Lattice field theories
Exercise sheet 3 – Fermionic Dispersion

Lectures: Jan Pawlowski j.pawlowski@thphys.uni-heidelberg.de
Felipe Attanasio pyfelipe@thphys.uni-heidelberg.de

Tutorials: Felipe Attanasio pyfelipe@thphys.uni-heidelberg.de

Institut für Theoretische Physik, Uni Heidelberg

Exercise 7: Fermionic dispersion relations on the lattice

We have seen in the lecture of 09.11.2021 different formulations of lattice fermions. Generally, the free fermionic action on the lattice can be written in the form

$$S_\psi = \sum_{x,y} \overline{\psi}_x K_{xy} \psi_y \,, \tag{1}$$

where spinor indices are omitted, and x and y are 4 dimension vectors of integers.

For **naïve** and **Wilson** fermions

- write the expression for K_{xy};
- use

$$\delta_{x,y} = \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)} \tag{2}$$

  to re-write K_{xy} as

$$K_{xy} = a^3 \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} K(p) e^{ip\cdot(x-y)} \,. \tag{3}$$

  What do you get for $K(p)$?

- Define the *lattice momentum* $\tilde{p}$ such that the expressions you have for $K(p)$ have the same form as the continuum Dirac operator in Euclidean spacetime, $(\gamma_\mu p_\mu + m)$. How does the lattice momentum look like for each case?
- We have seen on the first exercise sheet that continuum behaviour is retrieved at the zeros for p within the Brillouin zone, $p \in [-\pi/a, \pi/a]$, as the lattice spacing is sent to zero. How many zeros each of the different formulations have in the first Brillouin zone? What does that imply for the continuum limit?
- Optional & for fun: Devise a lattice Dirac operator with the continuum dispersion (SLAC fermions)?