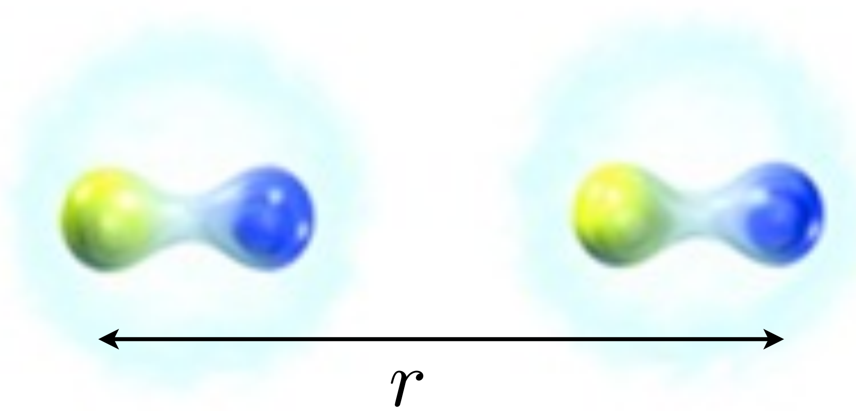
$$F_{q\bar{q}} \simeq -\frac{1}{r}$$



$$F_{q\bar{q}} \simeq \sigma r$$

string breaking at $\mathrm{r} \approx 1\mathrm{fm}$





$$F_{q\bar{q}} \simeq \mathrm{const.}$$

Order parameter $\sim {}'\langle q \rangle{}'$

$$\Phi = e^{-\frac{1}{2T} F_{q\bar{q}}(\infty)}$$

- **Confinement** $\Phi = 0$
- **Deconfinement** $\Phi \neq 0$

Polyakov loop

$$\Phi = \frac{1}{3} \langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0\, A_0\} \rangle$$

Confinement

Order parameters

Polyakov loop operator

$$L[A_0]=\frac{1}{N_c}\mathrm{tr}\mathcal{P}\quad e^{ig\int_0^{1/T}dt A_0}$$

$\Phi=\langle L[A_0]\rangle$

order parameter

$L[\langle A_0\rangle]$ **order parameter**

$$L[\langle A_0\rangle]=0\longleftrightarrow\langle L[A_0]\rangle=0$$

$$L[\langle A_0\rangle]\geq\langle L[A_0]\rangle$$

up to lattice renormalisation

Braun, Gies, JMP '07
Marhauser, JMP '08

$\langle A_0\rangle$ **order parameter**

$$\left.\frac{\partial V[A_0]}{\partial A_0}\right|_{A_0=\langle A_0\rangle}=0$$

$$V[A_0]=\frac{1}{\beta\,\mathrm{Vol}_3}\Gamma[A_0]$$

constant backgrounds

background Landau gauge

Confinement

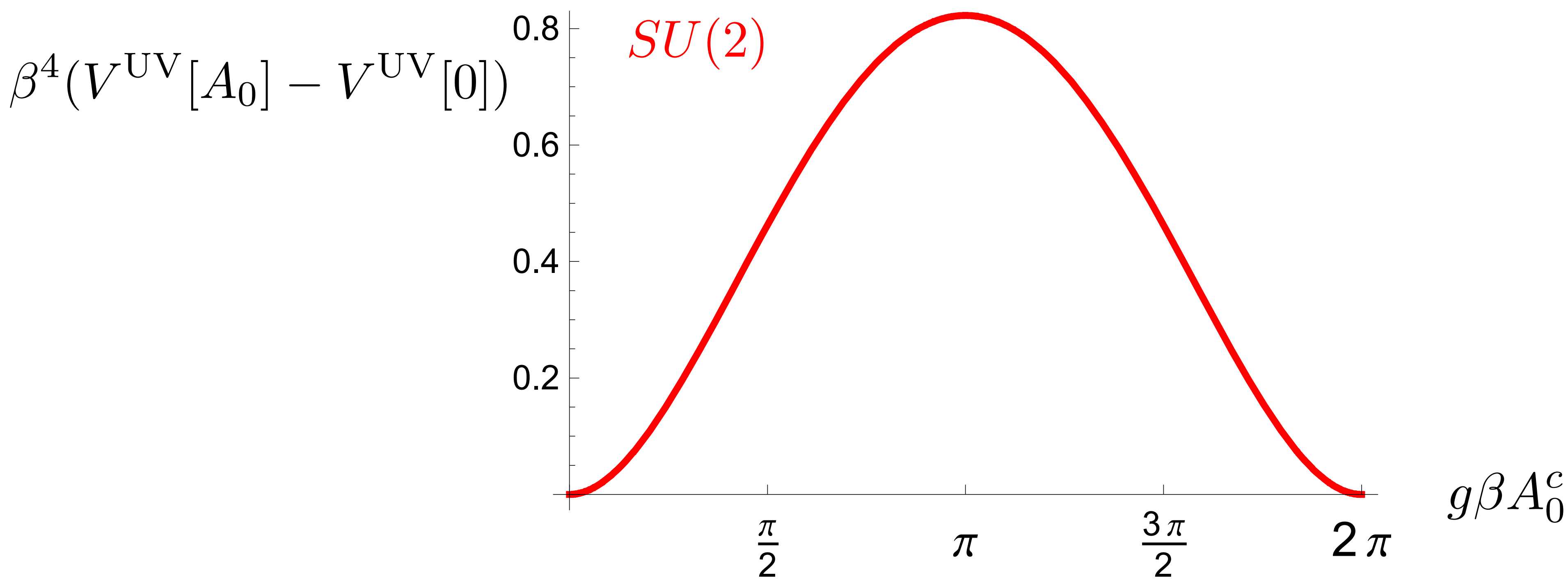
Effective Polyakov loop potential

One-loop

$$V^{\mathrm{UV}}[A_0] = \frac{1}{2\Omega}\,\mathrm{Tr}\log S^{(2)}_{AA}[A_0] - \frac{1}{\Omega}\,\mathrm{Tr}\log S^{(2)}_{C\bar{C}}[A_0]$$

free energy

Gross, Pisarski, Yaffe '81
Weiss '81



$$SU(2):\ \Phi[A_0] = \cos\frac{1}{2}\beta g A_0^c \qquad \text{with} \qquad A_0 = A_0^c\,\frac{\sigma_3}{2}$$

Non-perturbative effective potential

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

Confinement

Effective Polyakov loop potential

Non-perturbative effective potential

$$V[A_0] \simeq -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

flow

$$\frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t R_k = \frac{1}{2}\mathrm{Tr}\,\partial_t\log(\Gamma_k^{(2)}[\phi]+R_k) - \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t\Gamma_k^{(2)}[\phi]$$

Confinement

Effective Polyakov loop potential

Non-perturbative effective potential

$$V[A_0] \simeq -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

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Propagators

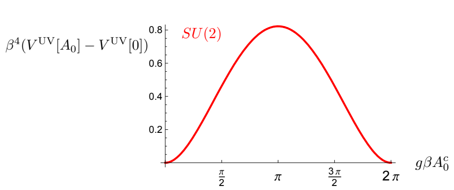
$$\langle AA\rangle[A_0] \simeq \frac{1}{-D_\mu^2(A_0)}\,\frac{1}{Z[-D_\mu^2(A_0)]}$$

Integrals & sums

$$\mathrm{Tr}\,f[-D_\mu^2(A_0)] = \sum\!\!\!\!\!\!\int_{\vec{p},\pm} f[(2\pi T)^2(n\pm\varphi)^2+\vec{p}^{\,2}] + \varphi-\text{indep.terms}$$

$$g\,A_0 = \frac{\varphi}{2\pi T}\tau^3_{\mathrm{ad}}$$

One-loop result



$$\beta^4 V^{UV}[A_0] = -2*3\left(\frac{\pi^2}{90} - \frac{2\pi^2}{3}\tilde{\varphi}^2(1-\tilde{\varphi})^2\right)$$

$$\tilde{\varphi} = \varphi \mod 1$$

Confinement

Effective Polyakov loop potential

Non-perturbative effective potential

$$V[A_0] \simeq -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

flow

$$\frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t R_k = \frac{1}{2}\mathrm{Tr}\,\partial_t \log(\Gamma_k^{(2)}[\phi]+R_k) - \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t\Gamma_k^{(2)}[\phi]$$

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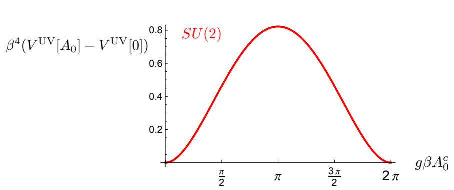
$$\beta^4 p_{\mathrm{SB}}$$

$$g\,A_0 = \frac{\varphi}{2\pi T}\tau^3_{\mathrm{ad}}$$

One-loop result



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free energy

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$$\frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t R_k = \frac{1}{2}\mathrm{Tr}\,\partial_t\log(\Gamma_k^{(2)}[\phi]+R_k) - \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t\Gamma_k^{(2)}[\phi]$$

Propagators

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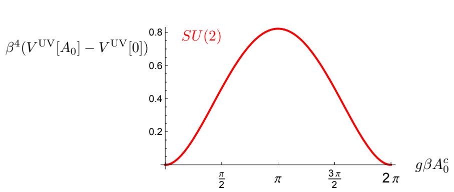
Integrals & sums

$$\mathrm{Tr}\,f[-D_\mu^2(A_0)] = \sum\!\!\!\!\!\!\int_{\vec{p},\pm} f[(2\pi T)^2(n\pm\varphi)^2+\vec{p}^{\,2}] + \varphi - \text{indep.terms}$$

$$N_c^2-1$$

$$g\,A_0 = \frac{\varphi}{2\pi T}\tau^3_{\mathrm{ad}}$$

One-loop result



$$\beta^4 V^{UV}[A_0] = -2 * 3\left(\frac{\pi^2}{90} - \frac{2\pi^2}{3}\tilde{\varphi}^2(1-\tilde{\varphi})^2\right)$$

$$\tilde{\varphi} = \varphi \mod 1$$

Confinement

Effective Polyakov loop potential

Non-perturbative effective potential

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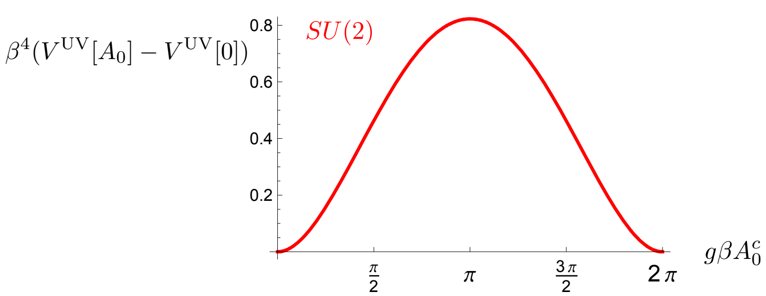
free energy

Braun, Gies, JMP '07

Fister, JMP '13



Confinement criterion

$$\beta^4 V^{UV}[A_0] = -2 * 3\left(\frac{\pi^2}{90} - \frac{2\pi^2}{3}\tilde{\varphi}^2(1-\tilde{\varphi})^2\right)$$

2 = 2 transversal physical polarisations+ 1 transversal (zero mode)+1 longitudinal - 2 ghosts

Confinement

Effective Polyakov loop potential

Non-perturbative effective potential

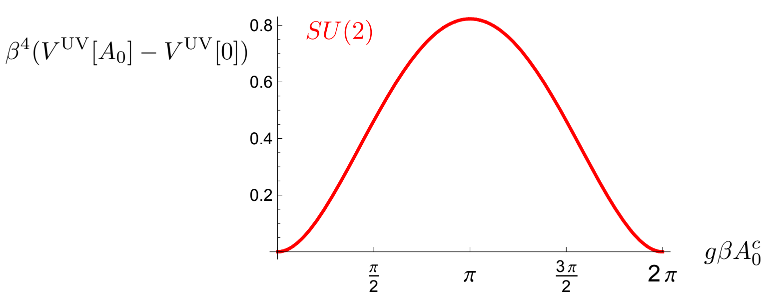
$$V[A_0] \simeq -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

Confinement criterion

Braun, Gies, JMP '07

Fister, JMP '13



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2 **= 2 transversal physical polarisations+ 1 transversal (zero mode)+1 longitudinal - 2 ghosts**

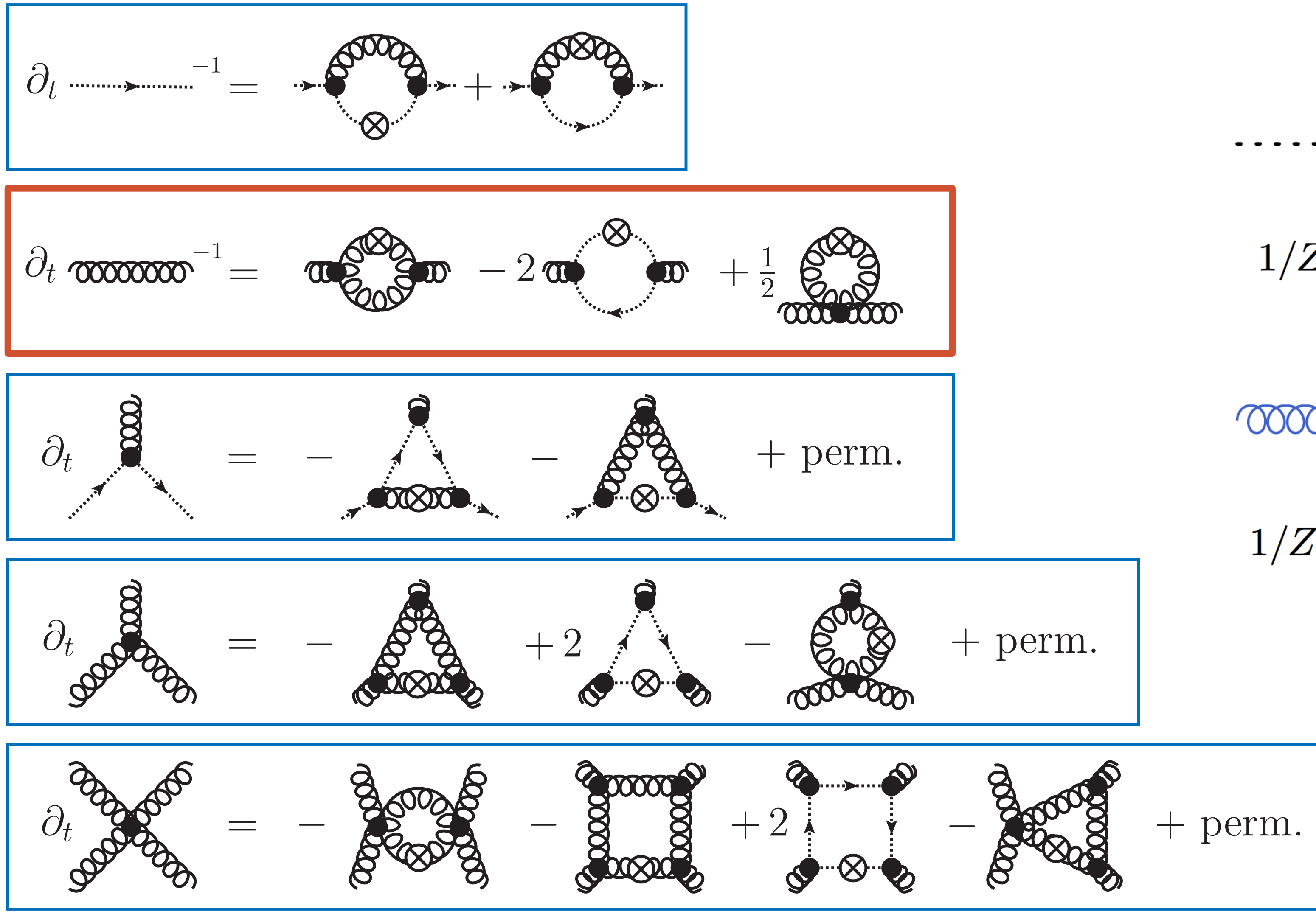
Gluon contribution deconfines

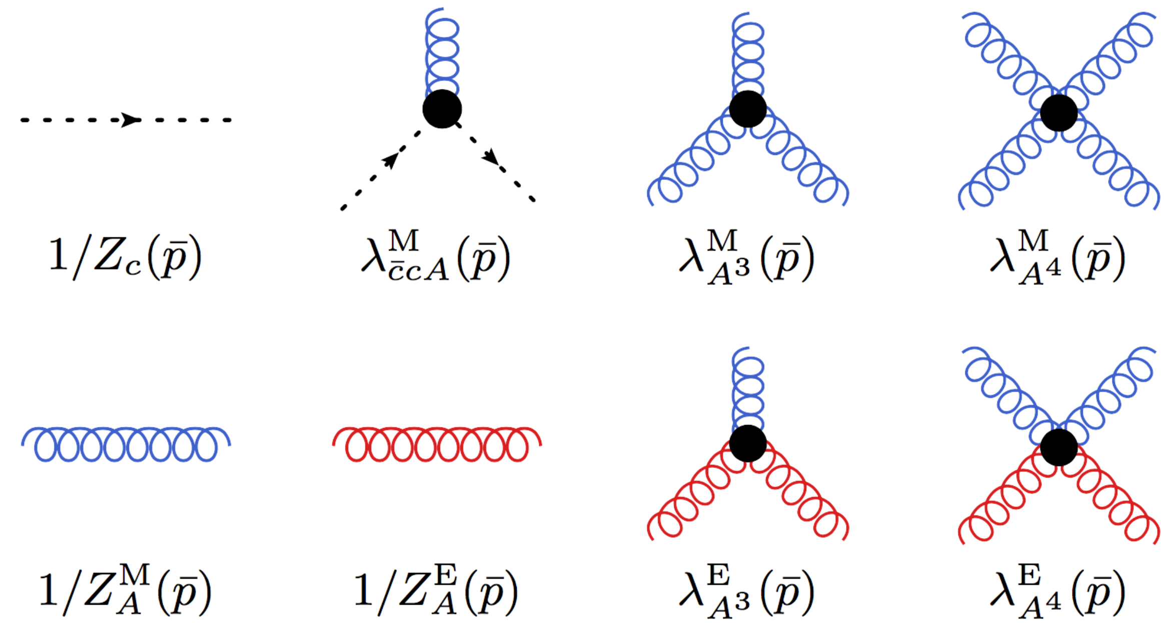
Ghost contribution confines

Confinement ⟷ suppression of the gluon relative to the ghost

Correlation functions at finite temperature

YM-theory: gluonic correlation functions

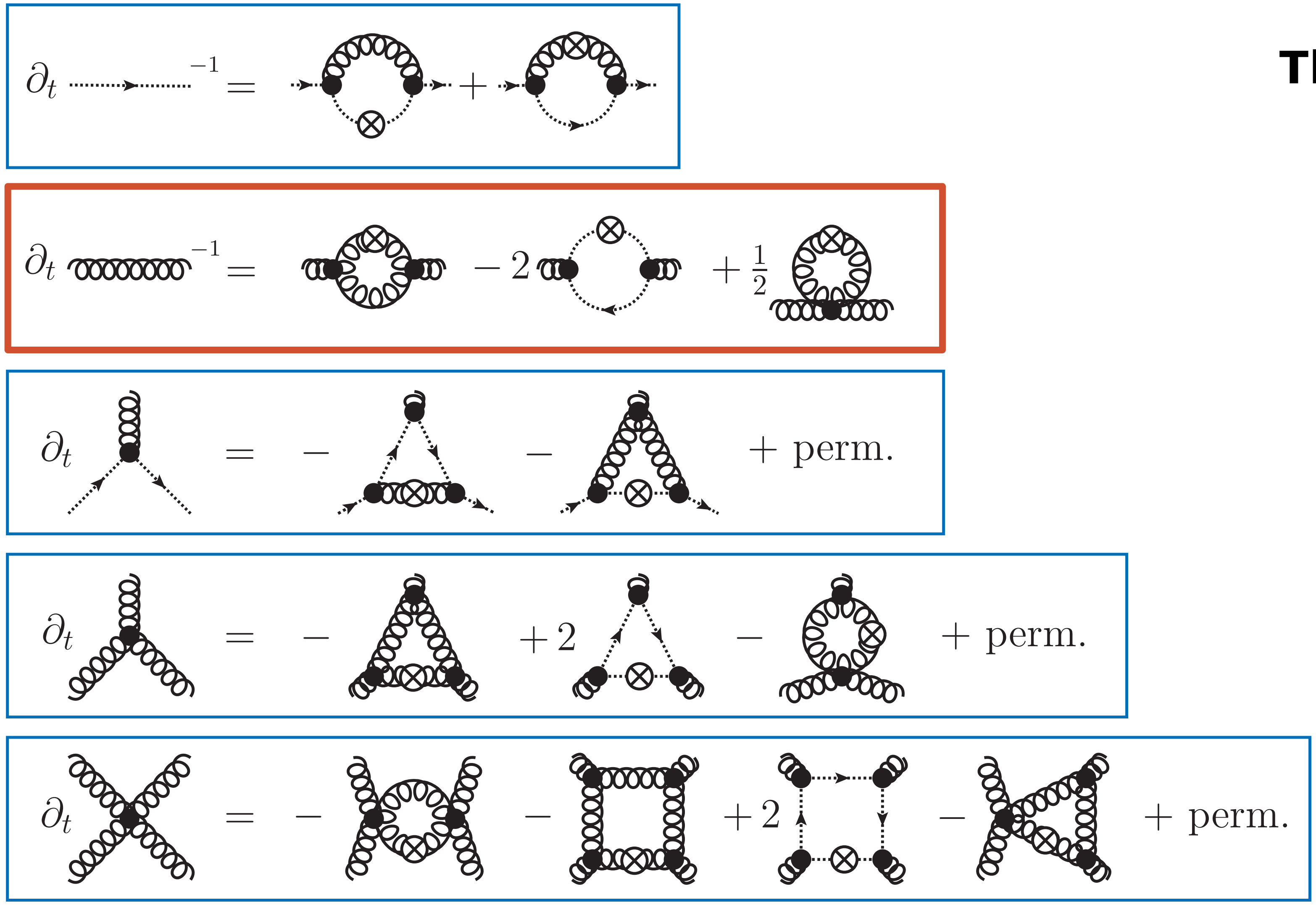




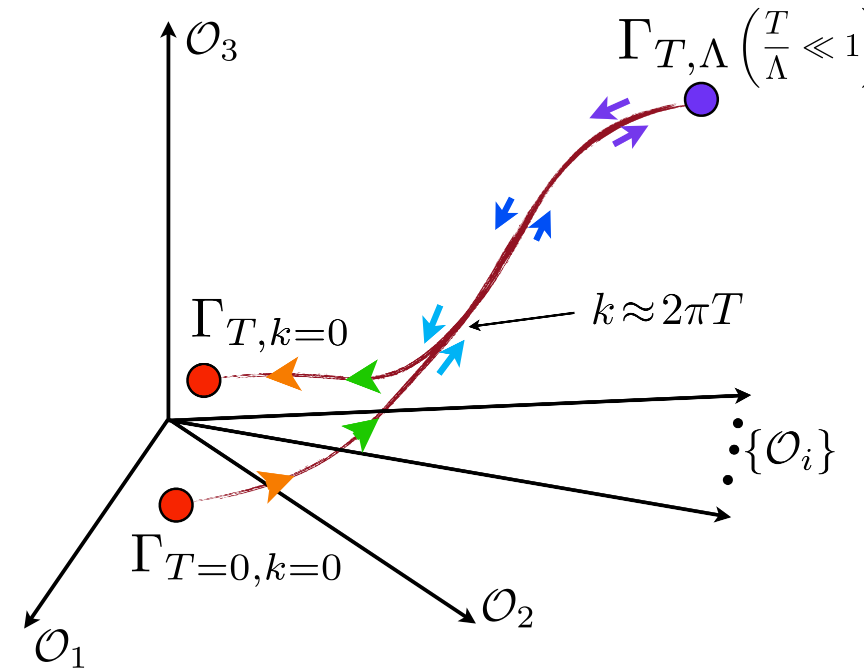
Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

YM-theory: gluonic correlation functions



Thermal flows

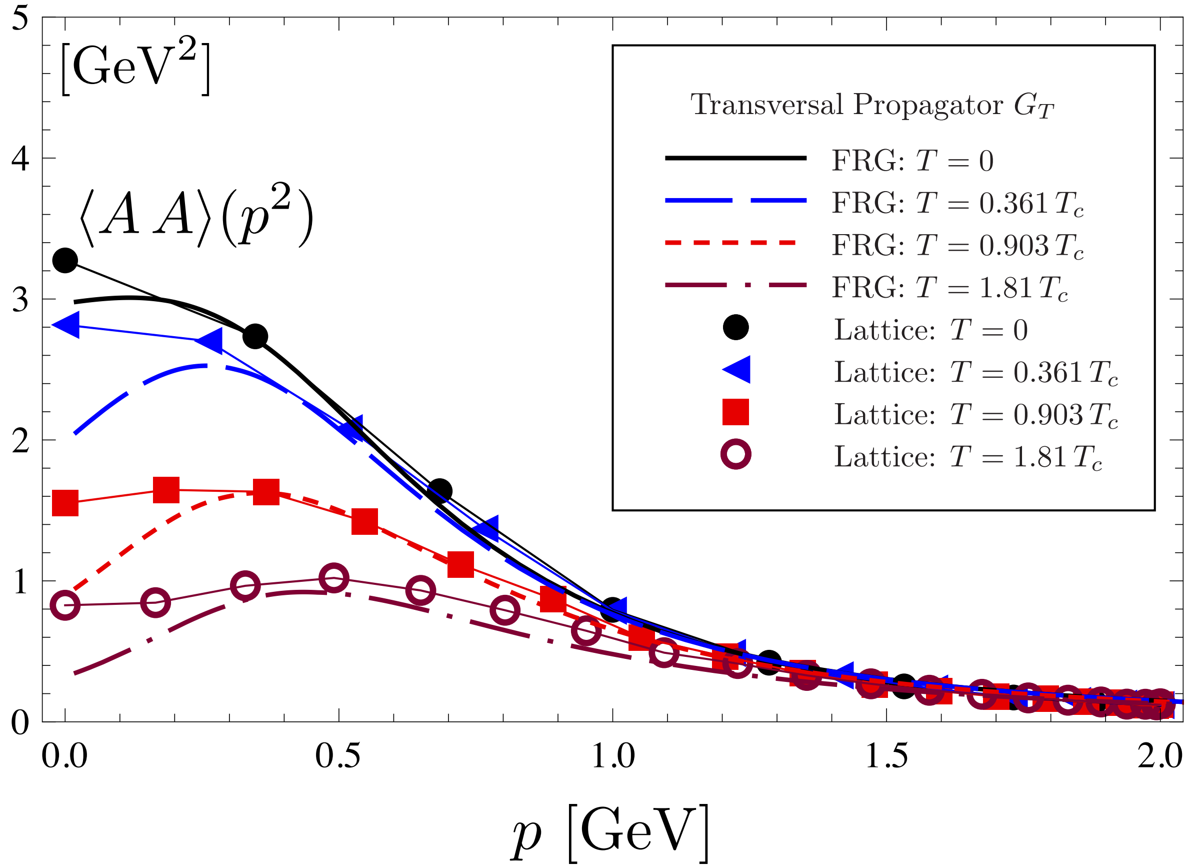


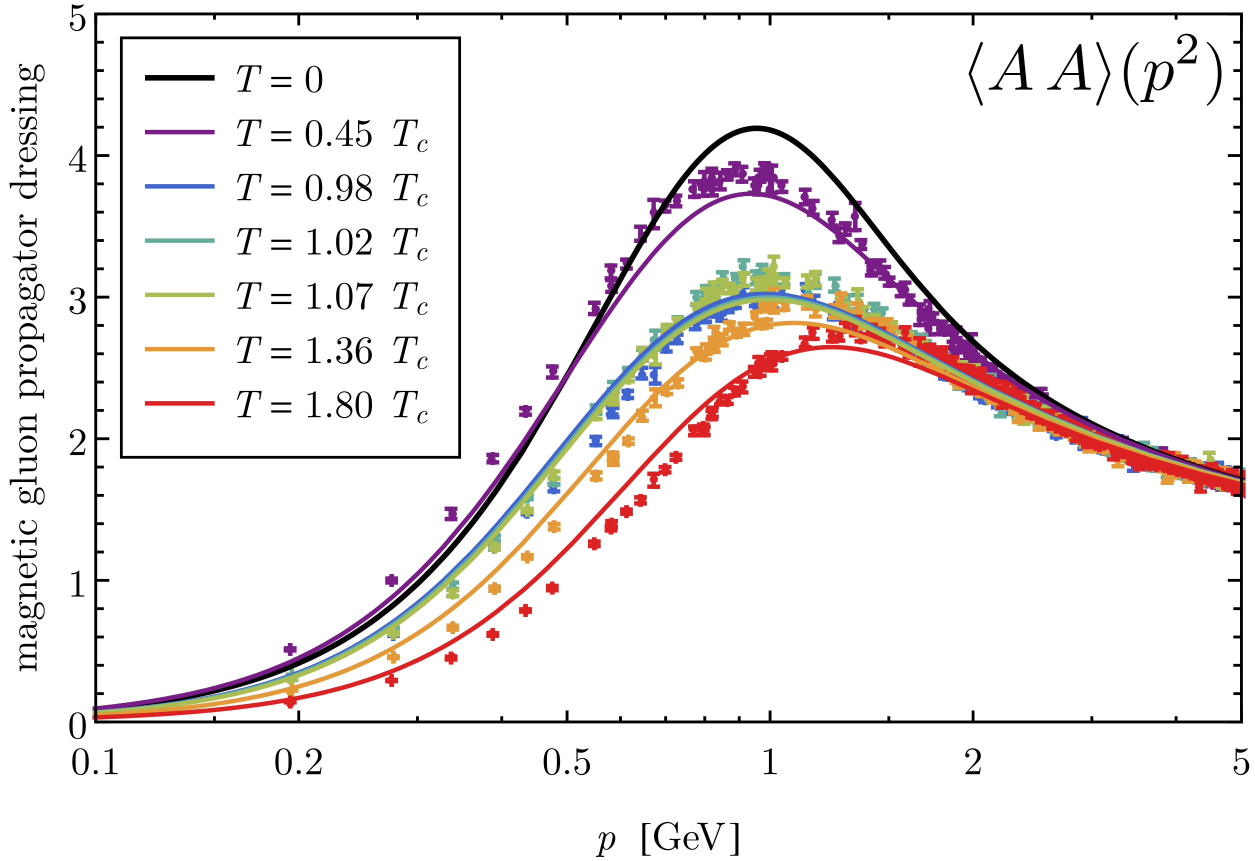
Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

Euclidean gluon propagator at finite T

chromo-magnetic propagator





Fister, JMP, arXiv:1112.5440

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

CF model: Reinosa, Serreau, Tissier, Tresmontant, PRD 95 (2017) 045014

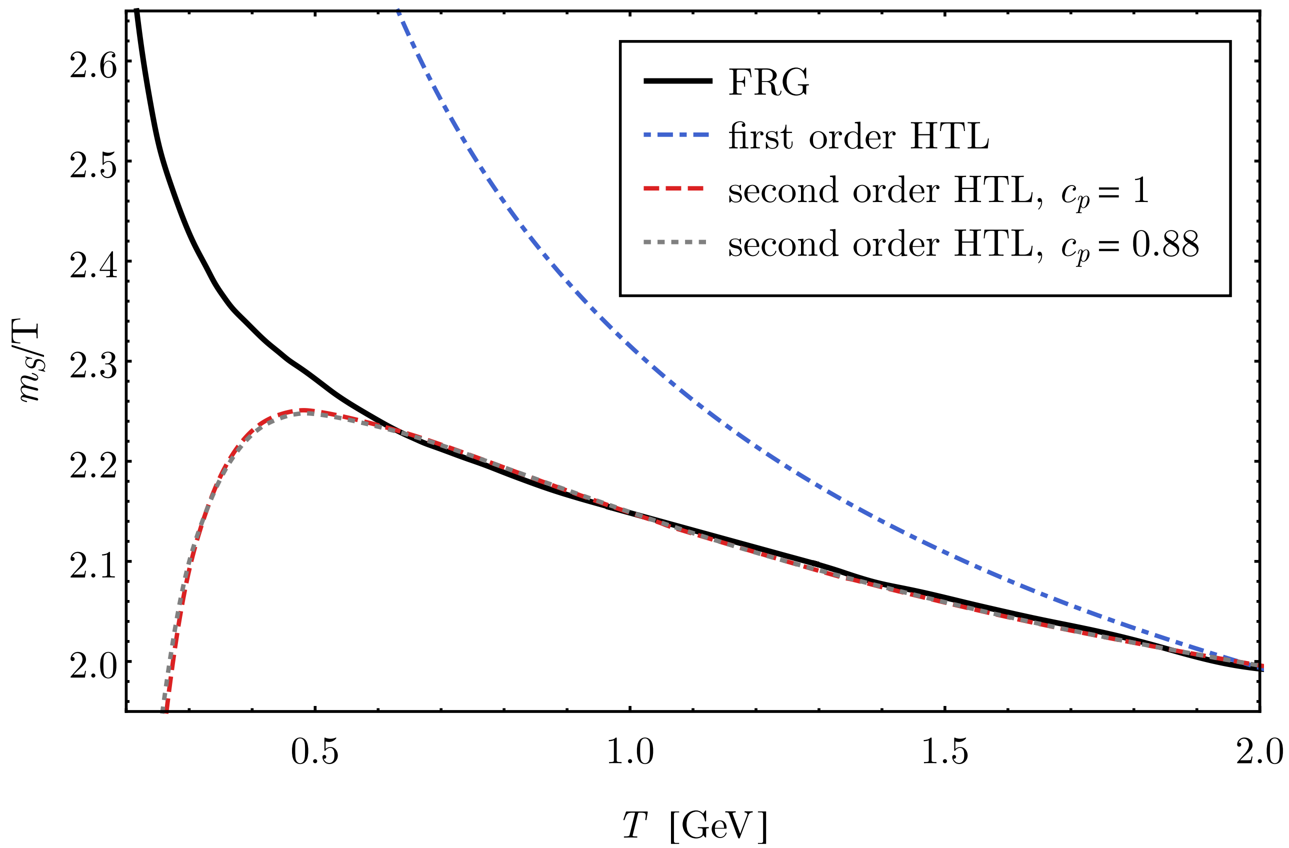
Lattice: Silva, Oliviera, Bicudo, Cardoso, PRD89 (2014) 7, 074503

Aiming at apparent convergence

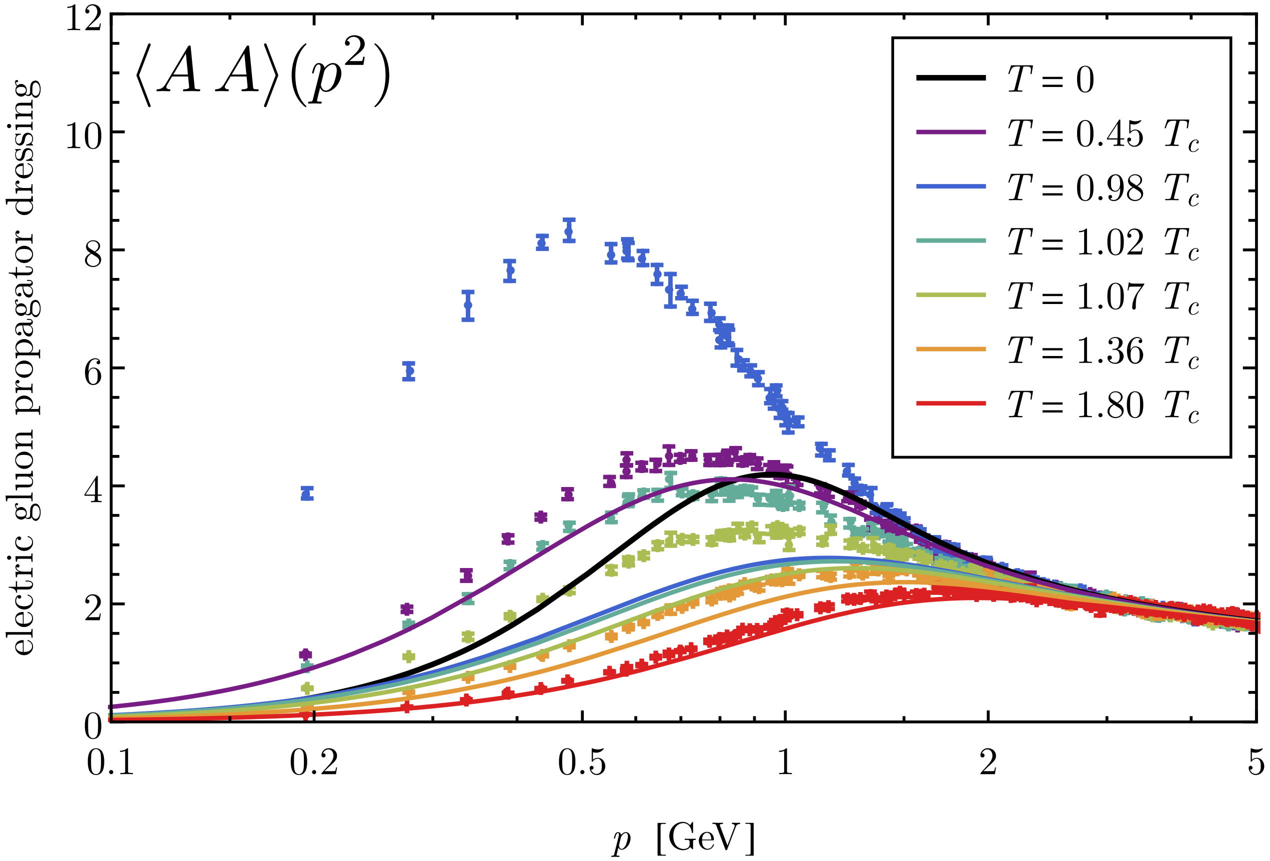
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 5, 054015

