II-2 Confinement & chiral symmetry breaking

Confinement

(i) What do we expect $\overset{?}{\Leftrightarrow}$ Truncation
" " " know

(ii) Compo

(iii) Relation to (quark) confinement

χ - sym. breaking

(i) What do we expect

(ii) (sketch of) computation

(iii) Relation to confinement

2.1 RG - Flows for IR - QCD

What do we expect?

Gribov - Zwanziger / Kugo - Ojima confinement crit.:
in Landau gauge

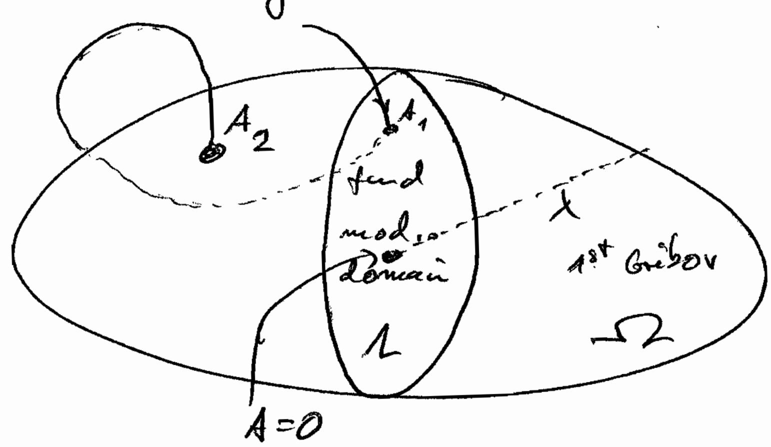
(i) path integral

$$Z[J] = \int dA \Big|_{-\partial_\mu D_\mu \geq 0} \; e^{-S[\underbrace{A, c, \bar{c}}_{\phi}] + \int J_i \phi_i}$$

$$S[\phi] = \frac{1}{4} \int d^d x \, F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} \int d^d x \, \partial_\mu A^a_\mu \partial_\nu A^a_\nu$$

$$- \int d^d x \, \bar{c}^a \partial_\mu D^{ab}_\mu c^b$$

1st Gribov region:



$$A_1^{\mathcal{U}} = A_2$$

Ω : compact space

as $\quad -\partial_\mu D^{ab}_\mu = -\partial^2_\mu \delta^{ab} - g f^{acb} A^c_\mu$

e.g. $SU(2)$: $f^{acb} \simeq \varepsilon^{acb}$ with positive/negative eigenvalues

$A_\mu \to \lambda A_\mu$: λ big enough

$\Rightarrow -\partial_\mu D_\mu$ has negative EV

(ii) ghost propagator: $\langle(-\partial_\nu D_\nu)^{-1}\rangle_c(p^2) = 1/\frac{\partial}{\partial\bar{c}}\frac{\partial}{\partial c}\Gamma\Big|_{\phi=c}(p^2) = 1/\Gamma_c^{(2)}$

- $p^2 \to 0$: low lying eigenspectrum, dominated by A with small EV of $-\partial_\nu D_\nu$
- weight of A with $\min(-\partial_\nu D_\nu) \to 0$ is 1

$$\Rightarrow \lim_{p\to 0} \langle(-\partial_\nu D_\nu)^{-1}\rangle(p^2) \cdot \frac{1}{(p^2)^{1+\kappa_c}}$$

see II-64a,b

with $\boxed{\kappa_c > 0}$

safe if $\boxed{\int e^{-S_{YM}[A]}\Big|_{\partial\Omega} \mathcal{J}\, dA \neq 0}$

- $\Gamma_c^{(2)}(p^2) \sim (p^2)^{1+\kappa_c}$

difficult to see as trivial peaces $\sim p^2$ to be <u>adjusted/fine-tuned</u>

heuristic argument:

(i) weight $d\mu$ of config A: $d\mu = \dfrac{[dA.\ e^{-S_{YM}[A]} \det(-\partial D)}{\int dA\ e^{-S_{YM}[A]} \det(-\partial_\mu D_\mu)}$

