2.3 Chiral phase transition

(1) without gluons

Lit: Braun, diss.

Nambu-Jona-Lasinio model (1 flavour, $N_f=1$)

$$S_{NJL}[\psi,\bar\psi] = \int d^dx \left\{ i\bar\psi(\not\partial + m_\psi)\psi + \frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2\right]\right\}$$

scalar

pseudo-scalar

non-renormalisable: $[\lambda_\sigma] = -2$

Flow of λ_ψ-coupling:

dimensionless coupling: $\hat\lambda_\psi = \lambda_\psi \cdot k^2$

$$\dot{\hat\lambda}_\psi = \dot\lambda_\psi \cdot k^2 + 2\hat\lambda_\psi$$

$$= 2\hat\lambda_\psi + \underbrace{A_\psi(T)}_{<0}\hat\lambda^2 + B_\psi(T)\cdot\hat\lambda_6$$

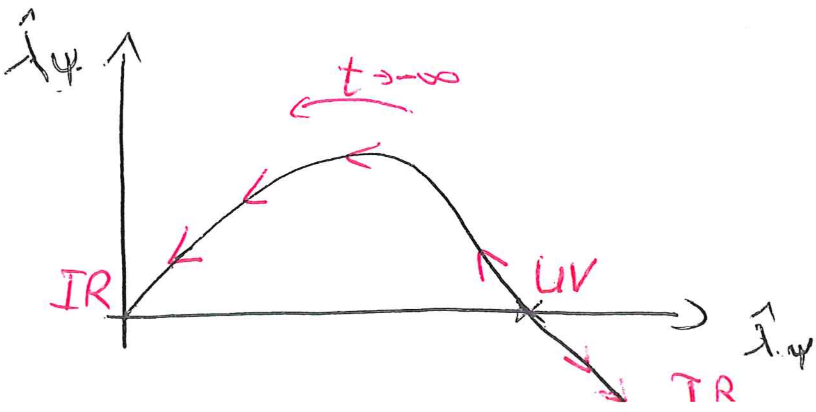


Fig 1

Chiral symmetry breaking:

$S[\psi,\bar\psi]$ with $\boxed{m_\psi = 0}$ is invariant under chiral rotations

$$\psi \to e^{i\alpha\gamma_5}\psi\,, \quad \bar\psi \to \bar\psi\, e^{i\alpha\gamma_5}$$

see p. II-79a

Phases:

$\langle\bar\psi\psi\rangle = 0$ χ-symmetric

$\langle\bar\psi\psi\rangle \neq 0$ χ-broken

The chiral condensate $\langle\bar\psi\psi\rangle$ is an order parameter for spont. chiral symmetry breaking; it is sensitive to chiral symmetry, $\bar\psi\psi \xrightarrow{e^{i\alpha\gamma_5}} \bar\psi\, e^{2i\alpha\gamma_5}\psi$ and 'measures' the dynamical quark mass.

Eg. 1-loop:
$$\int_x \langle\bar\psi\psi\rangle \simeq -\mathrm{Tr}\,\langle\psi(x)\,\bar\psi(x)\rangle \simeq -\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\,\frac{1}{\not p - im}$$
$$\simeq -\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\,\frac{\not p + im}{p^2+m^2} \simeq 4im\int d^4p\,\frac{1}{p^2+m^2}$$

chiral condensate requires renormalisation chiral symmetry: $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} 1\!\!1_2 & 0 \\ 0 & -1\!\!1_2 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} = \frac{1\!\!1 + \gamma_5}{2}\psi \ , \ \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} = \frac{1\!\!1 - \gamma_5}{2}\psi$$

chiral transformation:

$$\psi \to e^{i\alpha\gamma_5} \ , \ \bar{\psi} \to e^{+i\alpha\gamma_5}$$

$$\psi_R \to e^{-i\alpha}\psi_R \quad \psi_L \to e^{i\alpha}\psi_L$$

with

$$\{\gamma_5, \gamma_\mu\} = 0$$

$\bar{\psi} = \psi^\dagger\gamma_0 \to \psi^\dagger e^{-i\alpha\gamma_5}\gamma_0$

$\{\gamma_5, \gamma_\mu\} = 0 \longrightarrow \quad = \psi^\dagger\gamma_0 e^{i\alpha\gamma_5}$

It follows that

(a) $i\bar{\psi}\not{\partial}\psi \to i\bar{\psi}e^{i\alpha\gamma_5}\not{\partial}e^{i\alpha\gamma_5} = \bar{\psi}\underbrace{e^{i\alpha\gamma_5}e^{-i\alpha\gamma_5}}_{1\!\!1}\not{\partial}\psi$

(b) $i\bar{\psi}m\psi \to im\,\bar{\psi}e^{2i\alpha\gamma_5}\psi$

and $(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 = (\psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R)^2 - (\psi_R^\dagger\psi_L - \psi_L^\dagger\psi_R)^2$

$\gamma_0 = \begin{pmatrix} 0 & 1\!\!1 \\ 1\!\!1 & 0 \end{pmatrix}$ $\longrightarrow$ $= 2(\psi_R^\dagger\psi_L)(\psi_L^\dagger\psi_R)$

chiral rep.