Euclidean gluon propagator at finite T

Debye mass (chromo-electric)



chromo-electric propagator



Lattice: Silva, Oliveira, Bicudo, Cardoso, PRD89 (2014) 7, 074503

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 5, 054015

Euclidean gluon propagator at finite T

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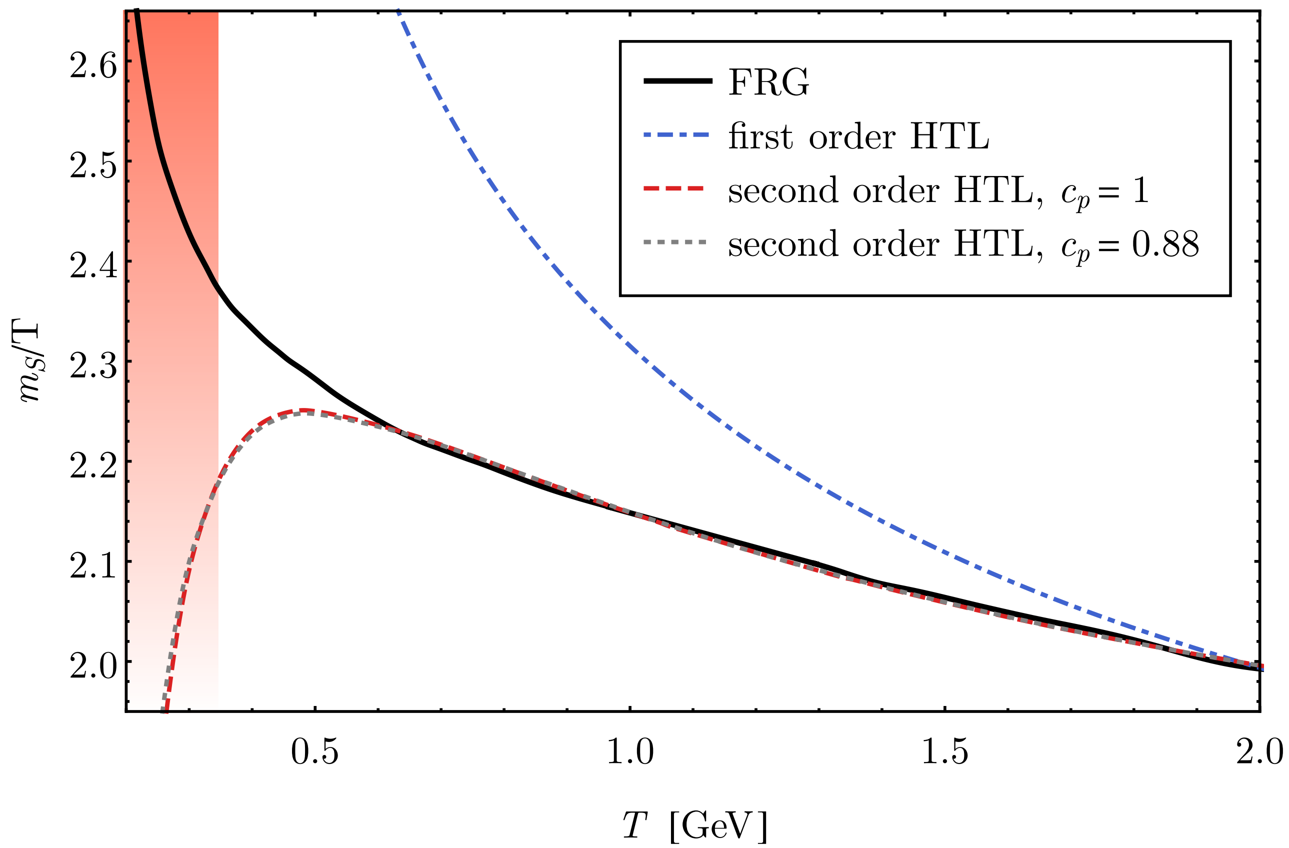
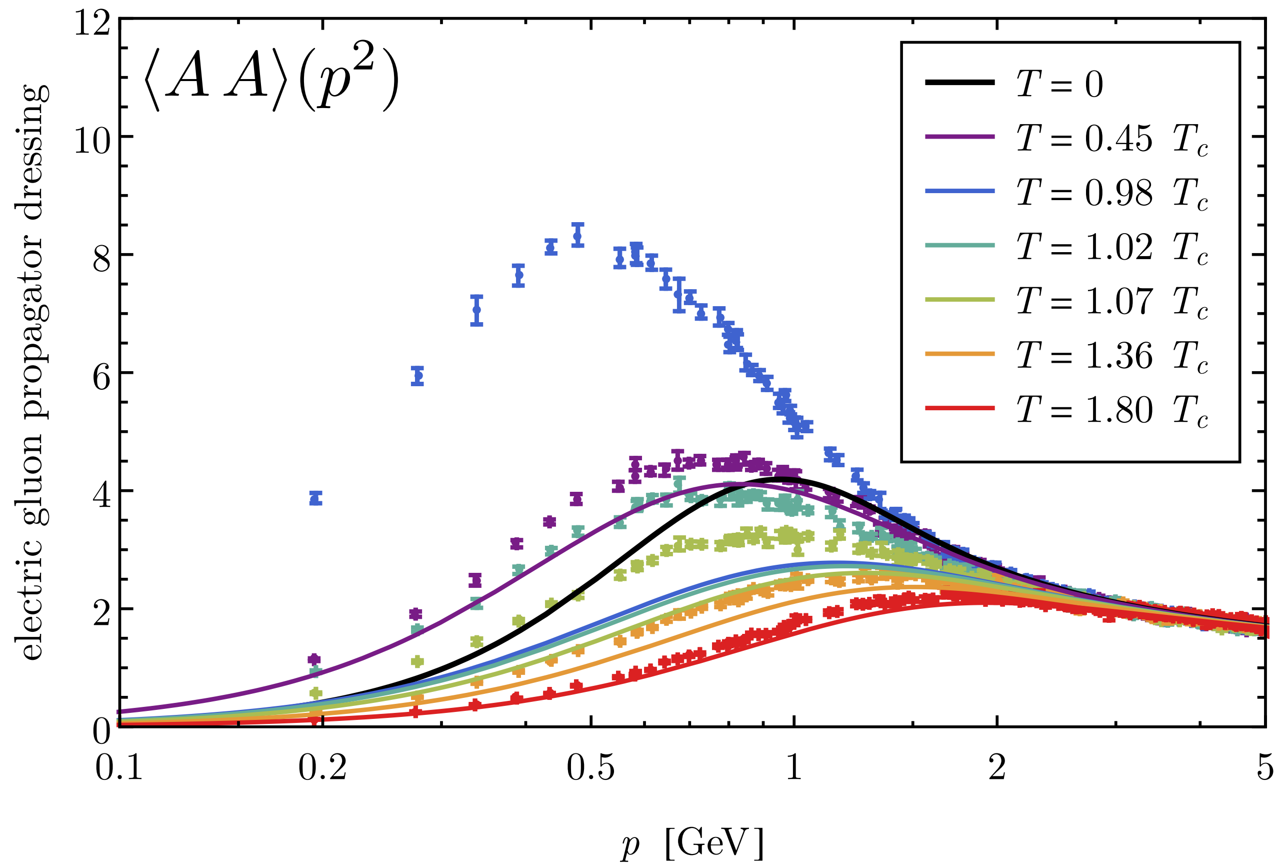


$\langle A_0 \rangle \neq 0$

chromo-electric propagator



Lattice: Silva, Oliveira, Bicudo, Cardoso, PRD89 (2014) 7, 074503

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 5, 054015

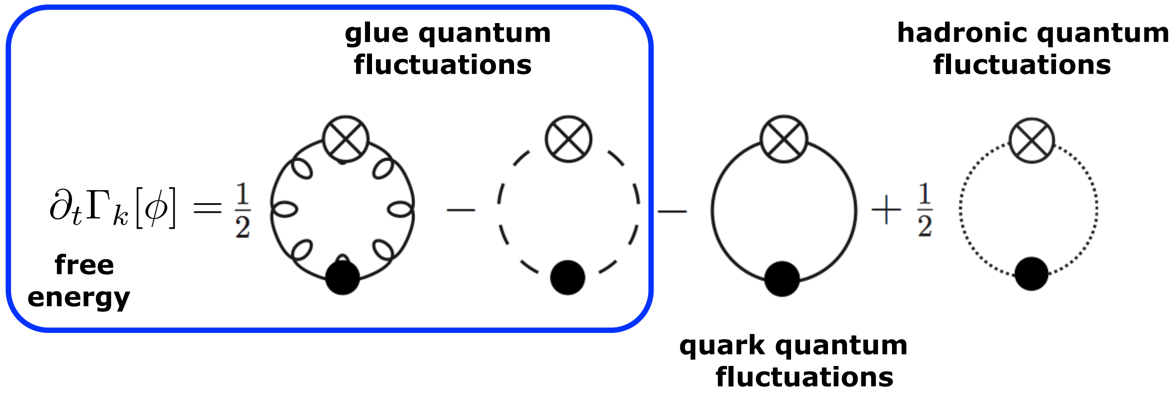
Polyakov loop from functional methods

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}} \mathbf{tr}\,\mathcal{P} \mathbf{e}^{\mathbf{i\,g}\int_\mathbf{0}^\beta \mathbf{A_0(x)}}$$

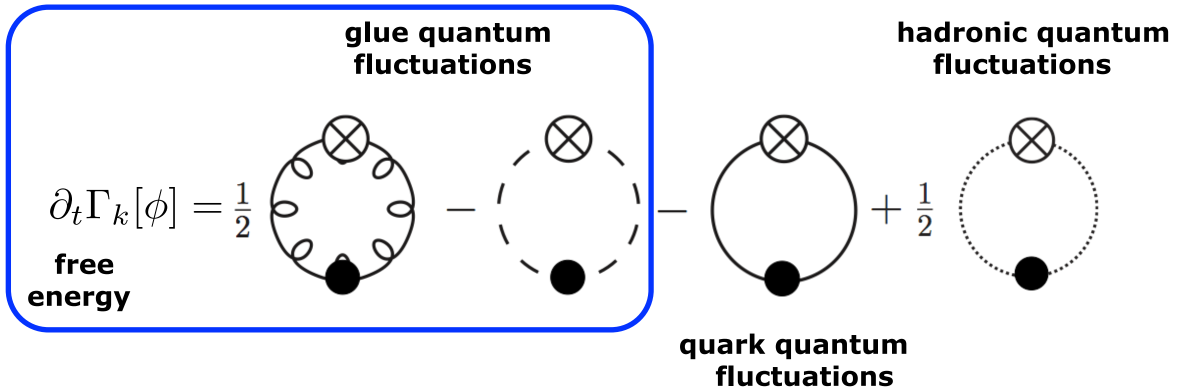


Confinement

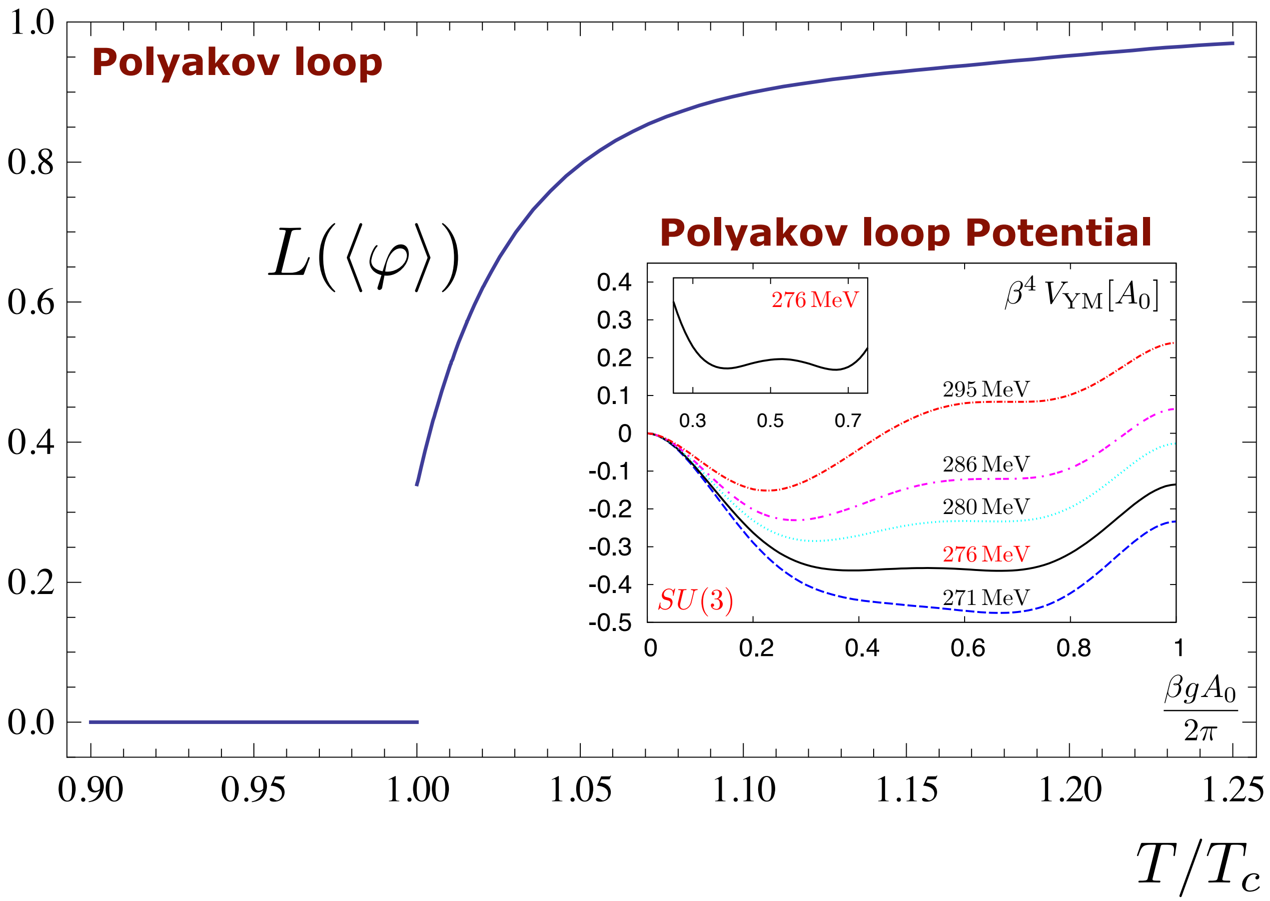
FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{i g \int_0^\beta A_0(x)}$$



$$\mathcal{P} e^{i g \int_0^\beta A_0(x)} = e^{i\varphi}$$



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

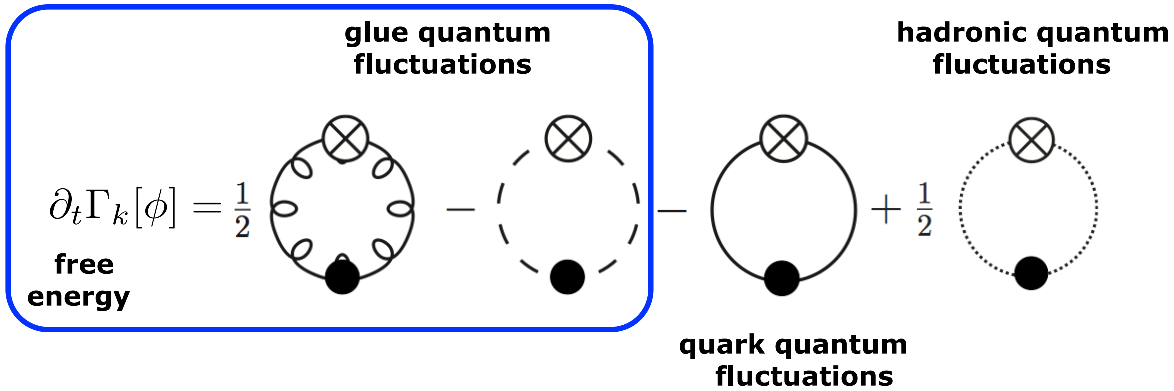
$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}}\,\mathbf{tr}\,\mathcal{P}\mathbf{e}^{\mathbf{i\,g}\int_0^\beta \mathbf{A_0(x)}}$$



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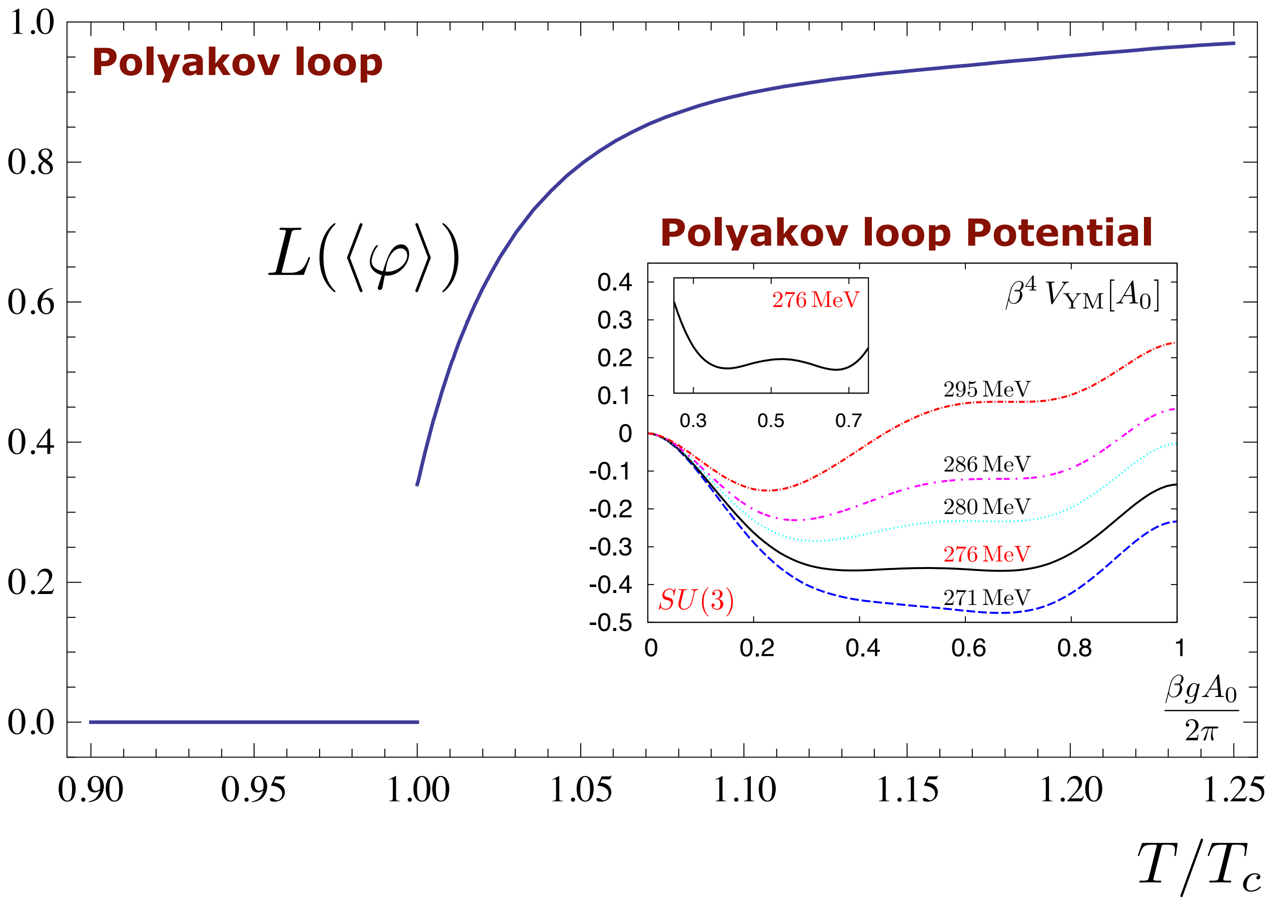
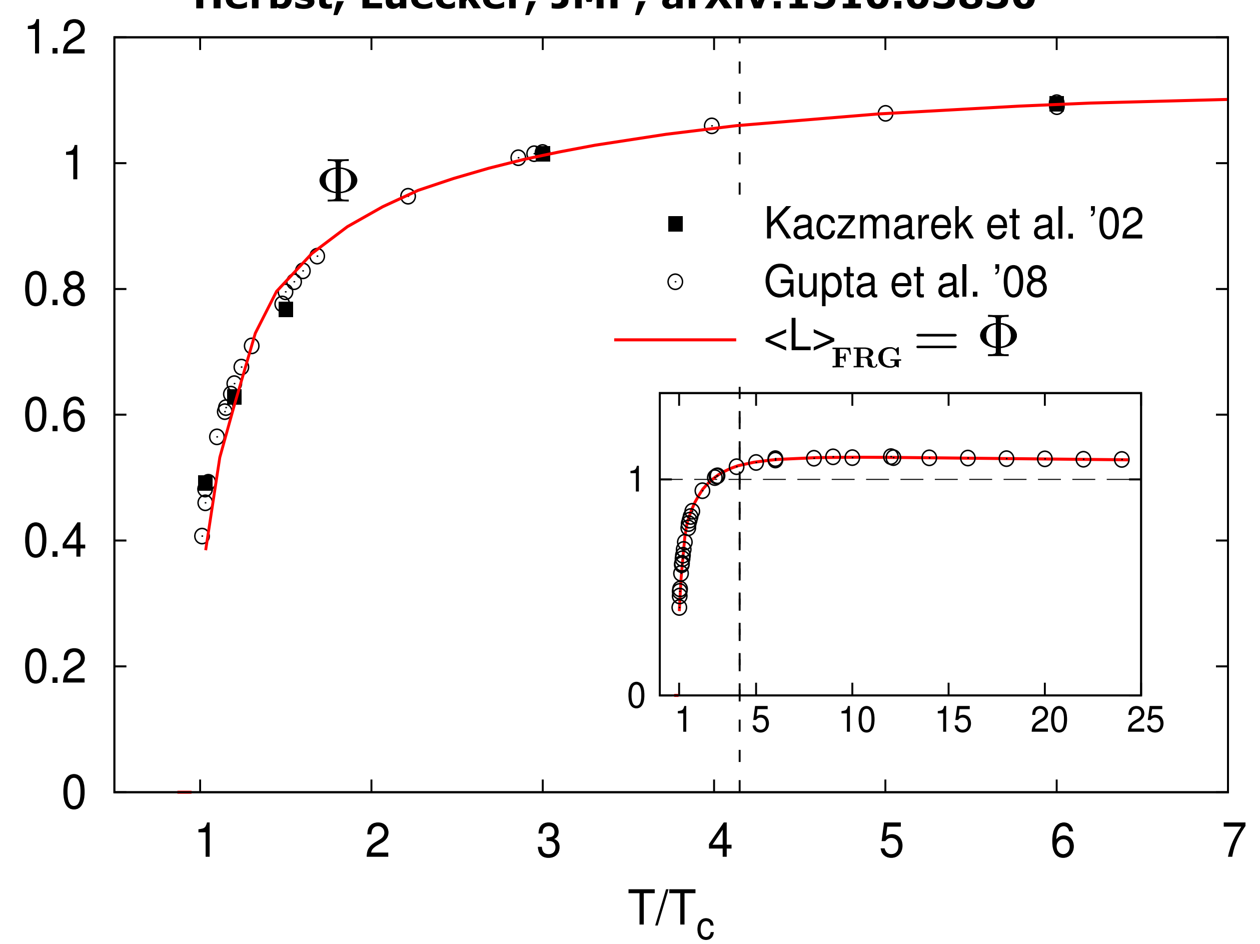


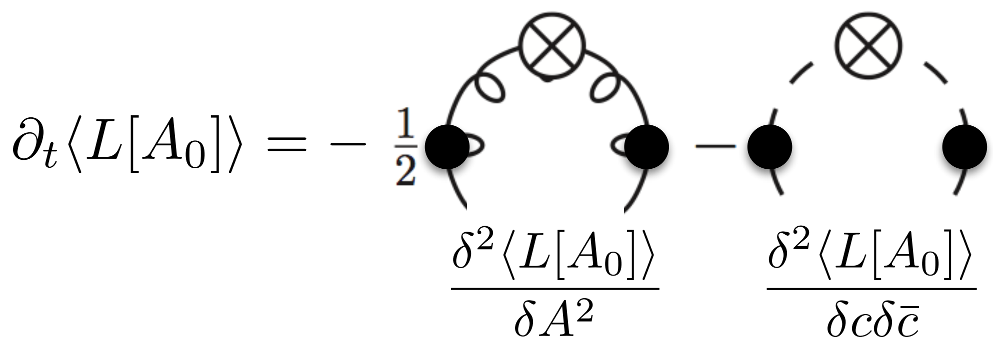
Herbst, Luecker, JMP, arXiv:1510.03830



Confinement

Herbst, Luecker, JMP, arXiv:1510.03830

Flow equation for the Polyakov loop expectation value



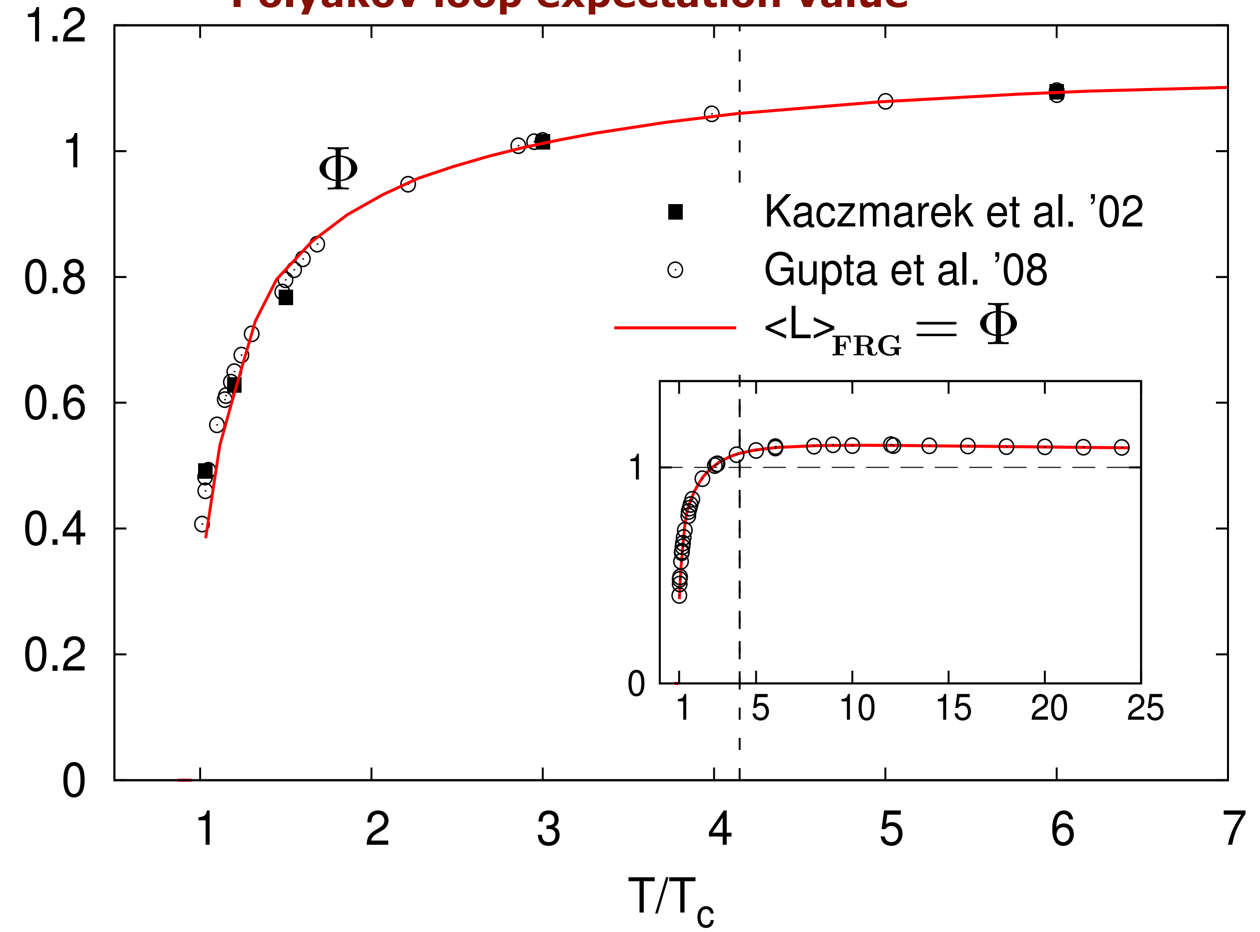
Flow equation for composite operators

JMP, AP 322 (2007) 2831

Igarashi, Itoh, Sonoda, PTP Suppl. 181 (2010) 1

Pagani, PRD 94 (2016) 045001

Polyakov loop expectation value

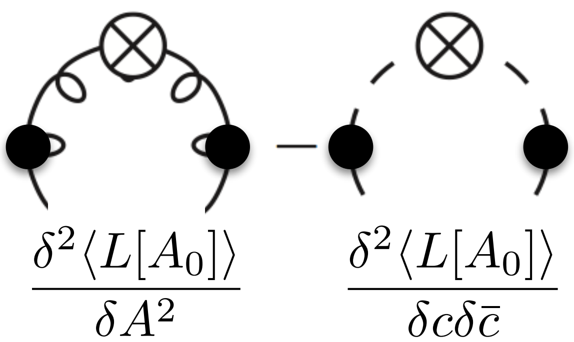


Confinement

Herbst, Luecker, JMP, arXiv:1510.03830

Flow equation for the Polyakov loop expectation value

$$\partial_t \langle L[A_0] \rangle = -\frac{1}{2}$$



Flow equation for composite operators

JMP, AP 322 (2007) 2831

Igarashi, Itoh, Sonoda, PTP Suppl. 181 (2010) 1

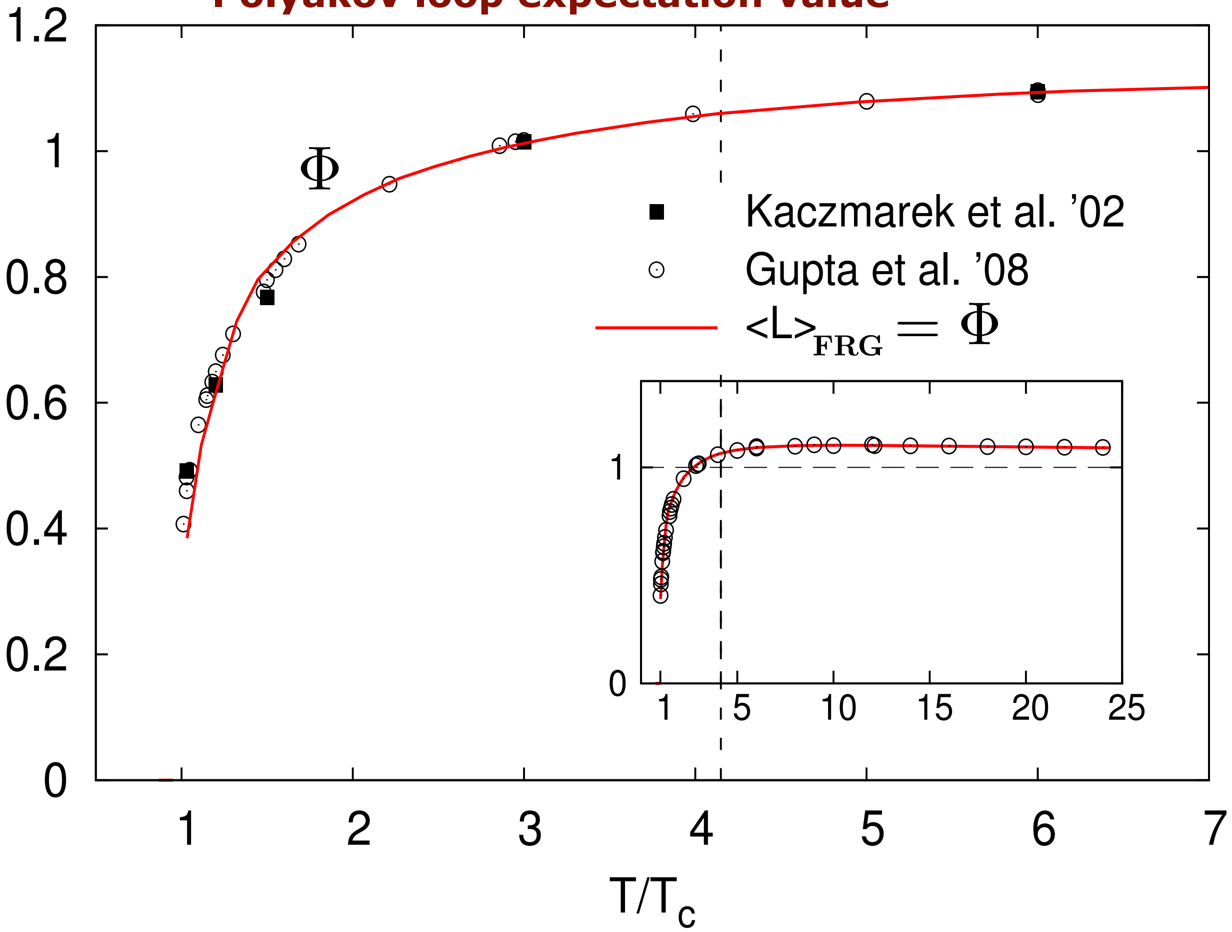
Pagani, PRD 94 (2016) 045001

Parameterisation

$$\langle L[A_0] \rangle = Z_L[\bar{A}, \phi] \cdot L[A_0]$$

with $\phi = (a_\mu, c, \bar{c})$

Polyakov loop expectation value



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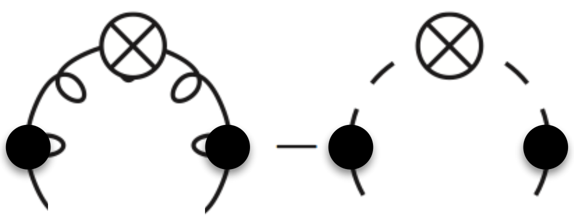


$$\frac{\delta^2 \langle L[A_0] \rangle}{\delta A^2} \qquad - \qquad \frac{\delta^2 \langle L[A_0] \rangle}{\delta c \delta \bar{c}}$$

Flow equation for composite operators

JMP, AP 322 (2007) 2831

Igarashi, Itoh, Sonoda, PTP Suppl. 181 (2010) 1

Pagani, PRD 94 (2016) 045001

Parameterisation

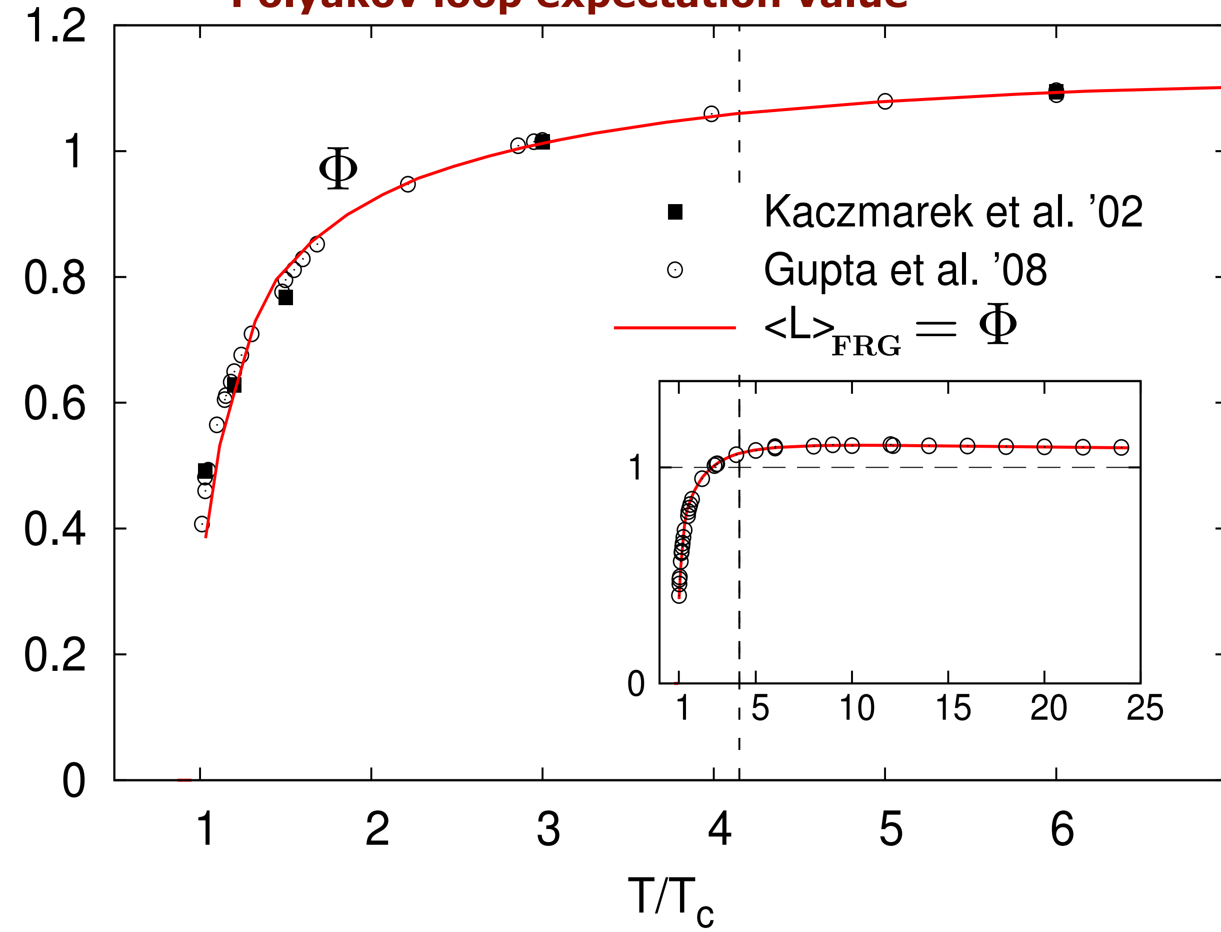
$$\langle L[A_0] \rangle = Z_L[\bar{A}, \phi] \cdot L[A_0]$$

with $\phi = (a_\mu, c, \bar{c})$

Flow for Polyakov loop wave function

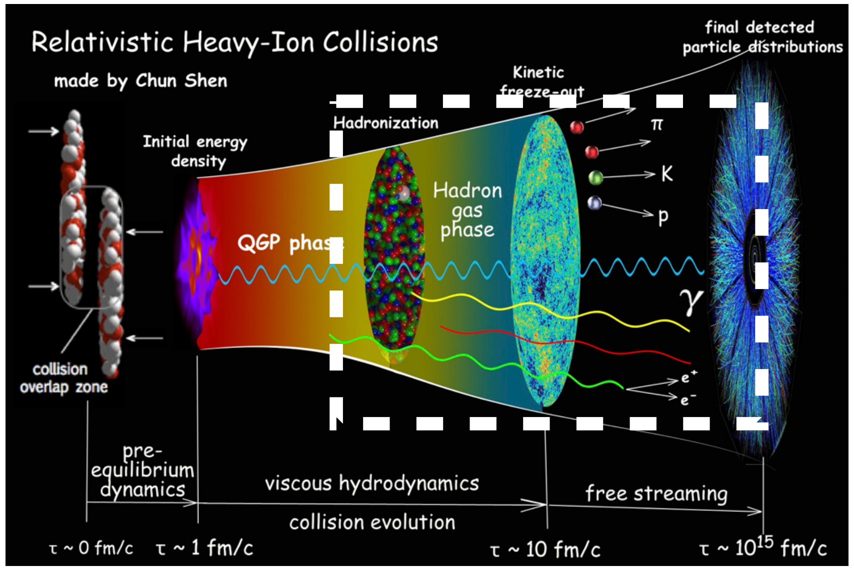
$$\partial_t Z_L[\bar{A}, \phi] = \mathrm{Flow}_{Z_L}[\bar{A}; Z_L, G_A, G_c, L[A_0]]$$

