I-3 Fixed points in the functional RG

Remark: (a) flow vanish identically for $k \to 0$:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \operatorname{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \dot{R}_k \Big|_{R_k \hat{=} 0} \hat{=} 0$$

(b) Fixed points have to be searched for in dimensionless quantities:

$$\boxed{\text{total rescaling of the theory}}$$

as $g(\text{scales} \to \lambda \cdot \text{scales}) = \lambda^{d_g} g(\text{scales})$

e.g. scalar theory in 4 d: $\hat{g}$ are dim-less

$$\lambda \to \hat{\lambda} = \lambda$$

$$m^2 \to \hat{m}^2 = m^2/k^2$$

$$\phi \to \hat{\phi} = \phi/k$$

$$V_k[\phi] \to \hat{V}_k[\hat{\phi}] = \frac{1}{k^d} \cdot V_k[\hat{\phi} \cdot k]$$

Fixed point: $\partial_t \hat{g}_i = \beta_i(\vec{\hat{g}})$

$$\beta_i(\vec{\hat{g}}_*) = 0$$

e.g. $\vec{\hat{g}} = (\hat{m}, \hat{\lambda})$

Stability: expansion about $\vec{\hat{g}}_*$:

$$\hat{g}_i = \hat{g}_{*i} + \delta\hat{g}_i$$

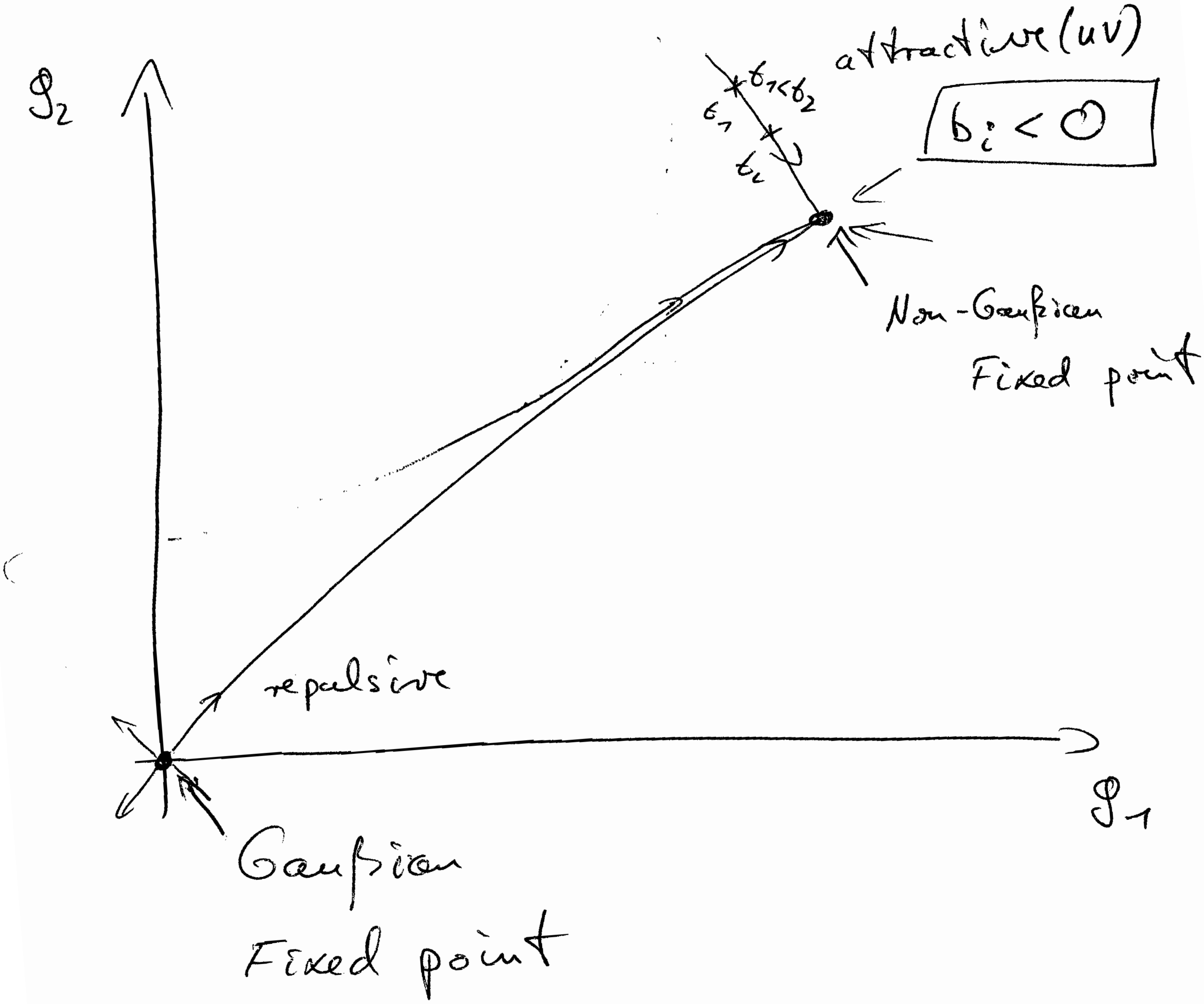
$$\Rightarrow \partial_t \hat{g}_i = \underbrace{\beta_i(\vec{\hat{g}}_*)}_{0} + B_{ij}(\vec{\hat{g}}_*)\,\delta\hat{g}_j + \mathcal{O}(\delta\hat{g}^2)$$

with $B_{ij} = \dfrac{\partial \beta_i}{\partial \hat{g}_j}$

Diagonalise $\delta\hat{g}$: $\partial_t \delta\bar{g}_i = b_i \delta\bar{g}_i$ (no sum)

with $B \cdot \vec{e}_i = b_i$, $\delta\vec{\hat{g}} = \sum_i \delta\bar{g}_i\, \vec{e}$

$$\Rightarrow \boxed{\delta\bar{g}_i = e^{b_i t}\,\delta\bar{g}_{0i}}$$


g_2
attractive (uv)
t_1 < t_2
t_1
t_2
b_i < 0
Non-Gaußian
Fixed point
repulsive
g_1
Gaußian
Fixed point

b_i complex