2.3 Chiral phase transition

(1) without gluons

Lit: Braun, diss.

Nambu-Jona-Lasinio model

$$S[\psi,\bar\psi] = \int d^dx \left\{ \bar\psi \not\partial \psi + \lambda_\sigma \Big[\underbrace{(\bar\psi\psi)^2}_{\text{scalar}} - \underbrace{(\bar\psi\gamma_5\psi)^2}_{\text{pseudo-scalar}} \Big] \right\}$$

non-renormalisable: $[\lambda_\sigma] = -2$

Flow of λ_σ-coupling:

$\partial_t \lambda_\sigma =$ 

dimensionless coupling: $\hat\lambda_\sigma = \lambda_\sigma \cdot k^2$

$$\dot{\hat\lambda}_\sigma = \dot\lambda_\sigma \cdot k^2 + 2\hat\lambda_\sigma$$

$$= 2\hat\lambda_\sigma - A_k(T)\,\hat\lambda_\sigma^2 + B_k(T)\cdot\hat\lambda_6$$

($-$: fermion loop)

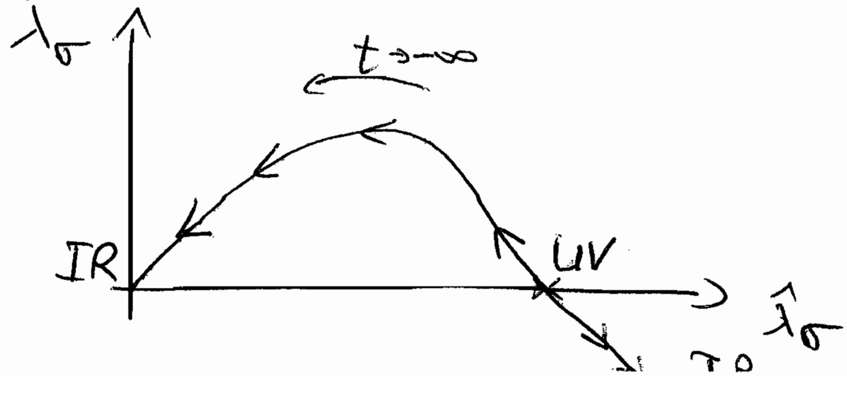


Fig 1

chiral symmetry breaking:

$$\langle \bar\psi\psi \rangle \neq 0 \quad : \sigma = 2\lambda_\sigma \bar\psi\psi : \langle\sigma\rangle = -2\lambda_\sigma \langle\bar\psi\psi\rangle$$

$$\pi = 2i\lambda_\sigma \bar\psi\gamma_5\psi : \langle\pi\rangle = -2i\lambda_\sigma \langle\bar\psi\gamma_5\psi\rangle$$

part.) Bosonisation: (Hubbard-Stratonovich)

$$S_{QM}[\psi,\bar\psi,\sigma,\pi] = \int d^dx \Big\{ \bar\psi i\partial\!\!\!/ \psi + h_\sigma \bar\psi(\sigma + i\gamma_5\pi)\psi$$

$$- \underbrace{\boxed{\frac{h_\sigma^2}{4\lambda_\sigma}}}_{\sim m_\sigma^2}(\sigma^2 + \pi^2) \Big\}$$

$\longrightarrow$ page II-79

+kinetic term $\quad + \frac{1}{2}\int d^dx \Big\{ \sigma(-\partial^2)\sigma + \pi(-\partial^2)\pi \Big\}$

in QCD iso-vector $(\pi^0, \pi^\pm)$

EoM: $\frac{\partial S}{\partial\sigma} = 0: \quad \sigma = 2\lambda_\sigma \bar\psi\psi / h_\sigma$

$\frac{\partial S}{\partial\pi} = 0: \quad \pi = 2i\lambda_\sigma \bar\psi\gamma_5\psi / h_\sigma$

$\Rightarrow$ on-shell $\quad S_{QM} = S_{NJL}$

$$-\frac{1}{4}\lambda(\sigma^2 + \pi^2 - v^2)^2 \quad - c\sigma$$

explicit symm. break

Integrate-out fermions: (non-) linear σ-model

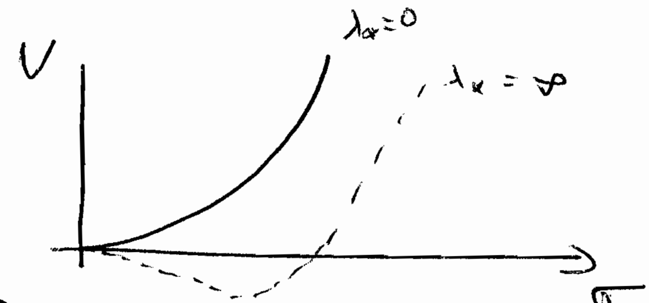
Fixed points:

$$\text{IR}: \quad \lambda_* = 0$$

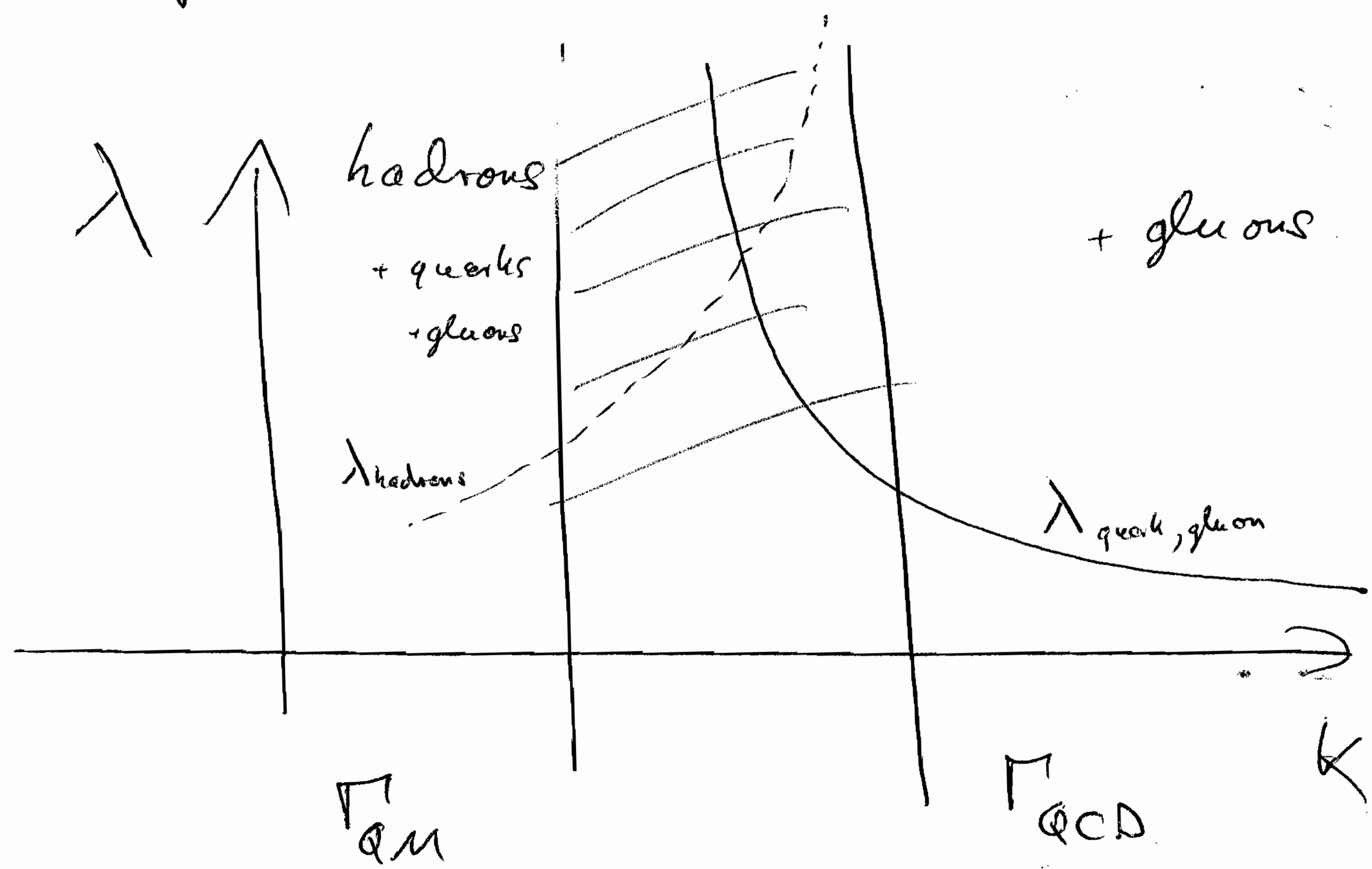
$$\lambda_* = \infty \quad \Leftarrow$$

$$\text{UV}: \quad \lambda_* = \lambda_*(\Lambda)$$

$$m_\sigma^2 \sim \frac{h_\sigma^2}{4\lambda_\sigma} \sim \begin{cases} 0 & \lambda_* = \infty \\ \infty & \lambda_* = 0 \end{cases}$$



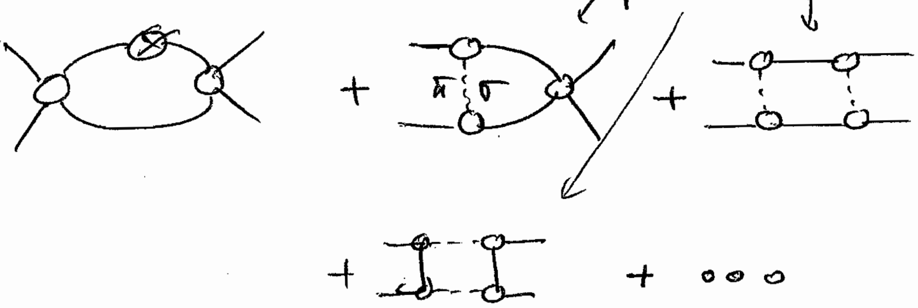
$\partial_\sigma^2 V \sim m^2$



Flow of λ_σ-coupling: (schematically)