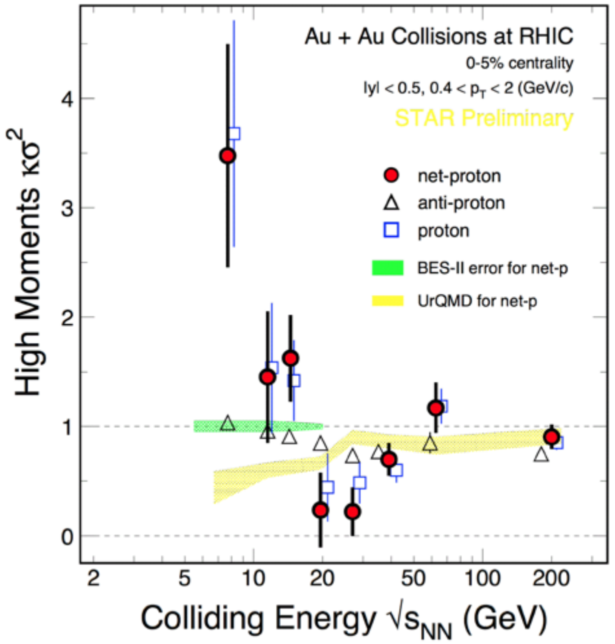
Polyakov loop expectation value



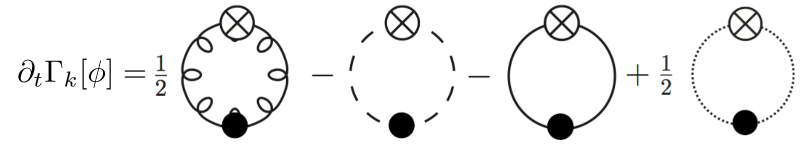
Phase structure of QCD and dynamics

QCD-assisted transport

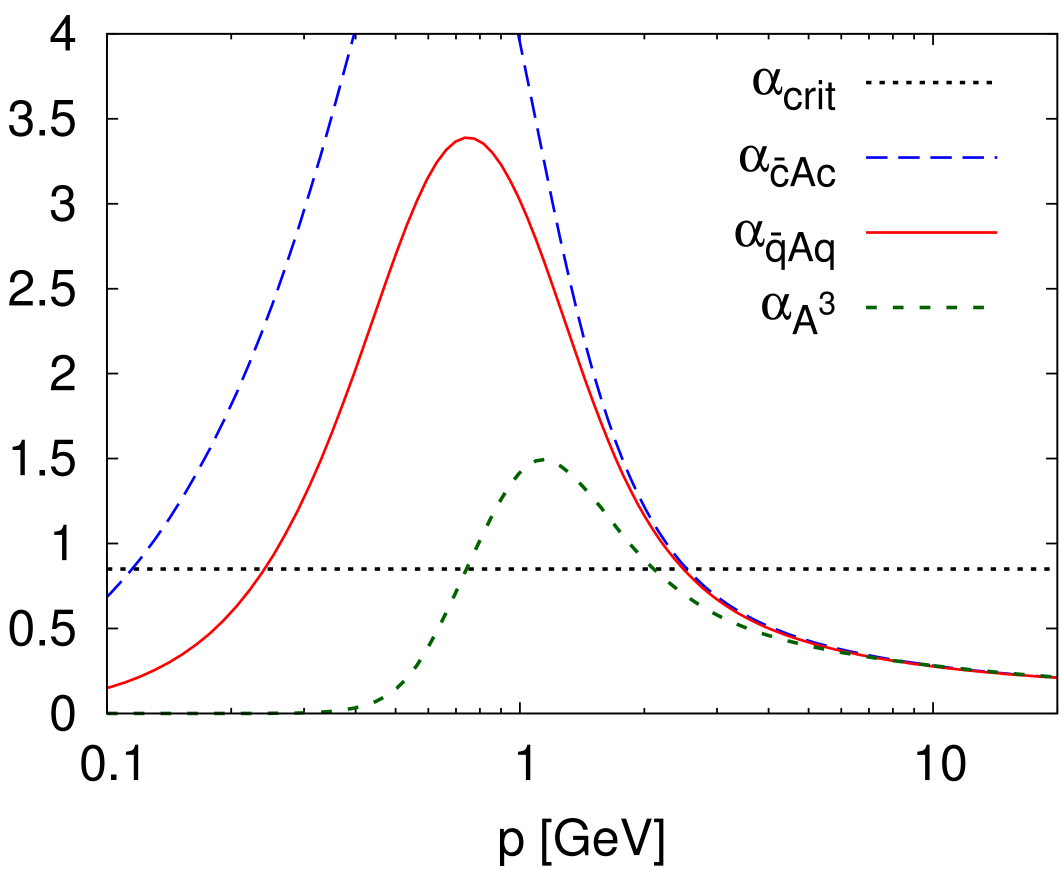




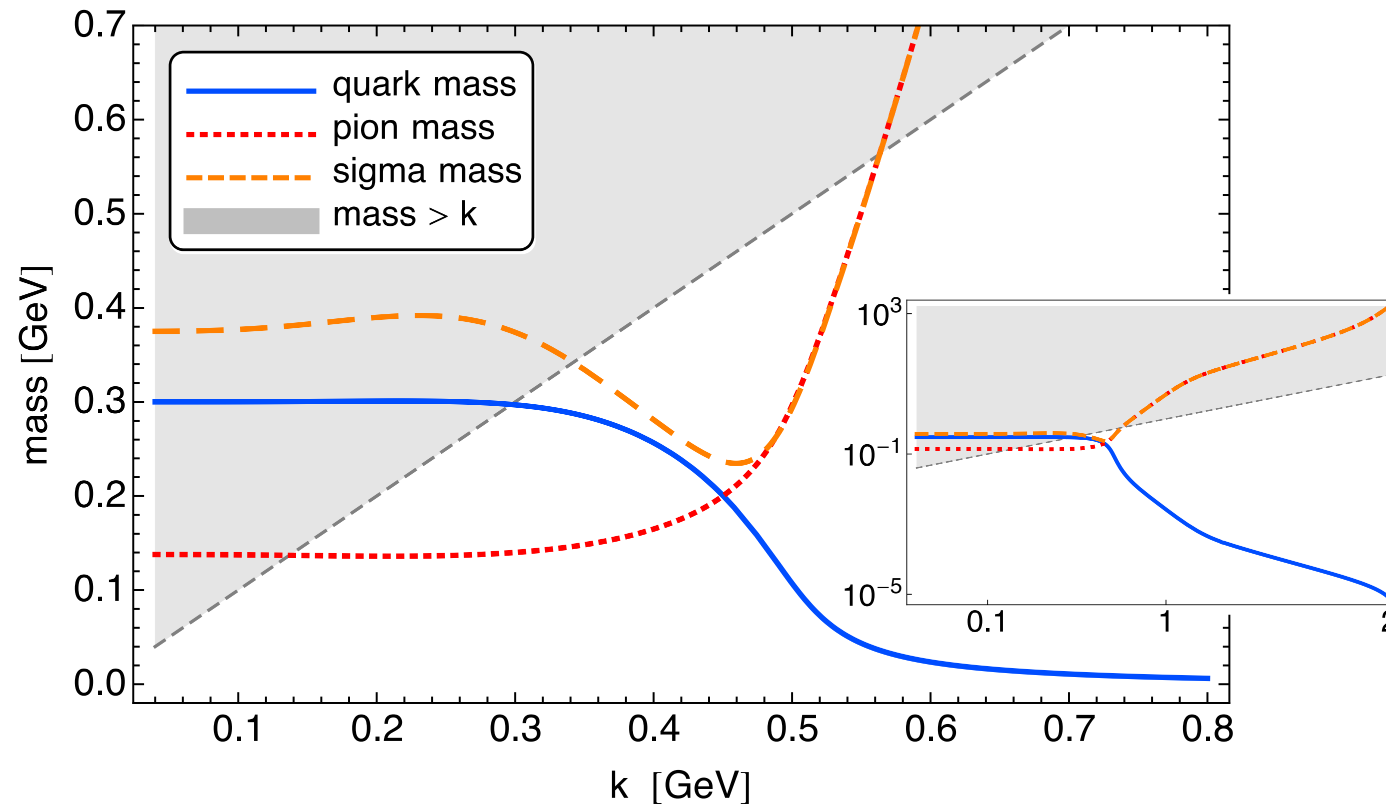
On the unreasonable effectiveness of low energy effective theories



Sequential decoupling of gluon, quark, sigma, pion fluctuations



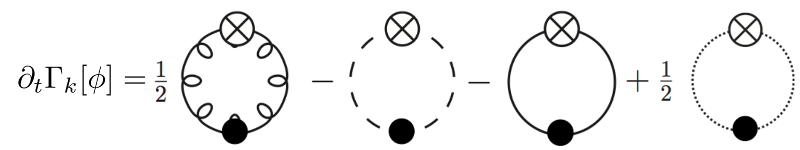
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035



Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Rennecke, PRD 92 (2015) 076012

On the unreasonable effectiveness of low energy effective theories



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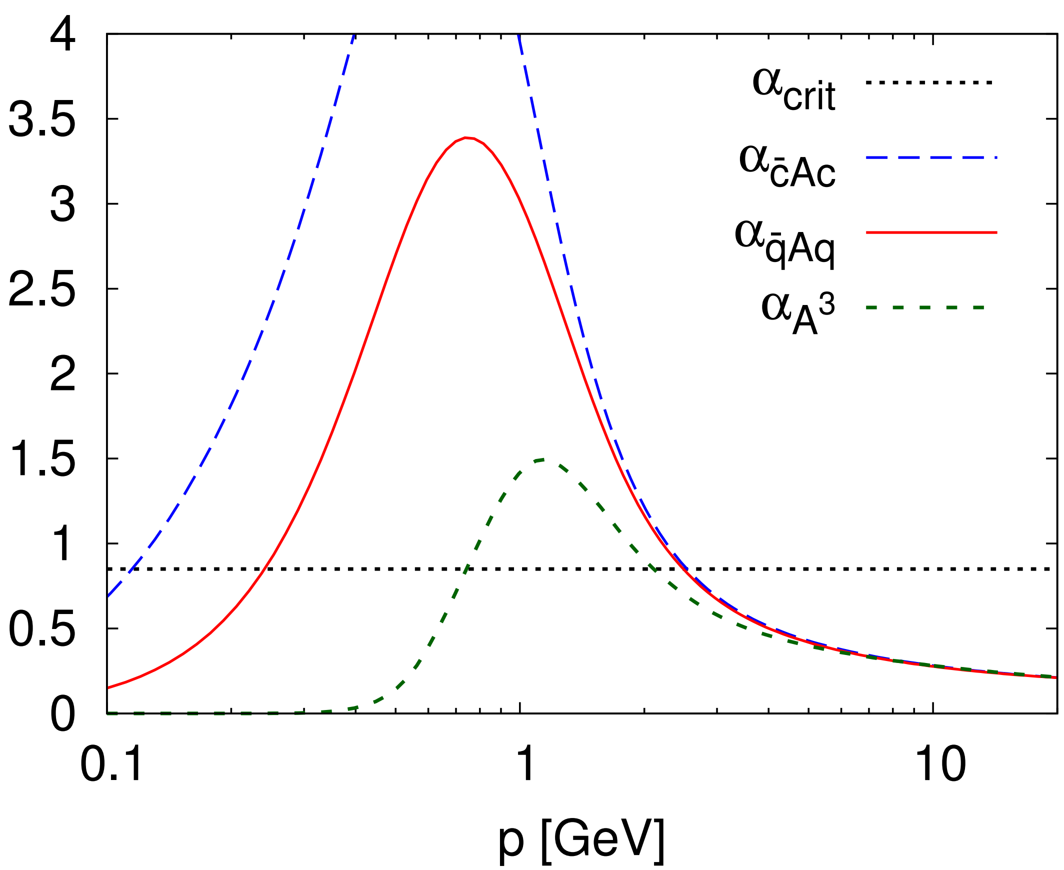
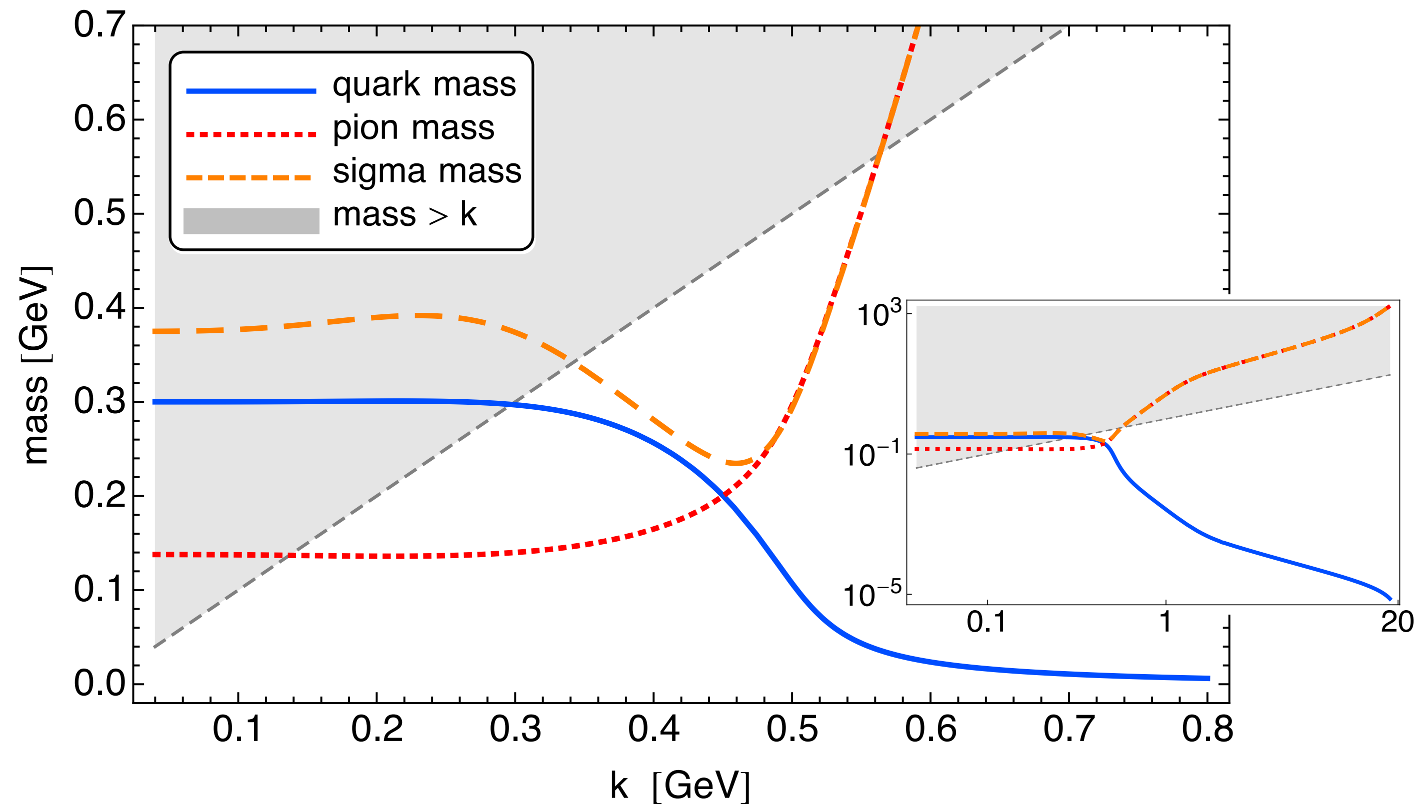
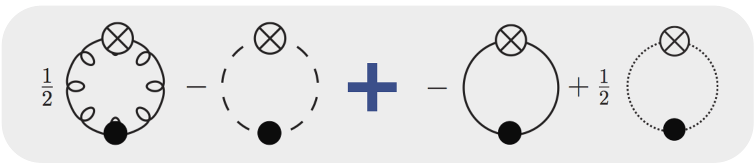
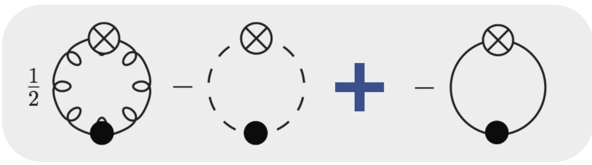




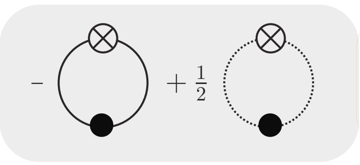
PQM-model



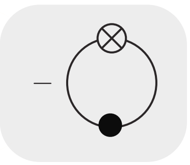
PNJL-model



QM-model

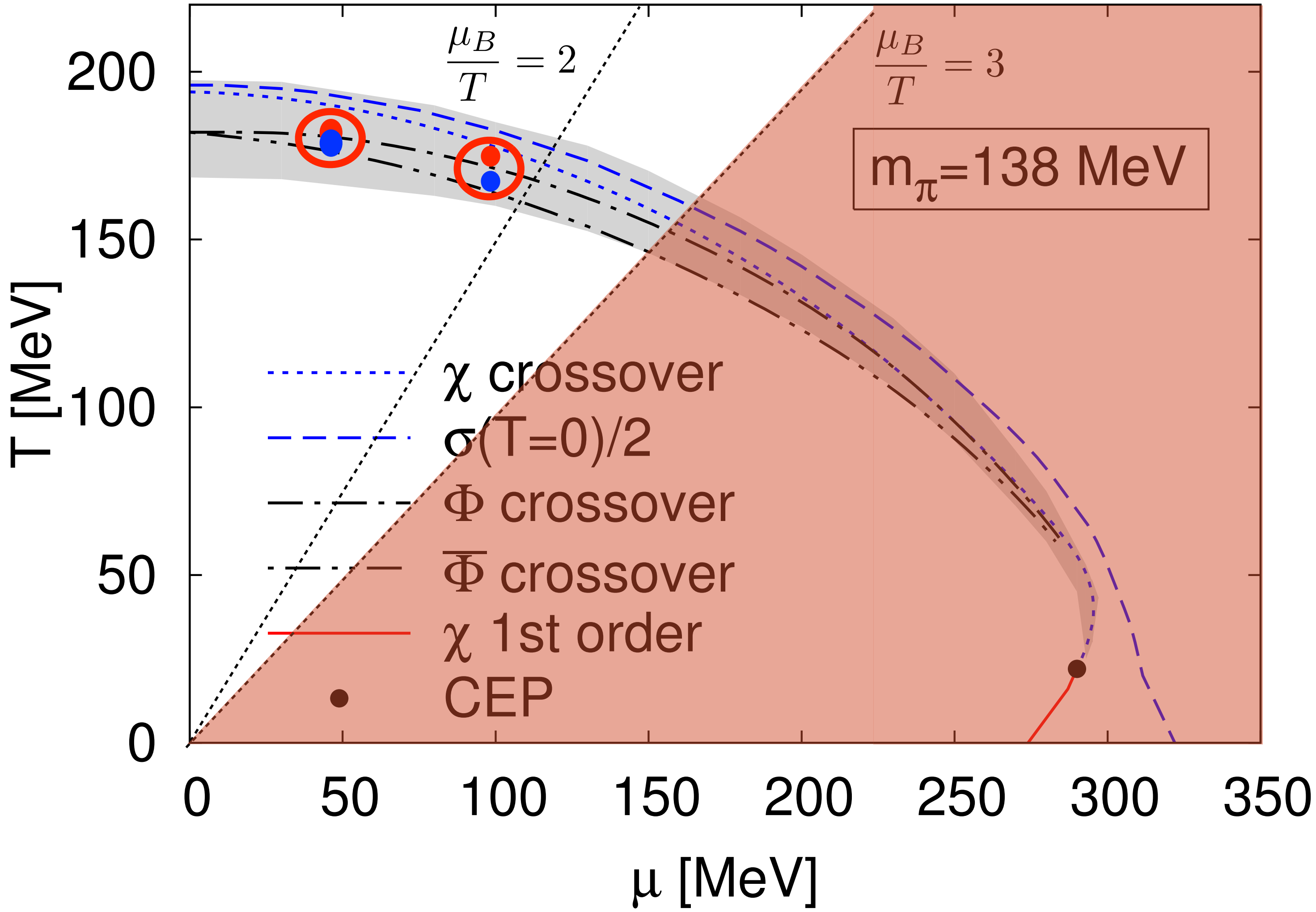


NJL-model

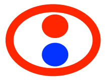


QCD at finite density

Phase diagram of the QCD-enhanced PQM



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



FRG QCD results at finite density

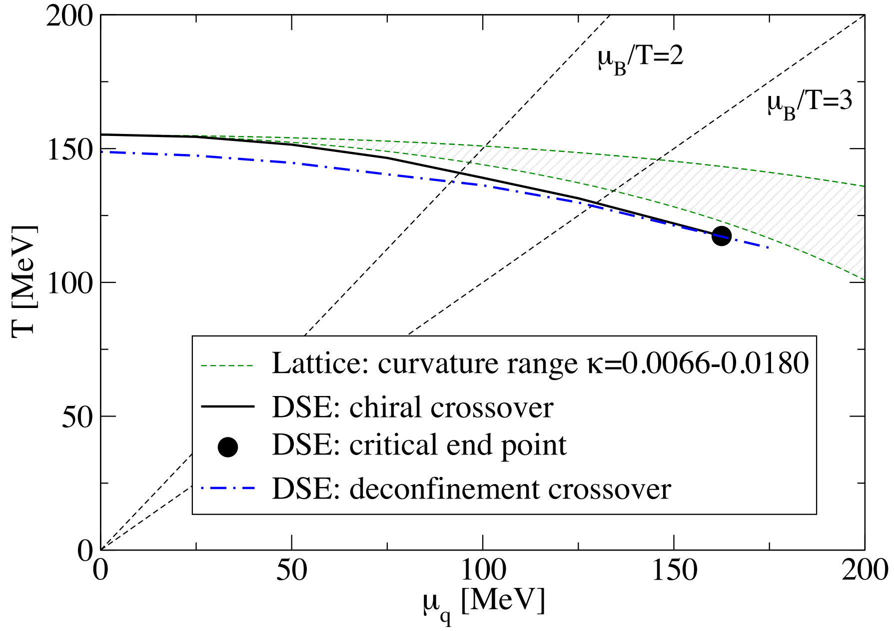
Haas, Braun, JMP ’09, unpublished

Extension of FRG QCD results at imaginary chemical potential

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

Phase structure at finite density

Phase diagram of 2+1 flavor QCD



Kaczmarek at al. '11
Endrodi, Fodor, Katz, Szabo '11
Cea, Cosmai, Papa '14

Fischer, Fister, Luecker, JMP, PLB732 (2014)

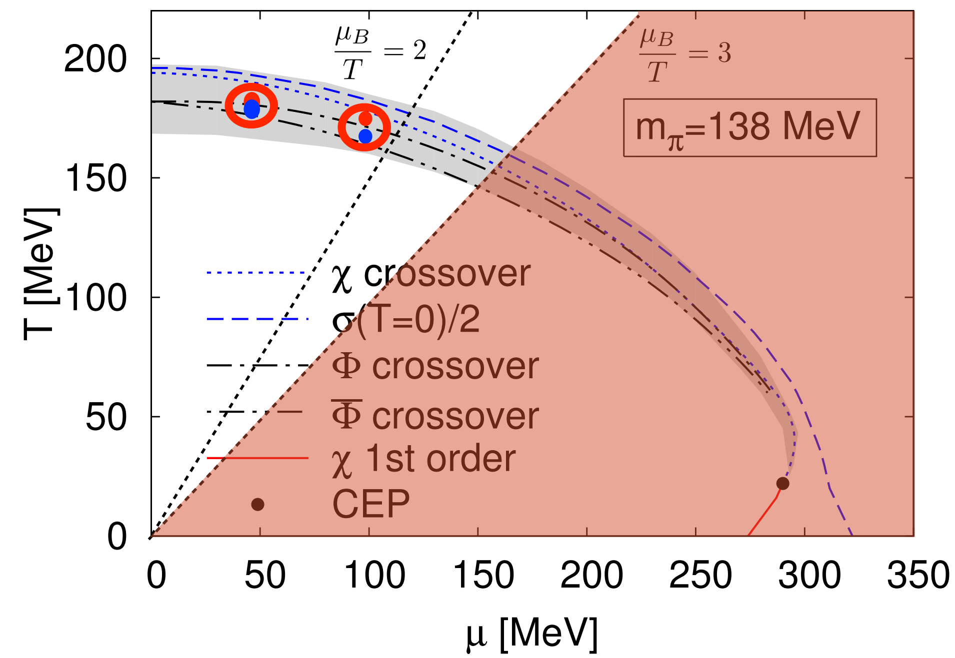
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

Eichmann, Fischer, Welzbacher, PRD 93 (2014) 034013

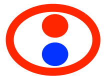
Chiral phase structure

Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301

Phase diagram of QCD-enhanced 2-flavor PQM-model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



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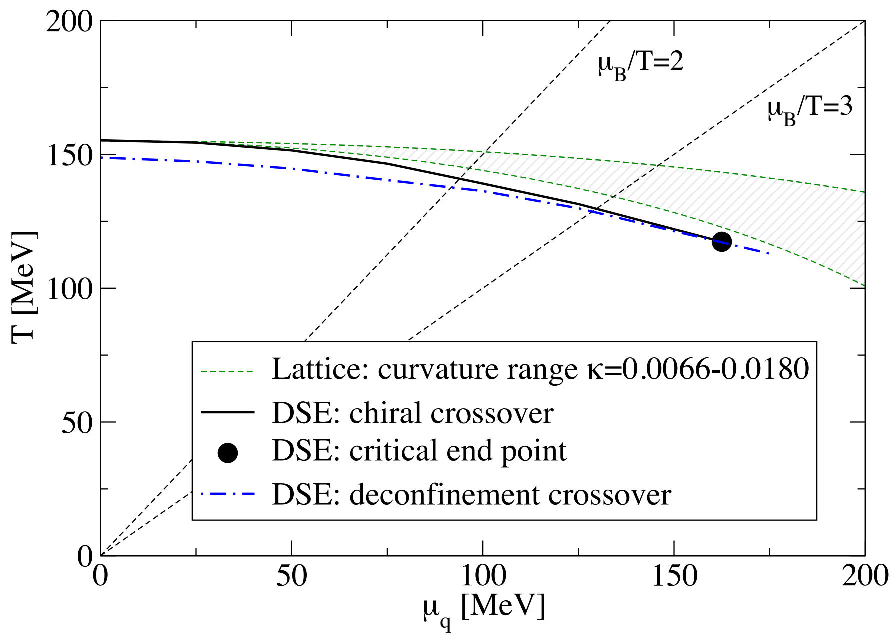

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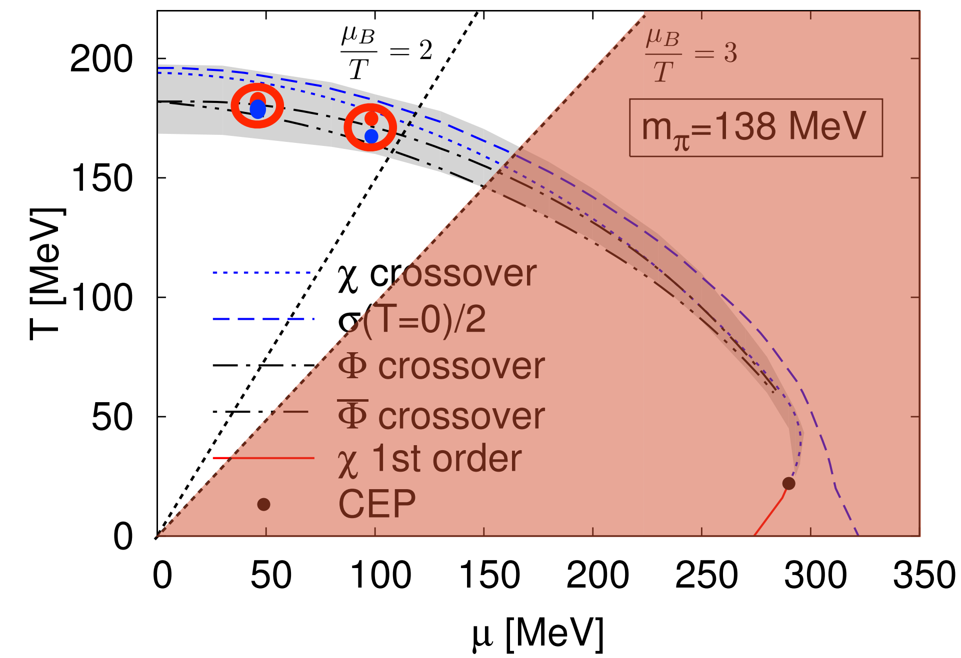
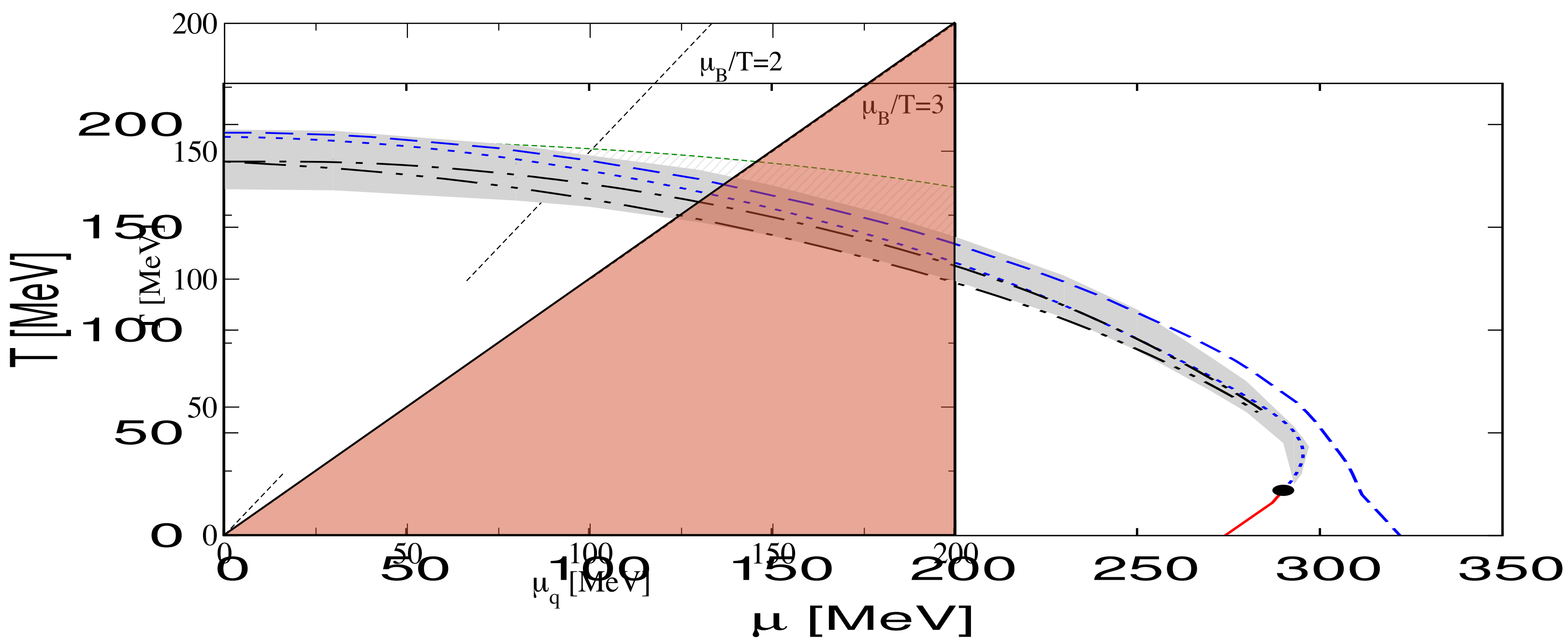


Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

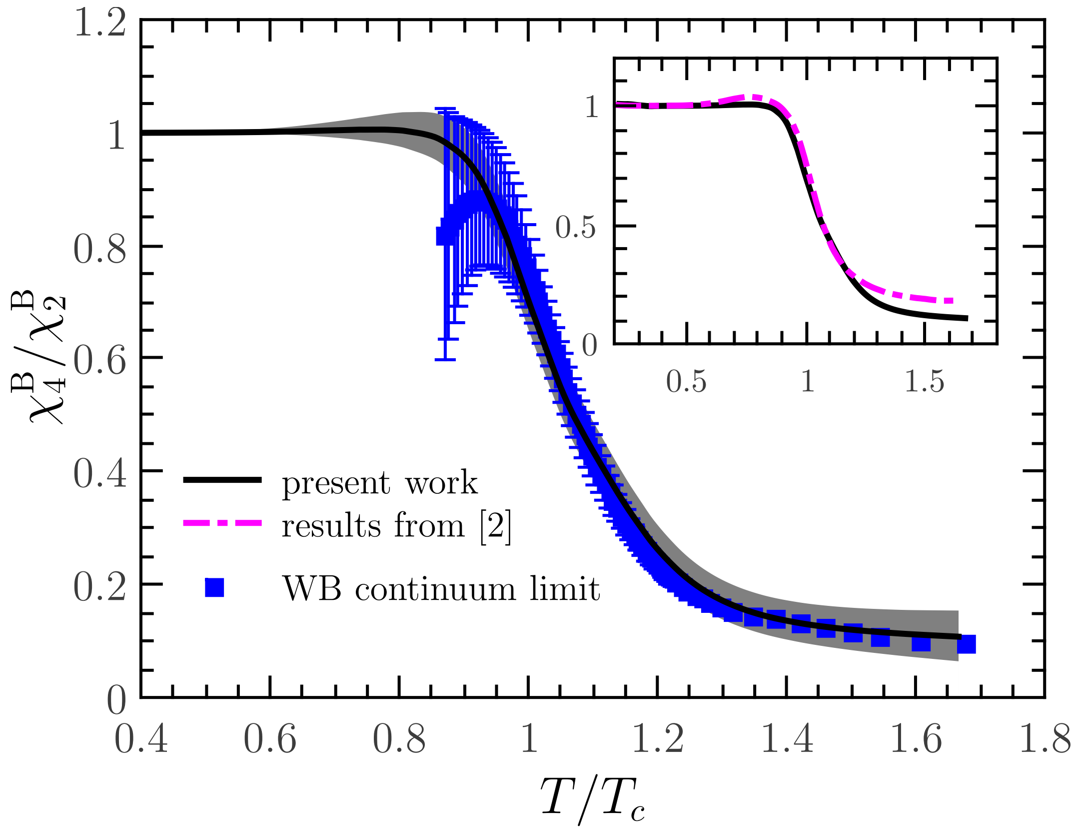
FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

Comparison with 2 flavor vs 2+1 flavor scale matching of T_c



Fluctuations as a measure of confinement

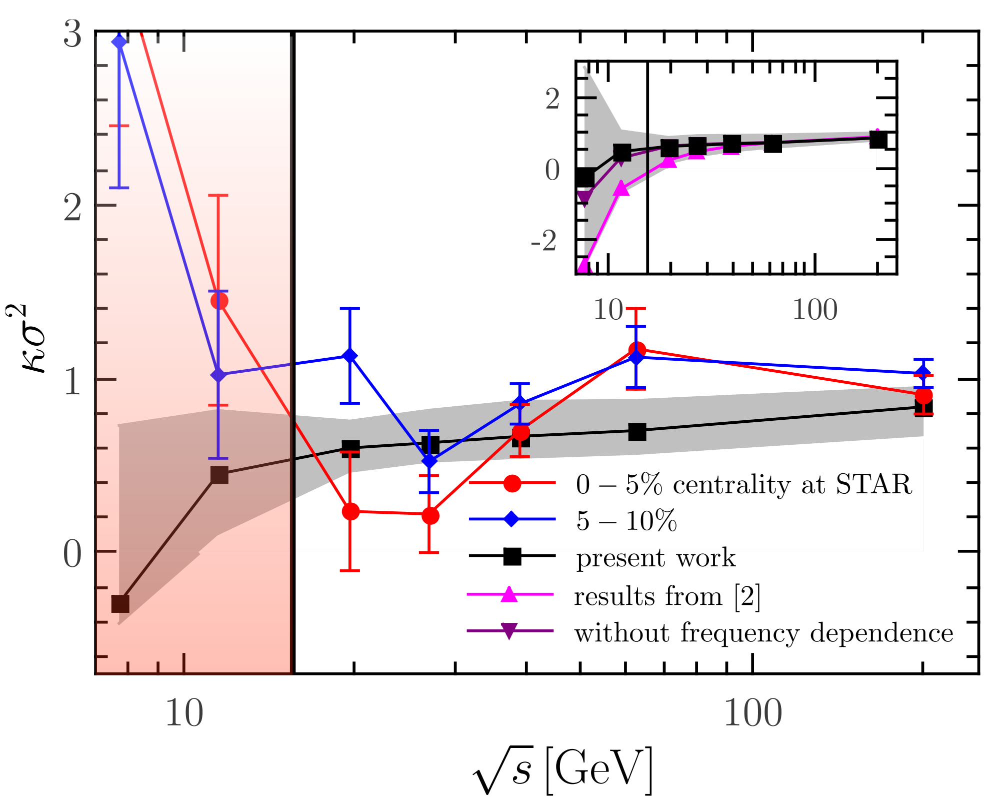


[2] Fu, JMP, PRD 92 (2015) 116006

$$\chi_n^{\mathrm{B}} = \frac{\partial^n}{\partial(\mu_{\mathrm{B}}/T)^n}\frac{p}{T^4}$$

Skewness, Kurtosis

$$\sigma^2 = VT^3\chi_2^{\mathrm{B}} \qquad S = \frac{\chi_3^{\mathrm{B}}}{\chi_2^{\mathrm{B}}\sigma} \qquad \kappa = \frac{\chi_4^{\mathrm{B}}}{\chi_2^{\mathrm{B}}\sigma^2}$$



[2] Fu, JMP, PRD 93 (2016) 091501

Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020

Karsch, Schaefer, Wagner, Wambach, PLB 698 (2011) 256

Friman, Karsch, Redlich, Skokov, EPJ C71 (2011) 1694

Schaefer, Wagner, PRD 85 (2012) 034027

Skokov, Friman, Redlich, PRC 88 (2013) 034911

Almasi, Friman, Redlich, Nucl.Phys. A956 (2016) 356-359

Transport approach to QCD

Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink

Time evolution of the critical (scalar) σ mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion noise field