(c) $\Rightarrow (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2 \longrightarrow (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2$

$\psi_R^\dagger\psi_L \to \psi_R^\dagger e^{-i\alpha}e^{i\alpha}\psi_L = \psi_R^\dagger\psi_L \ , \ \psi_L^\dagger\psi_R \to \psi_L^\dagger\psi_R$

Bosonisation: "Introducing the order parameter as a field"

σ - meson: $\sigma \sim \bar{\psi}\psi$

pion: $\pi \sim \bar{\psi}\gamma_5\psi$

Hubbard - Stratonovich

$$S_{QM}[\psi,\bar{\psi},\sigma,\pi] = \int d^4x \Big\{ i\bar{\psi}(\not{\partial}+m)\psi$$
$$+ ih\,\bar{\psi}(\sigma + i\gamma_5\pi)\psi$$
$$+ \frac{m_\sigma^2}{2}(\sigma^2+\pi^2)$$

kinetic term $\quad + \frac{1}{2}\int d^4x \big\{ \sigma(-\partial_\mu^2)\sigma + \pi(-\partial^2)\pi$

We have $S_{QM}[\psi,\bar{\psi},\sigma,\pi]\Big|_{EoM(\sigma,\pi)} = S_{NJL}[\psi,\bar{\psi}]$

EoM: $\frac{\partial S_{QM}}{\partial\sigma} = 0: \quad \sigma = -\frac{ih}{m_\sigma^2}\bar{\psi}\psi$

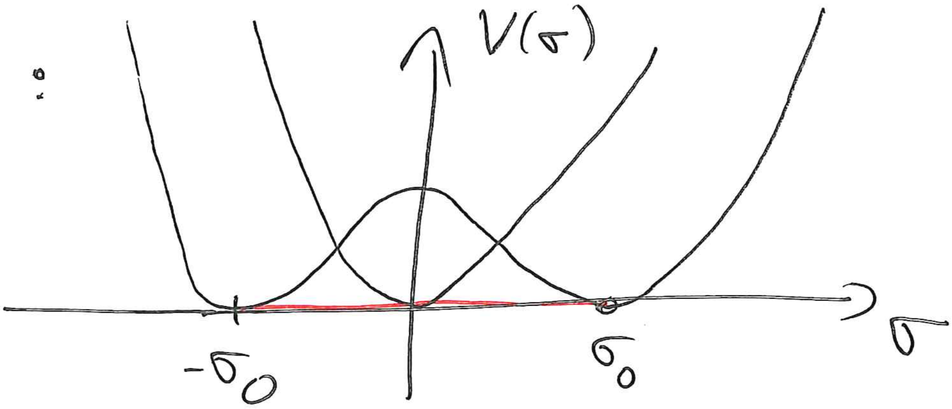
const. fields

$\frac{\partial S_{QM}}{\partial\pi} = 0: \quad \pi = \frac{h}{m_\sigma^2}\bar{\psi}\gamma_5\psi$

with $\boxed{\lambda_\psi = \frac{h^2}{m_\sigma^2}}$

Integrate out fermions $\rightarrow$ bosonic theory with $\underset{U(1)}{\underset{\cong}{O(2)}}$-symmetry.

Eff. pot:



Sym. breaking: $\langle \sigma \rangle = 0$ $\quad Z$ - sym.

$\langle \sigma \rangle \neq 0$ $\quad Z$-sym. breaking

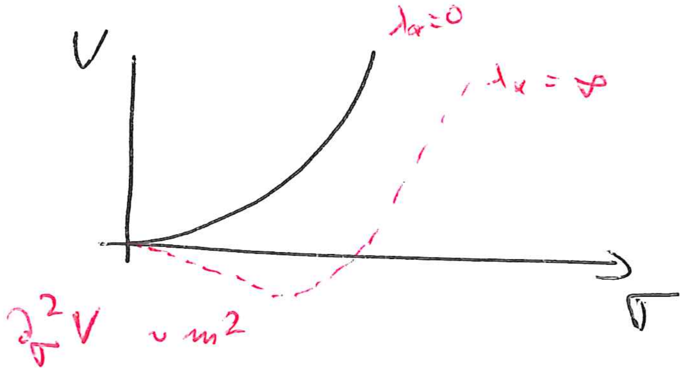
Fixed points:

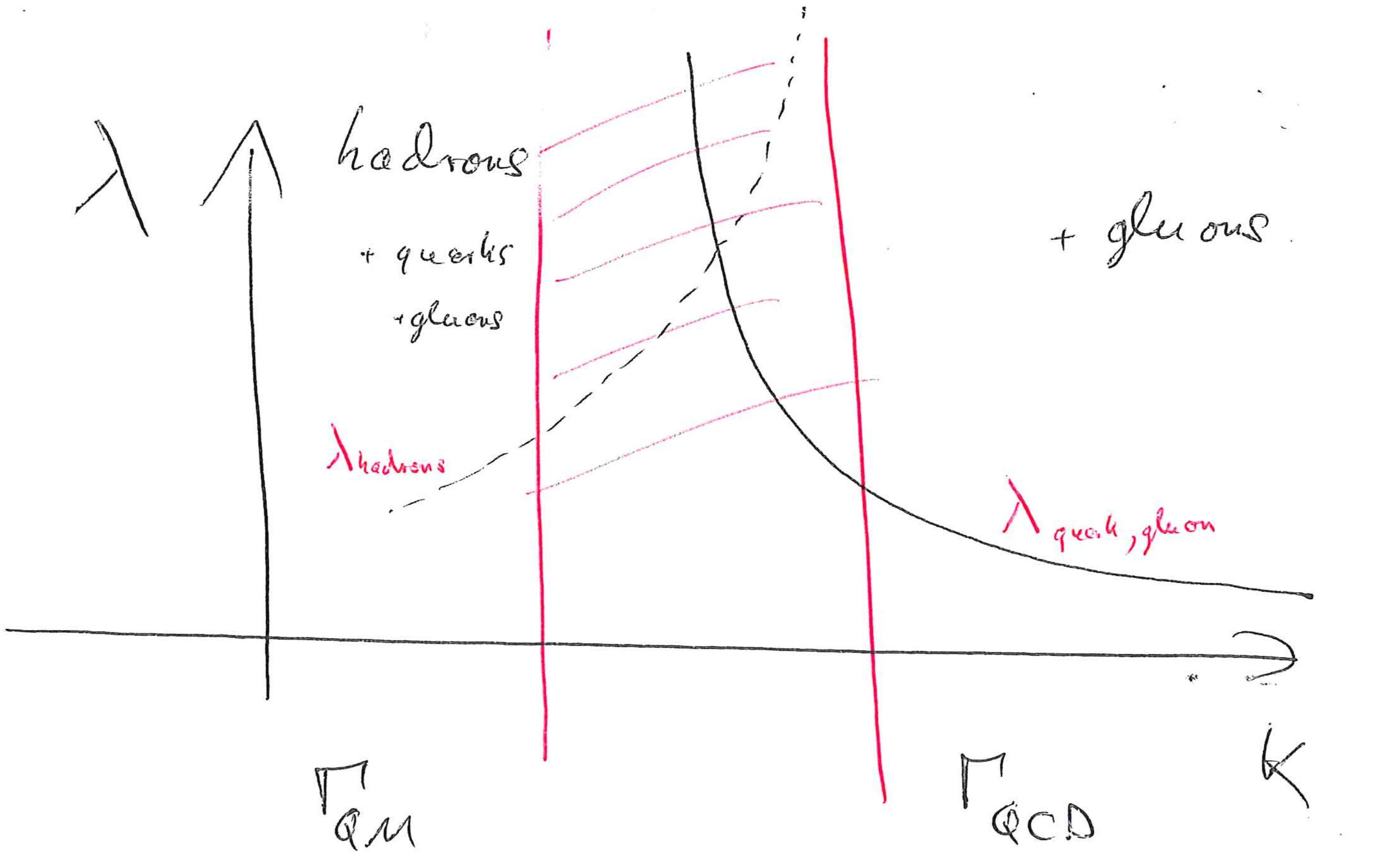
IR: $\lambda_* = 0$

$\lambda_* = \infty \quad \Leftarrow$

UV: $\lambda_* = \lambda_*(A)$

$$m_\sigma^2 \sim \frac{h^2}{\lambda_\varphi} \sim \begin{cases} 0 & \lambda_* = \infty \\ \infty & \lambda_* = 0 \end{cases}$$





chiral symmetry breaking:

$$\langle \bar\psi\psi \rangle \neq 0 \quad : \quad \sigma = 2\lambda_\psi \bar\psi\psi \; : \langle\sigma\rangle = -2\lambda_\sigma \langle\bar\psi\psi$$

$$\pi = 2i\lambda_\sigma \bar\psi\gamma_5\psi : \langle\pi\rangle = -2i\lambda_\sigma\langle\bar\psi\gamma_5$$