$\Rightarrow$ weight $\mu_0(\varepsilon)$ of configs. A with EV λ_0:

$$(-\partial_\mu D_\mu)\,\psi_0 = \lambda_0 \psi_0 \qquad \lambda_0 < \varepsilon$$

$$\mu_0(\varepsilon) = \int_{\partial\Omega_\varepsilon} d\mu \simeq 1$$

$\uparrow$ Ω bounded + infinite dim

(ii) $$\left\langle \frac{1}{-\partial_\mu D_\mu} \right\rangle (p^2) = \sum_i \int \frac{1}{\lambda_i} \left|\langle \psi_i | e^{ipx} \rangle\right|^2 d\mu$$

$$\int d\mu \sum_i \frac{1}{\lambda_i} \left|\langle \psi_i | e^{ipx}\right|^2 = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{1}{\lambda} \cdot f_\lambda(p^2)$$

$\rho(\lambda \to 0) \sim \lambda^{-K}$ $\qquad K > 0$ [pert: $= 0$]

$f_\lambda \sim \frac{\lambda}{(\lambda + p^2)^2}$

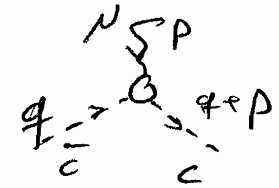
$$\Rightarrow \left\langle \frac{1}{-\partial_\mu D_\mu} \right\rangle (p^2) = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{1}{\lambda} \cdot f_\lambda(p^2)$$

$$\sim \int_0^\infty \frac{d\lambda}{\lambda} \cdot \frac{1}{\lambda^\alpha} \frac{\lambda}{(\lambda + p^2)^2}$$

$$\simeq \frac{1}{(p^2)^{1+1/2}}$$

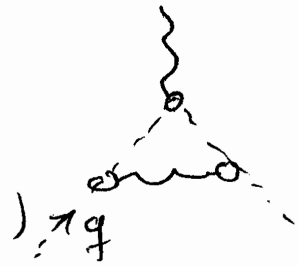
(iii) gluon propagator: $\langle A_\mu A_\nu \rangle (p^2) = 1/\frac{\delta^2}{\delta A_\mu \delta A_\nu}\Gamma^{(2)}(p$

Taylor '71: non-renormalisation of ghost-gluon vertex



$\sim g\, q_\mu$

1-loop:



$\sim q_\mu$

finite

n-loop:

$$\Rightarrow \boxed{\mu\partial_\mu Z_g Z_A^{1/2} Z_c = 0}$$

part. $\rightsquigarrow (p^2)^{\kappa_c} \cdot (p^2)^{\frac{1}{2}\kappa_A} \sim (p^2)^0$

$$\Rightarrow \boxed{\kappa_A = -2\kappa_c}$$

! beware of e^{-1/g^2}-corrections !

(a) the IR-scaling of at least <u>one</u> diagram has to agree!

(b) scaling applies to <u>all</u> vertices

beware decoupling (then there is <u>no</u> confinement (under dispute))

from (a): one vertex (at least) has to be trivial: ghost-gluon vertex!

$\Rightarrow$ $\boxed{\text{non-renormalisation}}$

Remark: valid for <u>general</u> theories in <u>scaling</u> regimes

Summary:

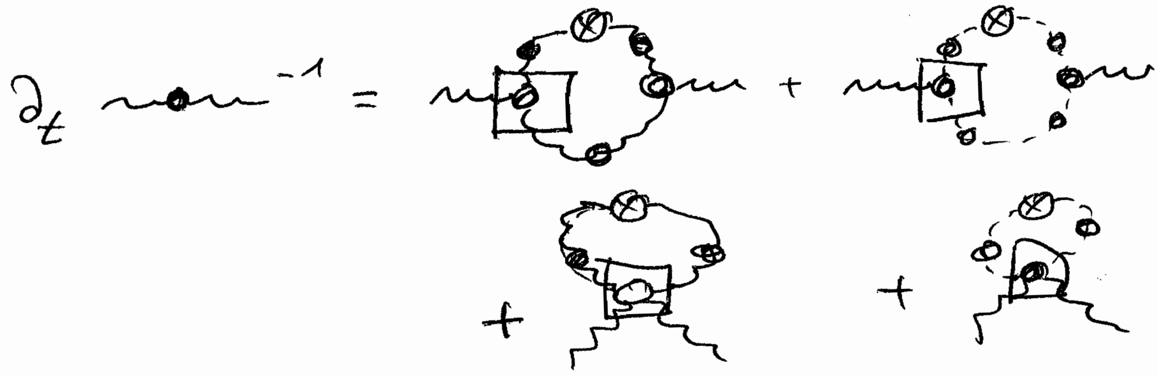
(1) propagators have non-trivial momentum-dep