Extension of mean-field version

Nahrgang, Leupold, Herold, Bleicher PRC84 (2011)

see also

Stephanov, Rajagopal, Shuryak PRL81 (1998)

Mukherjee, Venugopalan, Yin PRC92 (2015)

Herold, Nahrgang, Yan, Kobdaj PRC93 (2016)

Nahrgang, Bluhm, Schäfer, Bass arXiv:1804.05728

Transport approach to QCD

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Time evolution of the critical (scalar) σ mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion **noise field**

Input from equilibrium low energy effective action of QCD

$$\mathrm{Re}\,\Gamma^{(2)}_{\sigma}(\omega,\vec{p}) \qquad \mathrm{Im}\,\Gamma^{(2)}_{\sigma}(\omega,\vec{p}) \qquad U(\sigma)$$

kinetic term **diffusion term** $\eta\,\partial_t\sigma$ **effective potential**

Transport approach to QCD

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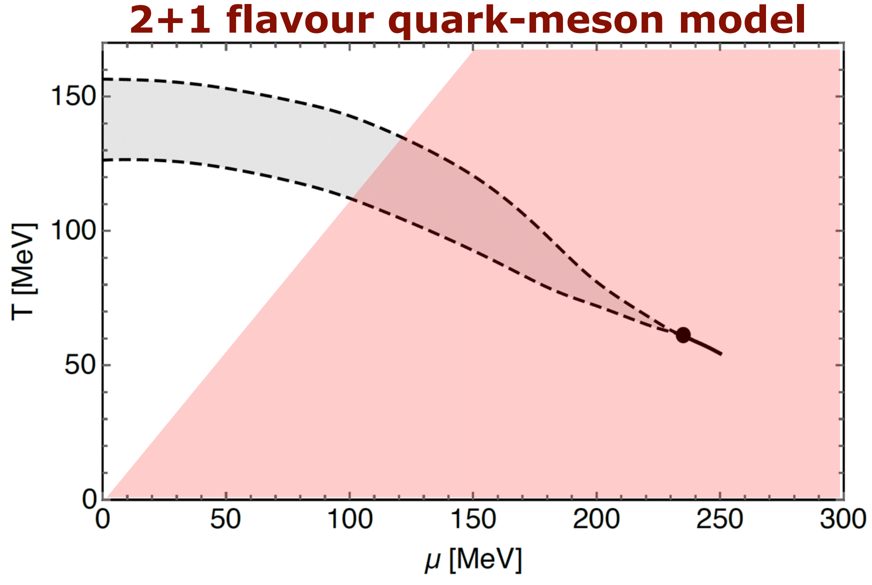
Phase structure of low energy QCD

2+1 flavour quark-meson model



Schaefer, Rennecke, PRD 96 (2017) 1, 016009

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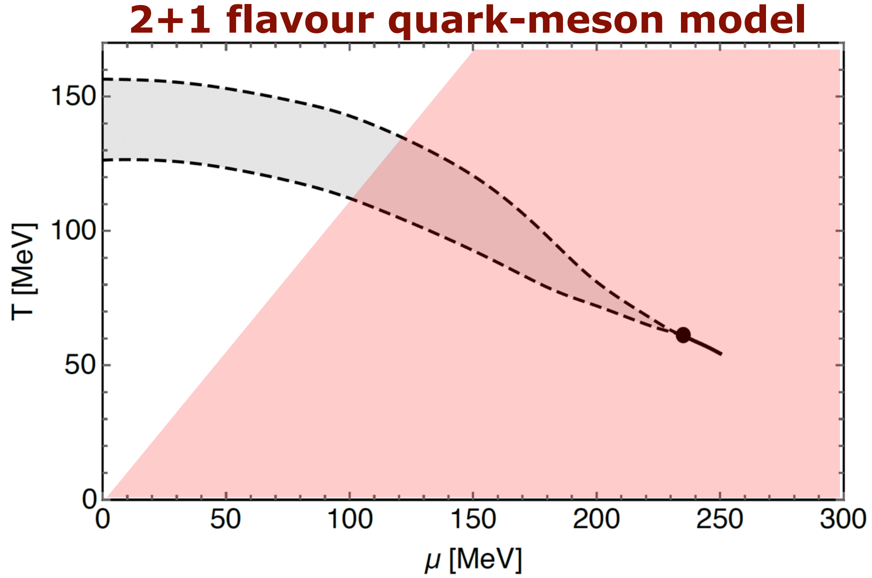
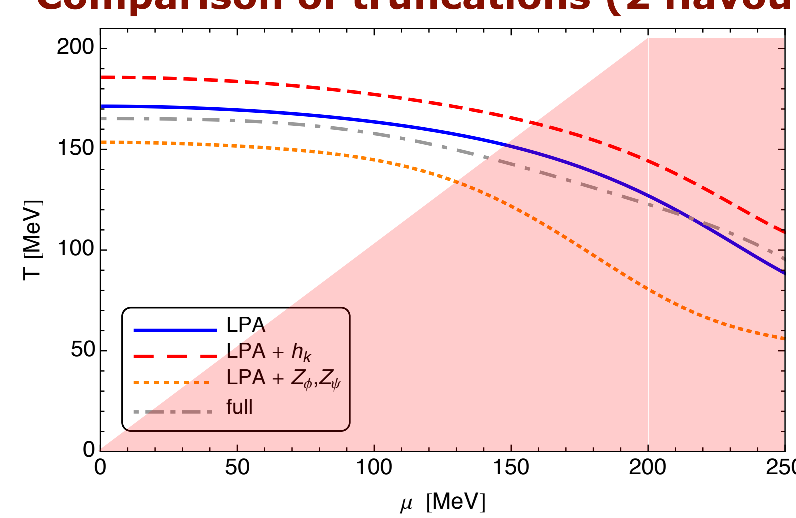


Schaefer, Rennecke, PRD 96 (2017) 1, 016009

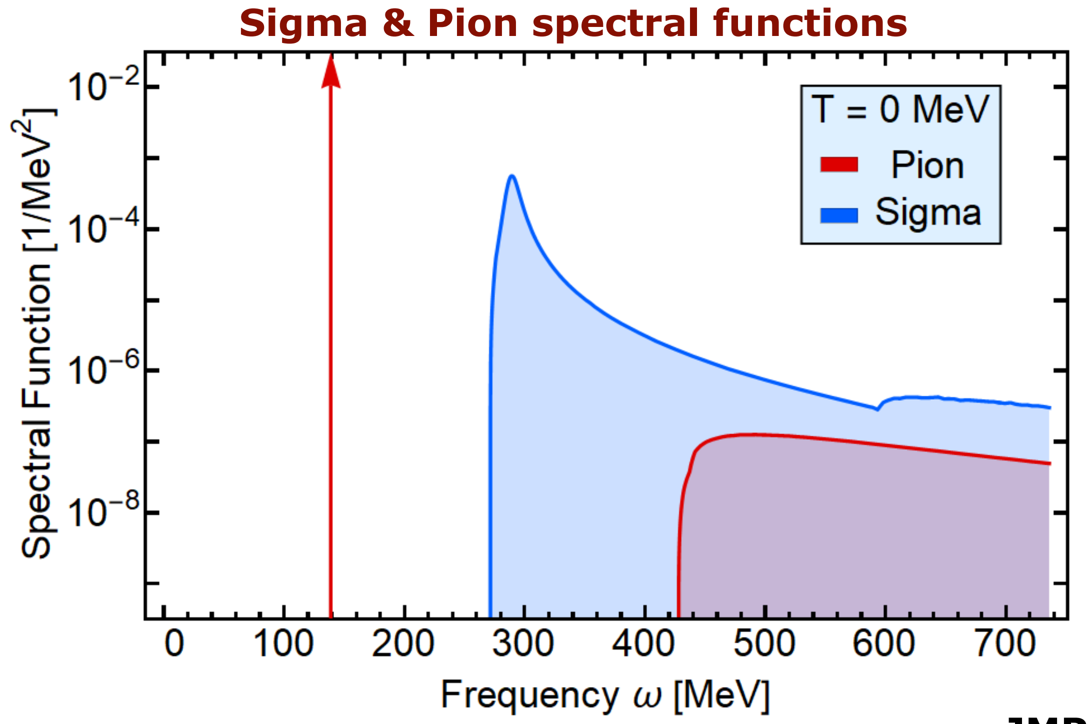
Comparison of truncations (2 flavours)



JMP, Rennecke, PRD 90 (2014) 7, 076002

Pion & sigma spectral functions

Show case in linear sigma model



JMP, Strodthoff, Wink, arXiv:1711.07444

Real-time FRG computations, e.g.

Flörchinger JHEP 1205 (2012) 021

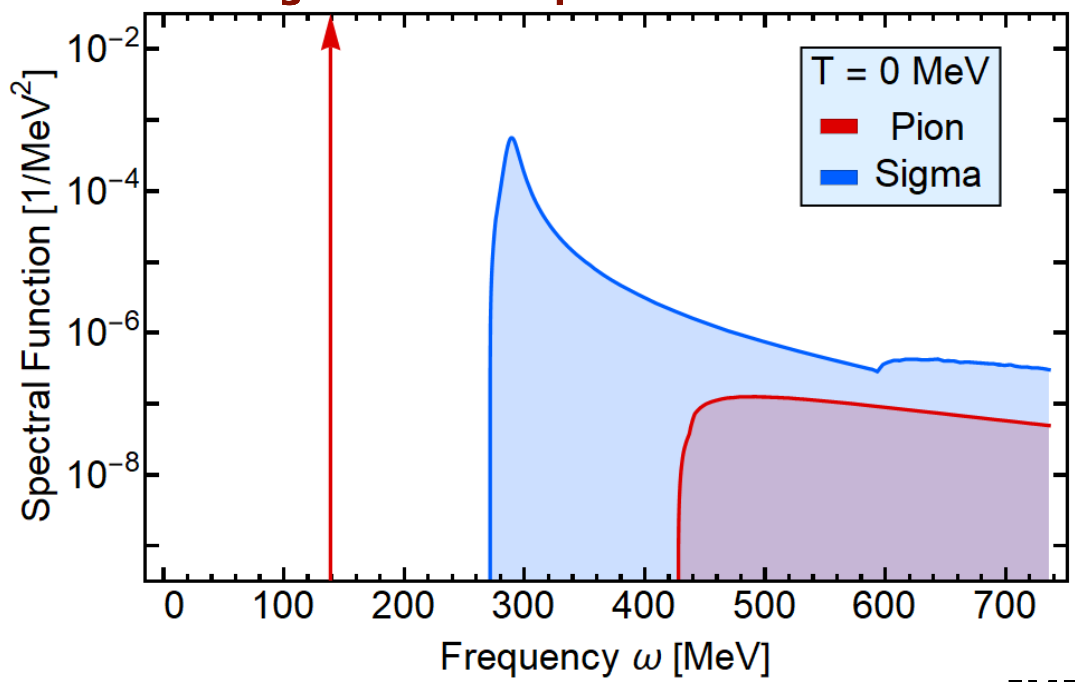
Kamikado, Strodthoff, von Smekal, Wambach, EPJC 74 (2014) 2806

JMP, Strodthoff, PRD 92 (2015) 094009

Pion & sigma spectral functions

Show case in linear sigma model

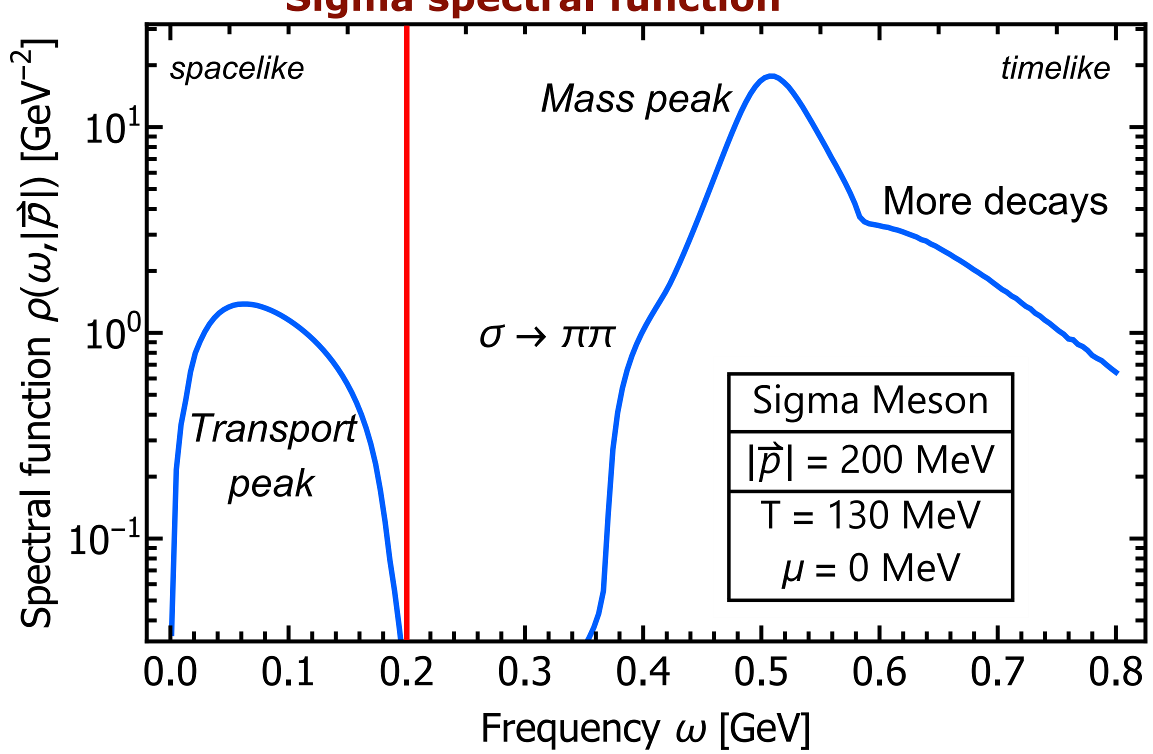
Sigma & Pion spectral functions



JMP, Strodthoff, Wink, arXiv:1711.07444

2+1 flavour quark-meson model sigma spectral function

Sigma spectral function

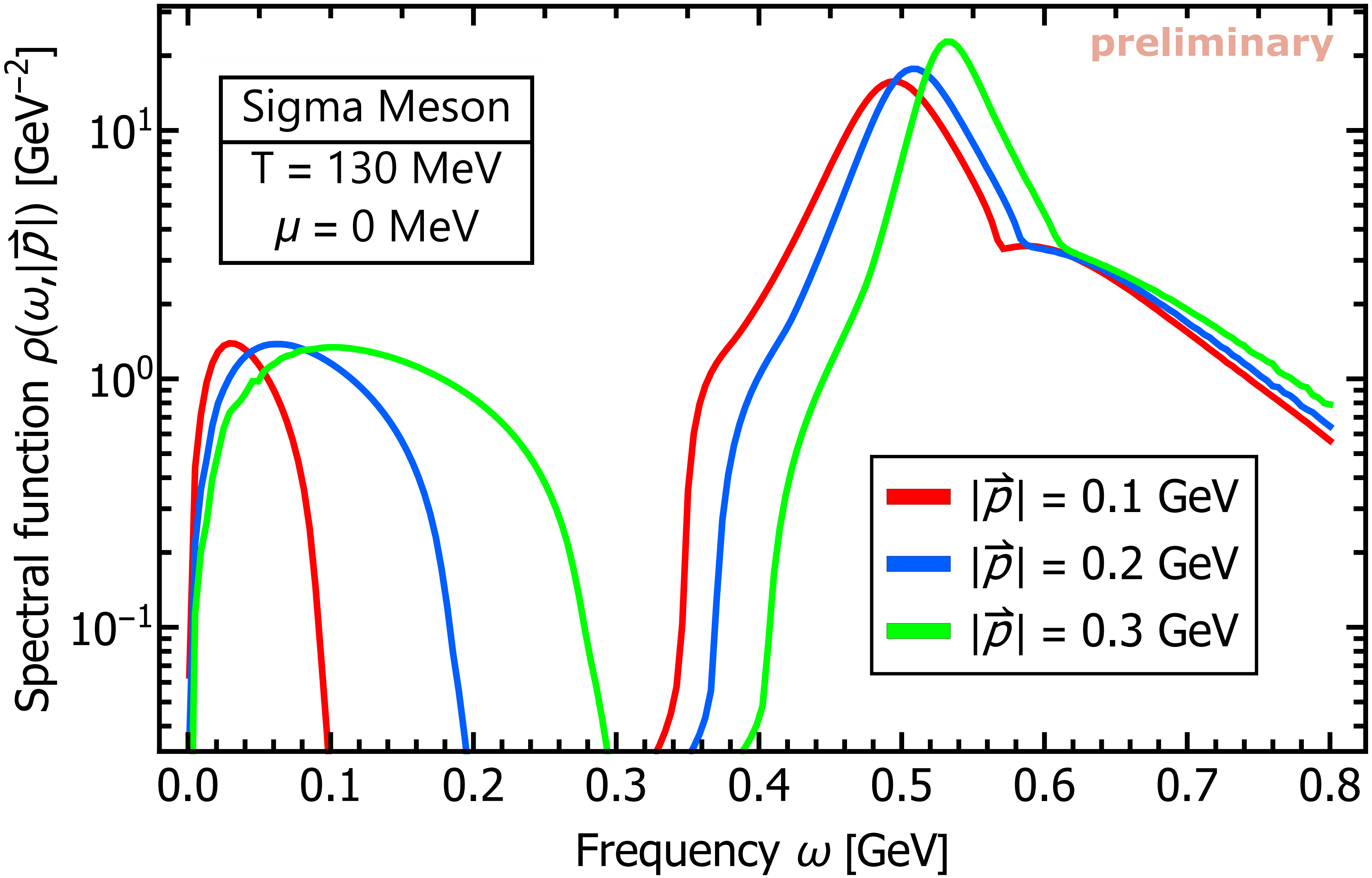


preliminary

JMP, Rennecke, Wink, in prep

Pion & sigma spectral functions

2+1 flavour quark-meson model sigma spectral function

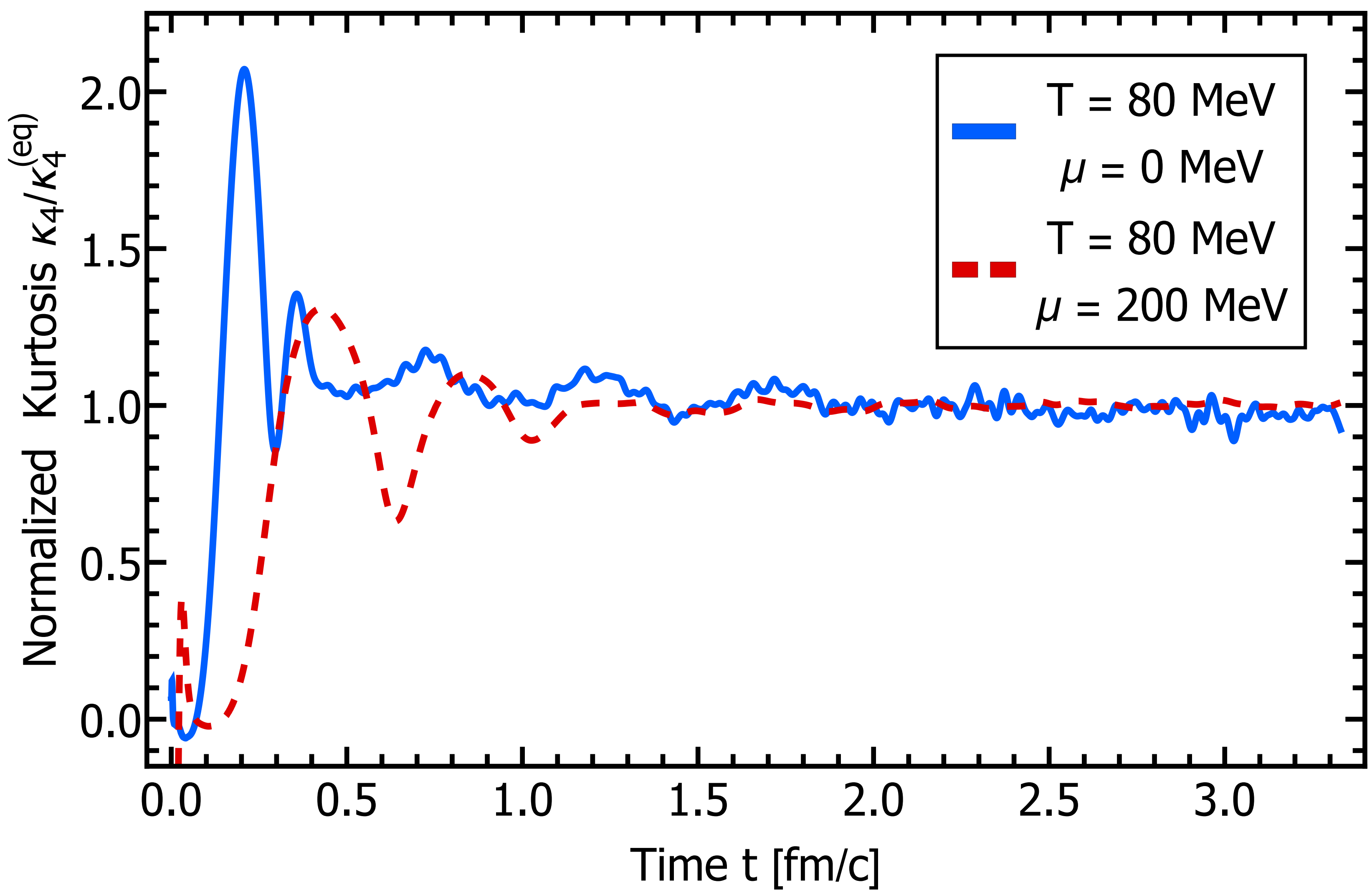


JMP, Rennecke, Wink, in prep

Time evolution of cumulants

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, in prep

Time evolution of kurtosis



nth central moment of the sigma field: χ_n

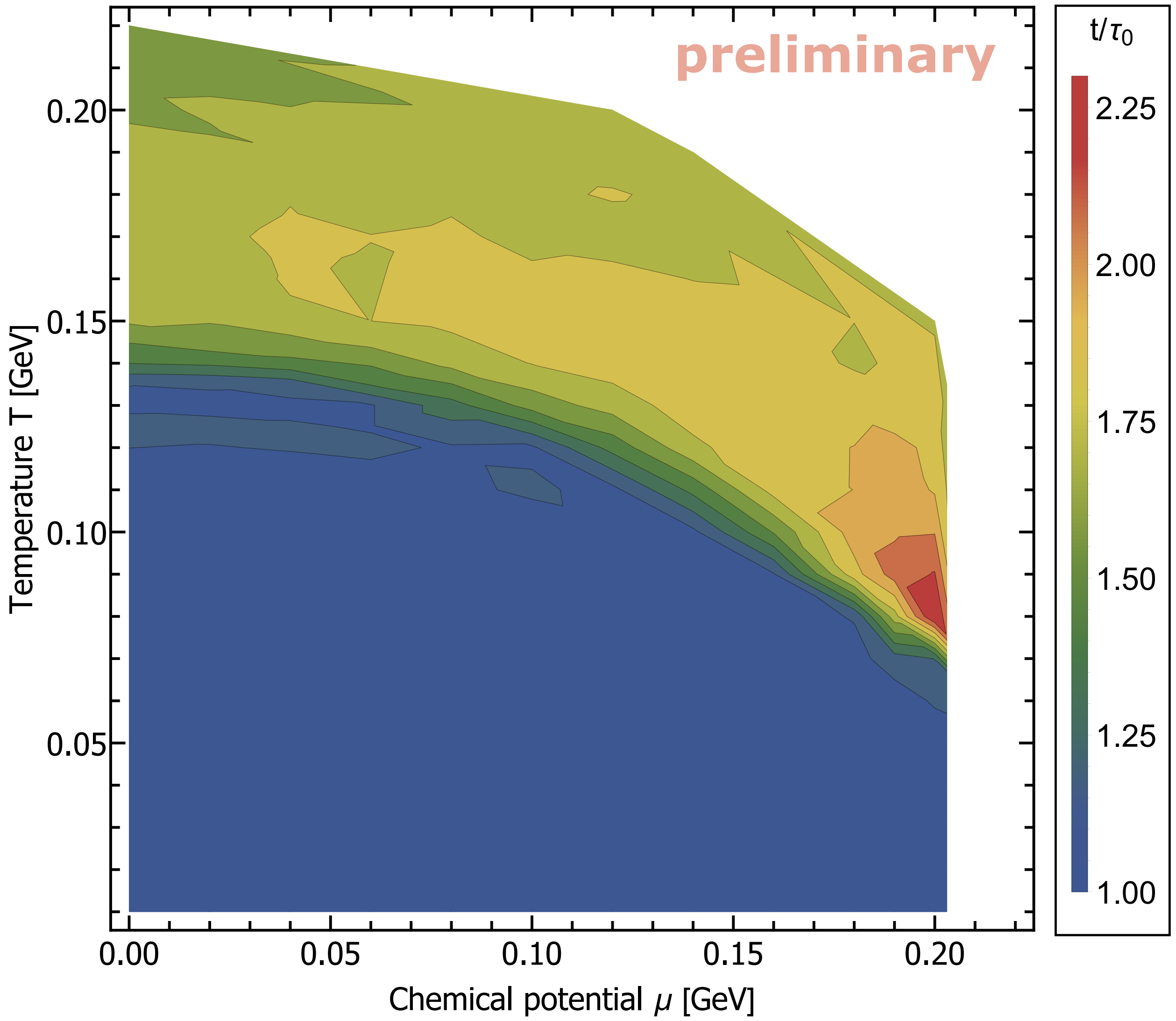
$$\chi_2 = \langle(\sigma - \langle\sigma\rangle)^2\rangle$$

kurtosis: $\kappa = \frac{\chi_4}{\chi_2^2} - 3$

Equilibration time phase structure

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, in prep

Equilibration time of sigma-kurtosis



nth central moment of the sigma field: χ_n

variance: $\chi_2 = \langle(\sigma - \langle\sigma\rangle)^2\rangle$

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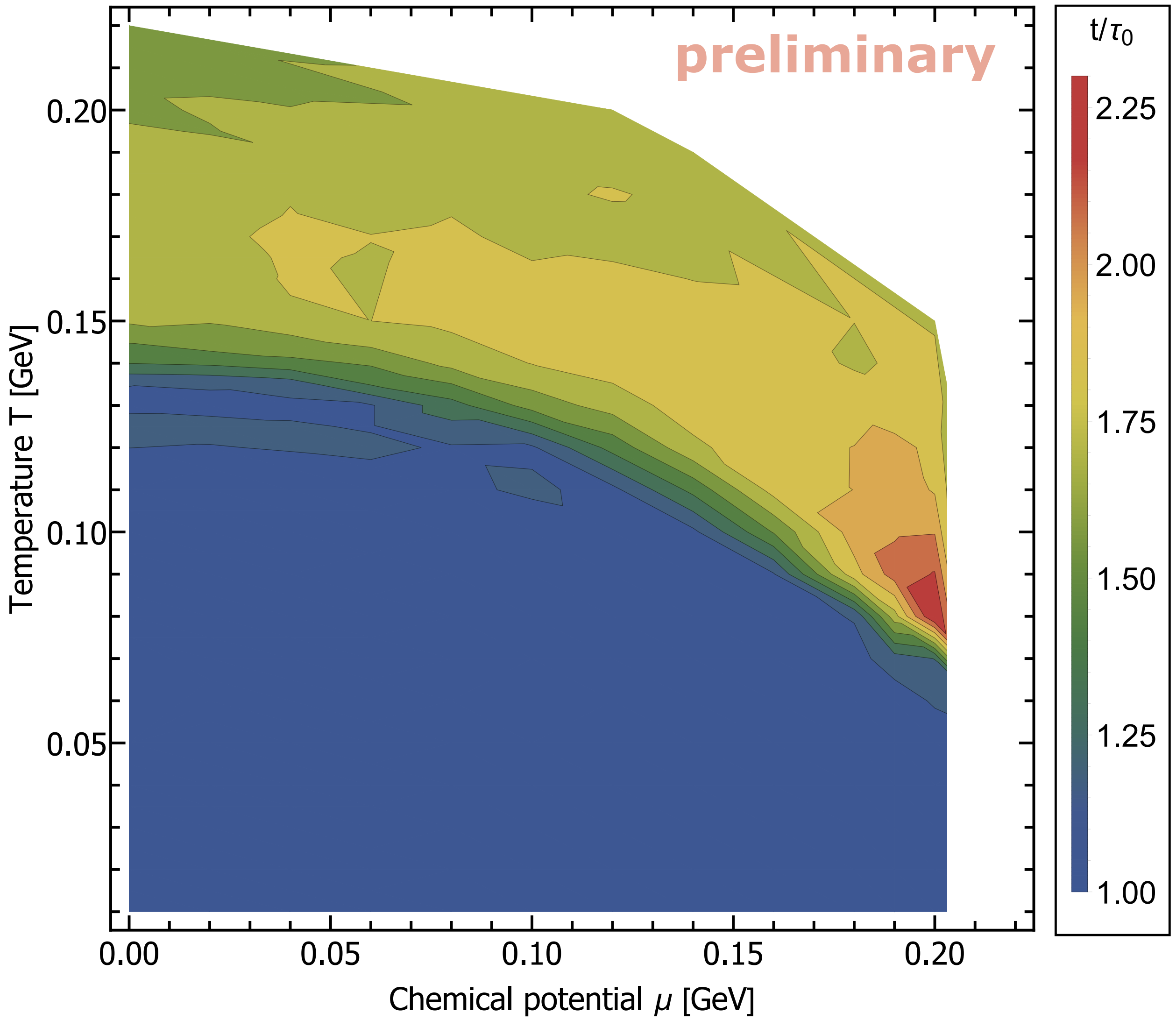
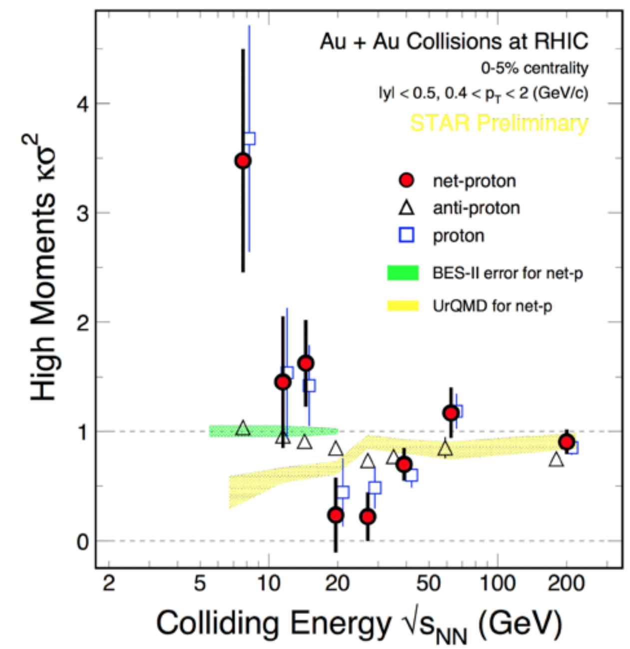
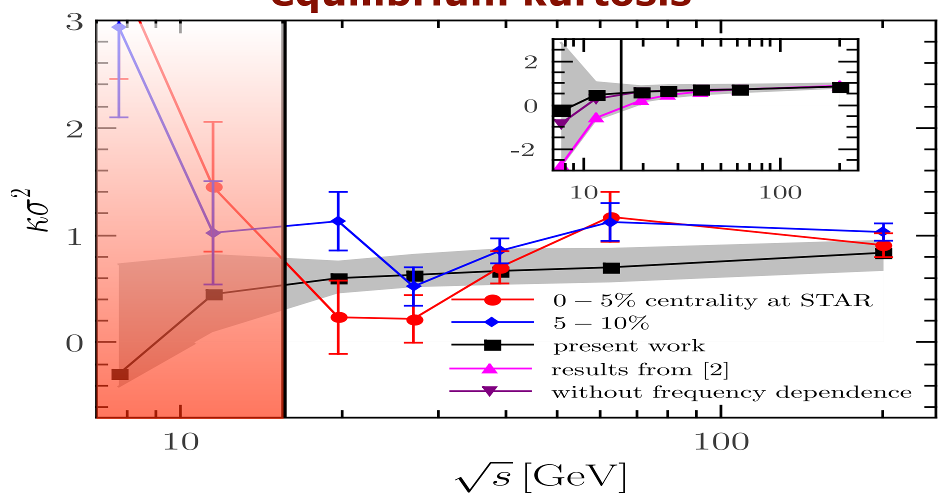


kurtosis of baryon number fluctuations



Luo, Cu, NST 28 (2017)

equilibrium kurtosis



Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 11, 116020

nth central moment of the sigma field: χ_n

variance: $\chi_2 = \langle (\sigma - \langle\sigma\rangle)^2 \rangle$

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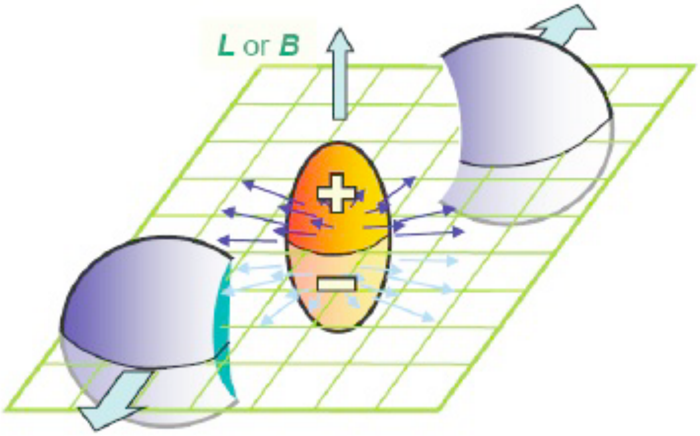
QCD-assisted hydrodynamics



QCD-assisted hydrodynamics

Dubla, Masciocchi, JMP, Schenke, Shen, Stachel, arXiv:1805.02985

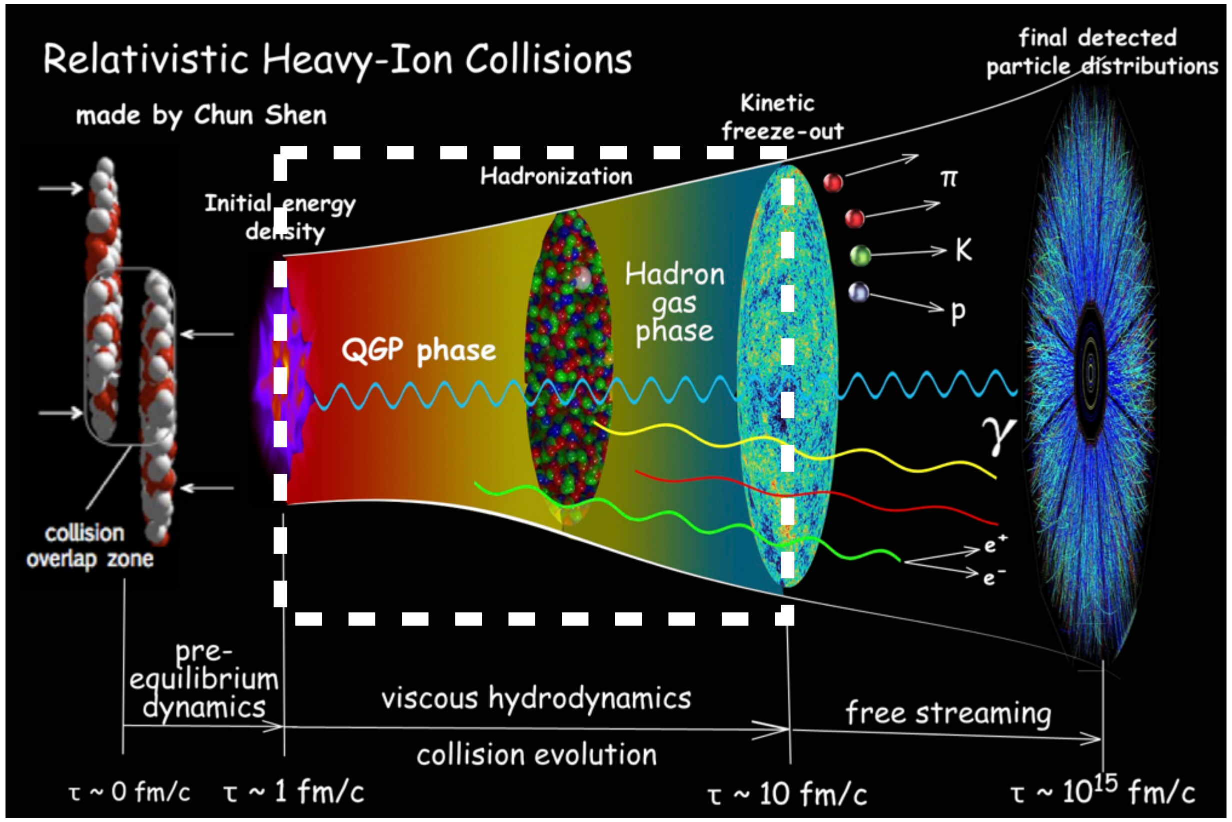
HIC 'phases'

