2.2 Confinement-Deconfinement Phase transition

Use of propagators for computing order parameter of confinement

Preparation:

Polyakov Loop: $\beta = 1/T$

$$L(\vec{x}) = \frac{1}{N_c} \, \mathrm{tr} \, \mathcal{P} e^{ig\int_0^\beta dx_0 A_0}$$

example:

$U(1)$: $e^{ig\int_0^\beta dx_0 A_0(x)} = e^{i\int_0^\beta d^4y \, A_0(y) j_0(y)}$

$$j_0(y) = \int_0^\beta d\tau \, \delta^{(4)}(y - x(\tau)) \quad , \; x_0(\tau) = \tau, \; \vec{x}(\tau) = \vec{x}$$

worldline of static charge

Quark-antiquark potential:

$$\langle q(x) \bar{q}(y) \rangle = \langle L(\vec{x}) L^+(\vec{y}) \rangle = e^{-F_{q\bar{q}}(|\vec{x}-\vec{y}|)}$$

Confinement: $\lim_{|\vec{x}-\vec{y}| \to \infty} \langle q(x) \bar{q}(y) \rangle \to 0$

$F_{q\bar{q}} \to \infty$

Deconfinement: $\lim_{|\vec{x}-\vec{y}| \to \infty} \langle q(x) \bar{q}(y) \rangle \neq 0$

$F_{q\bar{q}}$ finite

Declustering:

$$\lim_{|\vec{x}-\vec{y}|\to\infty} \left(\langle q(x)\, \bar{q}(y) \rangle - \langle q(\vec{x}) \rangle \langle \bar{q}(\vec{y}) \rangle \right) = 0$$

(or $\langle q(x) \rangle = e^{-F_q}$

confinement: $F_q = \infty$

Deconfinement: F_q finite)

Effective action approach:

- compute $\Gamma_k [A_0^c] = V_k [A_0^c]$
- $\langle A_0 \rangle = \bar{A}_0$ with $\left. \frac{\partial V_k}{\partial A_0^c} \right|_{\bar{A}_0} = 0$
- $\langle L(x) \rangle \leq 1/N_c \, \mathrm{tr} P e^{i \int_0^\beta \langle A_0 \rangle}$

Flow of V_k:

$$\dot{V}_k = \frac{1}{2} \, T \sum_{n\in\mathbb{Z}} \int d^3q \, \mathrm{tr} \frac{1}{\Gamma_k^{(2)}[A_0](q) + R_k(\vec{q}^{\,2})} \dot{R}_k(\vec{q}^{\,2})$$

with $q = (2\pi n T, \vec{q})$

Perturbation theory:

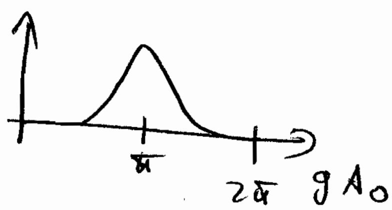
$$\dot{V}_{pert}[A_0] = \frac{1}{2}\, T \sum_n \int d^3q \, \frac{1}{\vec{q}^{\,2} + (2\pi n + gA_0)^2 + R(\vec{q}^{\,2})}\, \dot{R}(\vec{q}^{\,2})$$

$$\left(\frac{d-1}{2} - \frac{1}{2} \right)$$

$d = 4$

$$V_{pert}[A_0] = \left[\frac{1}{\beta^4} \left[\frac{\phi^2}{12} - \frac{\phi^3}{12\pi} + \frac{\phi^4}{48\pi^2} \right] \right]$$

with $\phi = g A_0$ mod 2π



$\Rightarrow$ $\boxed{\bar{A}_0 = 0 \;:\; L(\vec{x})|_{\bar{A}_0} = 1}$

IR : $\Gamma^{(2)}_{A/c} \simeq \left[-D(A_0) \right]^{1 + \kappa_{A/c}}$

$$\Rightarrow \quad \dot{V}_{IR}[A_0] = \left[\frac{d-1}{2}(1+\kappa_A) + \frac{1}{2} - (1+\kappa_c) \right] \dot{V}_{pert}[A_0]$$

$$\Rightarrow \quad V_{IR}[A_0] = \underbrace{\left[\frac{d-1}{2}(1+\kappa_A) + \frac{1}{2} - (1+\kappa_c) \right]}_{c(\kappa_A,\, \kappa_c)} V_{pert}[A_0]$$

(1) confinement : $c(\kappa_A, \kappa_c) < 0$: $\boxed{d - 2 + (d-1)\kappa_A - 2\kappa_c < 0}$

(2) deconfinement : $c(\kappa_A, \kappa_c) > 0$

Determination of $\Gamma_k^{(2)}[A_0]$:

(a) $\Gamma_k^{(2)}[A_0] = \Gamma^{(2)}_{k,\text{Landau}}\left(-D^2(A_0)\right) + A_0$- terms

$$\uparrow$$

$$\Gamma_k^{(2)}[A_0]\Big|_{A_0=0} = \Gamma^{(2)}_{k,\text{Landau}}(p^2)$$

Can we do better? ⇒ Background field gauge

(Landau gauge → Landau-DeWitt gauge

gauge: $\partial_\nu A_\nu = 0 \qquad\qquad D_\nu(\bar{A})\, a_\nu = 0$

FP: $-\partial_\nu D_\nu(A) \longrightarrow -D_\nu(\bar{A}) D_\nu(A)$

with $A_\nu = \bar{A}_\nu + a_\nu$; background field ↗, ↖ fluctuation field

• 'gauge invariant' gauge condition

background gauge trafos $\bar{\delta}$: $a_\nu \overset{1+\bar{\delta}}{\to} a_\nu - [a_\nu, \omega]$, $\bar{A}_\nu \overset{1+\bar{\delta}}{\to} \bar{A}_\nu - [D_\nu(\bar{A}), \omega]$ $\Big\}\ A_\nu \overset{1+\bar{\delta}}{\to} A_\nu - [D_\nu(A), \omega]$

fluct. field trafo: $a_\nu \overset{1+\delta}{\to} a_\nu - [D_\nu, \omega]$, $\bar{A}_\nu \overset{1+\delta}{\to} \bar{A}_\nu$ $\Big\}\ A_\nu \overset{1+\delta}{\to} A_\nu - [D_\nu(A), \omega]$

Gauge-inv.: $\delta(c, \bar{c}) = \bar{\delta}(c, \bar{c}) = -[\omega, (c, \bar{c})]$, $\phi = (a, c, \bar{c})$

$$\left.\begin{aligned} \bar{\delta}\, D(\bar{A}) a_\nu &= -[\omega, D_\nu(\bar{A}) a_\nu] \\ \bar{\delta}(-D_\nu(\bar{A}) D_\nu) &= -[\omega, -D_\nu(\bar{A}) D_\nu)] \end{aligned}\right\} \boxed{\bar{\delta} S[\phi, \bar{A}] = 0}$$

$$\Rightarrow \quad \bar{\delta}\, \Gamma_k[\phi, \bar{A}] = 0$$

$$\uparrow$$

e.g.: $\int d^dx\, a^a_\mu\, R^{ab}_{k\,\mu\nu}(-D^2(\bar{A}))\, a^b_\nu + \dots$

with $\boxed{\bar{\delta}\, R_k = -[\omega, R_k]}$

$$\Rightarrow \bar{\delta}\, \frac{\delta^2}{\delta a^2} \Gamma_k \Big|_{a=0} = -\left[\omega, \frac{\delta^2 \Gamma_k}{\delta a^2}\Big|_{a=0}\right]$$

$$\uparrow$$

$A = \bar{A}$

$$\Rightarrow \quad \frac{\delta^2 \Gamma_k}{\delta a^2}\Big|_{a=0} = \Gamma^{(2)}_{k,\,\text{Landau}}(-D^2(A)) + F_{\mu\nu}\text{-terms}$$

$$F_{\mu\nu}(A = A_0) = 0$$

[finite T: $+\ L(\vec{x})$-dep. terms]

⇒ Slides

$$V^{IR} = \left(\frac{1 + (d-1)\kappa_A - 2\kappa_C}{d-2}\right) V^{UV}$$

Criterion:

$$(d-1) + (d-1)\kappa_A - 2\kappa_C < 0$$

$\kappa_A = -2\kappa_C - \frac{4-d}{2}$: $\kappa_C > \frac{d-3}{4}$

4d: $2 + 3\kappa_A - 2\kappa_C < 0$

$\kappa_A = -2\kappa_C$: $\kappa_C > 1/4$

FRG / DSE: $\kappa_C \simeq \frac{93 - \sqrt{1201}}{98} = 0.595353\ldots$

$$2 + 3\kappa_A - 2\kappa_C \simeq -6.76 < 0$$

Lattice: $\kappa_A \simeq -1$, $\kappa_C \simeq 0$

$$\Rightarrow 2 + 3\kappa_A - 2\kappa_C \simeq -1 < 0$$

Numerics:

num. accuracy

SU(2): $T_c \simeq 276 \pm 10$ MeV $T_c/\sqrt{\sigma} = 0.614 \pm 0.023$

est. syst. error: $\gtrsim 20\%$ lattice: 0.709

est. syst. error: $\lesssim 10\%$

SU(3): $T_c \simeq 284 \pm 10$ MeV $T_c/\sqrt{\sigma} = 0.646 \pm 0.023$

num. accuracy lattice: 0.646

(blow-up)

(1) • $\kappa_A = -2\kappa_c$: $\boxed{\kappa_c > \frac{d-3}{4}}$

$d=4$: $\kappa_c > 1/4$

• FRG/DSE : $\kappa_c = 0.593$:

• lattice: : $\kappa_A \sim -1$, $\kappa_c \sim 0$

$$(4-2) + (4-1)\overset{\kappa_A}{(-1)} - 2 \cdot 0 \simeq -1 < 0$$

Numerics:

SU(2) : $T_c \simeq 276 \pm 10\,\text{MeV}$ $\quad T_c/\sqrt{\sigma} = 0.614 \pm 0.02$

lattice : $T_c/\sqrt{\sigma} = 0.709$

SU(3) : $T_c \simeq 284 \pm 10\,\text{MeV}$ $\quad T_c/\sqrt{\sigma} = 0.646 \pm 0.02$;

lattice : $T_c/\sqrt{\sigma} = 0.646$

Final Remarks:

(1) T-dep. of props
back reaction of $V[A_0]$; in part. for SU(2)

(2) Fermions : • chiral phase boundary $N_f \sim 12$
• hadronisation
• finite μ