

2.3 chiral phase transition

(1) without gluons

Lit: Braun, diss.

Nambu-Jona-Lasinio model

$$S[\psi, \bar{\psi}] = \int d^d x \left\{ \bar{\psi} \not{\partial} \psi + \lambda_\sigma \left[(\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 \right] \right\}$$

↑ scalar
↑ pseudoscalar

non-renormalisable: $[\lambda_\sigma] = -2$

Flow of λ_σ -coupling:



dimensionless coupling: $\hat{\lambda}_\sigma = \lambda_\sigma \cdot k^2$

$$\dot{\hat{\lambda}}_\sigma = \hat{\lambda}_\sigma \cdot k^2 + 2 \hat{\lambda}_\sigma$$

$$= 2 \hat{\lambda}_\sigma - \underbrace{A_k(\tau)}_{\text{fermion loop}} \hat{\lambda}_\sigma^2 + B_k(\tau) \cdot \hat{\lambda}_\sigma$$

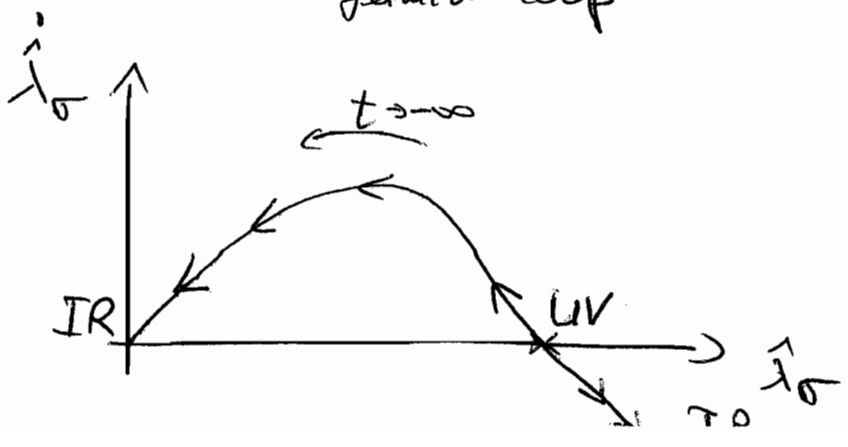


Fig 1

chiral symmetry breaking:

$$\langle \bar{\psi} \psi \rangle \neq 0 \quad \therefore \sigma = 2 \lambda_\sigma \bar{\psi} \psi \quad \langle \sigma \rangle = -2 \lambda_\sigma \langle \bar{\psi} \psi \rangle$$

$$\bar{\pi} = 2 i \lambda_\sigma \bar{\psi} \gamma_5 \psi \quad \langle \bar{\pi} \rangle = -2 i \lambda_\sigma \langle \bar{\psi} \gamma_5 \psi \rangle$$

part.) Bosonisation: (Hubbard-Stratonovich)

$$S_{QM}[\psi, \bar{\psi}, \sigma, \bar{\pi}] = \int d^d x \left\{ \bar{\psi} i \not{\partial} \psi + \frac{h_\sigma}{h_\sigma} \bar{\psi} (\sigma + i \gamma_5 \pi) \psi - \frac{\frac{h_\sigma^2}{4 \lambda_\sigma}}{v m_\sigma^2} (\sigma^2 + \bar{\pi}^2) \right\}$$