Far from equilibrium initial phase

Kinetic phase

Hydrodynamical phase

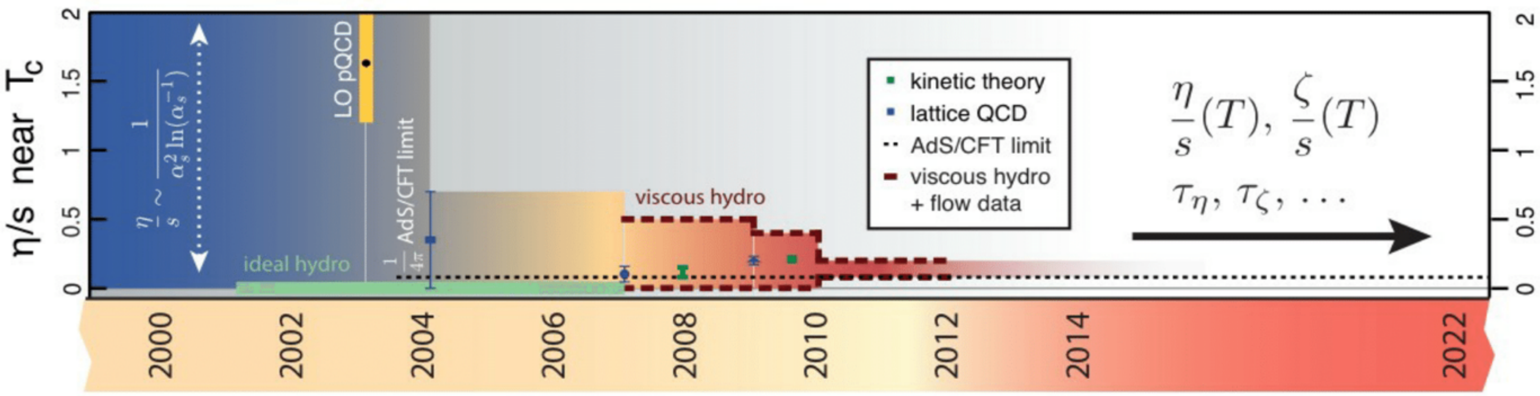
Hadronisation & freeze out



QCD-assisted transport

Hydro with QCD transport coefficients

Equilibrium transport coefficients



'Steady-state' hydro

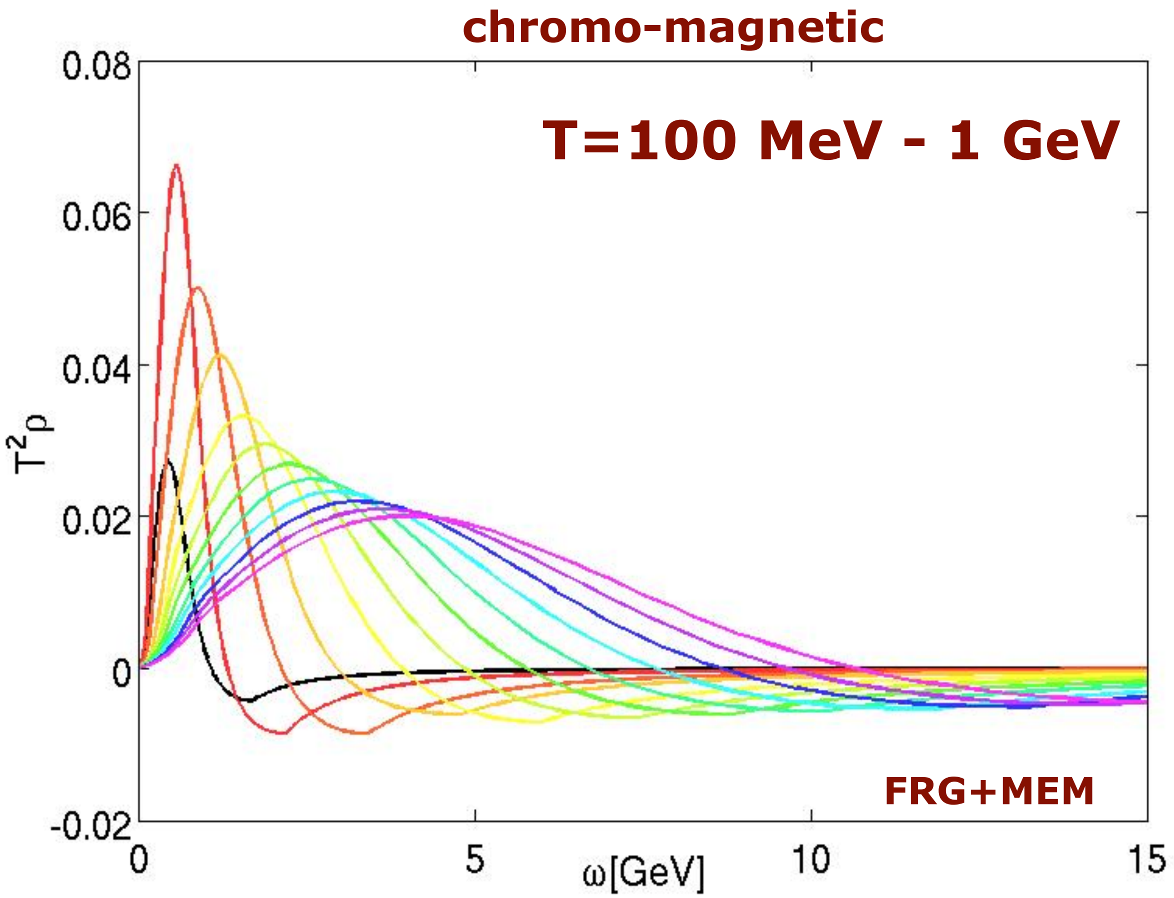
$$\pi^{\mu\nu} = \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\partial_\alpha u^\alpha - \tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta u^\sigma \partial_\sigma \pi^{\alpha\beta},$$

Constraints for the other phases

Single particle spectral functions

$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

Single particle spectral functions



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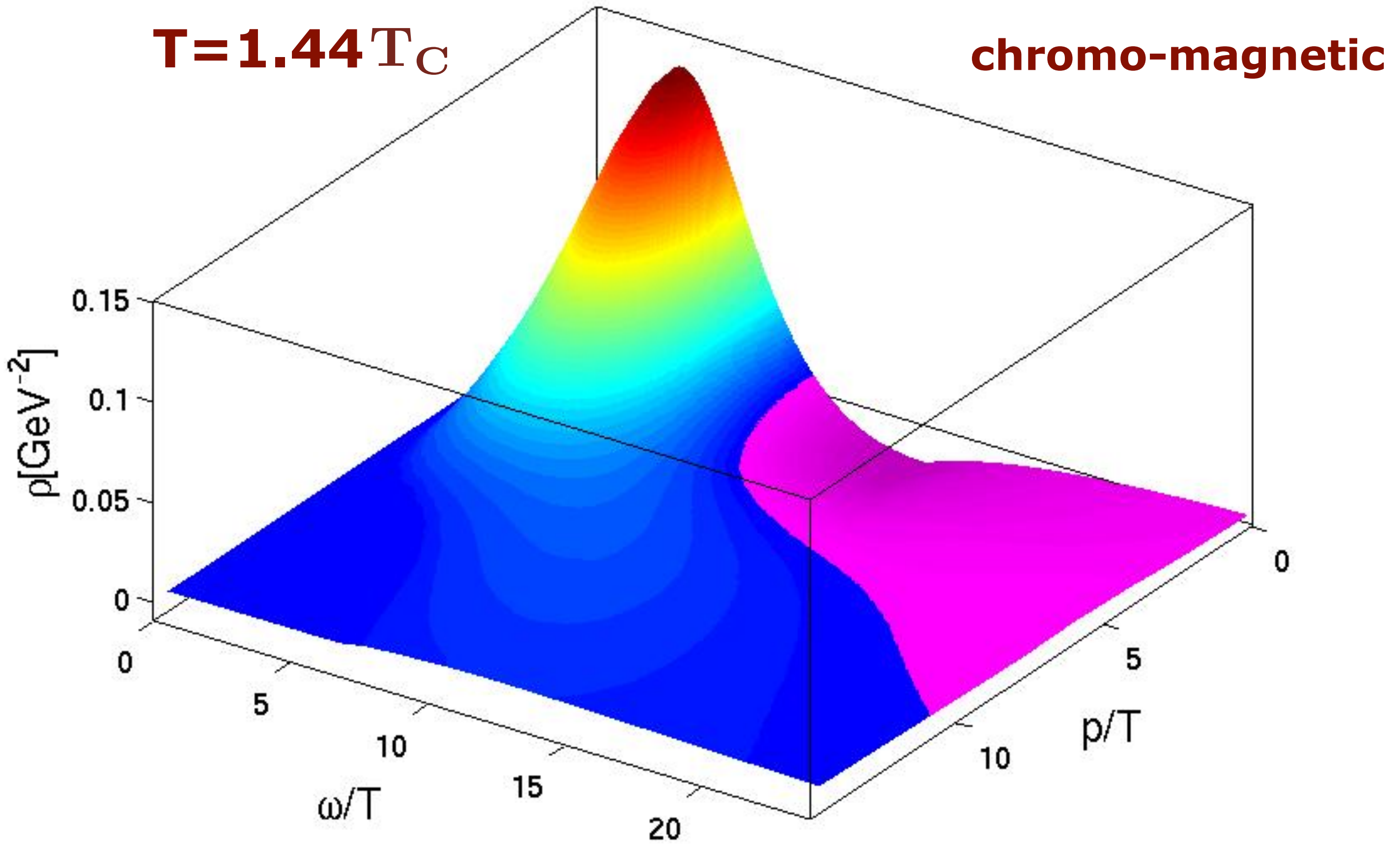
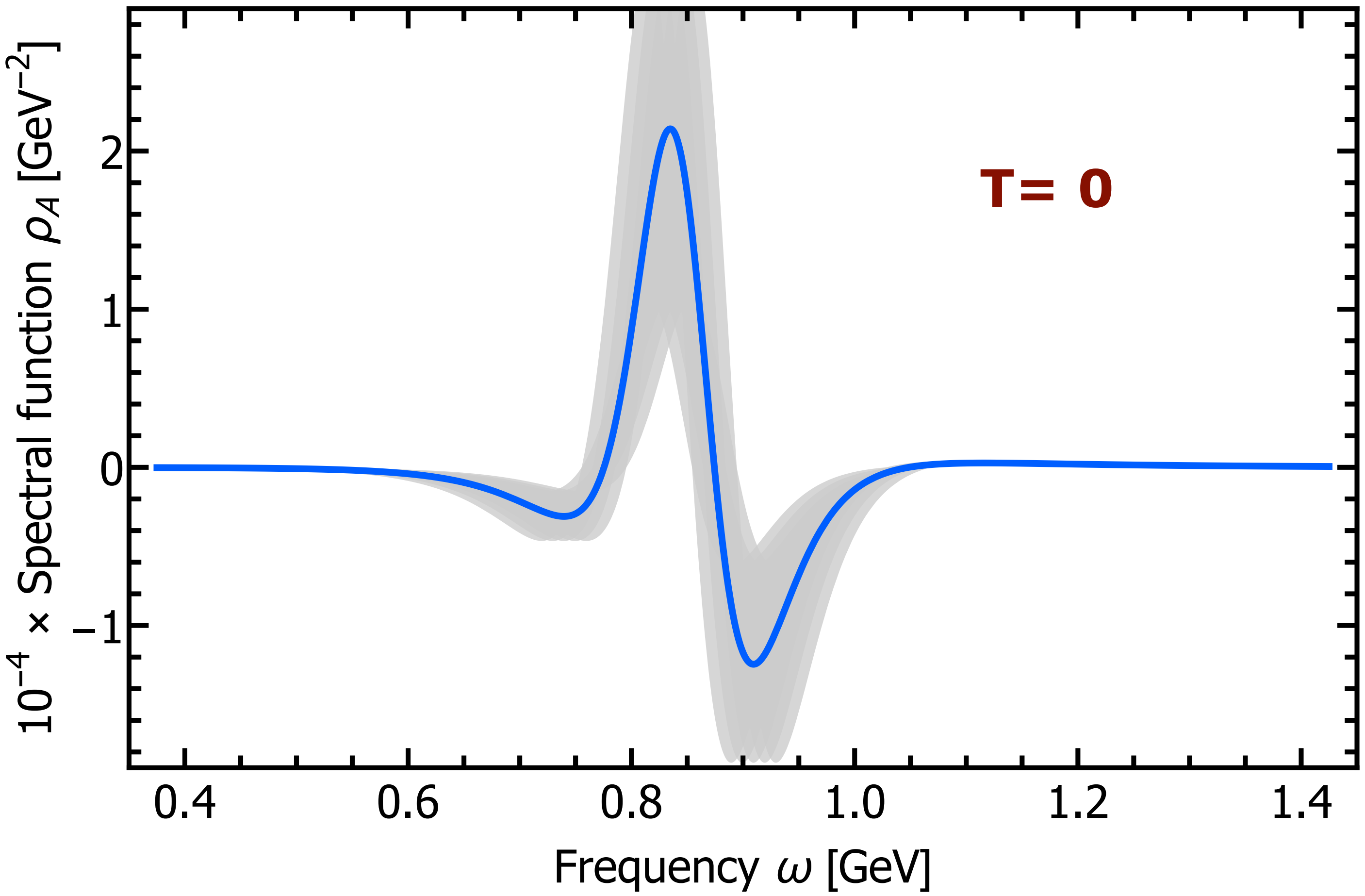


Haas, Fister, JMP, PRD 90 (2014) 091501

Single particle spectral functions

gluon spectral function at vanishing temperature



$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

novel analytic IR (& UV) behaviour and qualitatively refined reconstruction

Cyrol, JMP, Rothkopf, Wink, arXiv:1804.00945

Single particle spectral functions

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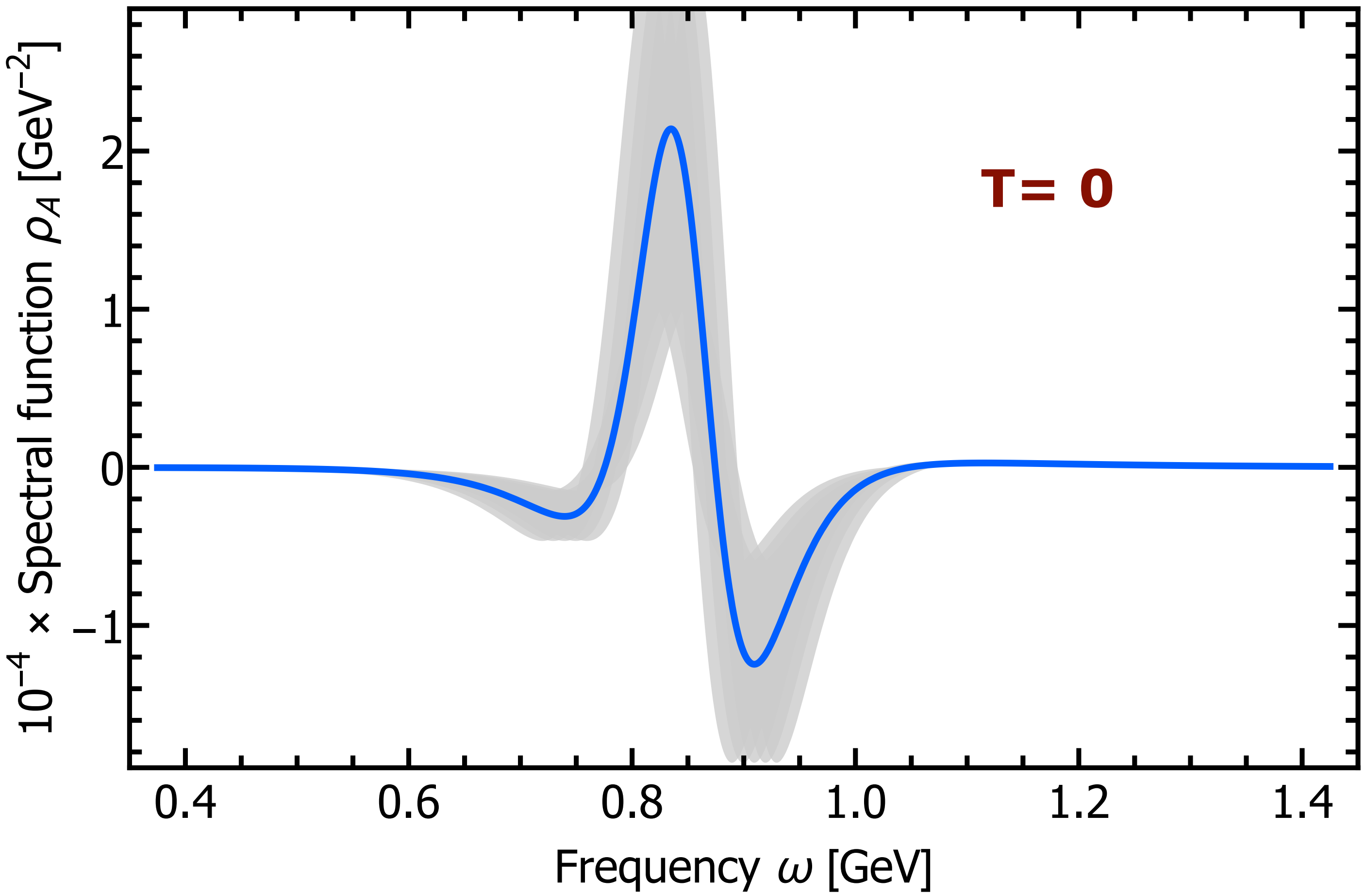


$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

novel analytic IR (& UV) behaviour and qualitatively refined reconstruction

'Those are my methods (principles), and if you don't like them...well, I have others'

Groucho Marx

direct computation

Real-time FRG: wait 5 minutes

Cyrol, JMP, Rothkopf, Wink, arXiv:1804.00945

Transport coefficients

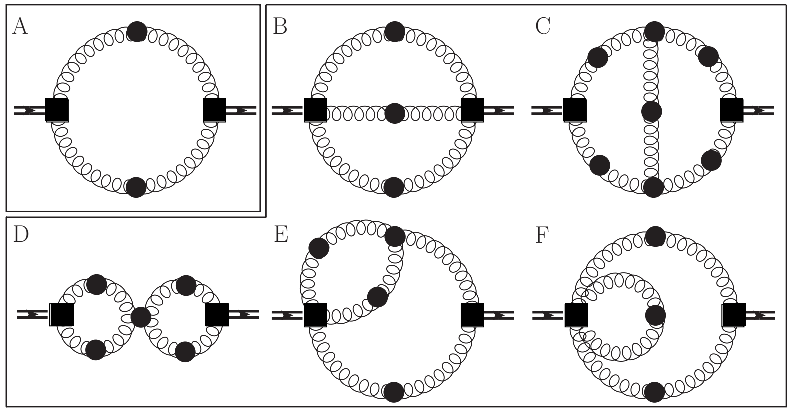
viscosity over entropy ratio in Yang-Mills theory

Kubo relation

$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

'3-loop' exact functional relation for $\rho_{\pi\pi}$

1 & 2-loop terms



Haas, Fister, JMP, PRD 90 (2014) 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

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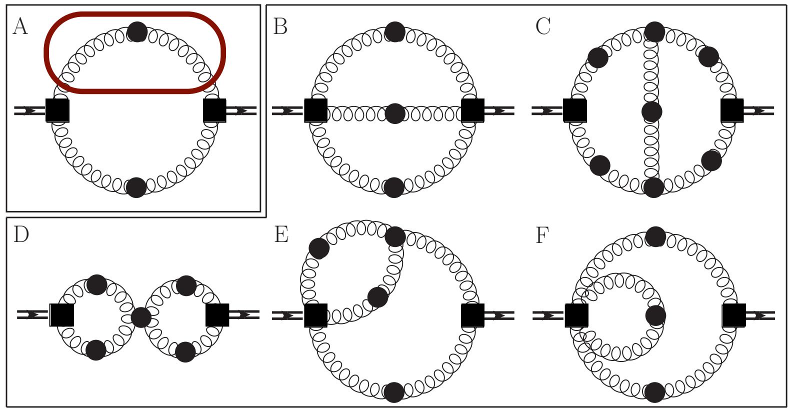
Gluon spectral function



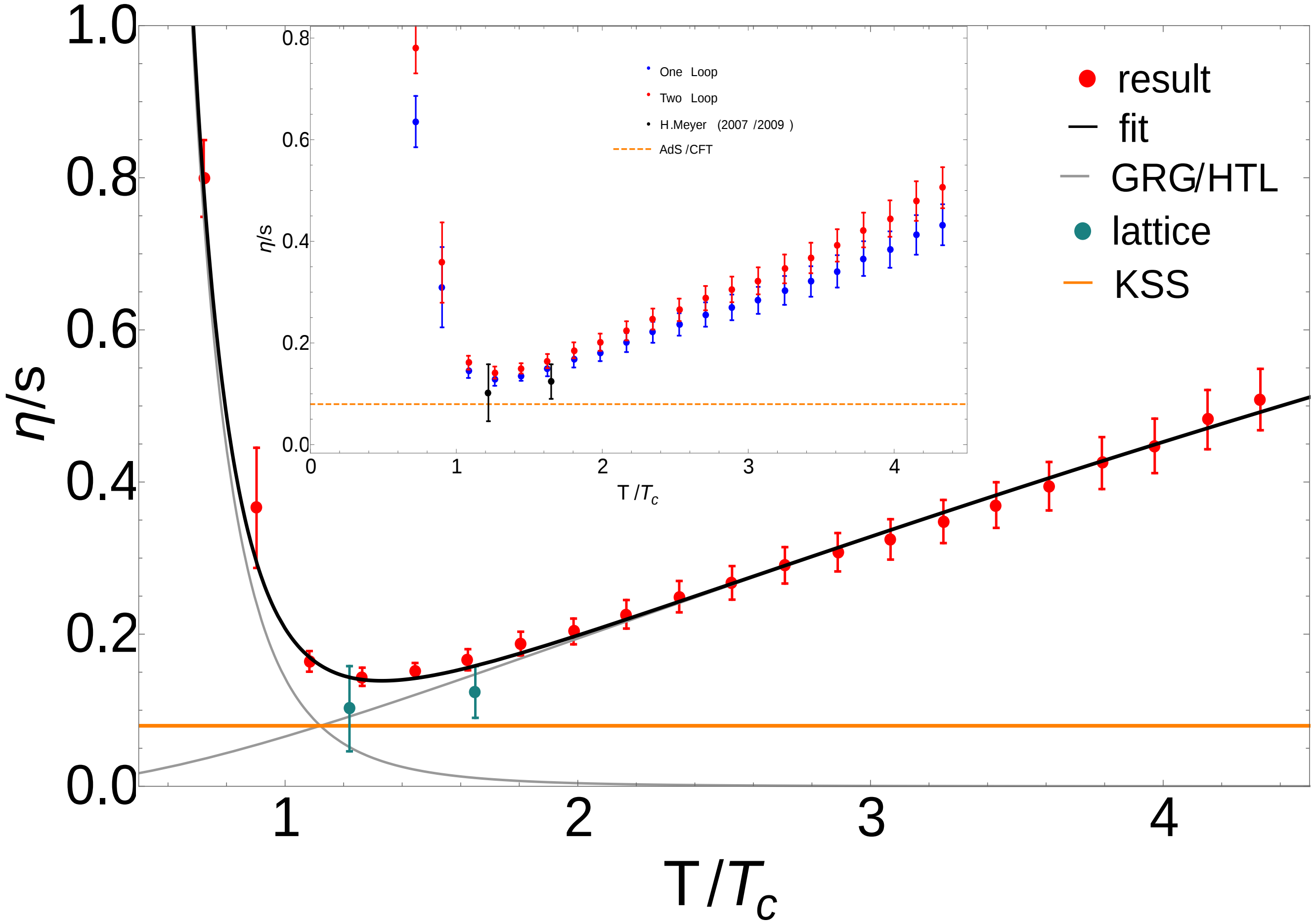
Haas, Fister, JMP, PRD 90 (2014) 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

Transport coefficients

viscosity over entropy ratio in Yang-Mills theory

Yang-Mills viscosity over entropy ratio



recent lattice results: Astrakhantsev, Braguta, Kotov, JHEP 1704 (2017) 101
arXiv:1804.02382

Aiming at apparent convergence

Kubo relation

$$\eta = \frac{1}{20}\,\frac{d}{d\omega}\bigg|_{\omega=0}\rho_{\pi\pi}(\omega,0)$$

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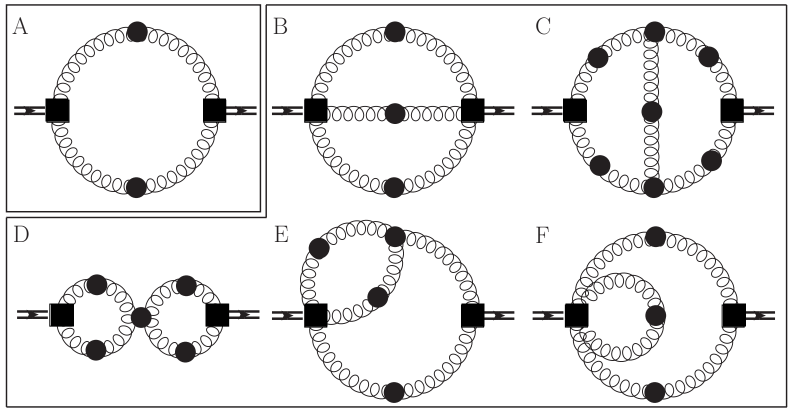


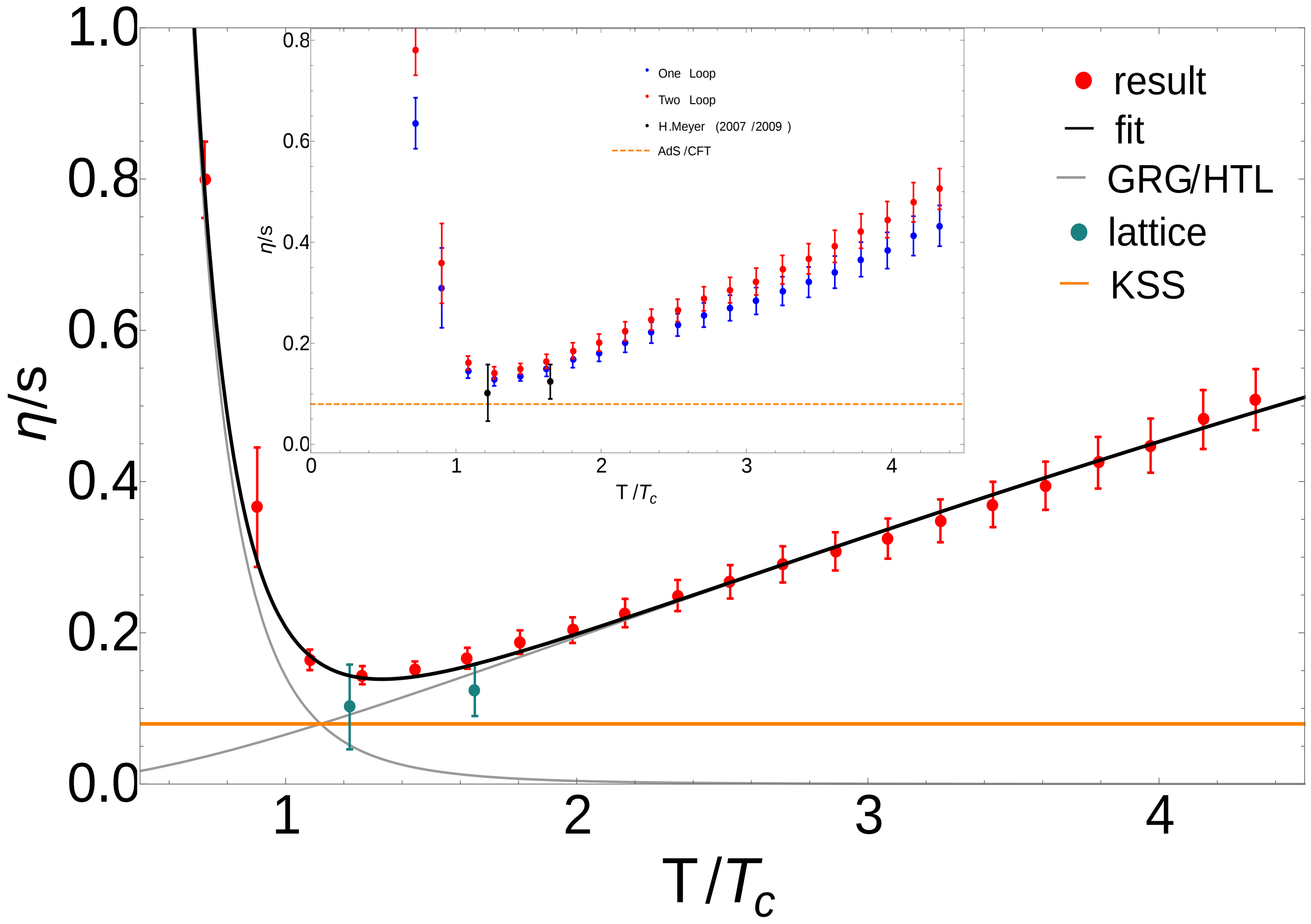
Haas, Fister, JMP, PRD 90 (2014) 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

Transport coefficients

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$

$$a_{\rm hrg} \approx 0.14$$

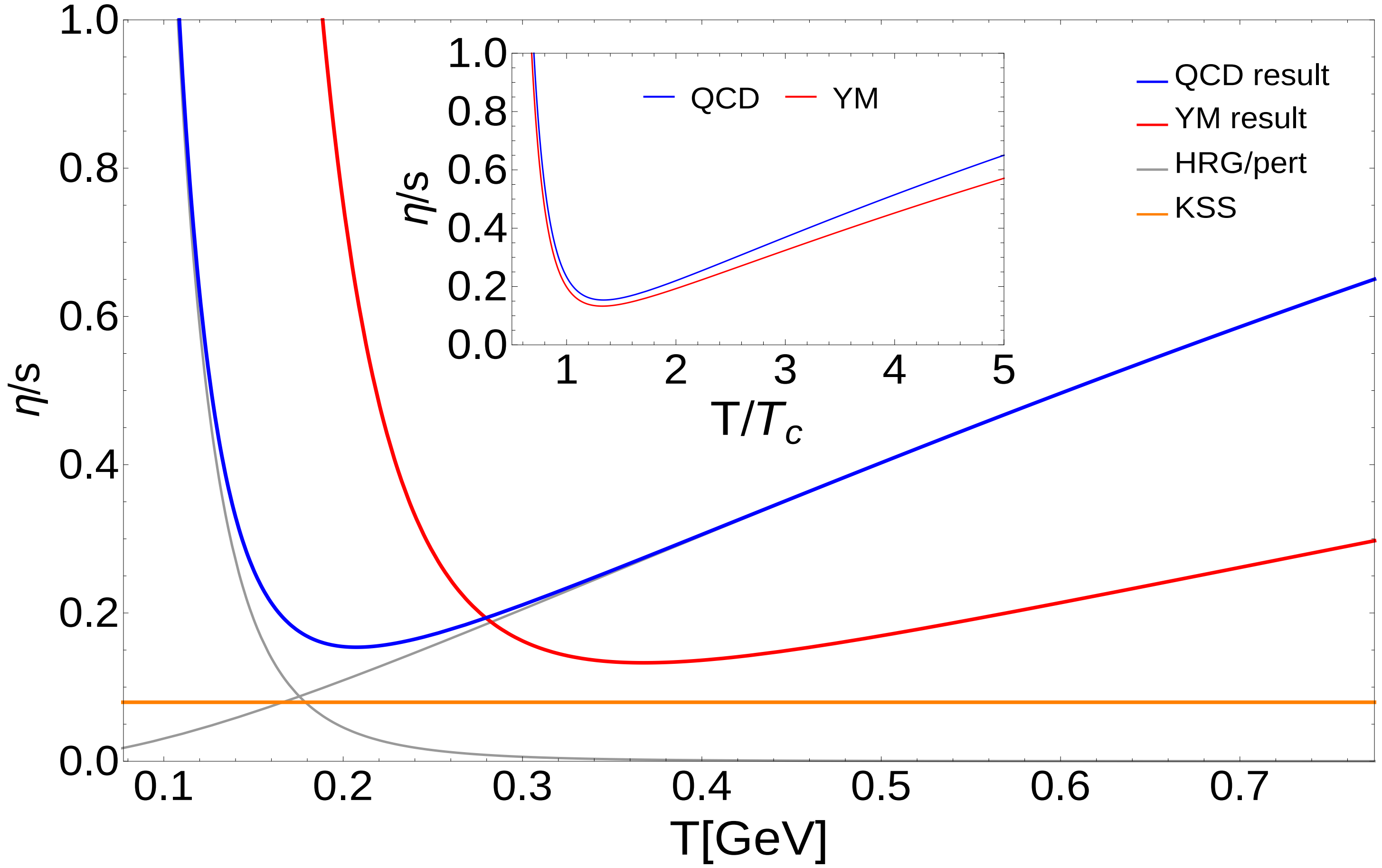
$$c \approx 0.66$$

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

Transport coefficients

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$a_{\rm qgp} \approx 0.2$$
$$a_{\rm hrg} \approx 0.16$$
$$c \approx 0.79$$

QCD

$$\gamma_{\rm grg} \approx 5$$
$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$
$$a_{\rm hrg} \approx 0.14$$
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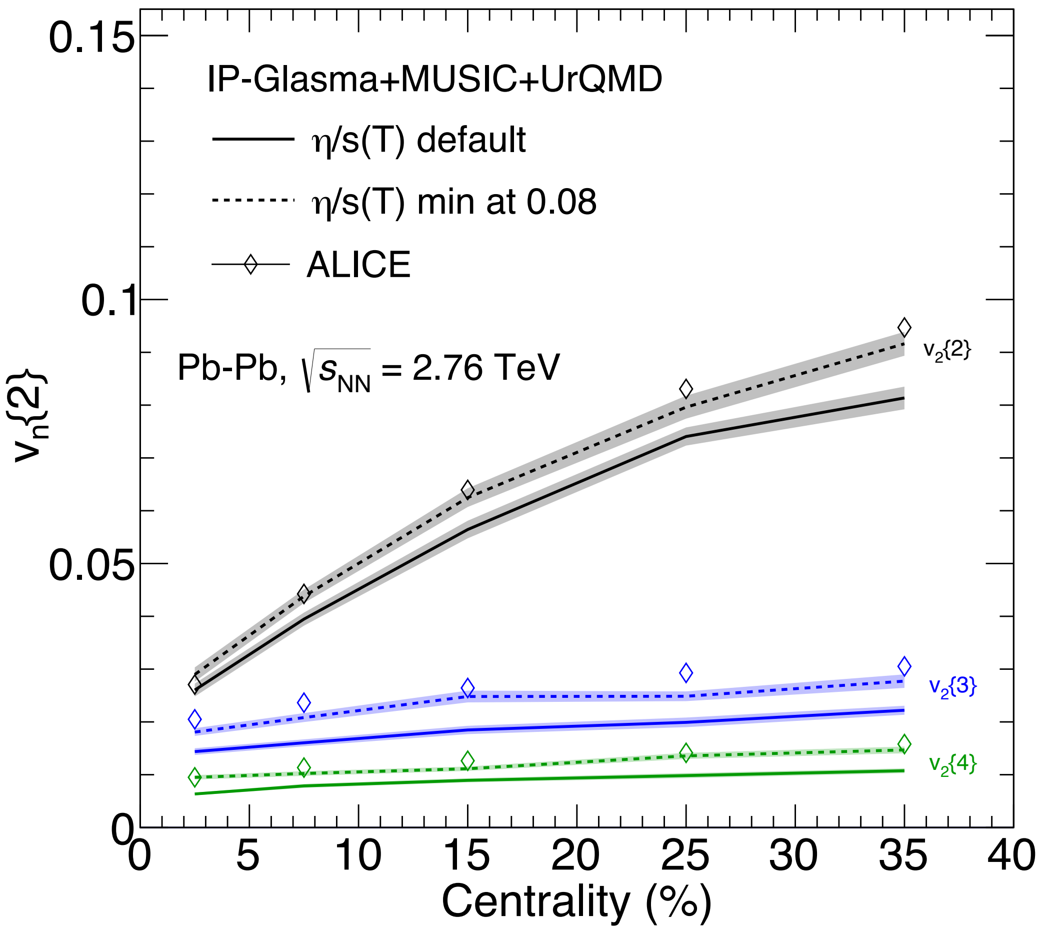
Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

QCD-assisted hydrodynamics

Dubla, Masciocchi, JMP, Schenke, Shen, Stachel, arXiv:1805.02985

IP-Glasma - MUSIC - UrQMD

v_n as function of centrality

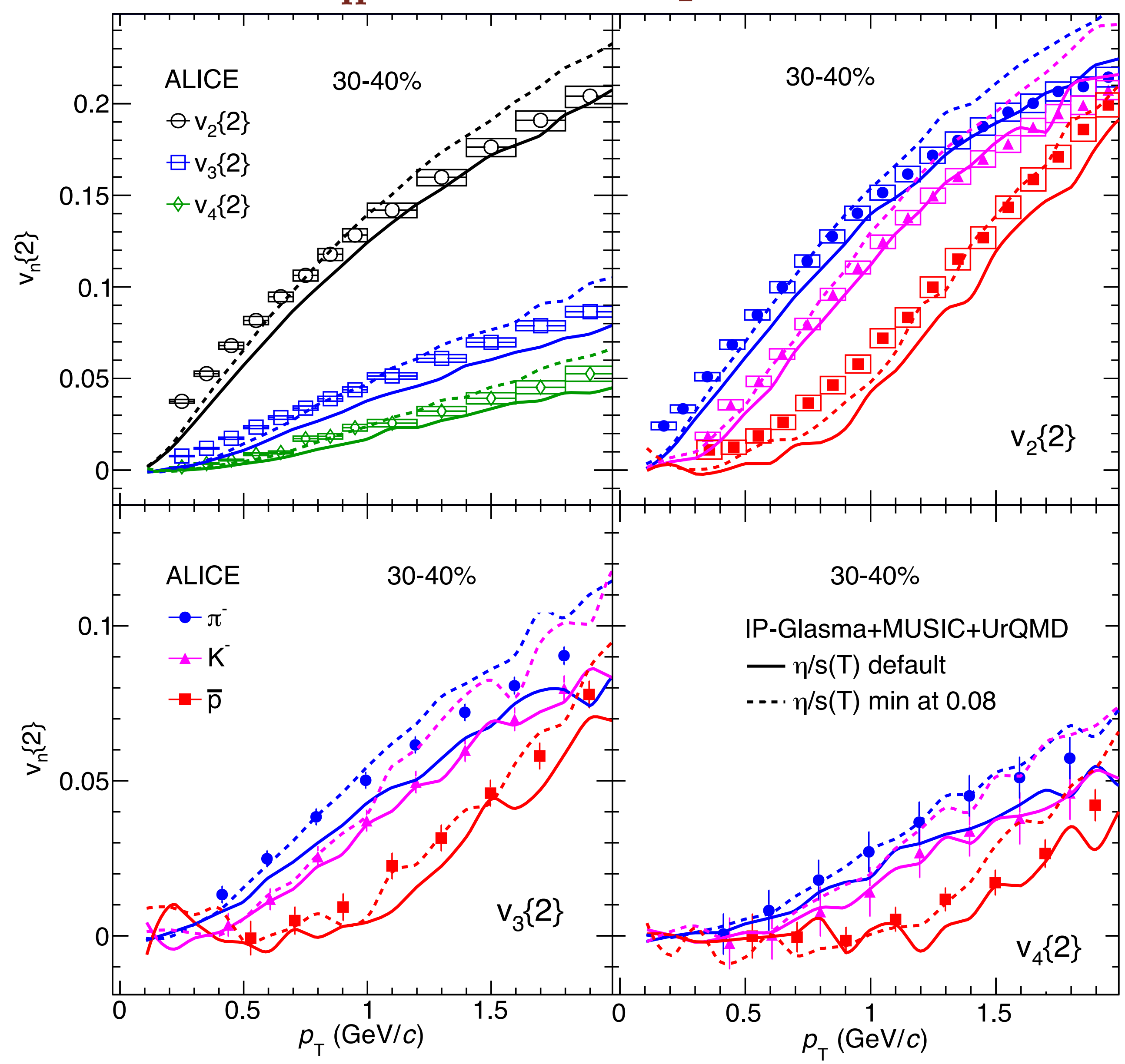


Test of systematic error

$$\eta/s(T) \rightarrow \eta/s(T) + d$$

$$d \in [-0.06, 0]$$

v_n as function of p_T



$$\frac{dN}{d(\varphi - \Psi_R)} = \frac{N_0}{2\pi}\left(1 + 2\sum_n \boxed{v_n} \cos[\,n\,(\varphi - \Psi_R)]\right)$$