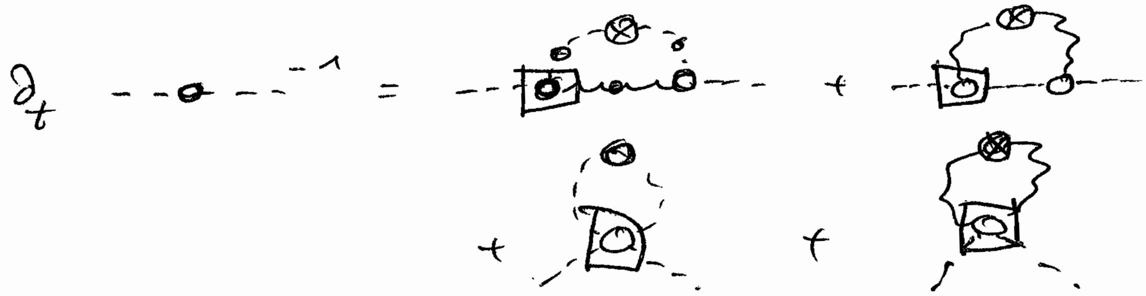
(2) IR-leading vertices are rather trivial

(3) so-called Gribov-Zwanziger / Kugo-Ojima conf. criterium: we shall prove quark confinement from A, C-props

non-pert. 'proof' with FRG + DSE   Fischer, Pawlowski '0?

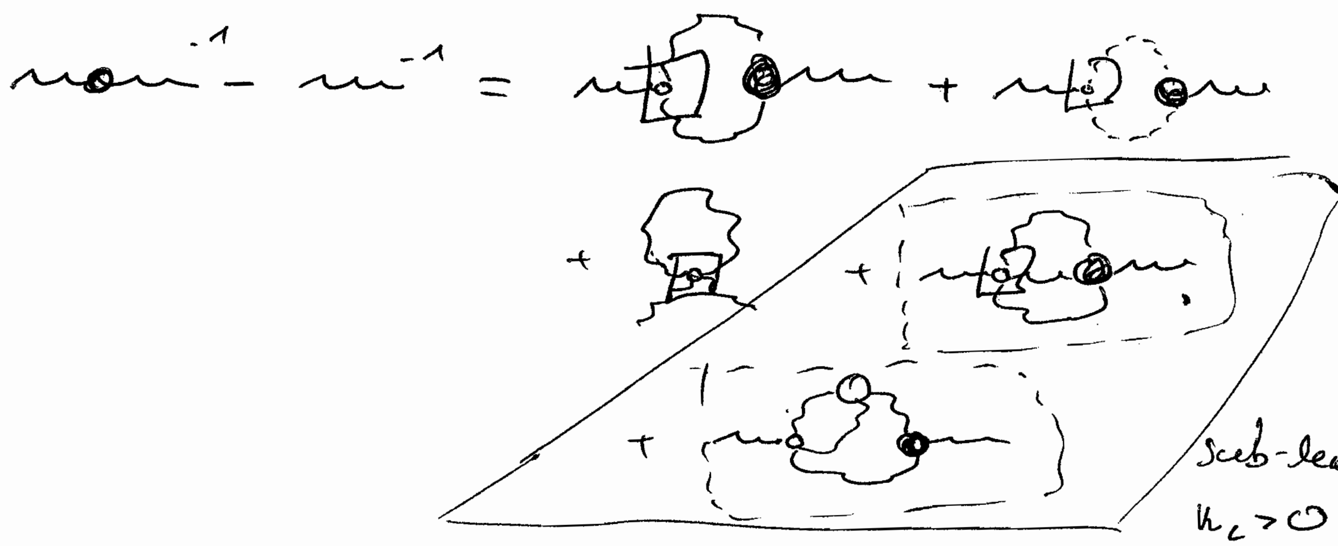
FRG:



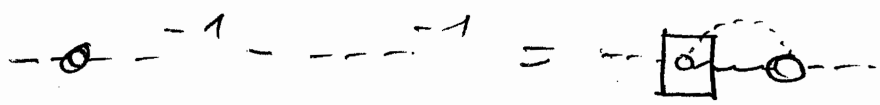


vertices

DSE:



sub-lead.
$k_c > 0$



vertices

Computations?
Truncation: (4-d)

$$S[\phi] = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\, A^a_\mu(p)\, \Gamma^{(2)\,ab}_{A\,\mu\nu}(p)\, A^b_\nu(q)$$

$$+ \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\, \bar{C}^a(p)\, \Gamma^{(2)\,ab}_{c}(p)\, C(q)$$

$$+ \frac{1}{3!}\int \prod_{i=1}^{3} \frac{d^4p_i}{(2\pi)^4}\, A^a_\mu(p_1)\, A^b_\nu(p_2)\, A^c_\rho(p_3)\; \Gamma^{(3)\,abc}_{A\,\mu\nu\rho}(p_1,p_2,p_3)$$

$$+ \frac{1}{4!}\int \prod_{i=1}^{4}\left[\frac{d^4p_i}{(2\pi)^4} A^{a_i}_{\mu_i}(p_i)\right] \Gamma^{(4)\,a_1\ldots a_4}_{A\,\mu_1\ldots\mu_4}(p_1,\ldots,p_4)$$

$$+ \int \prod_{i=1}^{3} \frac{d^4p_i}{(2\pi)^4}\, \bar{C}^{a_1}(p_1)\, A^{a_2}_\mu(p_2)\, C^{a_3}(p_3)\cdot \Gamma^{(3)\,a_1\ldots a_3}_{\bar{c}Ac\;\mu}$$

with $\Pi_{\mu\nu}(p) = \delta_{\mu\nu} - p_\mu p_\nu/p^2$

drops out of dyn. for $\xi = 0$

$$\Gamma^{(2)\,ab}_{A\,\mu\nu}(p) = \left(p^2 Z_A(p^2)\cdot \Pi_\perp(p^2) + p^2 \boxed{Z_{A_L}(p^2)}\, \Pi_L(p^2)\right)\delta^{ab}$$

general momentum dep.

$$\Gamma^{(2)\,ab}_{c}(p) = p^2\, Z_c(p^2)\, \delta^{ab}$$

$$\Gamma^{(n>2)}(p_1,\ldots,p_n) \simeq S^{(n>2)}_{cl}(p_1,\ldots p_n)\cdot \text{RG-improvements}$$

$$\sim \left(Z_\phi^{1/2}\right)^n \to \text{next page}$$

What about $Z_{A_L}(p^2)$? $\boxed{\text{mSTI - check}}$

$$S^{(3)\,a_1a_2a_3}_{A\;\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = (2\pi)^4\,\delta(p_1+p_2+p_3)\cdot f^{a_1a_2a_3}$$

$$\cdot\, i\Big[(p_1-p_2)_{\mu_3}\,\delta_{\mu_1\mu_2} - (2p_1+p_2)_{\mu_2}\,\delta_{\mu_1\mu_3}$$

$$+ (p_1+2p_2)_{\mu_1}\,\delta_{\mu_2\mu_3}\Big]\cdot Z_{A^3_k}\big[p_1^2,p_2^2,(p_1+p_2)^2\big]$$

$$S^{(4)\,a_1\cdots a_4}_{A\;\mu_1\cdots\mu_4}(p_1,\dots,p_4) = (2\pi)^4\,\delta(p_1+p_2\cdots+p_4)\cdot f_{a_1a_2c}\,f_{a_3a_4}$$

$$\cdot\Big[g_{\mu_1\mu_3}\,g_{\mu_2\mu_4}+g_{\mu_1\mu_4}\,g_{\mu_2\mu_3}-2g_{\mu_1\mu_2}\,g_{\mu_3\mu_4}\Big]$$