(drop ⊗ insertions)

$\partial_t \lambda_\sigma =$



$$\dot{\hat{\lambda}}_\sigma = 2\hat{\lambda}_\sigma - A_k\, \hat{\lambda}_\sigma^2 + \dots \Rightarrow \text{Fig 1 page}$$

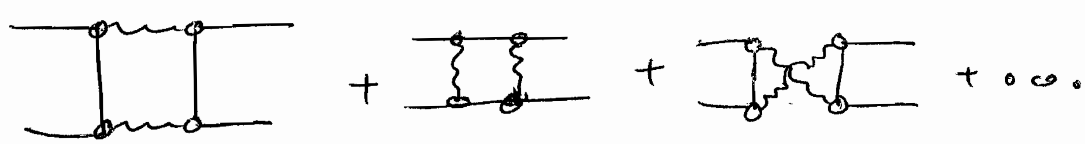
Warning: potential overcounting

Rebosonisation: ⟶ page II-80a

QCD: (1 flavour)

$$S_{QCD}[\phi, \psi, \bar\psi] = S[\phi] + \int d^dx\; \bar\psi\, i\not{D}\, \psi$$

with $i\not{D} = i\not\partial + ig\not{A}$



$$\Rightarrow \Gamma = S_{QCD} + \int d^dx\, \lambda_\sigma \left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2 \right] + \dots$$

Rebosonisation:

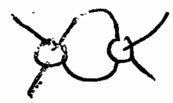
Idea: Gies, Wett '01

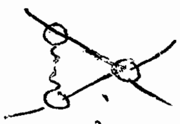
bosonise at each flow-step, e.g.

$$(\lambda_{\sigma_k}, h_{\sigma_k}, m_{\sigma_k}, \lambda_k) \to (\lambda_\sigma, \bar{h}_{\sigma_k}, \bar{m}_{\sigma_k}, \bar{\lambda}_k)$$

- absorb running of λ_σ in running of Yukawa coupling h_σ, mass m_σ, ϕ^4-coupl. λ.
- moves bound state/condensate fractions in λ_σ to bosonic dofs.

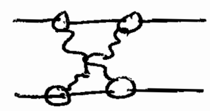
Flow of $\hat{\lambda}_\sigma$:

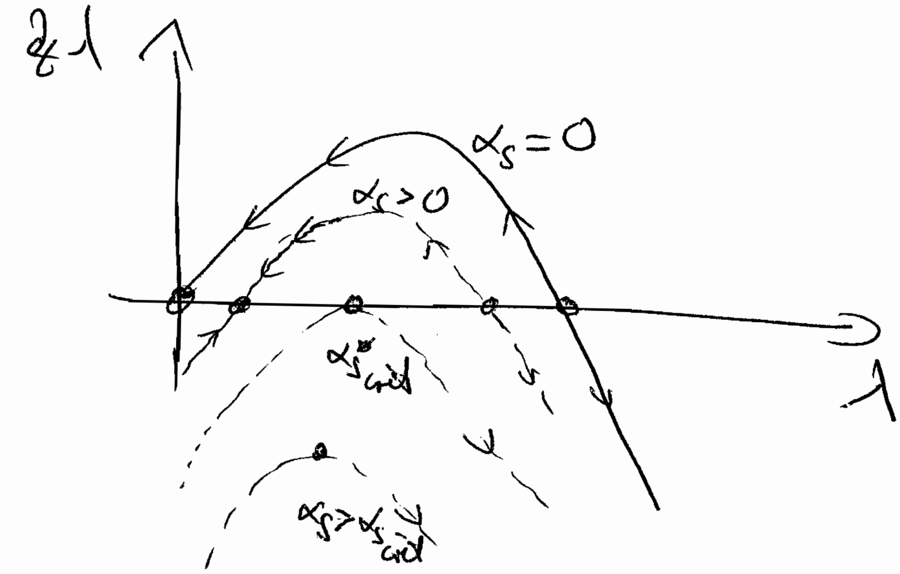




$$\dot{\hat{\lambda}}_\sigma = 2\hat{\lambda}_\sigma - A_k(T)\,\hat{\lambda}_\sigma^2 - B_k(T)\,\lambda\,\alpha_s$$

$$- C_k(T)\,\alpha_s^2 + \dots$$





FRG: $\boxed{\alpha_{s_{crit}} \simeq 0.85}$ $(3 = N_f = N_c)$, $T = 0$ Gies, Jäckel '05

Landau gauge: $\alpha_s^* \sim 3$

$T > 0$: $\boxed{\alpha_{s_{crit}}(T) > \alpha_{s_{crit}}(0)}$

Remark: low energy QCD successfully modelled with Quark-Meson model

Bonus: dynamical creation of Hadrons

Challenges:

(1) QCD flow at <u>all</u> scales

(2) finite μ

$\Rightarrow$ bound state spectrum of QCD

(3) Non-equilibrium