QCD-assisted hydrodynamics

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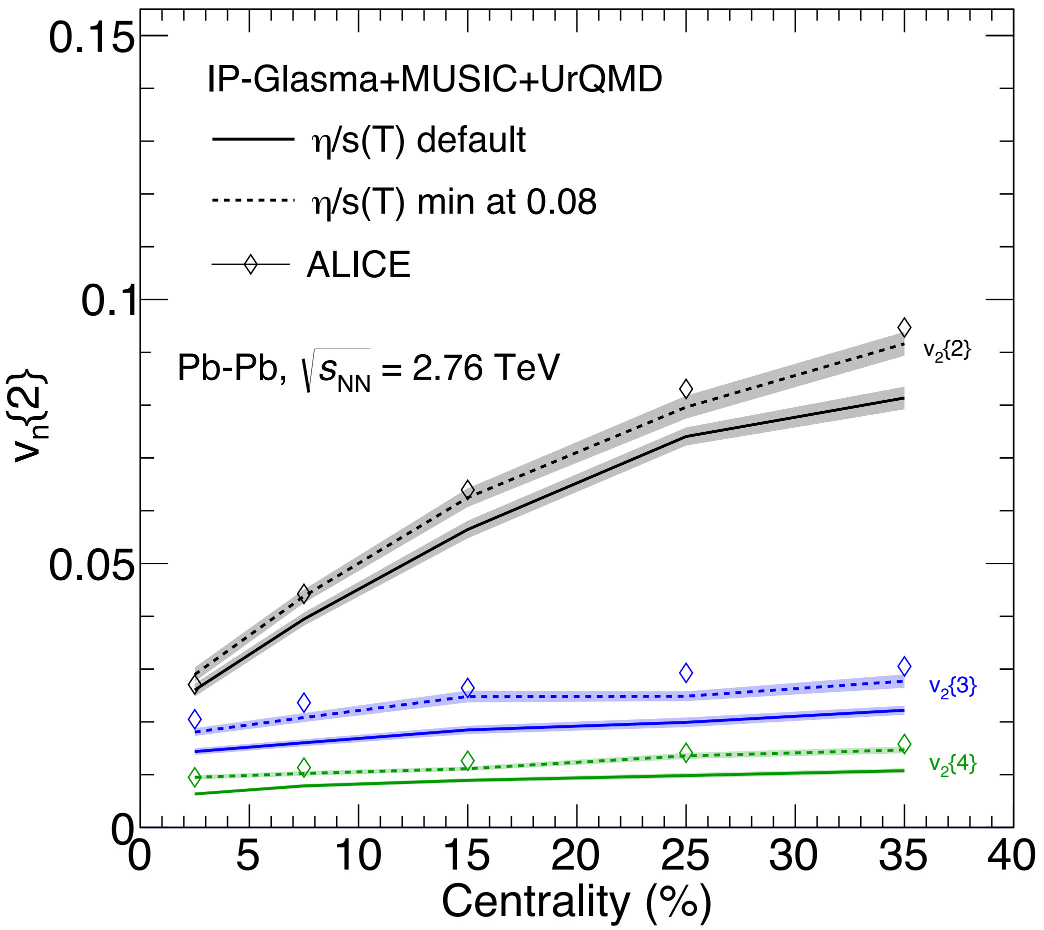

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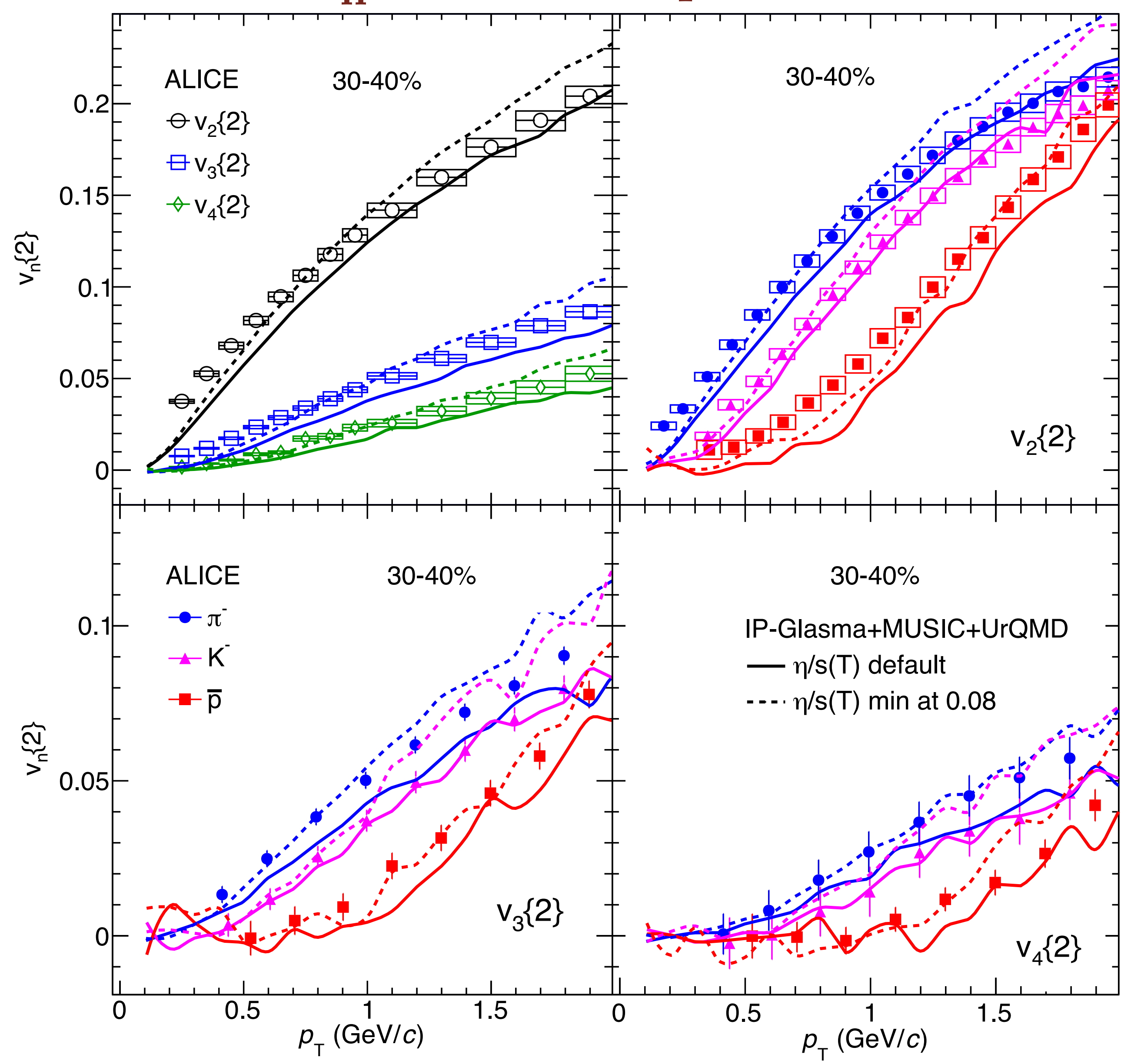

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Normalisation!?

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Dubla, Masciocchi, JMP, Schenke, Shen, Stachel, arXiv:1805.02985

IP-Glasma - MUSIC - UrQMD

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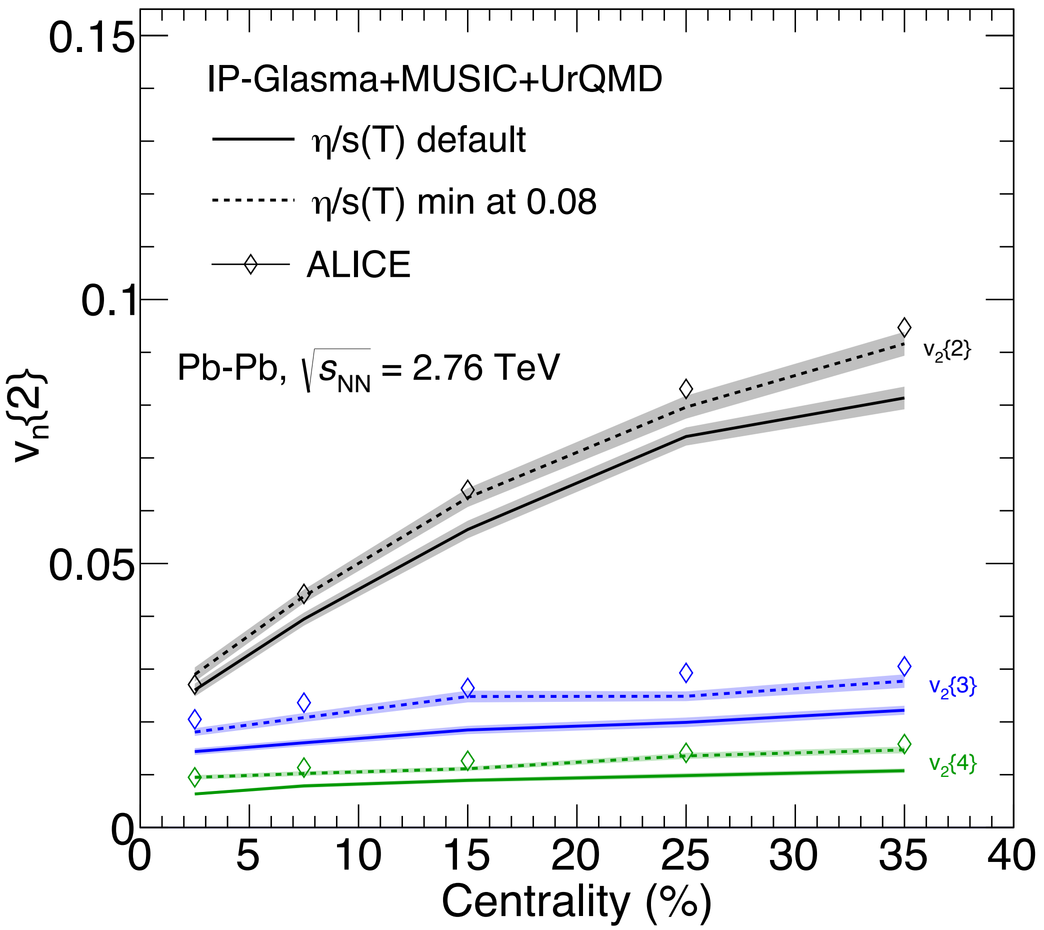
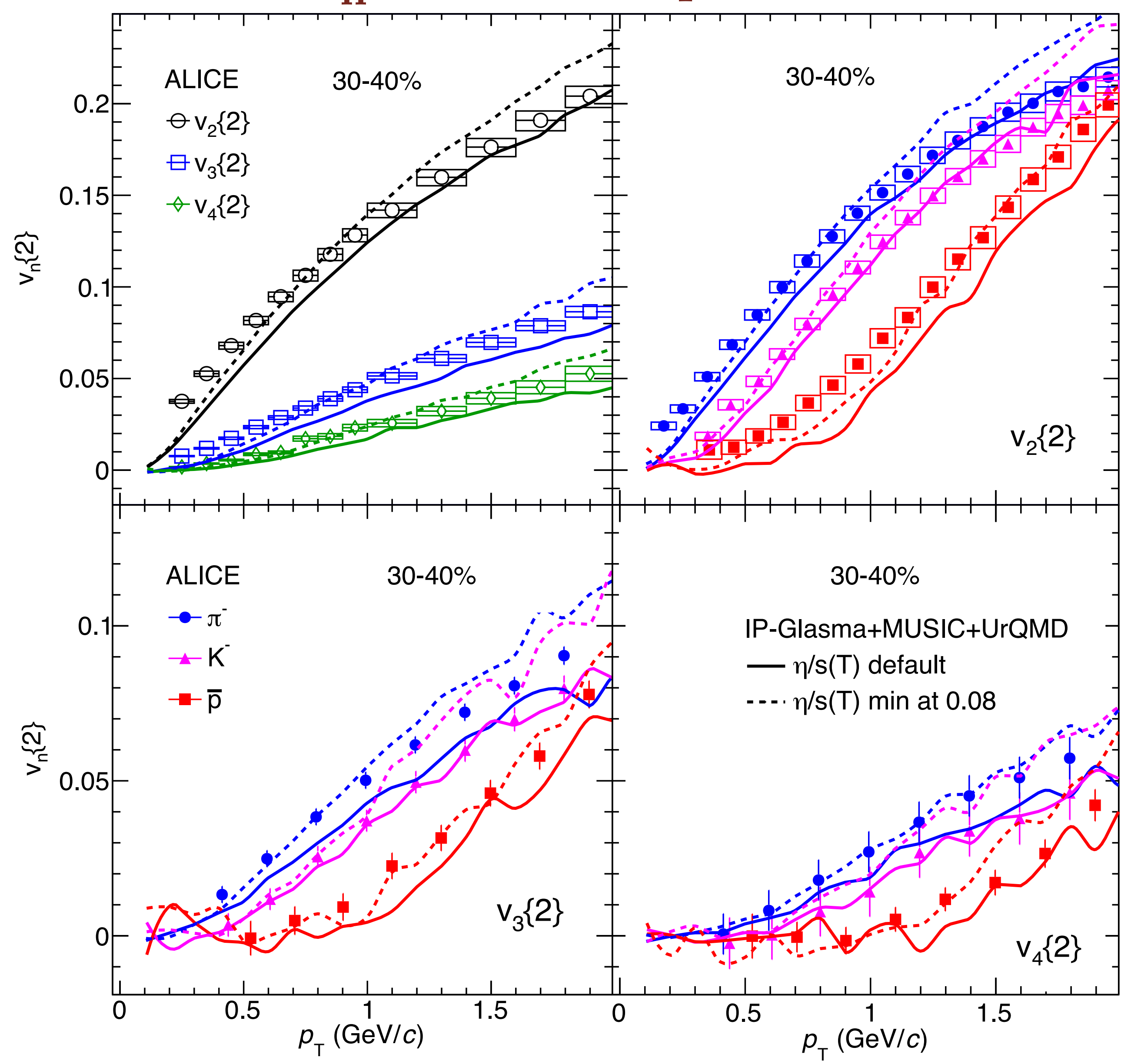


v_n as function of p_T



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Initial state fluctuations?

Normalisation!?

Kinetic phase?

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IP-Glasma - MUSIC - UrQMD

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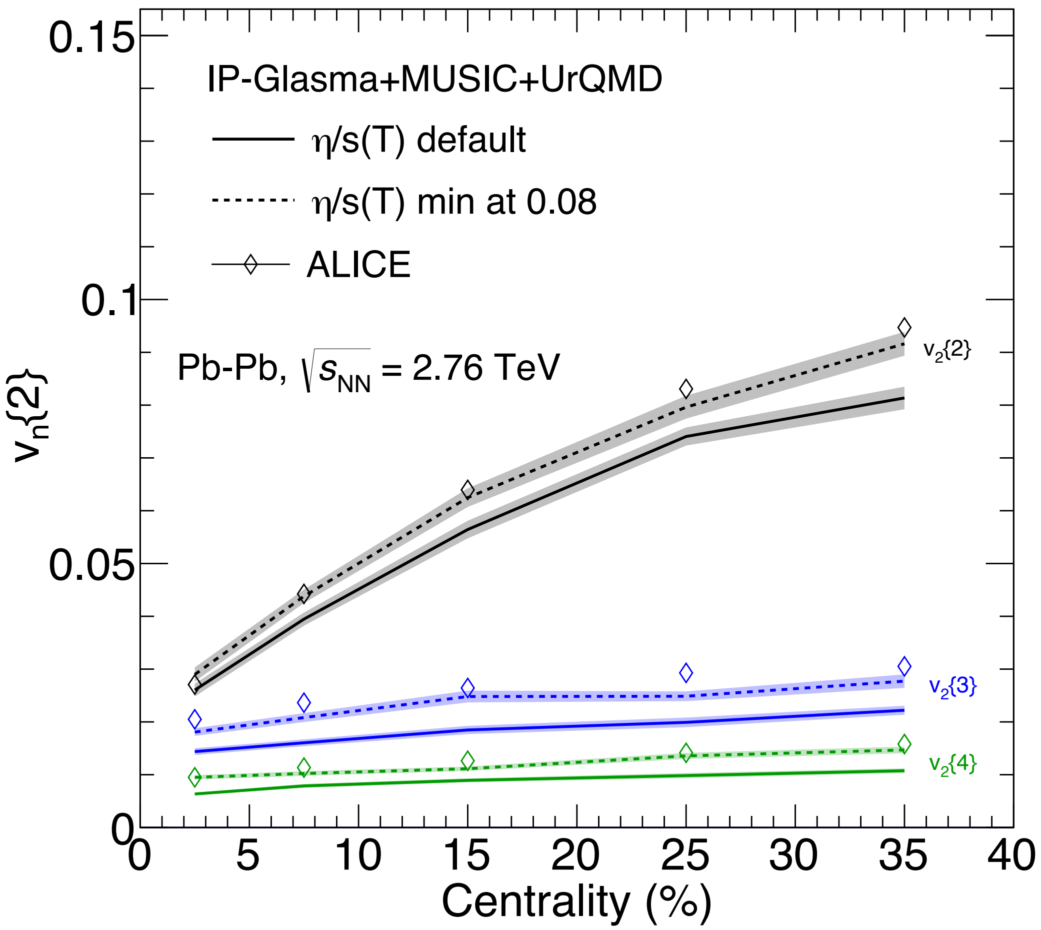

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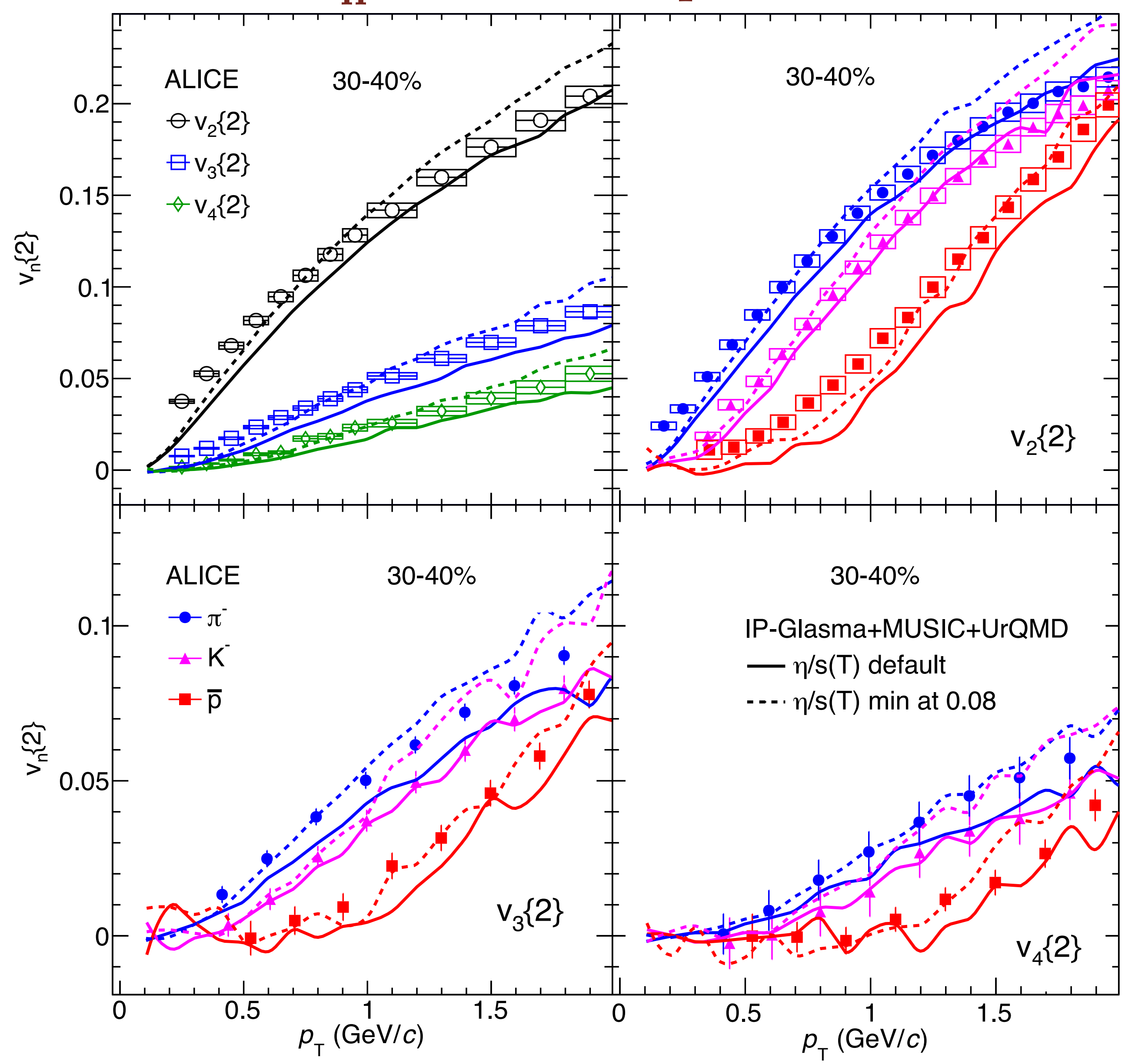

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Initial state fluctuations?

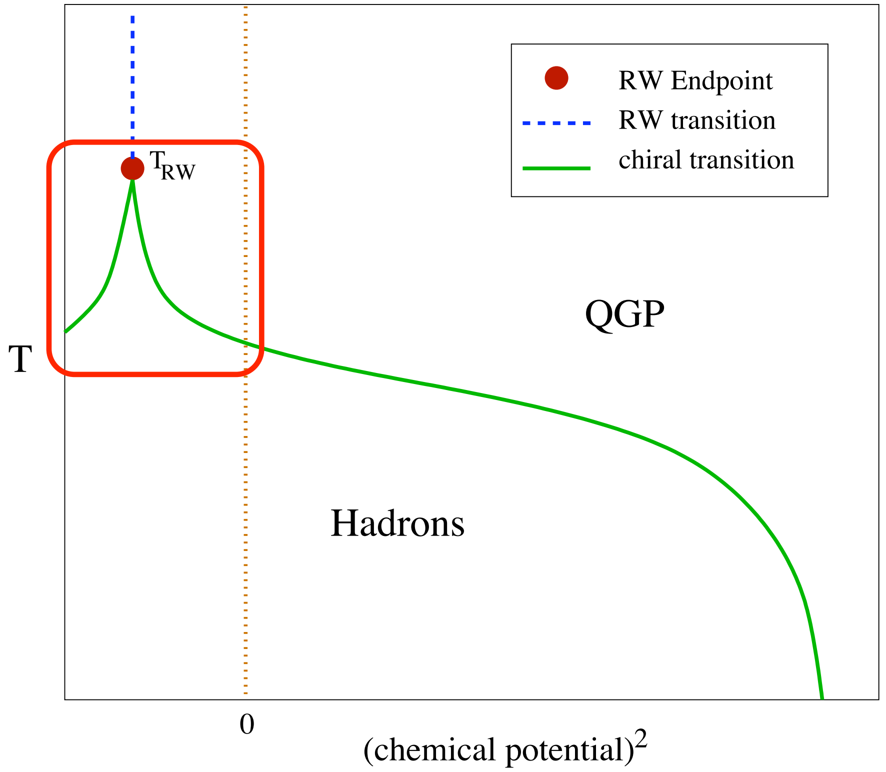
'Steady-state' hydro?

Normalisation!?

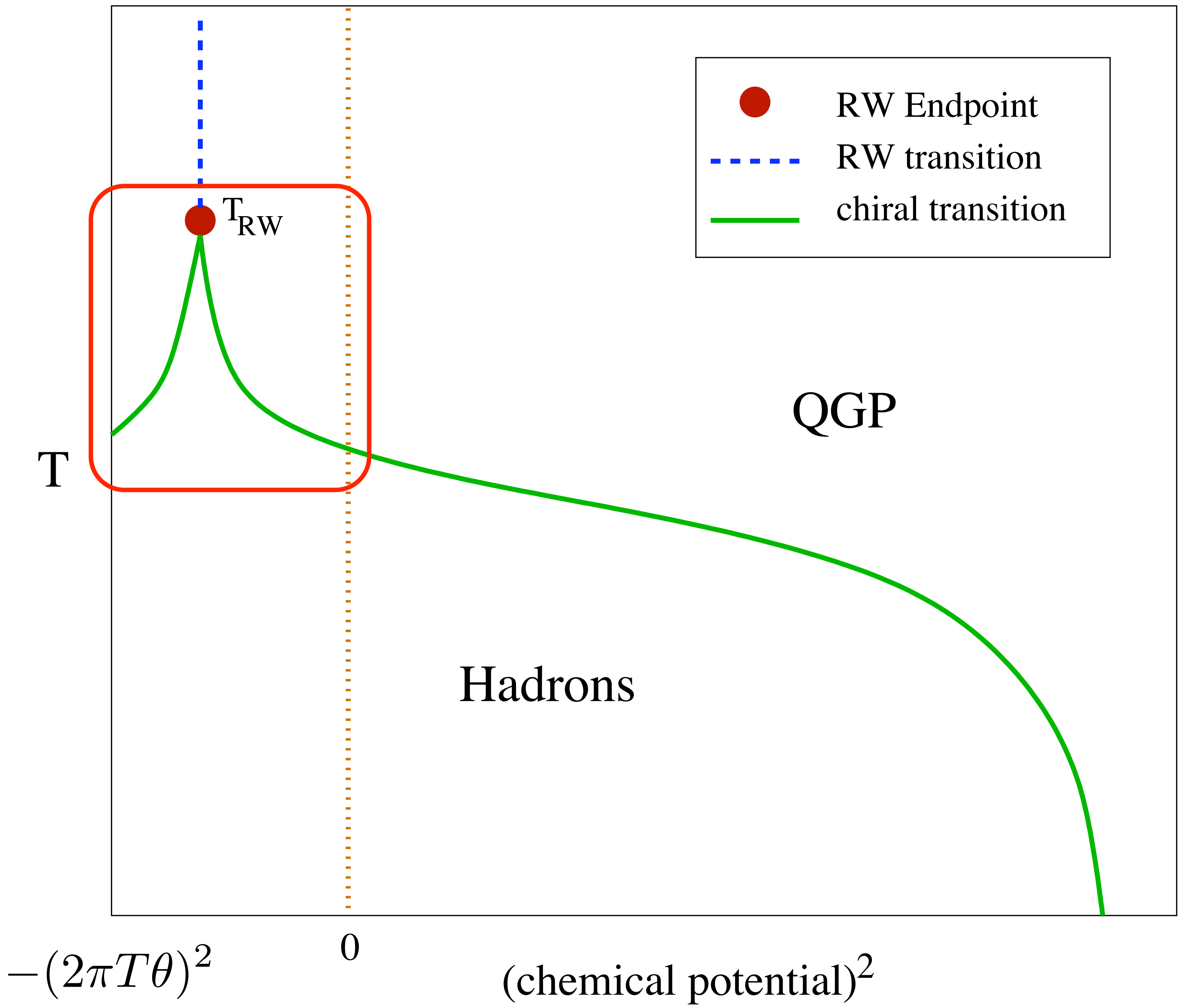
Kinetic phase?

Hadronisation & freeze-out?

QCD at imaginary chemical potential



Imaginary chemical potential



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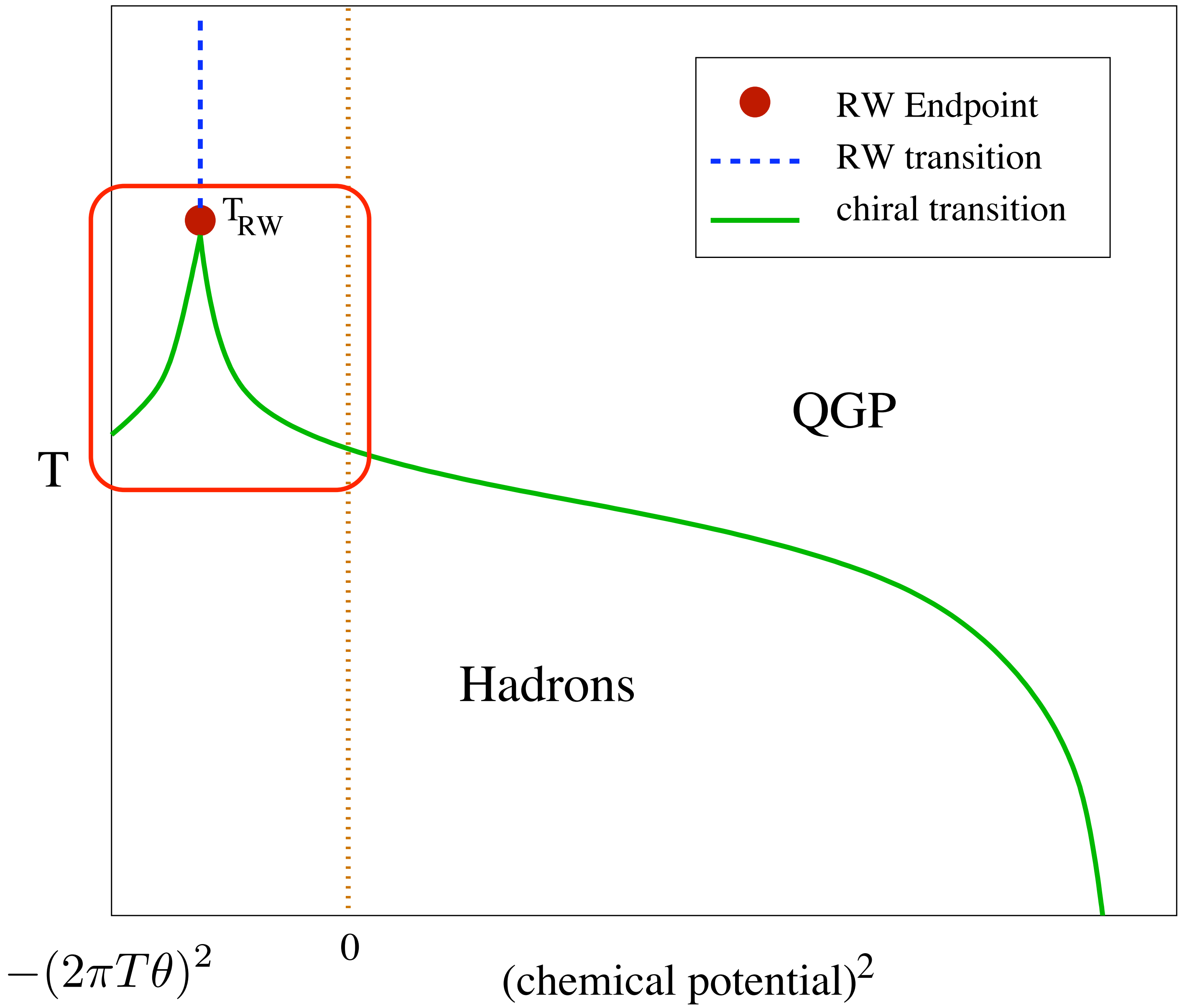


Dirac term

$$\int_x \bar{q} \cdot (i\not{D} + i\, m_\psi + i\mu\gamma_0) \cdot q$$

$$\mu = 2\pi T\theta\, i$$

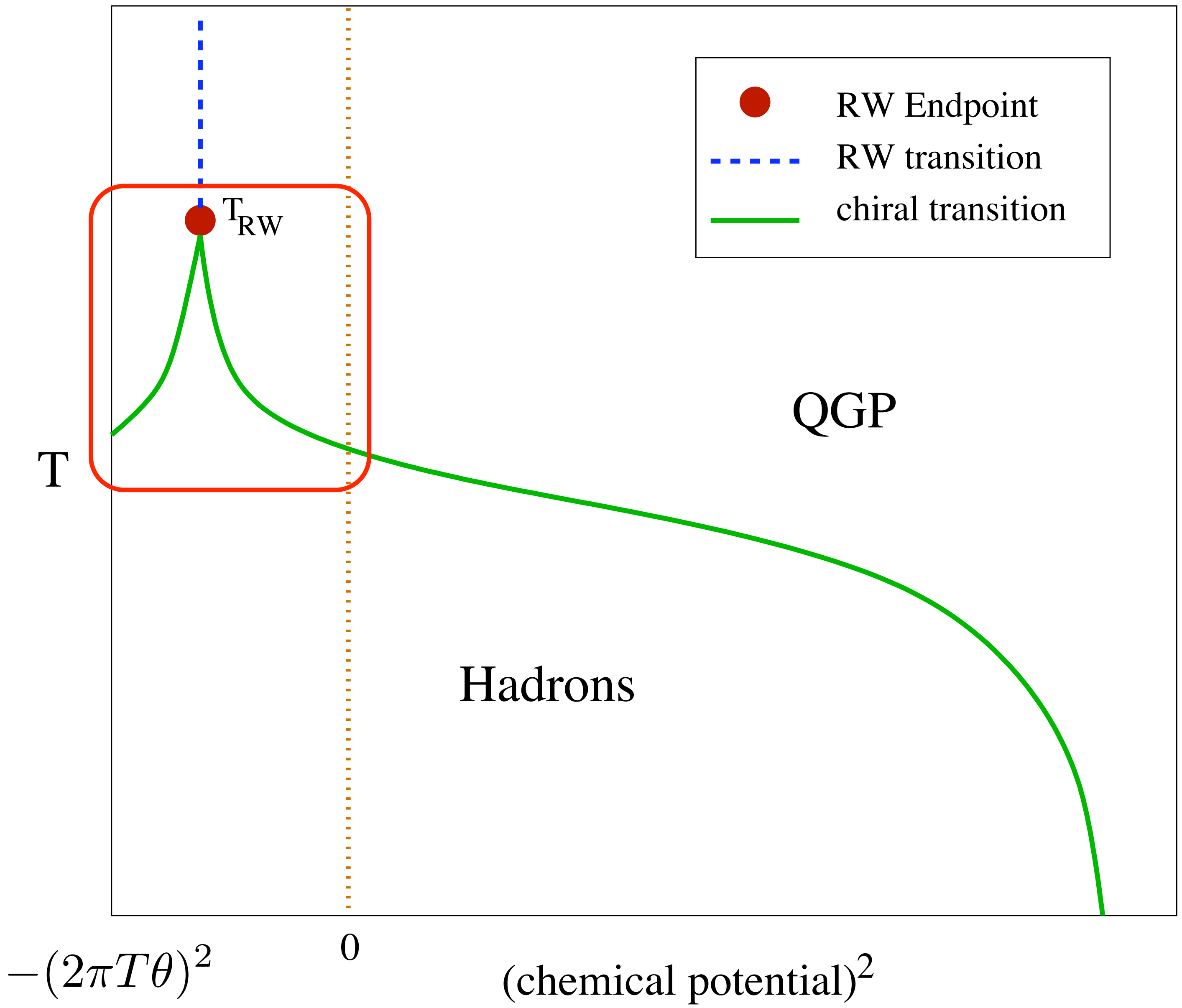
$$\int_x \bar{q}_\theta \cdot (i\not{D} + i\, m_\psi) \cdot q_\theta$$

$$q_\theta(t, \vec{x}) = e^{2\pi T\theta\, i\, t} q(t, x)$$

Periodicity

$$q_\theta(t + \beta, \vec{x}) = -e^{2\pi\theta\, i} q_\theta(t, x)$$

Imaginary chemical potential



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Roberge-Weiss symmetry

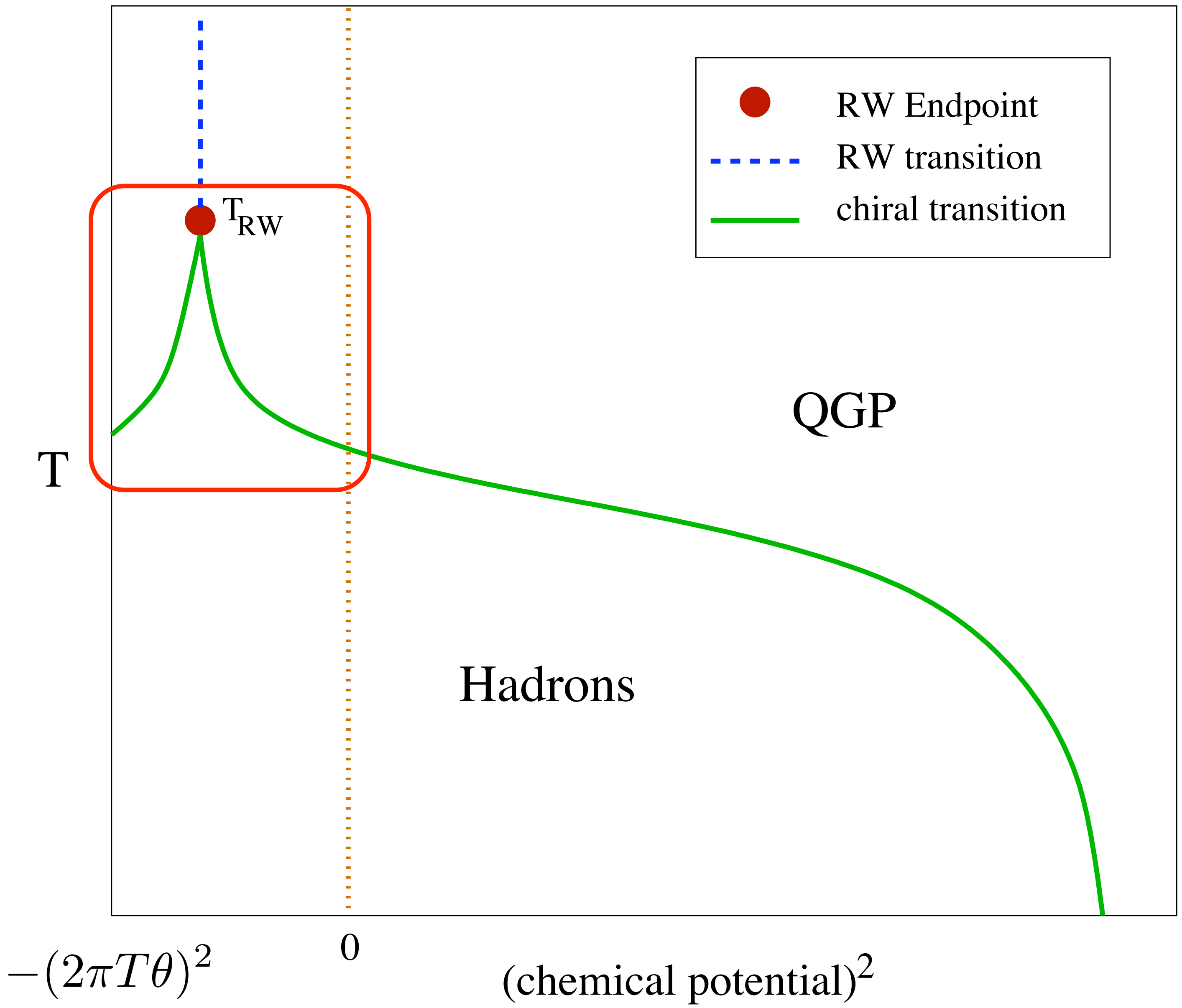
$$Z_\theta = Z_{\theta+1/3}$$

Partition function

via a center transformation $e^{\frac{2}{3}\pi\, i}\mathbb{1} \in \text{center}[SU(3)]$

gauge field insensitive to center transformations

Imaginary chemical potential



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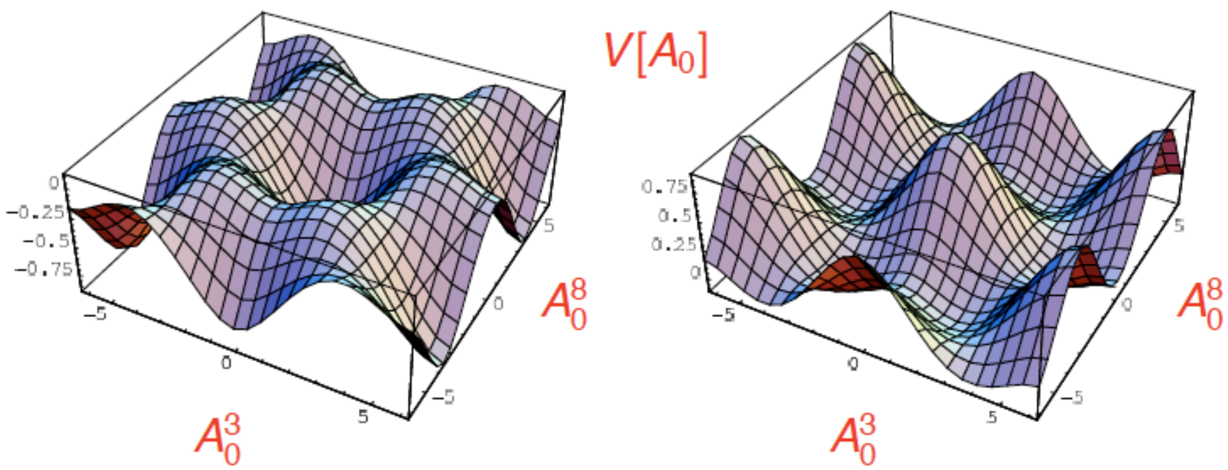
Polyakov loop potential