$$\cdot\, Z_{A^4}\big[p_1^2,\dots,(p_1+p_2+p_3)^2\big]$$

$$S^{(3)\;\overset{\bar c}{\downarrow}\!a_1\,\overset{A}{\downarrow}\!a_2\,\overset{c}{\downarrow}\!a_3}_{\bar cAc\;\;\mu}(p_1,p_2,p_3) = (2\pi)^4\,\delta(p_1+p_2+p_3)\cdot f^{a_1a_2a_3}$$

$$-i\,p_{1\mu}\;Z_{\bar cAc}\big[p_1^2,p_2^2,(p_1+p_2)^2\big]$$

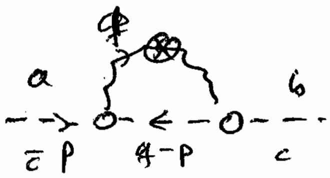
gluon propagator:

$$: \sim g^2 \int \frac{d^4 q}{(2\pi)^4} \Big\{ \overbrace{f^{cad} f^{dbc}}^{tr_{ad}\, t^a t^b = -N_c \delta^{ab}} \cdot \Big[q_\mu (q+p)_\nu \cdot \frac{1}{q^2 Z_c(q^2) + R_c(q^2)} \dot{R}_c(q^2) \frac{1}{q^2 Z_c(q^2) + R_c(q^2)} \cdot \frac{1}{(q+p)^2 Z_c((q+p)^2) + R_c((q+p)^2)} + \left(\begin{array}{c} \mu \to \nu \\ p \to -p \end{array} \right) \Big]$$

$$\frac{1}{3} \Pi_{\perp\mu\nu}(p) \, [\quad J_{\mu\nu} = I_A[q,p]\, \delta^{ab}$$

gluonic diagrams 'excessive'

ghost propagator: $\Pi_{\perp\mu\nu}(q) = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$, $\xi = 0$



$$: \sim g^2 \int \frac{d^4 q}{(2\pi)^4} \Big\{ \overbrace{f^{acd} f^{bdc}}^{tr_{ad}\, t^a t^b = -N_c} \Big[p_\mu \Pi_{\perp\mu\nu}(q) (q-p)_\nu \cdot \frac{1}{q^2 Z_A(q^2) + R_A(q^2)} \dot{R}_A \frac{1}{q^2 Z_A(q^2) + R_A(q^2)} \cdot \frac{1}{(q-p)^2 Z_A((q-p)^2) + R_c((q-p)^2)} + (p \to -p) \Big]$$

$$[\quad J = I_c(q,p)\, \delta$$

$+$  $+$ 

IR – analysis:

take momenta and cut-off

$$p^2, k^2 \ll \Lambda^2_{QCD}$$

physics:

$$k^2 \ll p^2 \ll \Lambda^2_{QCD}$$

IR-regularised:

$$p^2 \ll k^2 \ll \Lambda^2_{QCD}$$

In this region: $\Gamma^{(2)}_{A/c} \simeq p^2 \, Z_{A/c}(p^2)$

with $Z_{A/c}(p^2) = z_{A/c}(p^2)^{1+\kappa_{A/c}}\left(1 + \delta Z_{A/c}(p^2/k^2)\right)$

δZ with limits: $x = p^2/k^2)$

IR-reg: $\delta Z_A(x \to 0) \simeq -1 + c_A \, x^{-(1+\kappa_A)} + O(x^{-\kappa_A})$

$\delta Z_c(x \to 0) \simeq -1 + c_c \, x^{-\kappa_c} + O(x^{1-\kappa_c})$

phys.: $\delta Z_{A/c}(x \to \infty) \to 0$

Integrated flow: $\Gamma^{(2)}_{A/c} = z_{A/c} (p^2)^{1+\kappa_{A/c}} \left(1 + \delta Z_{A/c}(p^2/k^2)\right)$

$$\int_k^0 \frac{dk'}{k'} \partial_t \Gamma^{(2)}_{A/c} = z_{A/c} (p^2)^{1+\kappa_{A/c}} \delta Z_{A/c}(p^2/k^2)$$