+ kinetic term $\quad + \frac{1}{2} \int d^d x \left\{ \sigma (-\partial^2) \sigma + \bar{\pi} (-\partial^2) \bar{\pi} \right\}$

in QCD iso-vector $(\bar{\pi}^0, \bar{\pi}^\pm)$

$$E_0 \mu: \frac{\partial S}{\partial \sigma} = 0: \quad \sigma = 2 \lambda_\sigma \bar{\psi} \psi / h_\sigma$$

$$\frac{\partial S}{\partial \bar{\pi}} = 0: \quad \bar{\pi} = 2 i \lambda_\sigma \bar{\psi} \gamma_5 \psi / h_\sigma$$

$$\Rightarrow \text{on-shell} \quad S_{QM} = S_{NJL}$$

$$- \frac{1}{4} \lambda (\sigma^2 + \bar{\pi}^2 - v^2)^2 - c \sigma$$

explicit symbol

Integrate-out fermions: (non-) linear σ -model

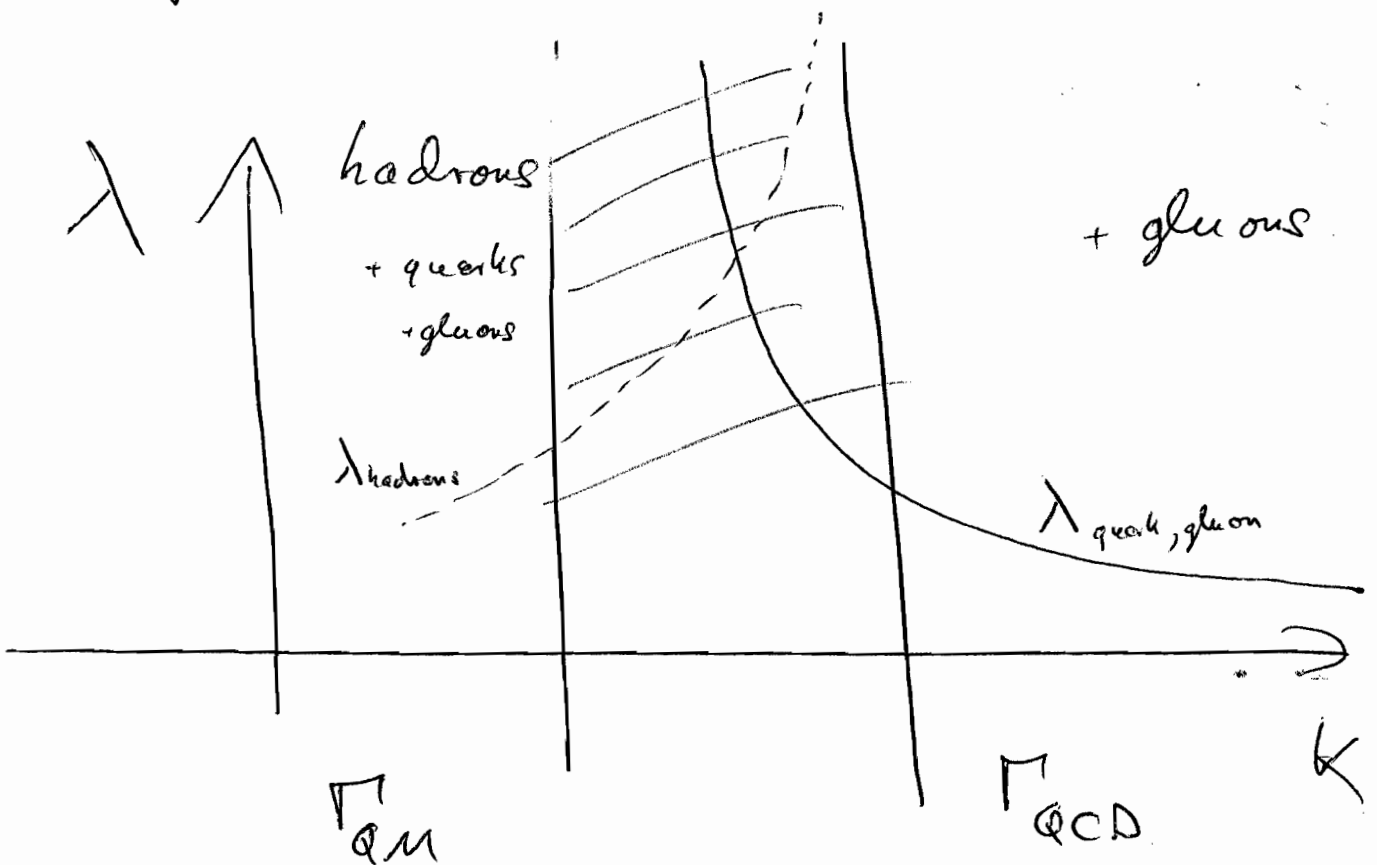
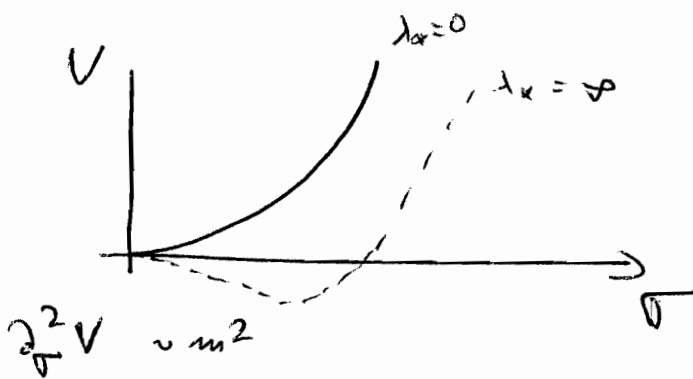
Fixed points :