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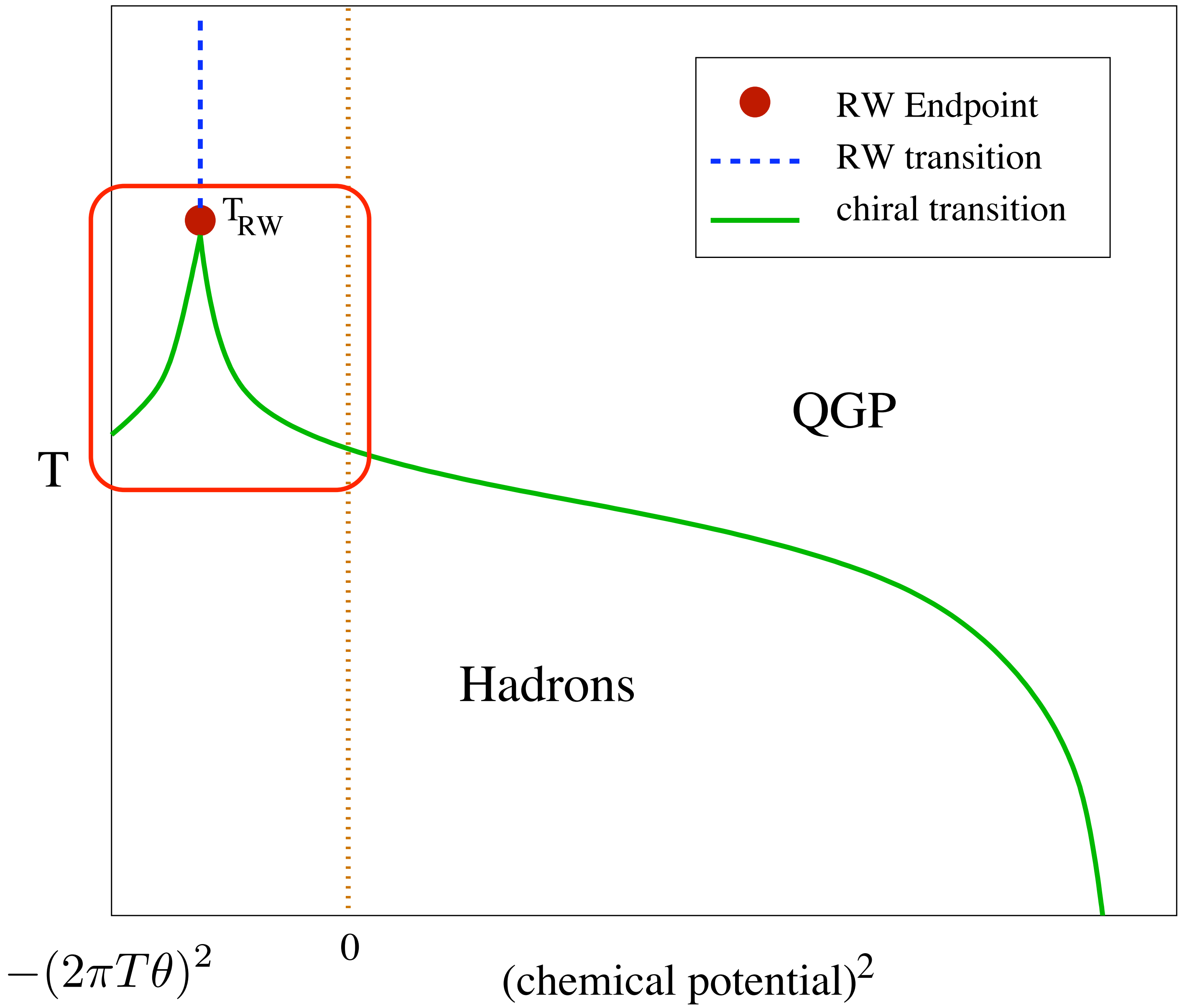

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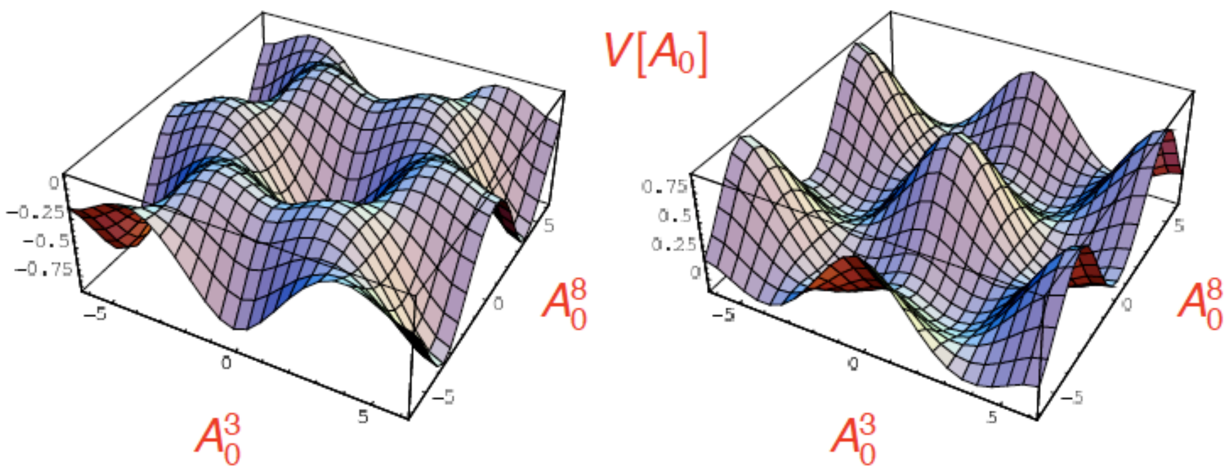

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gauge field insensitive to center transformations

Imaginary chemical potential

confinement order parameters

$$q_\theta(t+\beta,\vec{x}) = -e^{2\pi\theta\, i} q_\theta(t,x)$$

Center-sensitive observables

$$\mathcal{O}_\theta = \langle O[q_\theta]\rangle_\theta$$

$$\mathcal{O}_\theta = \langle O[q_\theta]\rangle_{\theta=0}$$

at imaginary chemical potential

at vanishing chemical potential

Dual order parameters

$$\tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

$$\tilde{\mathcal{O}} \xrightarrow{z} z\tilde{\mathcal{O}}$$

$$z = 1,\, e^{\frac{2}{3}\pi i},\;\, e^{\frac{4}{3}\pi i}$$

Imaginary chemical potential

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vanishing chemical potential

Lattice

Gattringer '06
Synatschke, Wipf, Wozar '07
Bruckmann, Hagen, Bilgici, Gattringer '08

FunMethods

Fischer '09
Fischer, Maas, Müller '10

Imaginary chemical potential

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imaginary chemical potential

average over diff. theories

$$\tilde{\mathcal{O}}[\langle A_0\rangle_{\theta=0}]$$

breaking of RW-symmetry

Braun, Haas, Marhauser, JMP '09

vanishing chemical potential

Lattice

Gattringer '06
Synatschke, Wipf, Wozar '07
Bruckmann, Hagen, Bilgici, Gattringer '08

FunMethods

Fischer '09
Fischer, Maas, Müller '10

Imaginary chemical potential

confinement order parameters

$$\tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

	at imaginary chemical potential			at vanishing chemical potential	
	FRG	**DSE**		**FRG**	**DSE**
	1-loop	1-loop	dual quark propagator	2-loop	3-loop
	1-loop	–	dual pressure	–	–
	1-loop	1-loop	dual density	2-loop	3-loop
	1-loop	1-loop	dual susceptibilities	2-loop	3-loop

dual pressure=-T dual density

dual susceptibility= T dual density

Imaginary chemical potential

confinement order parameters

$$\tilde{\mathcal{O}} = \int_0^1 d\theta \, \mathcal{O}_\theta e^{-2\pi i\theta}$$

at imaginary chemical potential			at vanishing chemical potential	
FRG	DSE		FRG	DSE
1-loop	1-loop	dual quark propagator	2-loop	3-loop
1-loop	–	dual pressure	–	–
1-loop	1-loop	dual density	2-loop	3-loop
1-loop	1-loop	dual susceptibilities	2-loop	3-loop

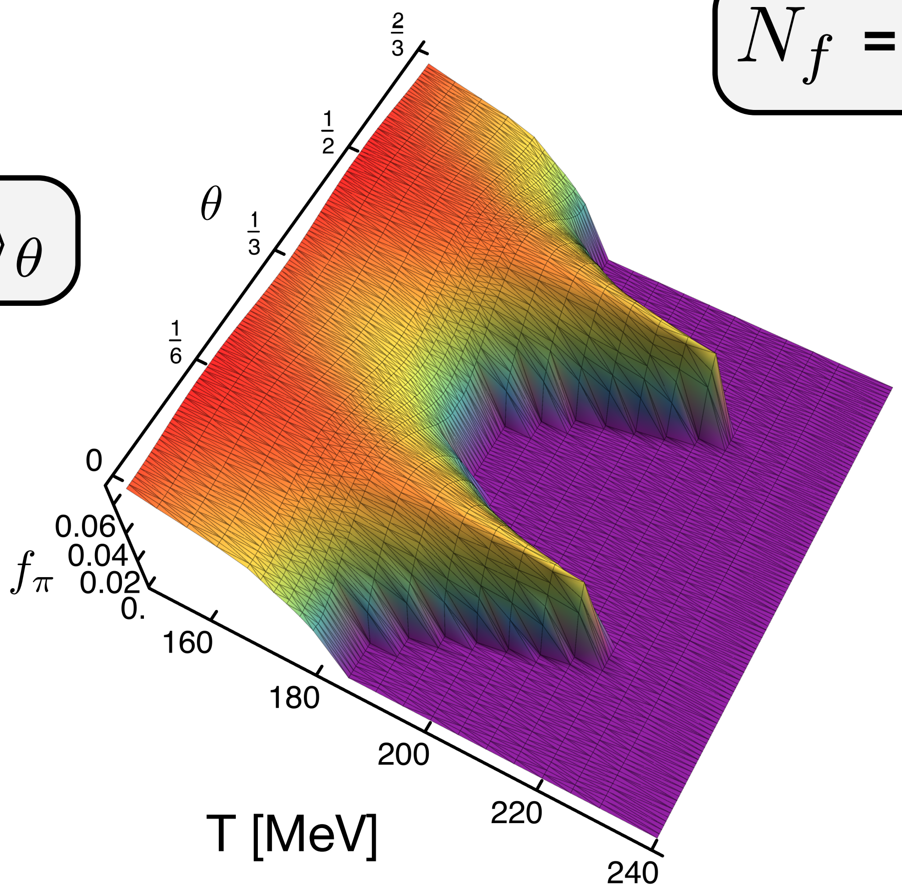
$$\frac{1}{2\pi T i} \int_0^1 d\theta \, (\partial_\theta \mathcal{O}_\theta) e^{-2\pi i\theta} = \frac{1}{T} \int_0^1 d\theta \, \mathcal{O}_\theta e^{-2\pi i\theta}$$

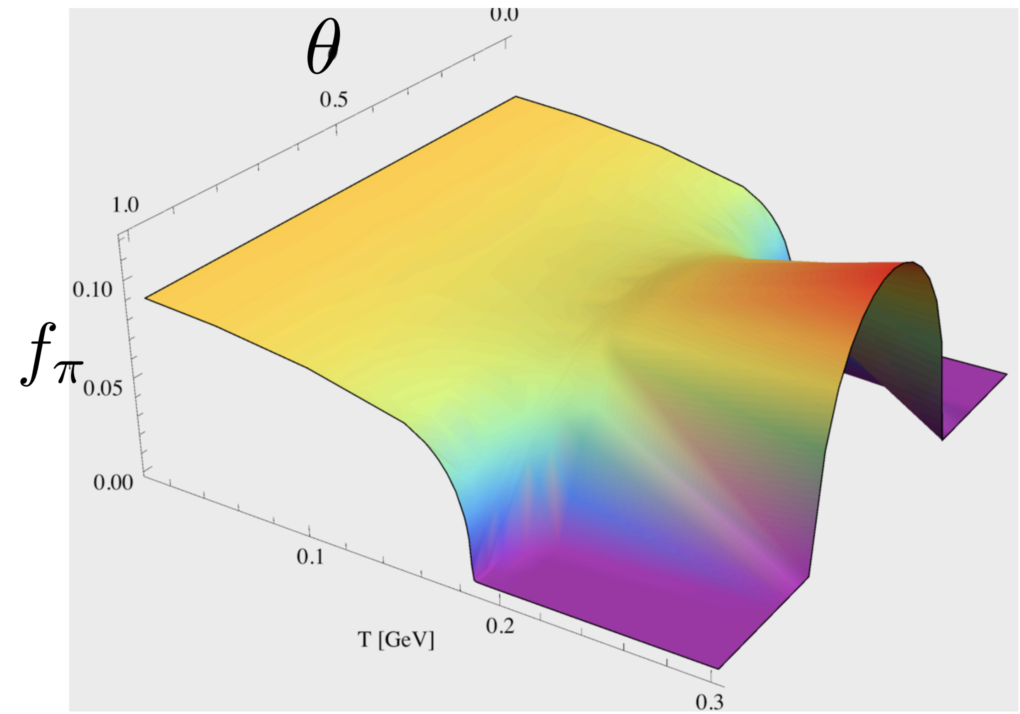
Imaginary chemical potential

N_f =2 & chiral limit

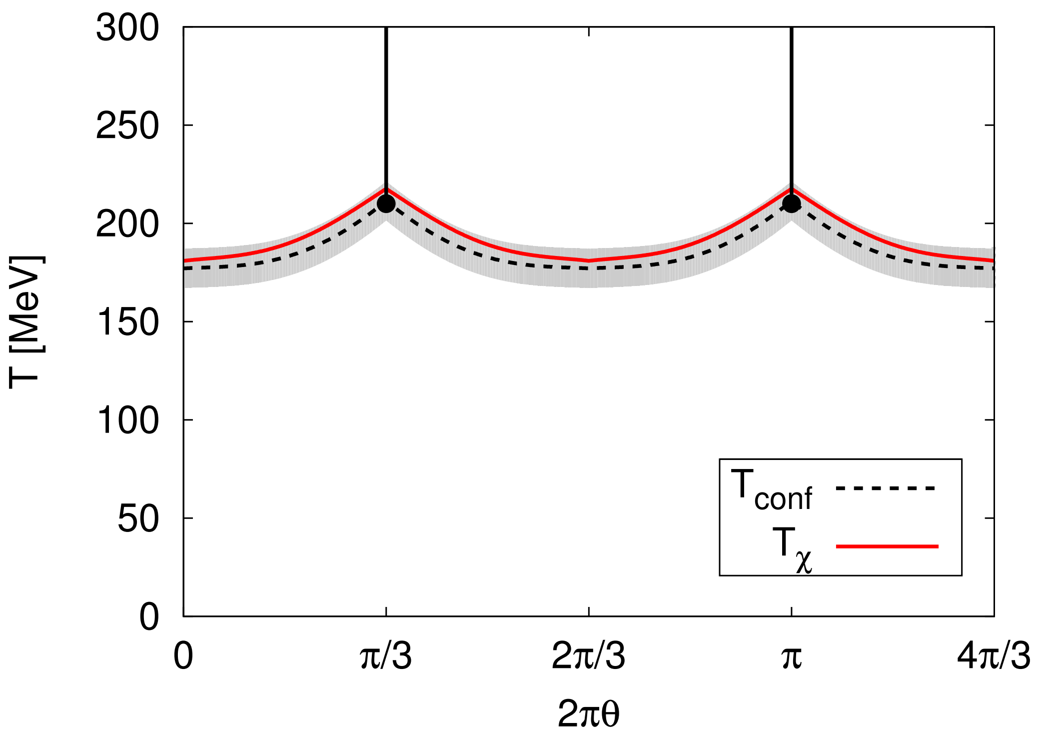
Braun, Haas, Marhauser, JMP '09

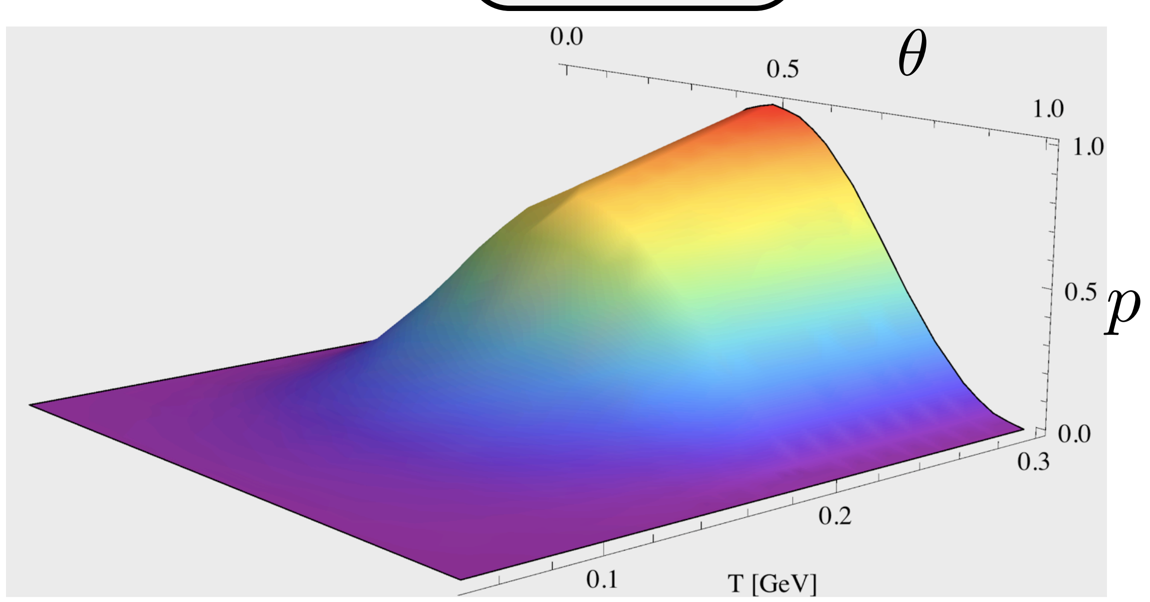
$\langle A_0 \rangle_\theta$





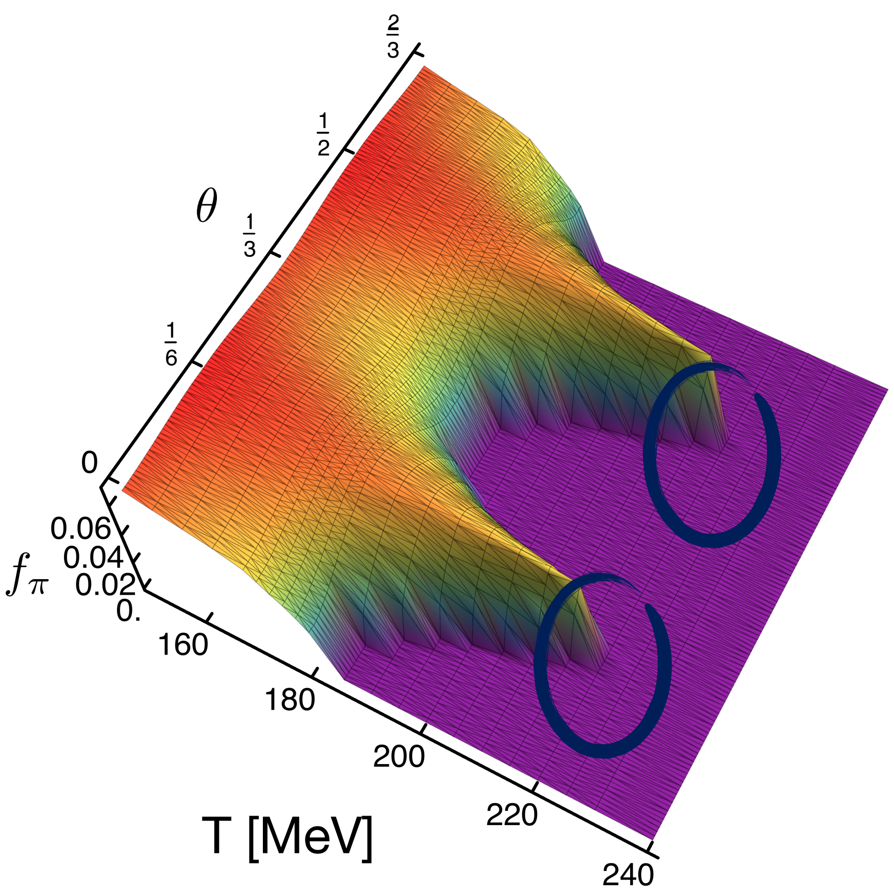
$\langle A_0 \rangle_{\theta=0}$

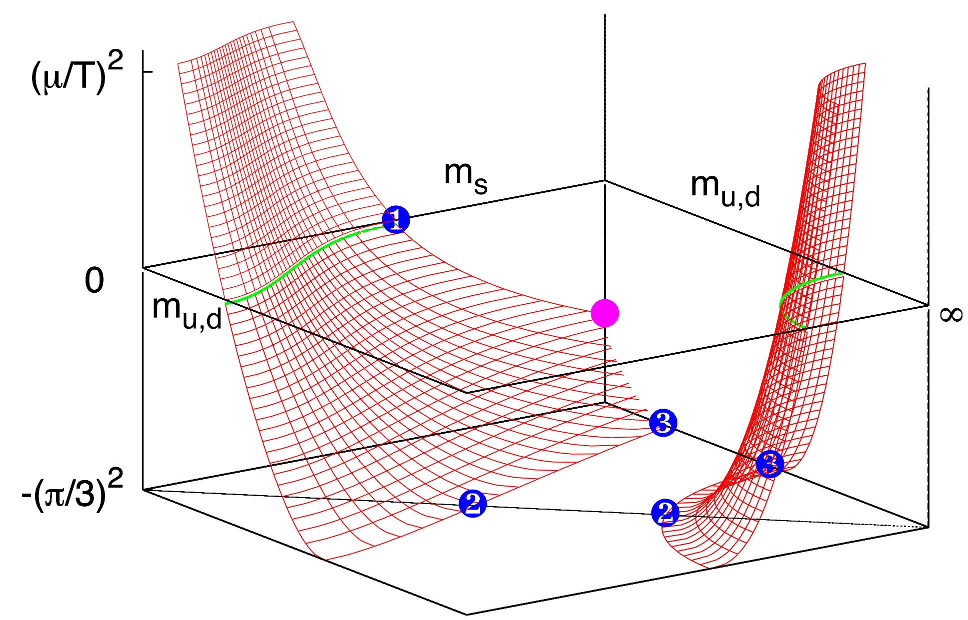




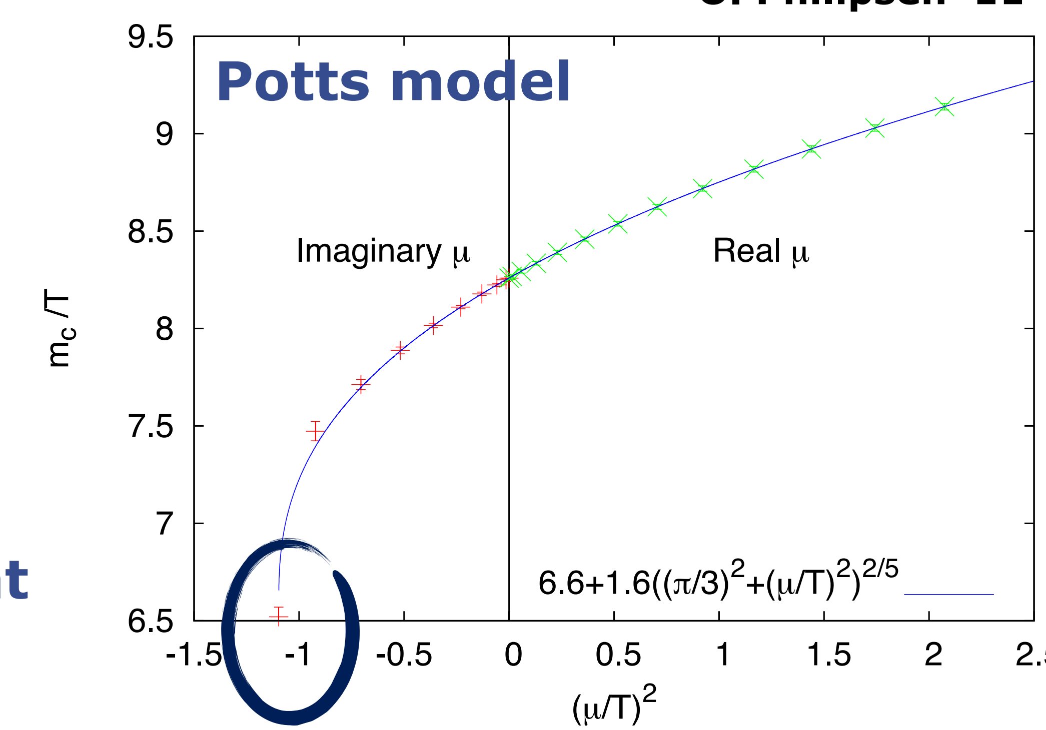
Imaginary chemical potential

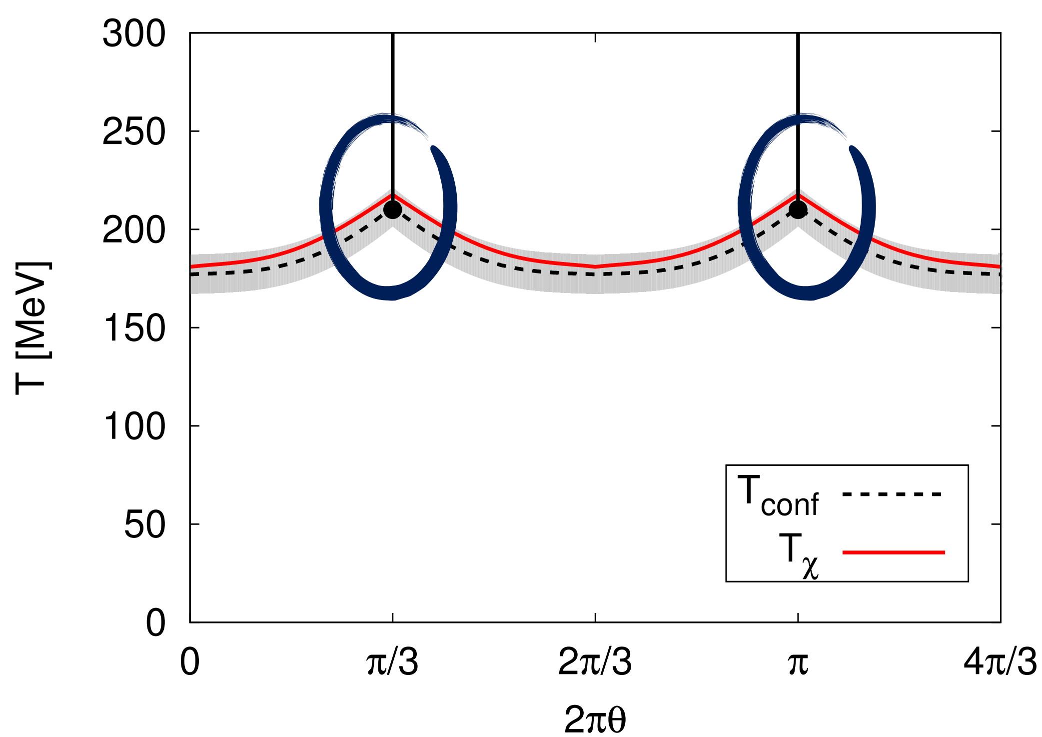
Nature of the RW endpoint





O. Philipsen '11





RW endpoint

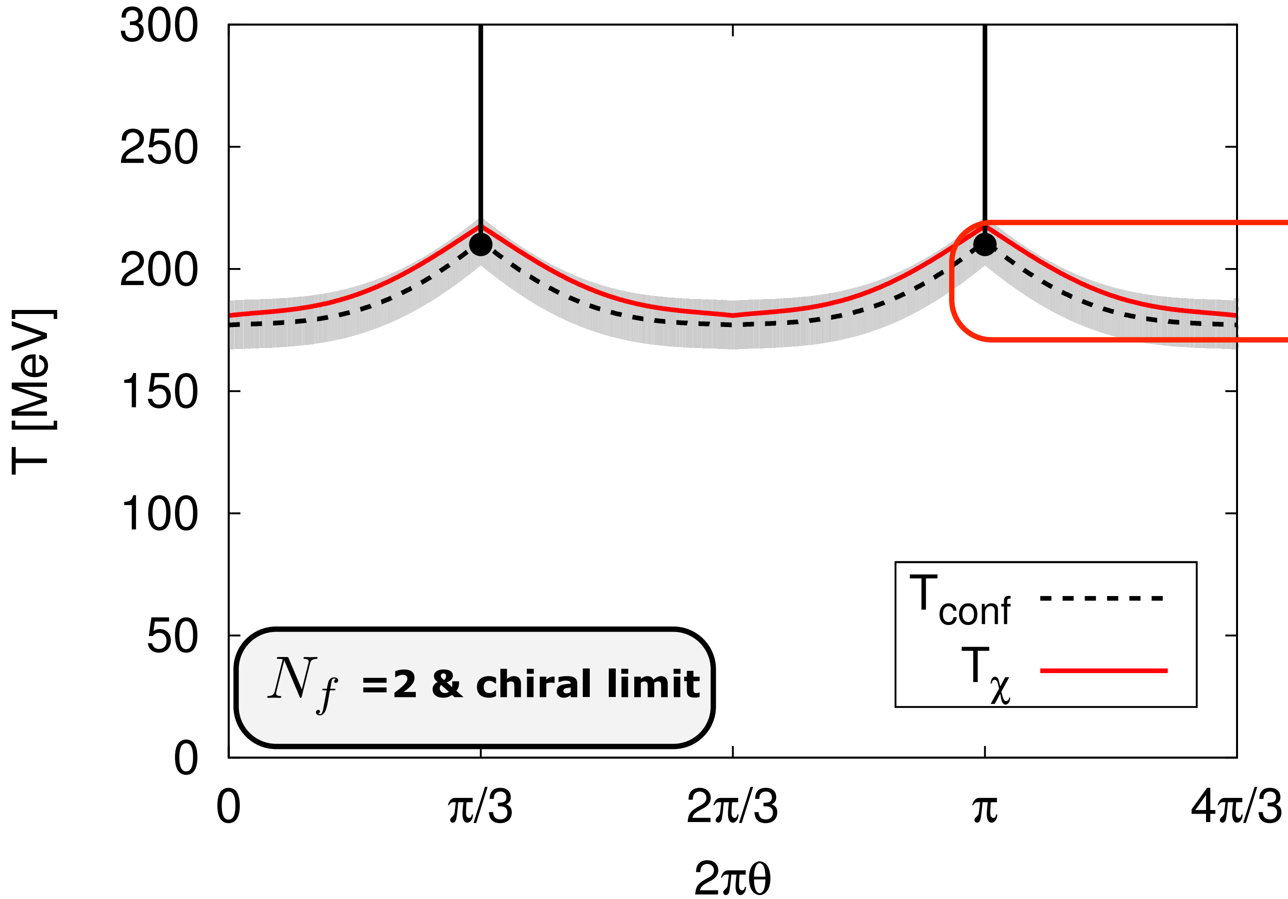
Nature of RW endpoint

lattice: D'Elia, Sanfilippo '09
de Forcrand, Philipsen '10
...