(part.) Bosonisation: (Hubbard-Stratonovich)

$$S_{QM}[\psi,\bar\psi,\sigma,\pi] = \int d^dx \left\{ \bar\psi i\partial\!\!\!/ \psi + i h_\sigma \bar\psi(\sigma + i\gamma_5 \pi)\psi - \boxed{\frac{h_\sigma^2}{4\lambda_\sigma}}(\sigma^2+\pi^2) \right\}$$

$\sim m_\sigma^2$ $\longrightarrow$ page II-79

+ kinetic term $\quad + \frac{1}{2}\int d^dx \left\{ \sigma(-\partial^2)\sigma + \pi(-\partial^2)\pi \right\}$

in QCD iso-vector $(\pi^0, \pi^\pm)$

EoM: $\frac{\partial S}{\partial\sigma} = 0$: $\quad \sigma = 2\lambda_\sigma \bar\psi\psi / h_\sigma$

$\frac{\partial S}{\partial\pi} = 0$: $\quad \pi = 2i\lambda_\sigma \bar\psi\gamma_5\psi / h_\sigma$

$\Rightarrow$ on-shell $\quad S_{QM} = S_{NJL}$

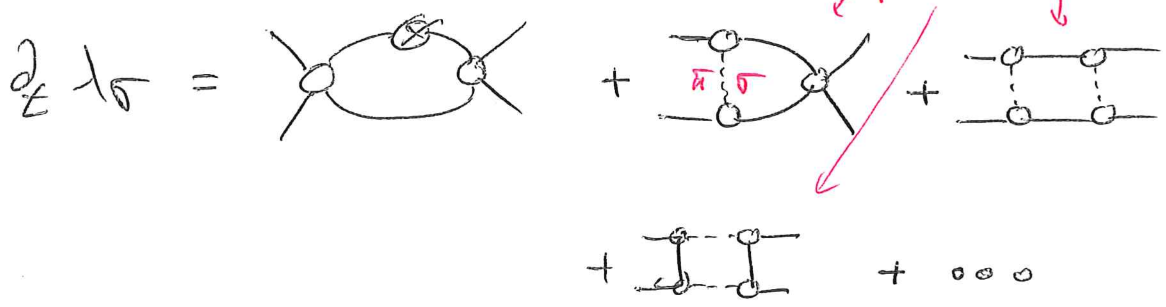
$$-\frac{1}{4}\lambda(\sigma^2+\pi^2-v^2)^2 \quad - c\sigma$$

explicit sym. break.

Integrate-out fermions: (non-) linear σ-model

Flow of λ_σ-coupling: (schematically)

(drop $\otimes$ insertions)



$$\dot{\hat\lambda}_\sigma = 2\hat\lambda_\sigma - A_k\,\hat\lambda_\sigma^2 + \dots \Rightarrow \text{Fig 1 page}$$

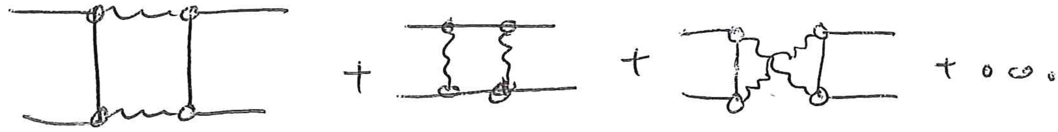
1) Warning: potential overcounting

2) Rebosonisation: → page II-80a

QCD: (1 flavour)

$$S_{QCD}[\phi,\psi,\bar\psi] = S[\phi] + \int d^dx\,\bar\psi\, i\not{D}\,\psi$$

$$\text{with } i\not{D} = i\not\partial + ig\not{A}$$



$$\Rightarrow \Gamma = S_{QCD} + \int d^dx\,\lambda_\sigma\left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2\right] + \dots$$

Rebosonisation:

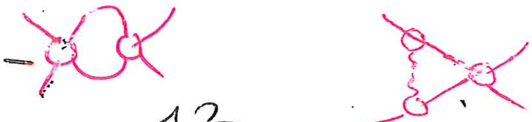
Idea: Gies, Wett '01

bosonise at each flow-step, e.g.

