Lattice field theories
Exercise sheet 8 – Quenched Yukawa theory

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Consider 2-dimensional fermionic and scalar fields coupled via a Yukawa interaction

$$S = \int d^2x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 + \overline{\psi}\left(\gamma_\mu\partial_\mu + m_f + g_Y\phi\right)\psi \right] , \tag{1}$$

where $m^2, m_f, g, g_Y \in \mathbb{R}$, $\mu \in \{1, 2\}$, ϕ is a real scalar field, and ψ is a fermion field.

Exercise 17: Making it suitable for computers

Show that the same theory is described by the effective action

$$S_{\text{eff}} = \int d^2x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 - \ln\det(D(\phi)) \right] , \tag{2}$$

where $D(\phi) = (\gamma_\mu\partial_\mu + m_f + g_Y\phi)$, and the fermions have been integrated out.

Exercise 18: Shifts to the fermion mass

In order to simplify the simulations, and quite substantially reduce simulation costs, we work in the *quenched* approximation, where the $\det D(\phi)$ term in the effective action is taken to be 1. In this scenario, the fermions “feel” a background of scalar fields. Due to the interaction with this background field the fermion mass shifts. This approxmation also neglects the possibility of a scalar creating a fermion–anti-fermion pair.

Use the Metropolis algorithm from exercise sheet number 2 to simulate this quenched Yukawa model. Use a lattice of dimensions $N_t = 40$ and $N_x = 20$. For the scalar field, take $\lambda = 0.02$ and two values of κ, one in the symmetric and one in the broken phase (e.g., 0.2 and 0.3). For each (decorrelated!!) configuration in the Markov chain, compute the fermion mass for different values of the fermion bare mass (e.g., $m_f = 0.8$ and $m_f = 1.0$) and $0.0 \leq g_Y \leq 0.3$. Use the same method of exercise sheet number 4, taking care to include the effects of the scalar fields on the fermion matrix.

Note that, unlike in exercise sheet 4, excited states can appear here. Therefore the correlation function $C(\delta)$ should not exhibit a sinh shape, and the data will be noisier than the free case, since the field ϕ fluctuates. To counteract this, it is convenient to compute the *effective mass*,

$$m_{\text{eff}}(\delta + \tfrac{1}{2}) = \ln \frac{C(\delta)}{C(\delta + 1)}\,. \tag{3}$$

Hint: For small time separations δ excited states can still be present, while for $\delta \sim N_t/2$ the signal might be very weak. It is common to look for a "good window" where m_{eff} is constant and perform a constant fit. You may want to do this visually.

Hint: If your results for $C(\delta)$ are too noisy, check whether the precision of your Conjugate Gradient algorithm is "too large". Remember that CG does the comparison $|r|^2 < \epsilon$, and not $|r| < \epsilon$.

Supplemental material

Please note that the runs used in these plots may suffer from "low" statistics and/or ergodicity problems. In other words, they were rather short. Take these plots with a grain of salt.