PNJL: Sakai et al '10,
Morita et al '11

Imaginary chemical potential

Braun, Haas, Marhauser, JMP '09

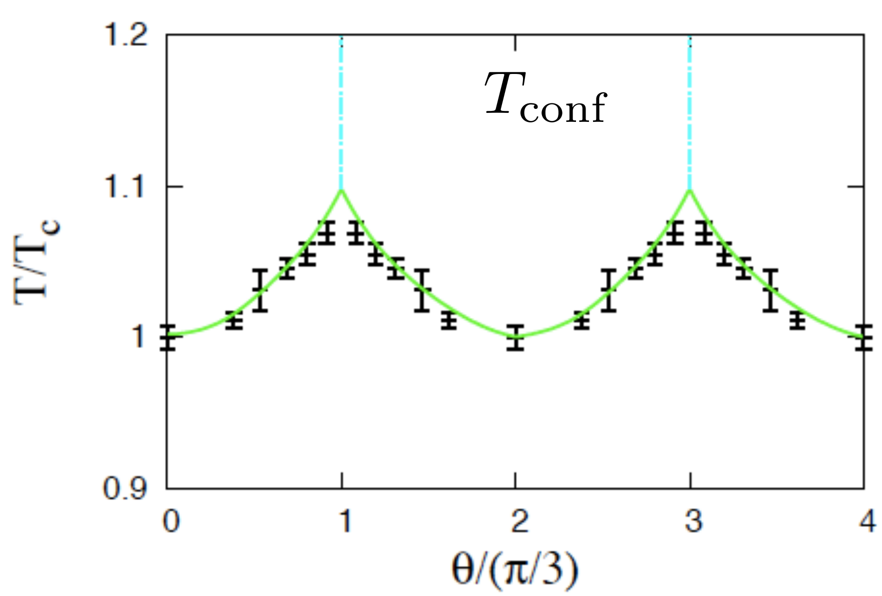


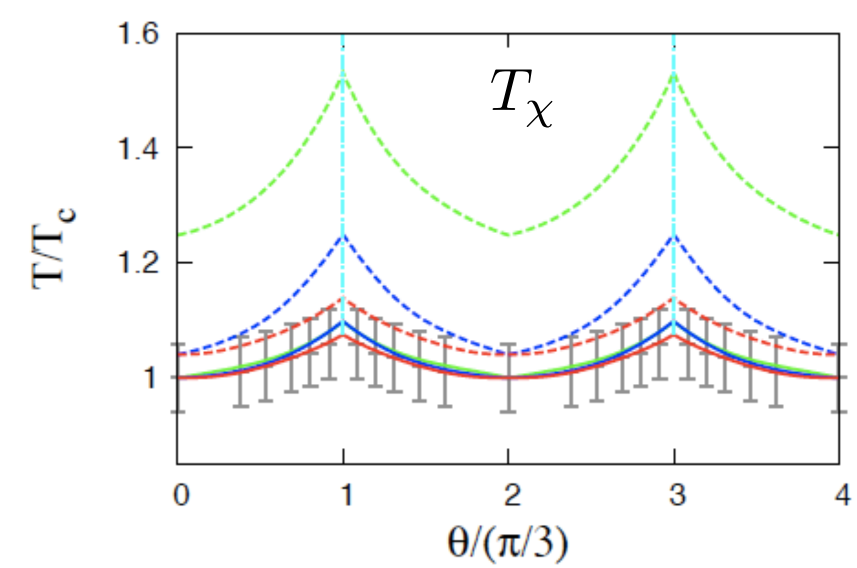
$\gtrsim |T_{\rm conf} - T_\chi|$

Dual order parameters

compatibility

lattice results, e.g.
Kratochvila et al '06, Wu et al '06
& D'Elia et al '07, Fromm et al '11



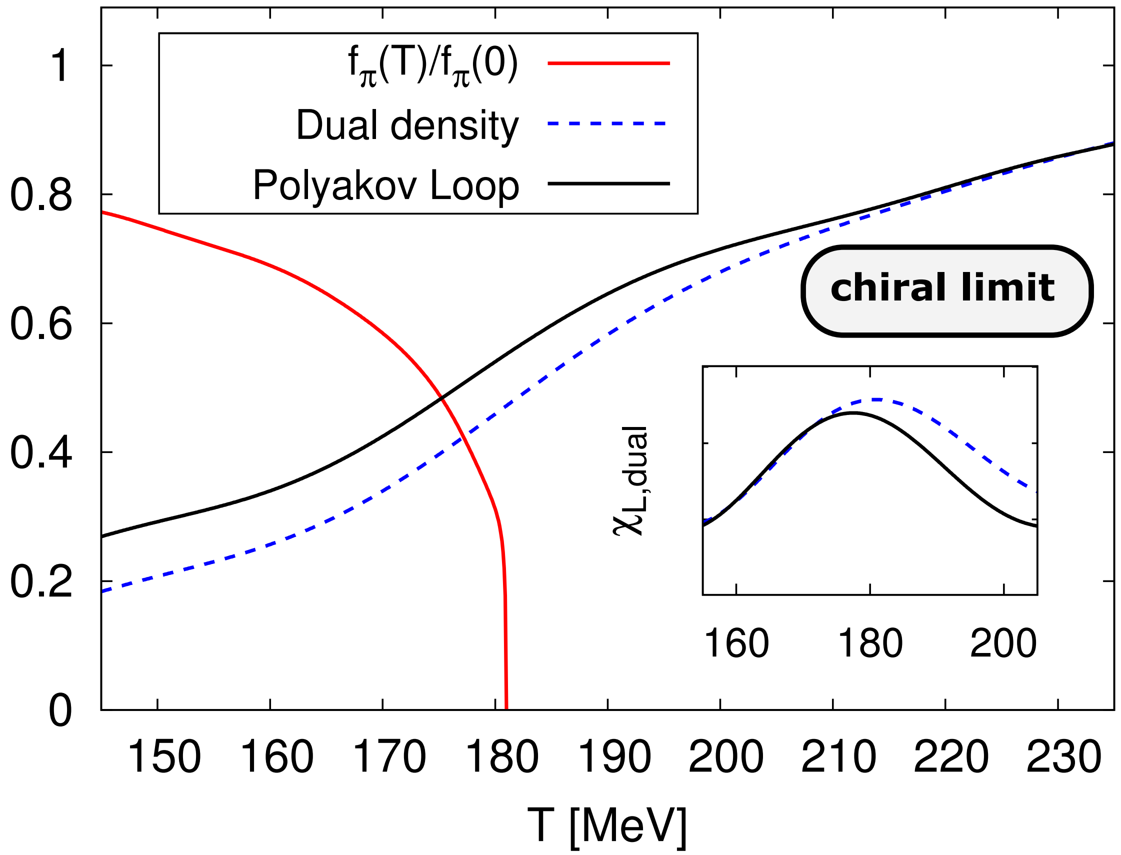


PNJL: Sakai et al '09

adjust 8-fermi interaction

Full dynamical QCD: $N_f = 2$ & chiral limit

Phase structure



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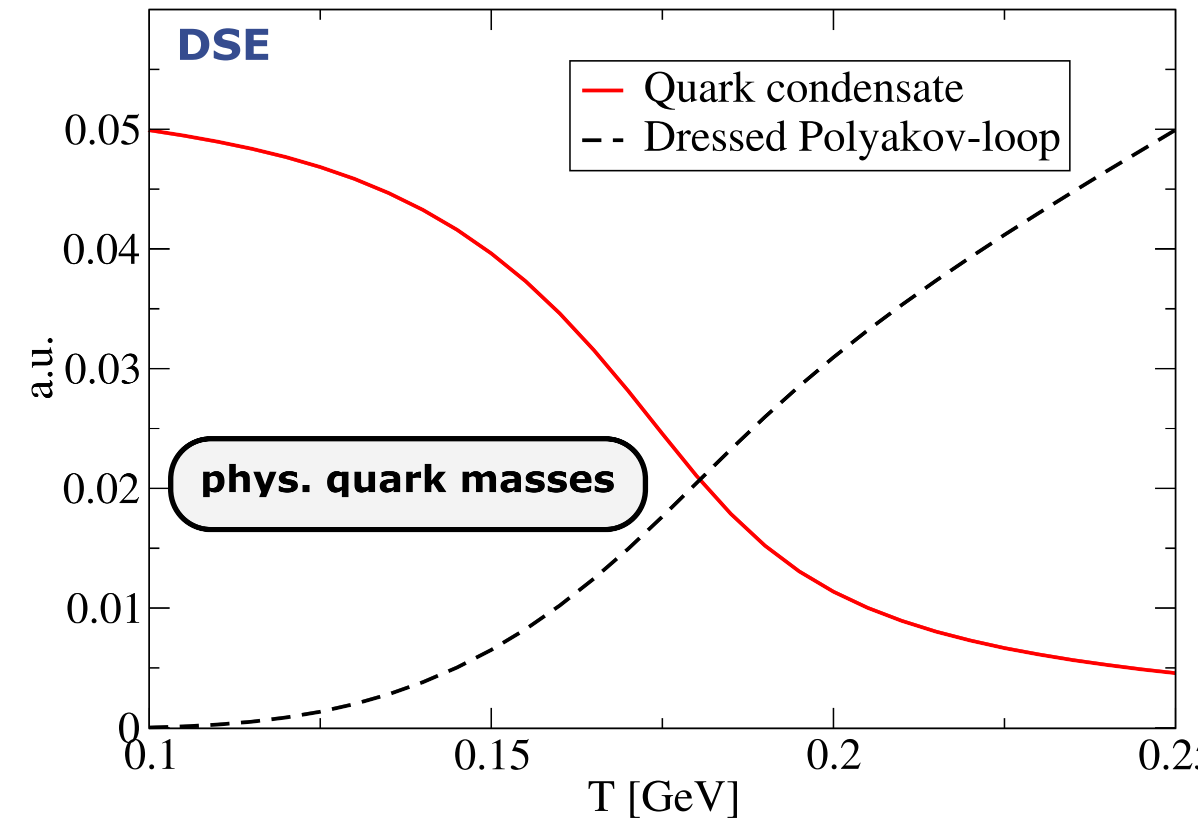
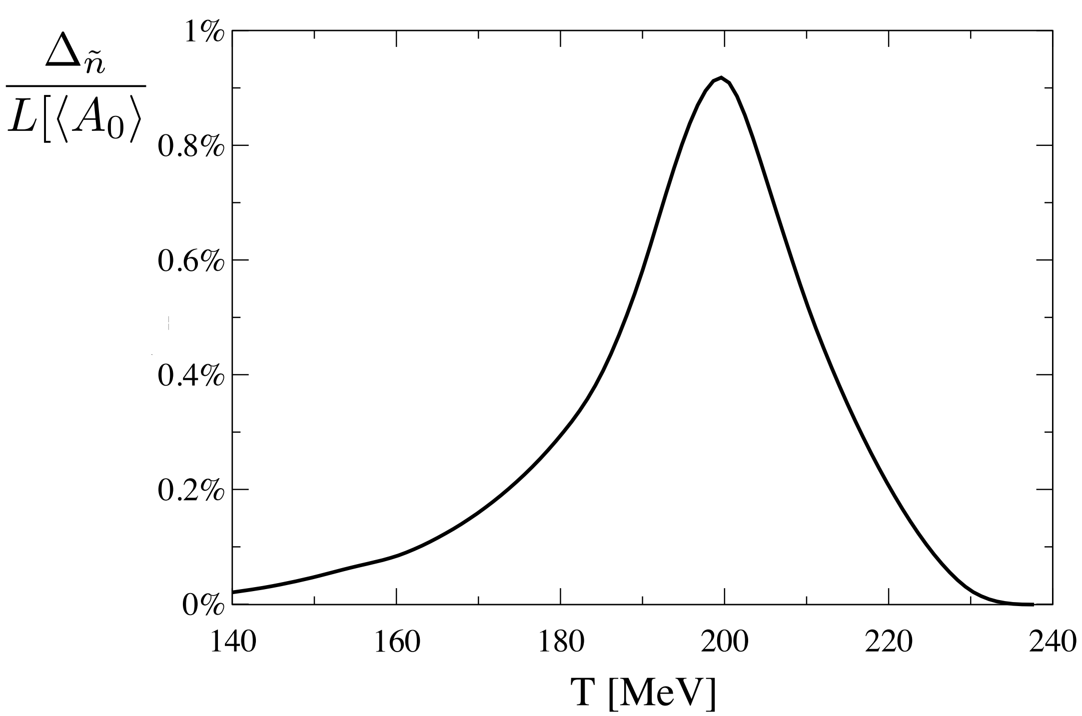


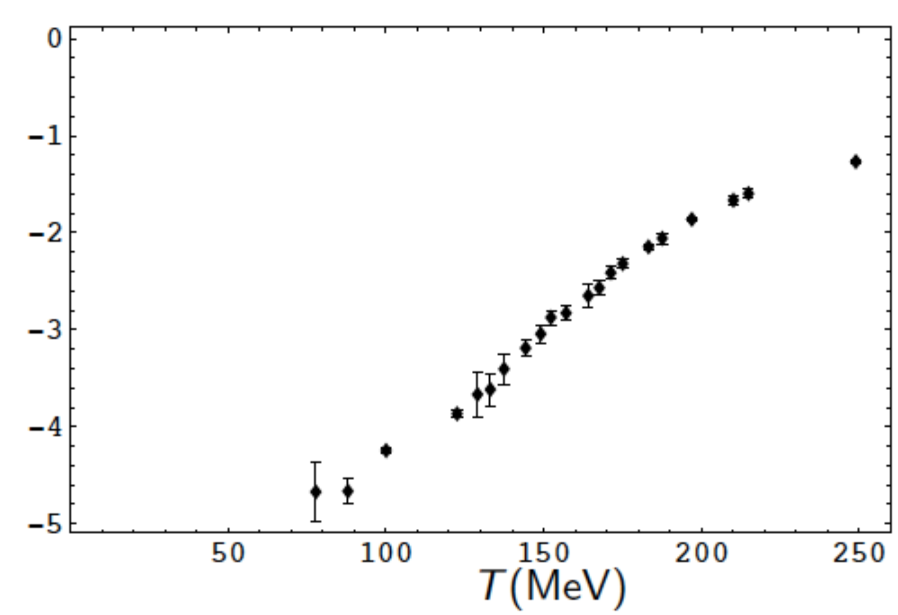
Fischer, Lücker, Müller '11

factorisation property of dual density



$$\Delta_{\tilde{n}} = \frac{\tilde{n}[\langle A_0 \rangle]}{\tilde{n}[0]} - L[\langle A_0 \rangle]$$

Log of dual condensate, m=60 MeV



Zhang, Bruckmann, Gattringer, Fodor, Szabo '10

Confinement

Effective Polyakov loop potential

Learning by diffusion

Non-perturbative effective potential

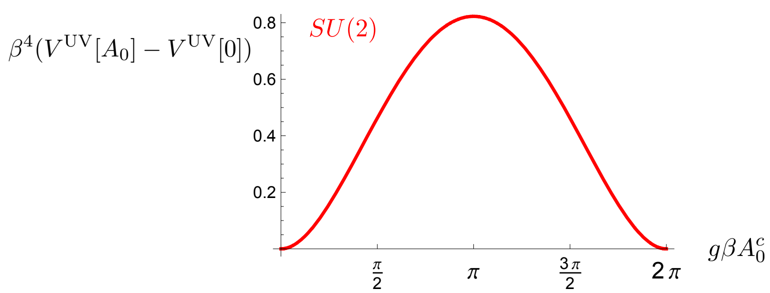
$$V[A_0] \simeq -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

Confinement criterion

Braun, Gies, JMP '07

Fister, JMP '13



$$\beta^4 V^{UV}[A_0] = -2 * 3\left(\frac{\pi^2}{90} - \frac{2\pi^2}{3}\tilde{\varphi}^2(1-\tilde{\varphi})^2\right)$$

2 **= 2 transversal physical polarisations+ 1 transversal (zero mode)+1 longitudinal - 2 ghosts**

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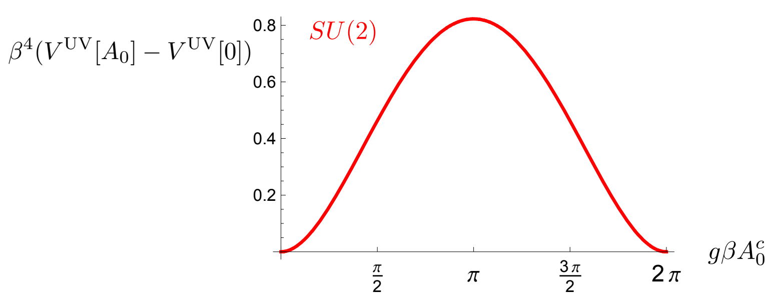

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Gluon contribution deconfines

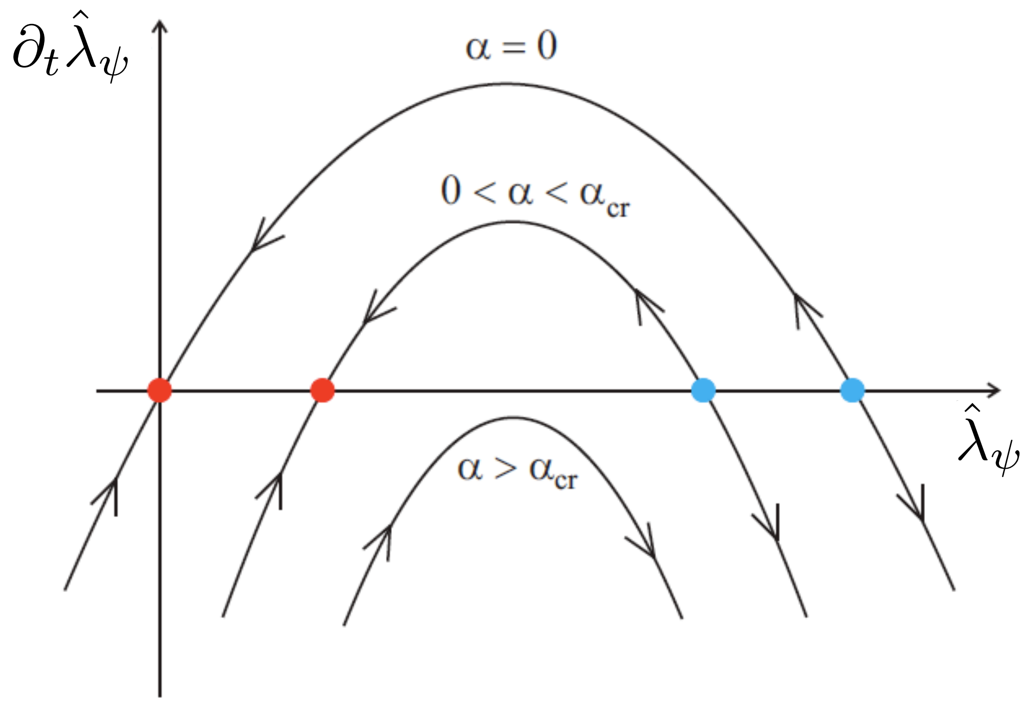
Ghost contribution confines

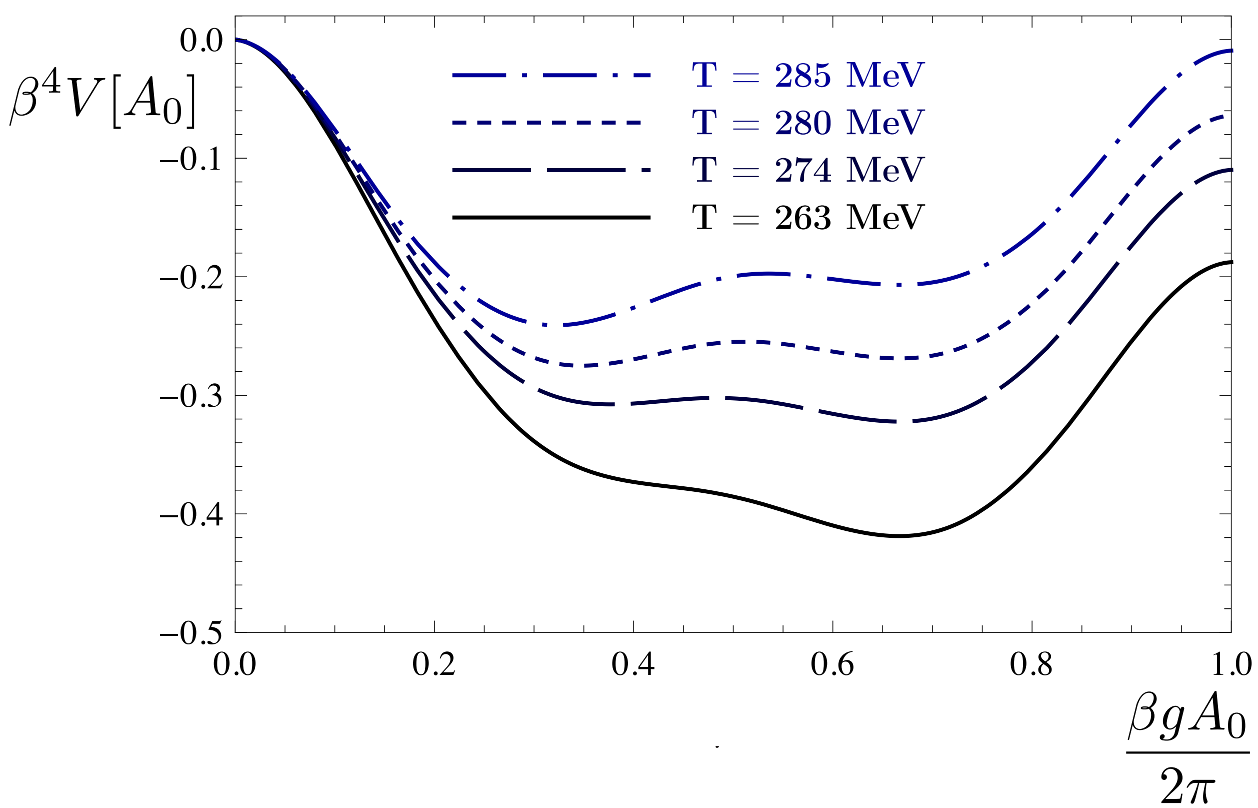
Confinement ⟷ suppression of the gluon relative to the ghost

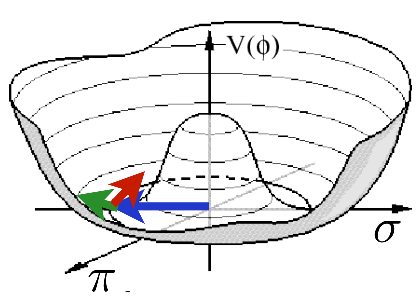
Full dynamical QCD: $N_f = 2$ & chiral limit

Phase structure

Reminder





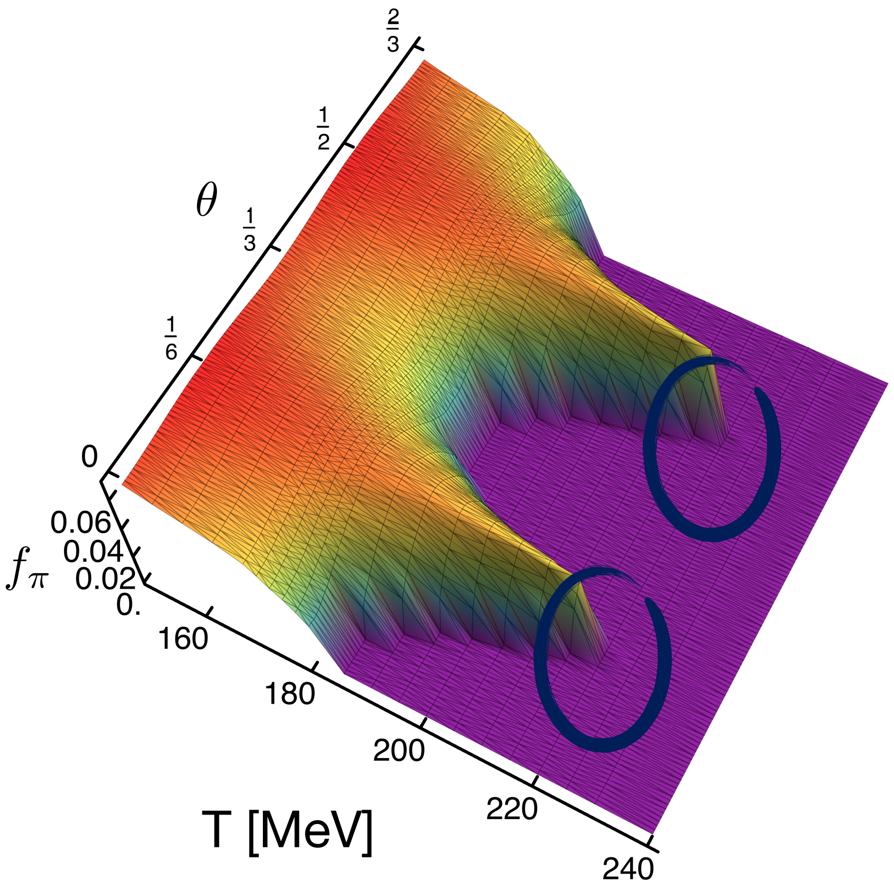


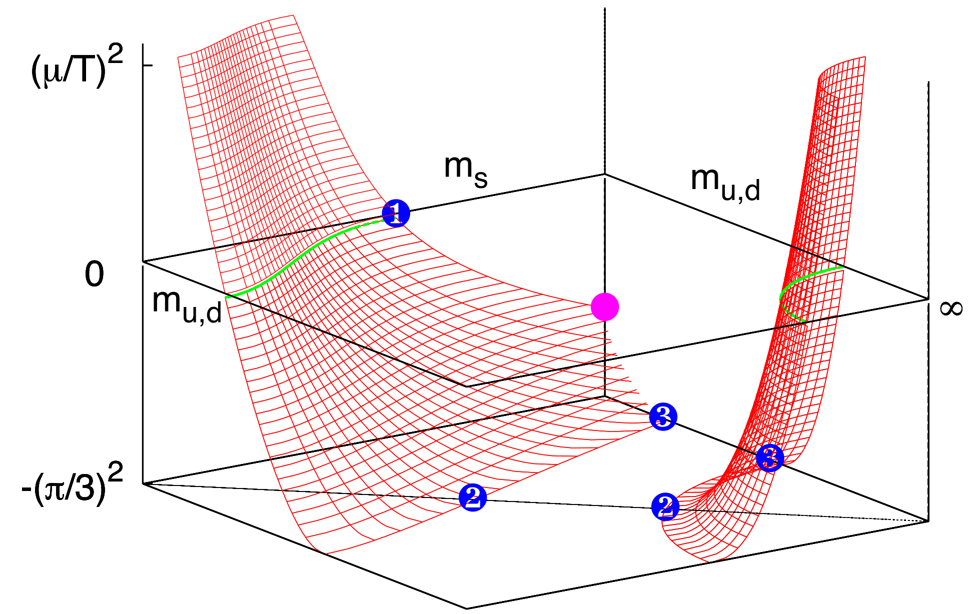
chiral symmetry breaking ⟷ $\alpha_s > \alpha_{s,cr}$

Confinement ⟷ **suppression of the gluon relative to the ghost**

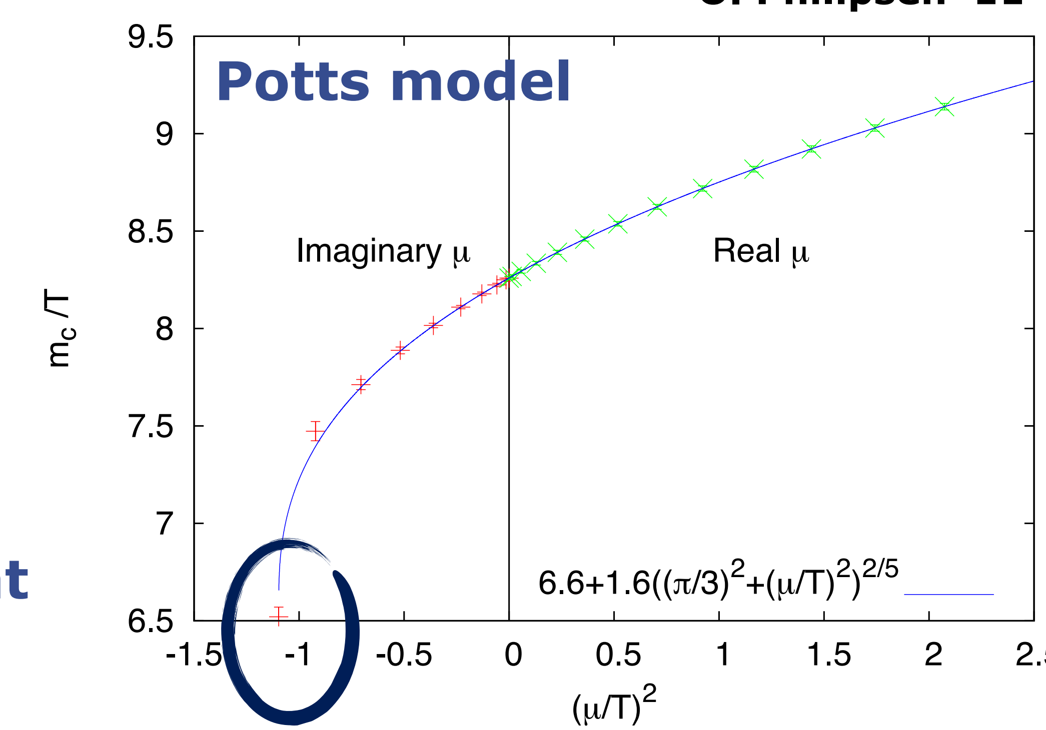
Imaginary chemical potential

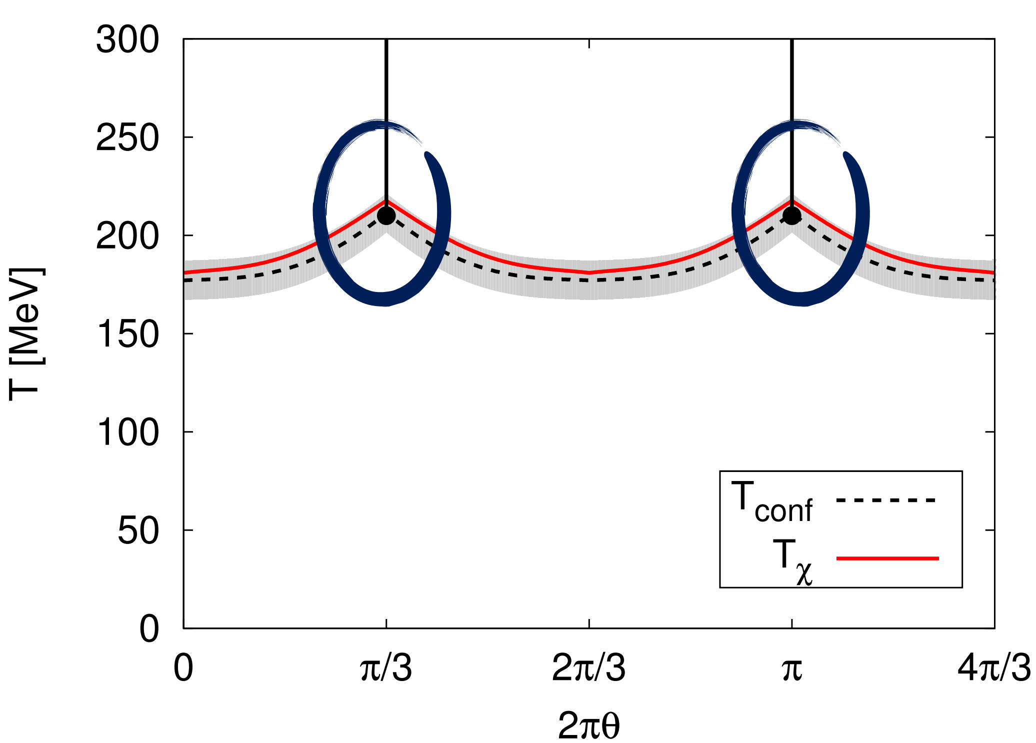
Nature of the RW endpoint





O. Philipsen '11





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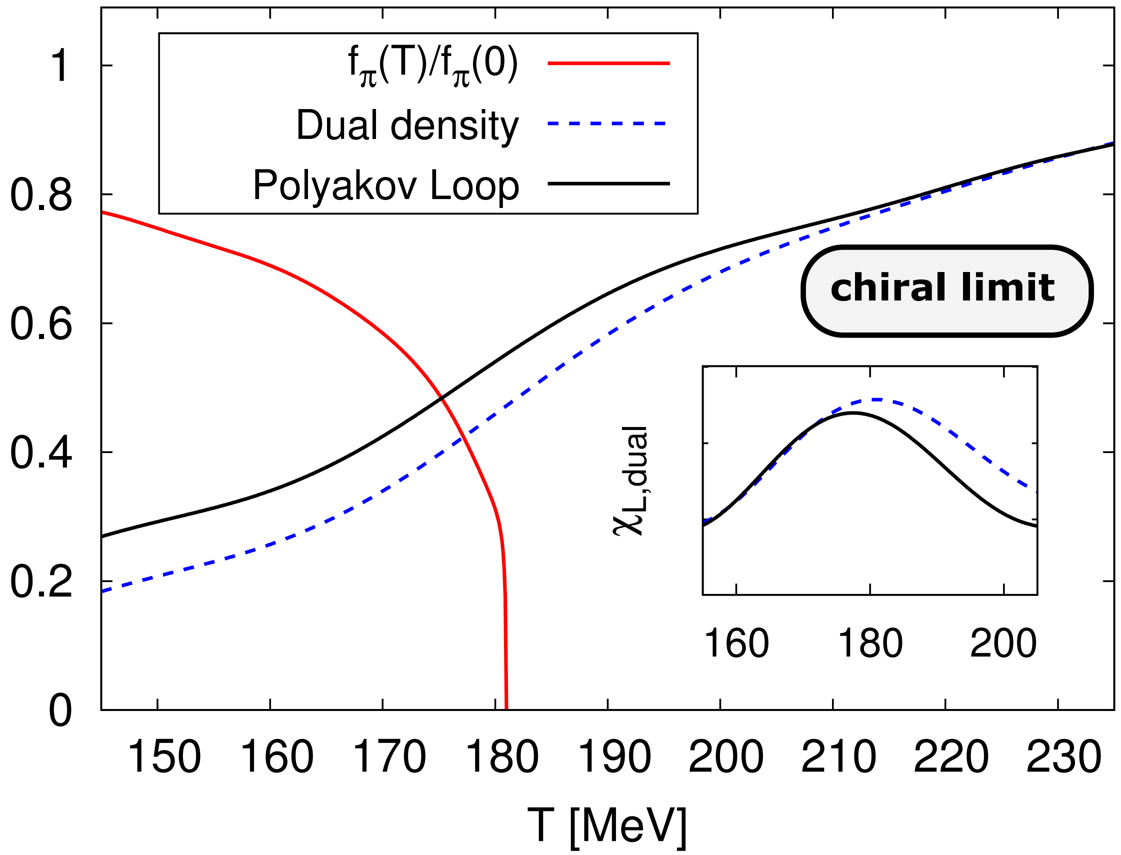
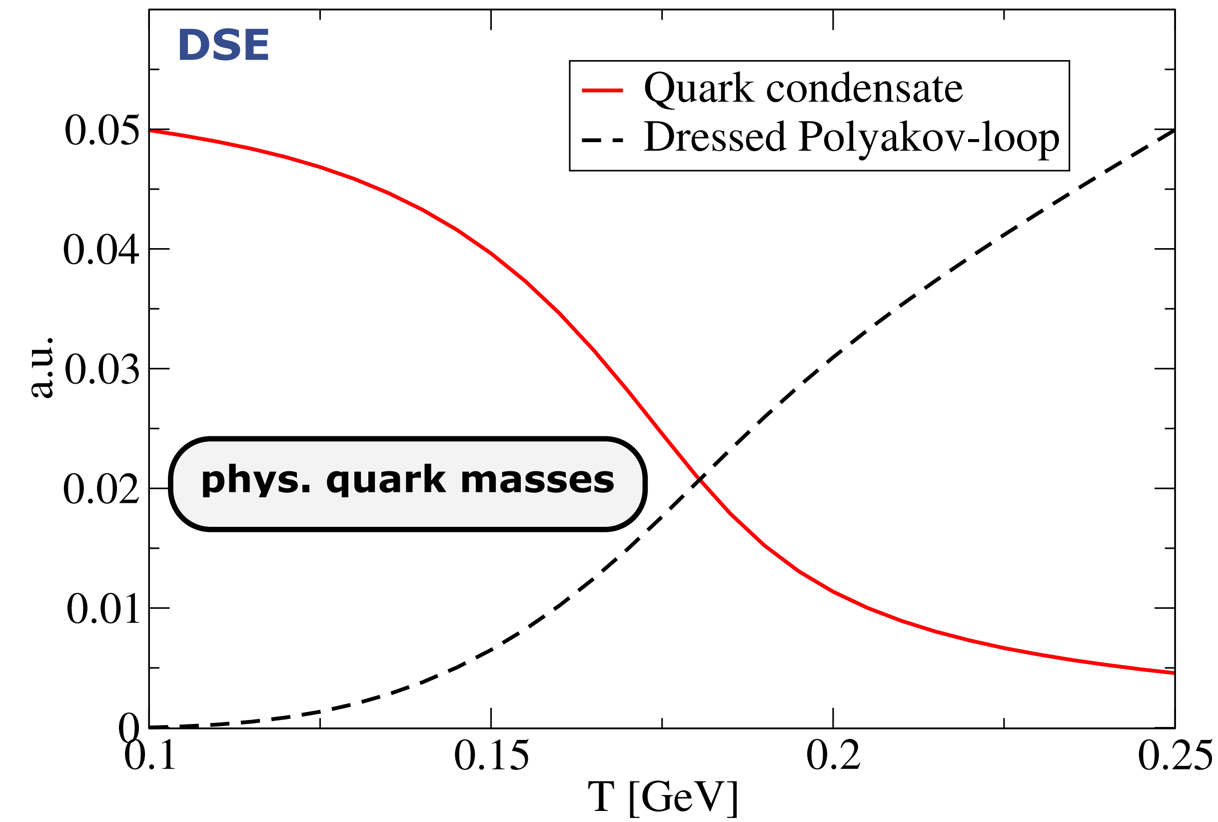


Braun, Haas, Marhauser, JMP '09



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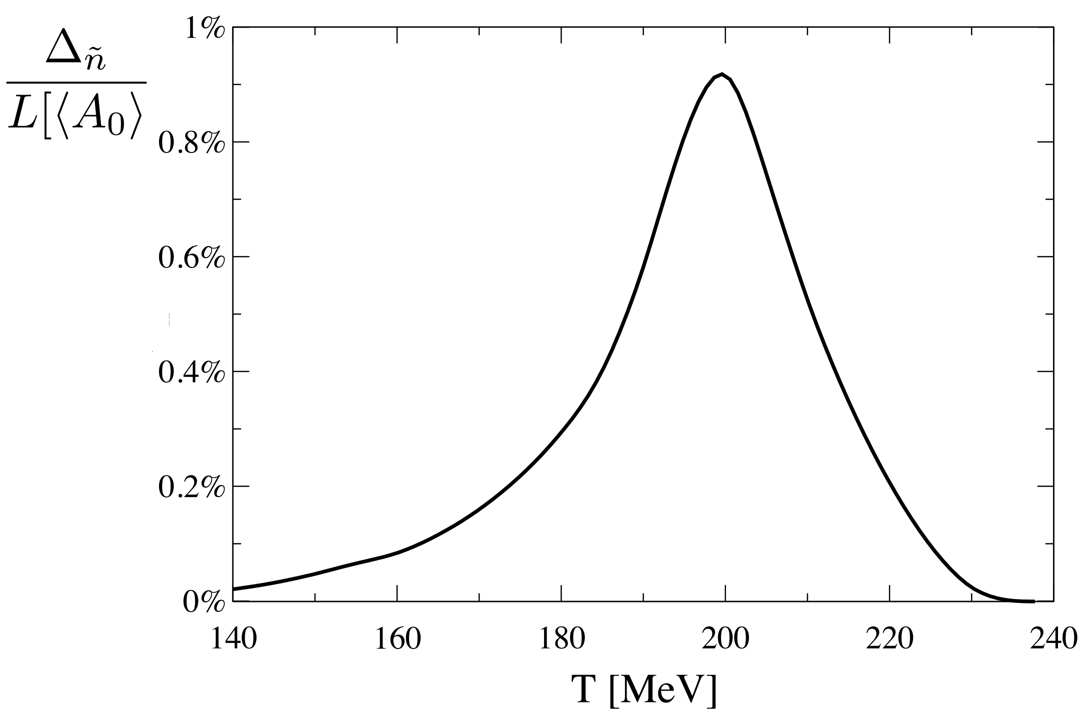

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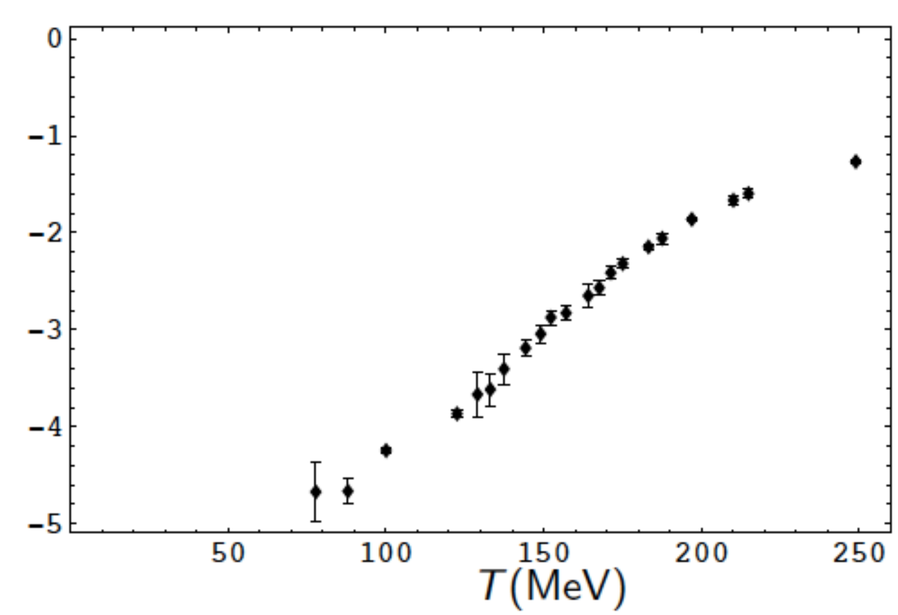

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