IR : $\lambda_* = 0$

$\lambda_* = \infty \iff$

UV : $\lambda_* = \lambda_*(A)$

$m_\sigma^2 \sim \frac{h_\sigma^2}{4\lambda_\sigma} \sim \begin{cases} 0 \\ \infty \end{cases} \quad \begin{matrix} \lambda_* = \infty \\ \lambda_* = 0 \end{matrix}$



Flow of λ_σ -coupling: (schematically)

$$2\hat{\lambda}_\sigma = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \dots$$

(drop & insertions)

The diagrams represent:
 1. A bubble diagram with two external lines and a loop containing a σ particle.
 2. A diagram with two external lines and a loop containing a σ particle, with a dashed line representing a fermion loop.
 3. A diagram with two external lines and two vertices connected by a dashed line, with a σ particle loop.
 4. A diagram with two external lines and two vertices connected by a dashed line, with a fermion loop.

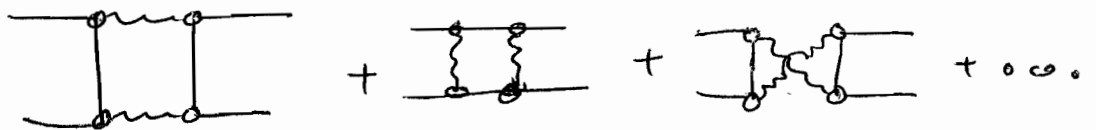
$$\dot{\hat{\lambda}}_\sigma = 2\hat{\lambda}_\sigma - A_\psi \hat{\lambda}_\sigma^2 + \dots \Rightarrow \text{Fig 1 page}$$

Warning: potential overcounting

1. Rebosonisation: \longrightarrow page II-80a
 QCD @ (1 flavour)

$$S_{\text{QCD}}[\phi, \psi, \bar{\psi}] = S[\phi] + \int d^d x \bar{\psi} i \not{D} \psi$$

with $i \not{D} = i \not{\partial} + i g A$



$$\Rightarrow \Gamma = S_{\text{QCD}} + \int d^d x \lambda_\sigma [(\bar{\psi}\psi)^2 - (\bar{\psi} \gamma_5 \psi)^2] + \dots$$

Rebosonisation:

Idea:

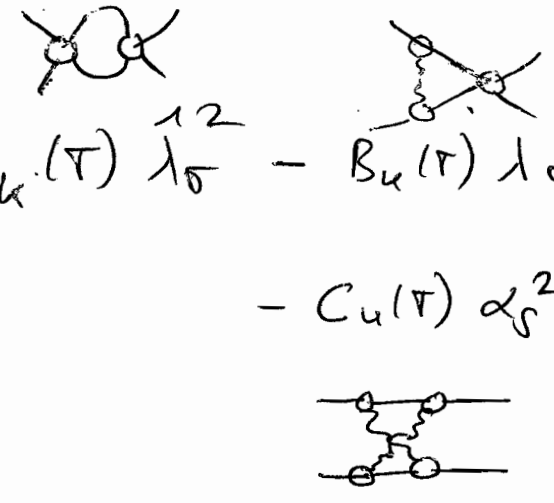
Gies, Wett '01

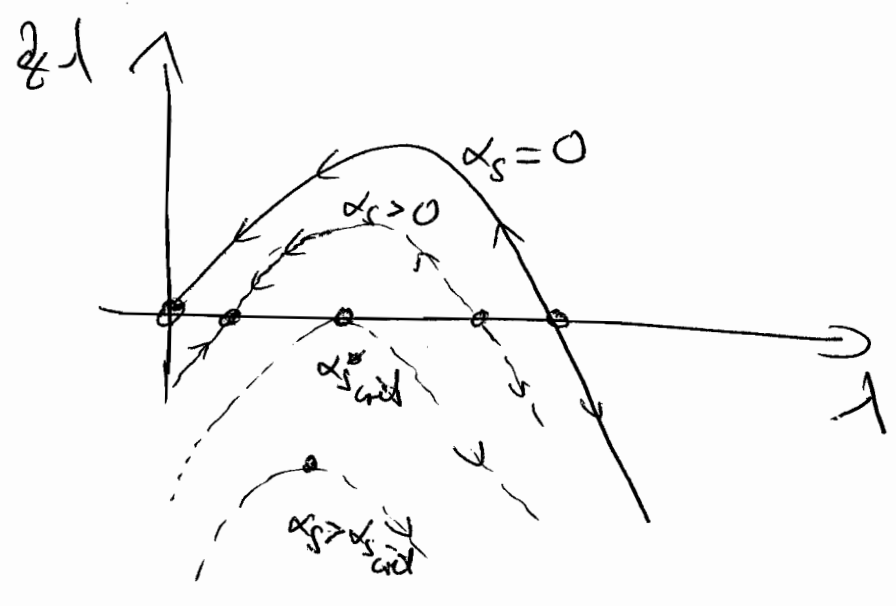
bosonise at each flow-step, e.g.

$$(\lambda_{\sigma_k}, h_{\sigma_k}, m_{\sigma_k}, \lambda_k) \rightarrow (\lambda_\sigma, \bar{h}_{\sigma_k}, \bar{m}_{\sigma_k}, \bar{\lambda}_k)$$

- absorb running of λ_σ in running of Yukawa coupling h_σ , mass m_σ , ϕ^4 -coupl. λ .
- moves bound state/condensate fractions in λ_σ to bosonic dofs.

Flow of $\hat{\lambda}_0$:

$$\hat{\lambda}_0 = 2 \hat{\lambda}_0 - A_U(\tau) \hat{\lambda}_0^2 - B_U(\tau) \lambda \alpha_s - C_U(\tau) \alpha_s^2 + \dots$$




FRG: $\alpha_{s\text{crit}} \sim 0.85$ ($3 = N_f$), $T=0$ Gies, Jädel '05
 $= N_c$

Landau gauge: $\alpha_s^* \sim 3$

$T > 0$: $\alpha_{s\text{crit}}(T) > \alpha_{s\text{crit}}(0)$

Remark: low energy QCD successfully modelled with Quark-Meson model

Bonus: dynamical creation of hadrons

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Challenges:

(1) QCD flow at all scales

(2) finite μ

\Rightarrow bound state spectrum of QCD

(3) Non-equilibrium