$$\Rightarrow \boxed{\delta Z_{A/c}(x) = \underbrace{\left(\frac{g^2}{4\pi Z_A Z_c^2}\right)}_{\alpha_S} \underbrace{\int_x^\infty \frac{dx'}{x'} \frac{1}{x'^{1+\kappa_{A/c}}} f_{A/c}(x')}_{F_{A/c}}} \quad (*)$$

$$\int_k^0 \frac{dk'}{k'} = -\frac{1}{2}\int_x^\infty \frac{dx'}{x'}$$

with $\quad f_{A/c}(x') = (+2\pi) N_c \int \frac{d^4 \hat{q}}{(2\pi)^4} \cdot I_{A/c}(\hat{q}, p/k')$

$$\boxed{\hat{q} = q/k}$$

Remarks:

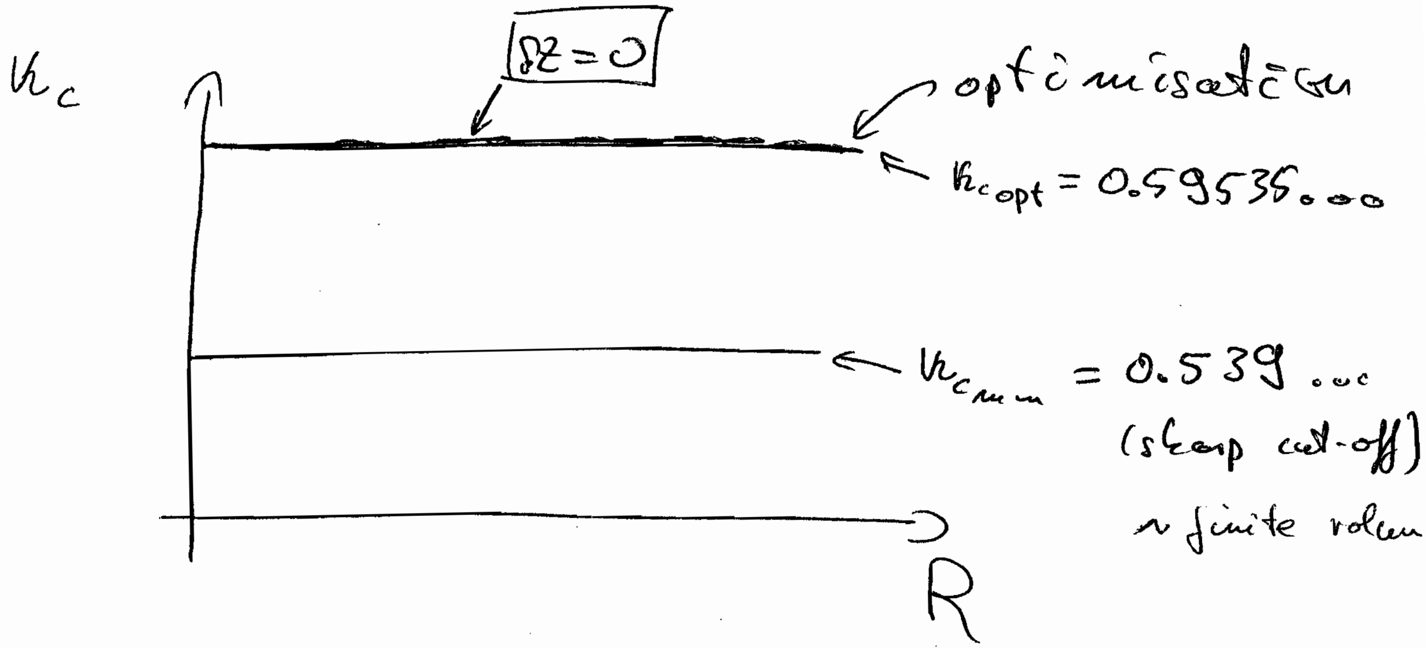
- $\delta Z_{A/c}(0) = -1 \quad \leftarrow \delta Z_c(x \to 0) = -1 + c_c/x^{\kappa_c} + \frac{d_c x}{x^{\kappa_c}}$

$$\left.\begin{array}{l} \Rightarrow \quad -1 = \alpha_S \cdot F_{A/c}(0) \\ \kappa_A = -2\kappa_c \end{array}\right\} (\alpha_s, \kappa_c)$$