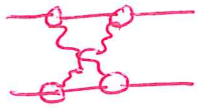
$$(\lambda_{\sigma_k}, h_{\sigma_k}, m_{\sigma_k}, \lambda_k) \to (\lambda_\sigma, \bar{h}_{\sigma_k}, \bar{m}_{\sigma_k}, \bar{\lambda}_k)$$

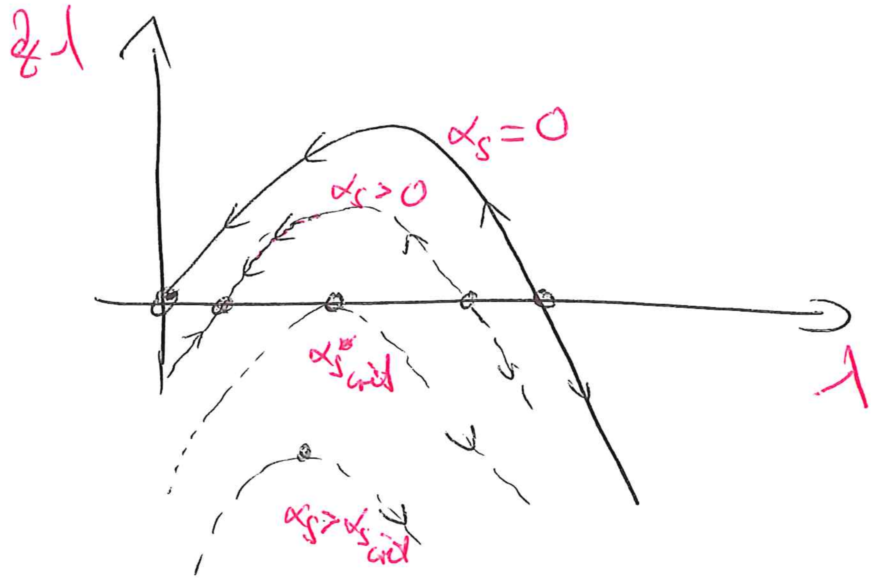
- absorb running of λ_σ in running of Yukawa coupling h_σ, mass m_σ, ϕ^4-coupl. λ.
- moves bound state/condensate fractions in λ_σ to bosonic dofs.

Flow of $\hat{\lambda}_4$:



$$\dot{\hat{\lambda}}_4 = 2\hat{\lambda}_4 + \underbrace{A_4(T)}_{<0}\hat{\lambda}^2 + \underbrace{B_4(T)}_{<0}\lambda\alpha_s + \underbrace{C_4(T)}_{<0}\alpha_s^2 + \ldots$$





FRG: $\boxed{\alpha_{s_{crit}} \sim 0.85}$ $\underset{= N_c}{(3 = N_f)}$, $T=0$ Gies, Jäckel '05

Landau gauge: $\alpha_s^* \sim 3$

$T>0$: $\boxed{\alpha_{s_{crit}}(T) > \alpha_{s_{crit}}(0)}$

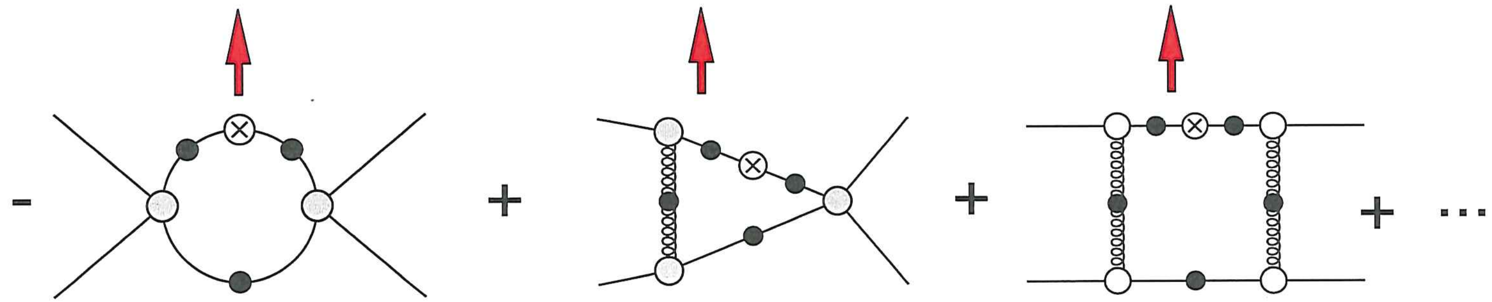
Remark: low energy QCD successfully modelled with Quark-Meson model