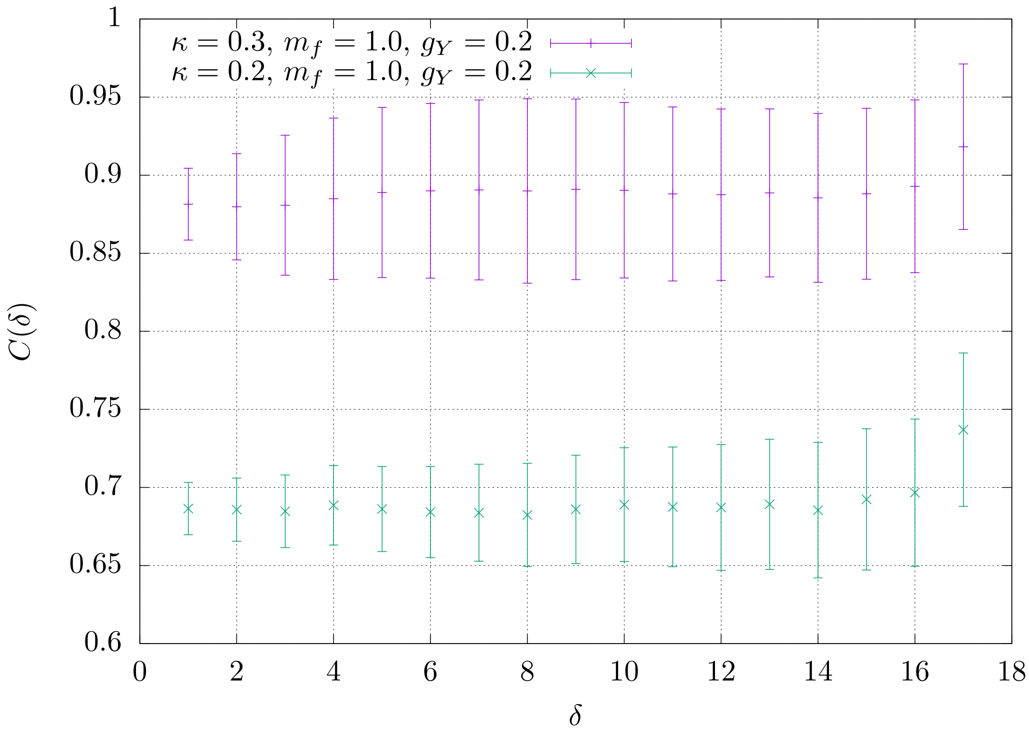

Figure 1: Fermion correlation function as a function of the time separation for different values of the scalar field's hopping parameter.

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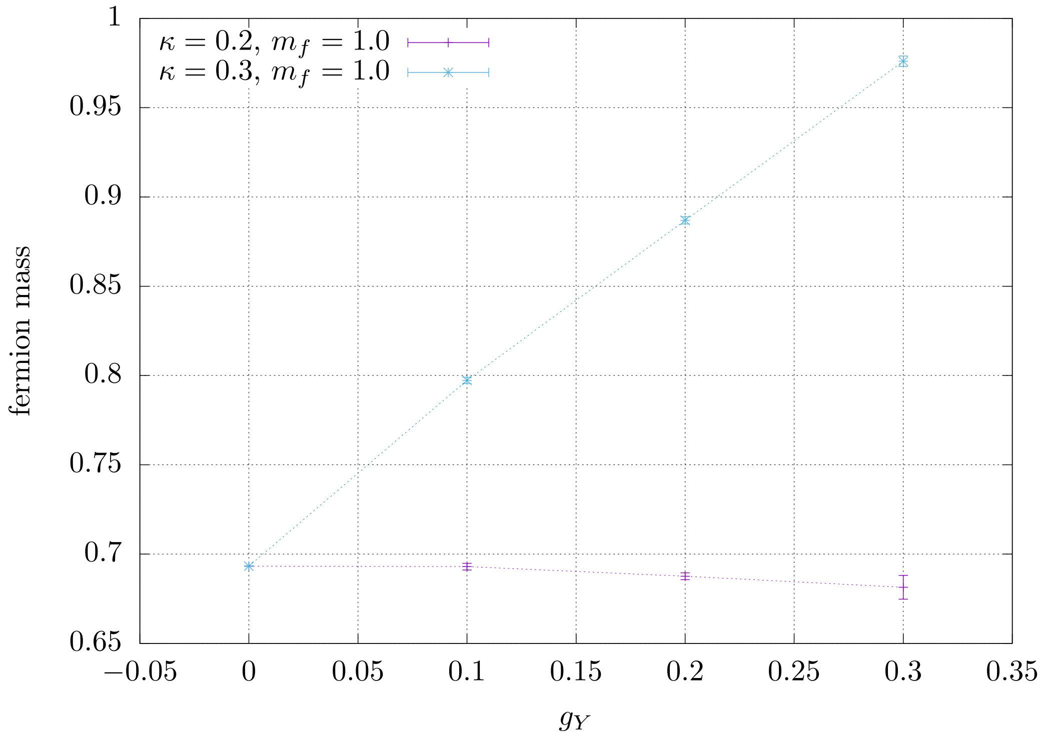

Figure 2: Fermion mass change as a function of the Yukawa coupling for different values of the scalar field's hopping parameter.