- iterate (*) about $\delta Z_{A/c} \stackrel{!}{=} 0$

- optimisation: $F_{A/c}\big|_{\delta Z = 0}$

Results : α_s big variations



(i) $(\kappa_{c\,opt}, \alpha_{s\,opt}) = (\kappa_{c\,DSE}, \alpha_{s\,DSE})$

with classical $\Gamma^{(3)}_{\bar{c}Ac}$

(ii) $\boxed{\int \text{Flow}_{opt} = \text{renormalised DSE}}$

iii) inclusion of tadpole diagrams in FRG

+ DSE for [diagram] , [diagram] , [diagram] + classical $\Gamma^{(3)}_{\bar{c}Ac}$

$$\frac{\delta}{\delta R}(\kappa_c, \alpha_s) = 0$$

$$(\kappa_c, \alpha_s) = (\kappa_{c\,opt}, \alpha_{s\,opt})$$

full momentum range:

(i) optimisation:

$$R_k \simeq \left[\Gamma_0^{(2)}(k^2) - \Gamma_k^{(2)}(p^2)\right] \Theta[\quad]$$

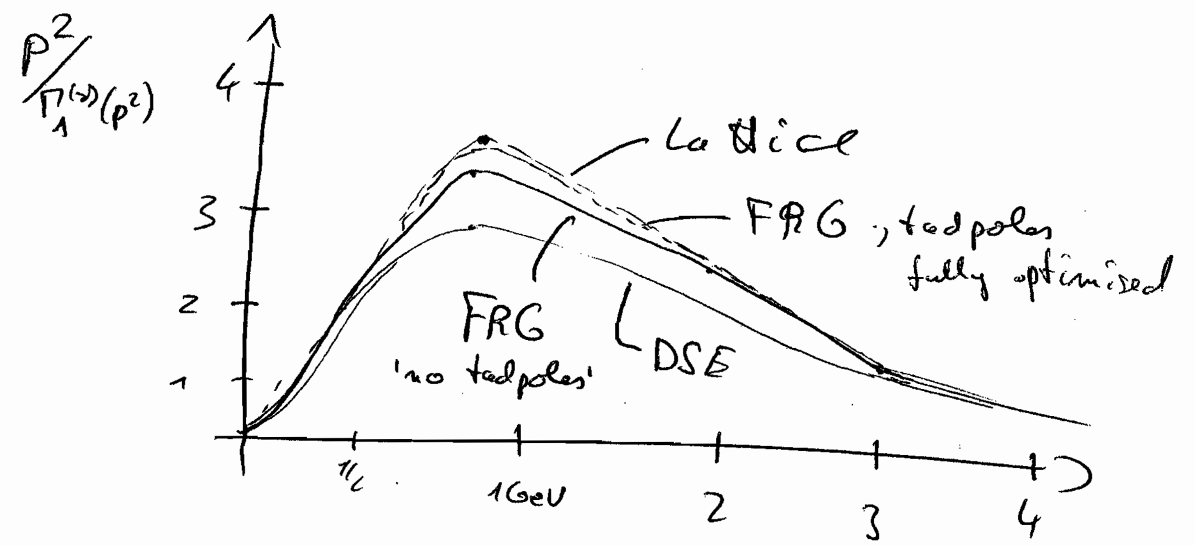
Φ_k with $\boxed{\partial_t \Gamma_k^{(2)}(p^2 > k^2) = 0}$