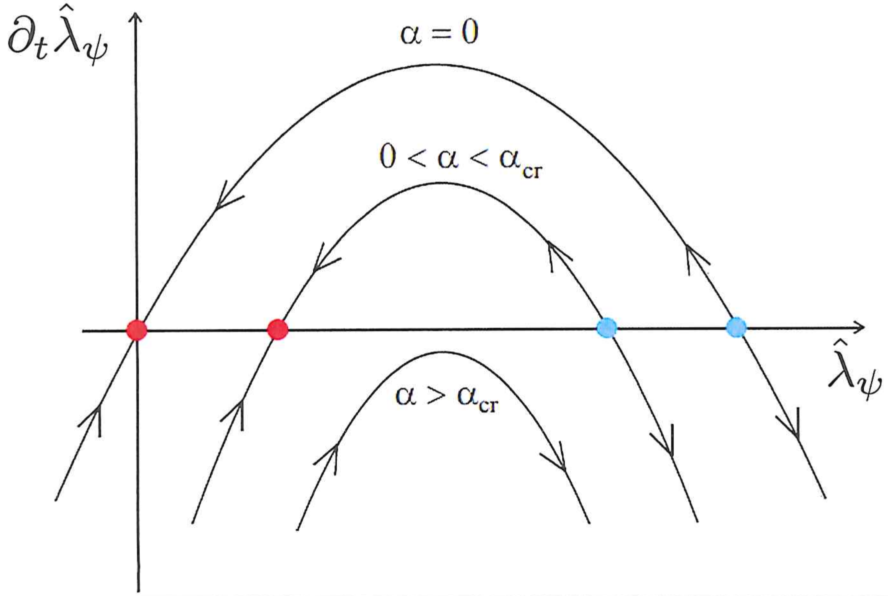
Chiral symmetry breaking

A glimpse at chiral symmetry breaking in QCD within the FRG

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi + A\left(\frac{T}{k}\right)\hat{\lambda}_\psi^2 + B\left(\frac{T}{k}\right)\hat{\lambda}_\psi\,\alpha_s + C\left(\frac{T}{k}\right)\alpha_s^2 + \cdots$$



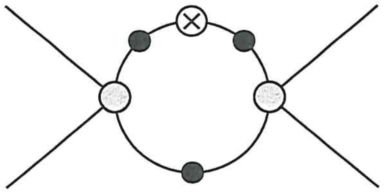
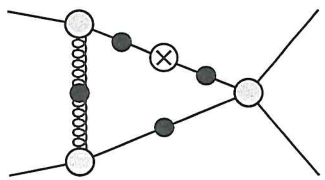
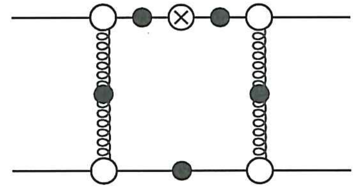


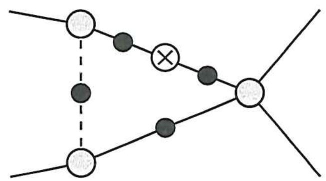
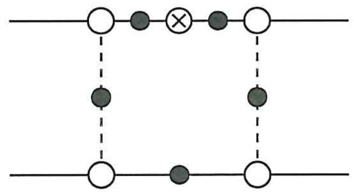
$T = 0$: $\alpha_{cr} \sim 1$

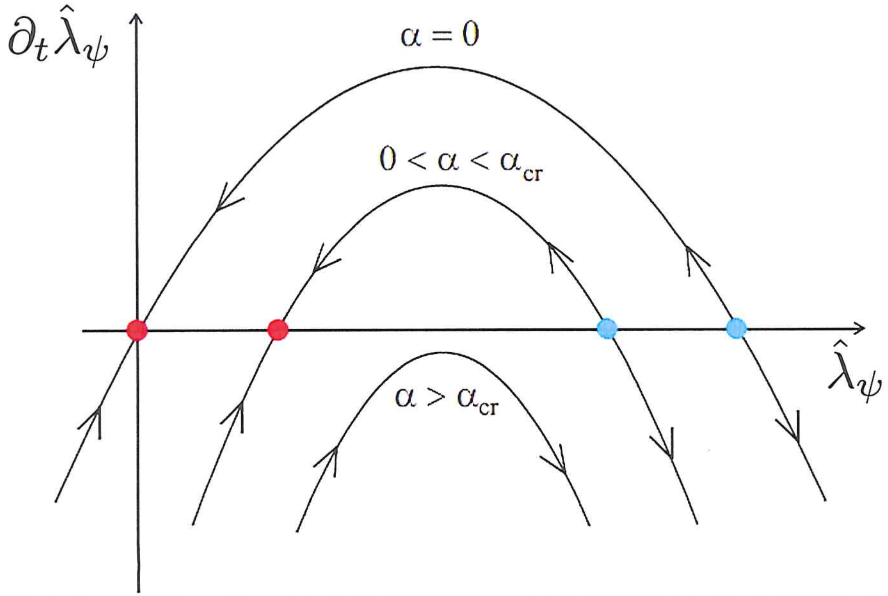
$T > 0$: $\alpha_{cr}(T) > \alpha_{cr}(0)$

Dynamical hadronisation

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$k\partial_k \hat{\lambda}_\psi \;=\; 2\hat{\lambda}_\psi$ -  +  + 

\+  + 



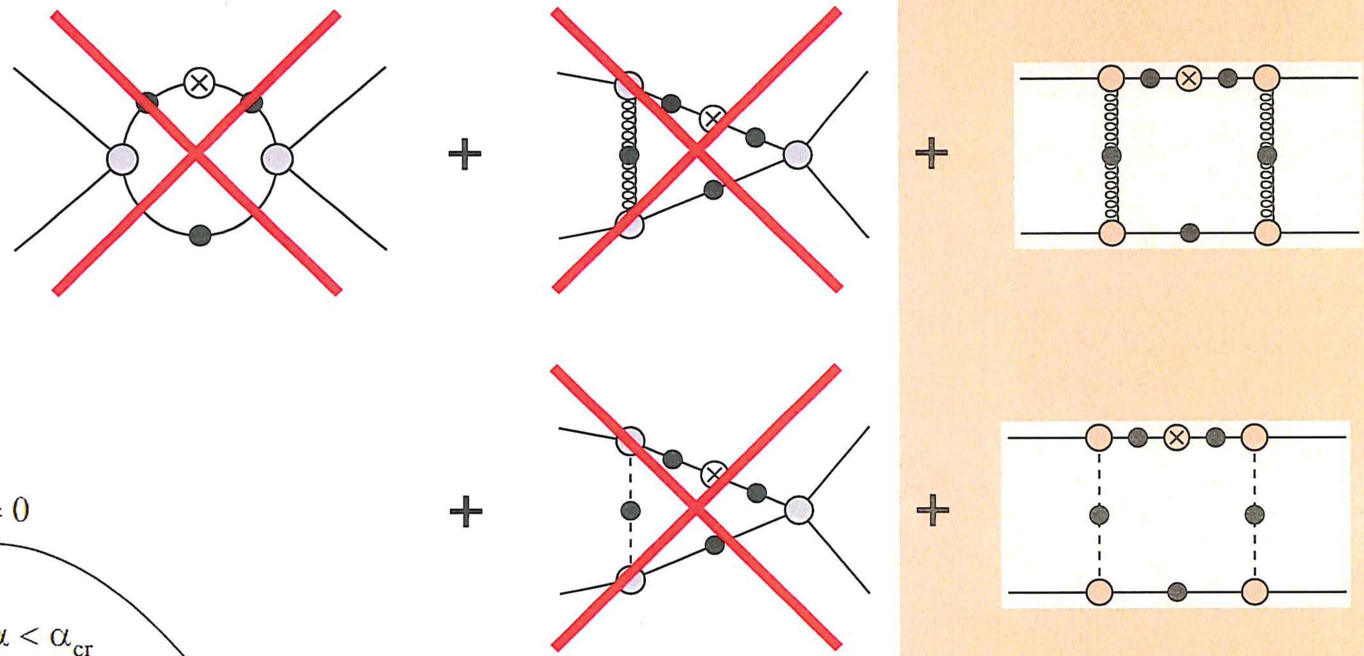
$+ \; \partial_t \dfrac{h^2}{m^2}$ **- terms**

$+ \; \cdots$

Dynamical hadronisation

Full bosonisation $\hat{\lambda}_\psi = 0$

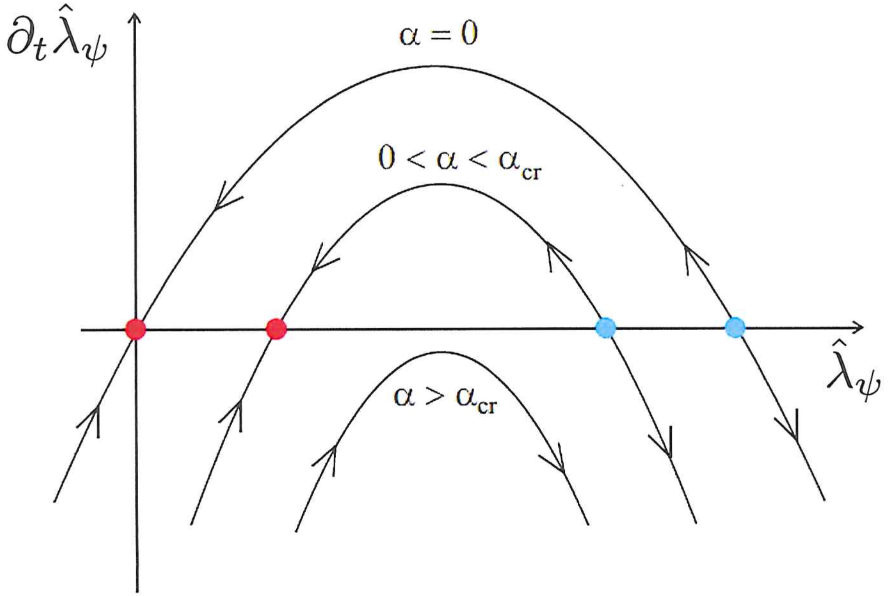
$\cancel{k\partial_k \hat{\lambda}_\psi - 2\hat{\lambda}_\psi}$ -