(ii) iteration about $\partial_t \Gamma_k^{(2)} = 0$

(a) Flow = Flow$(\Gamma_0^{(2)})$ 1st iteration

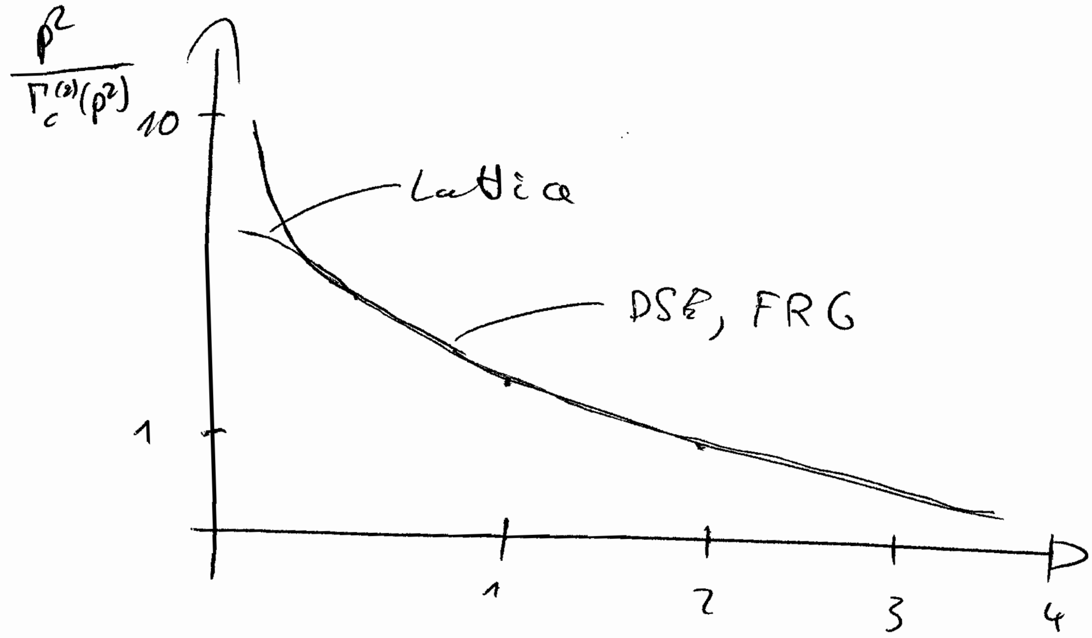
(b) $\int$ Flow = "DSE" with <u>local</u> integrand

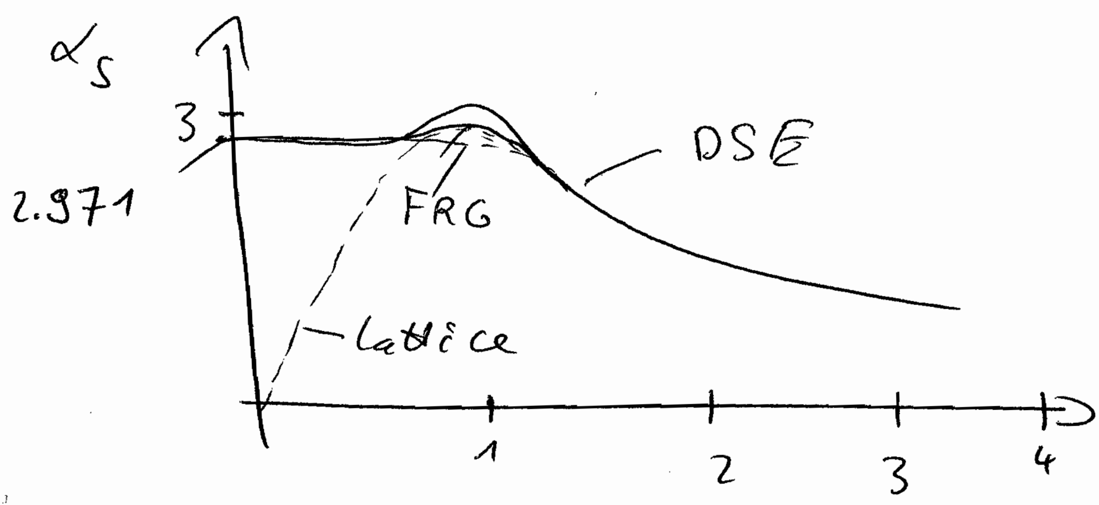
Results:



$$\boxed{\text{FRG-tadpoles} \gtrsim \text{2-loop diagrams in DSE}}$$

! momentum-dep !





Open questions:

(i) $\kappa_A = -2\kappa_c$

(ii) $\kappa_c > 0$

(iii) direct proof of confinement

problems:

(a) gauge fixing   Lattice

(b) truncations   Functional Methods

(c) [illegible]