$+ \partial_t \frac{h^2}{m^2}$ - terms

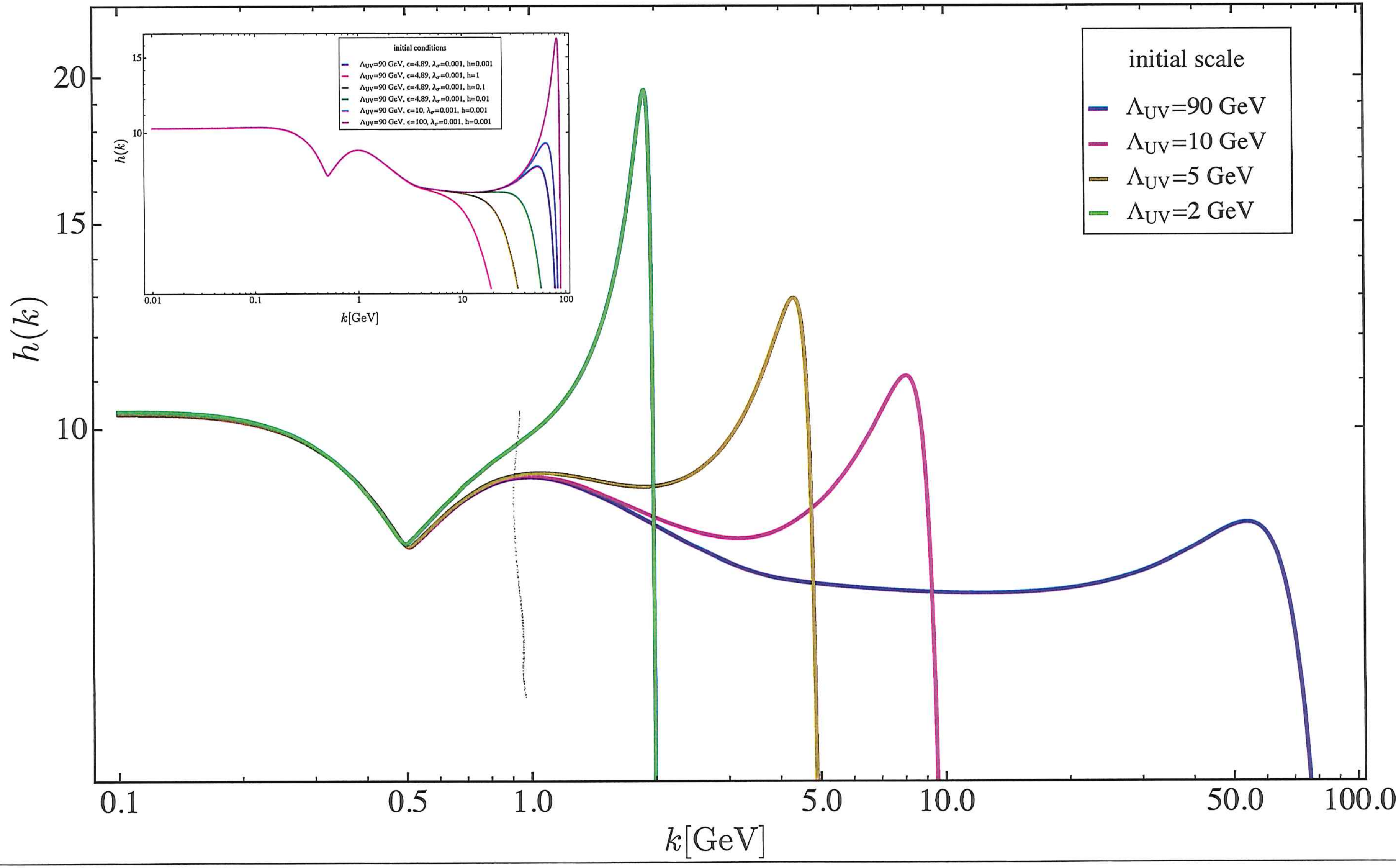
$+ \cdots$

$= 0$

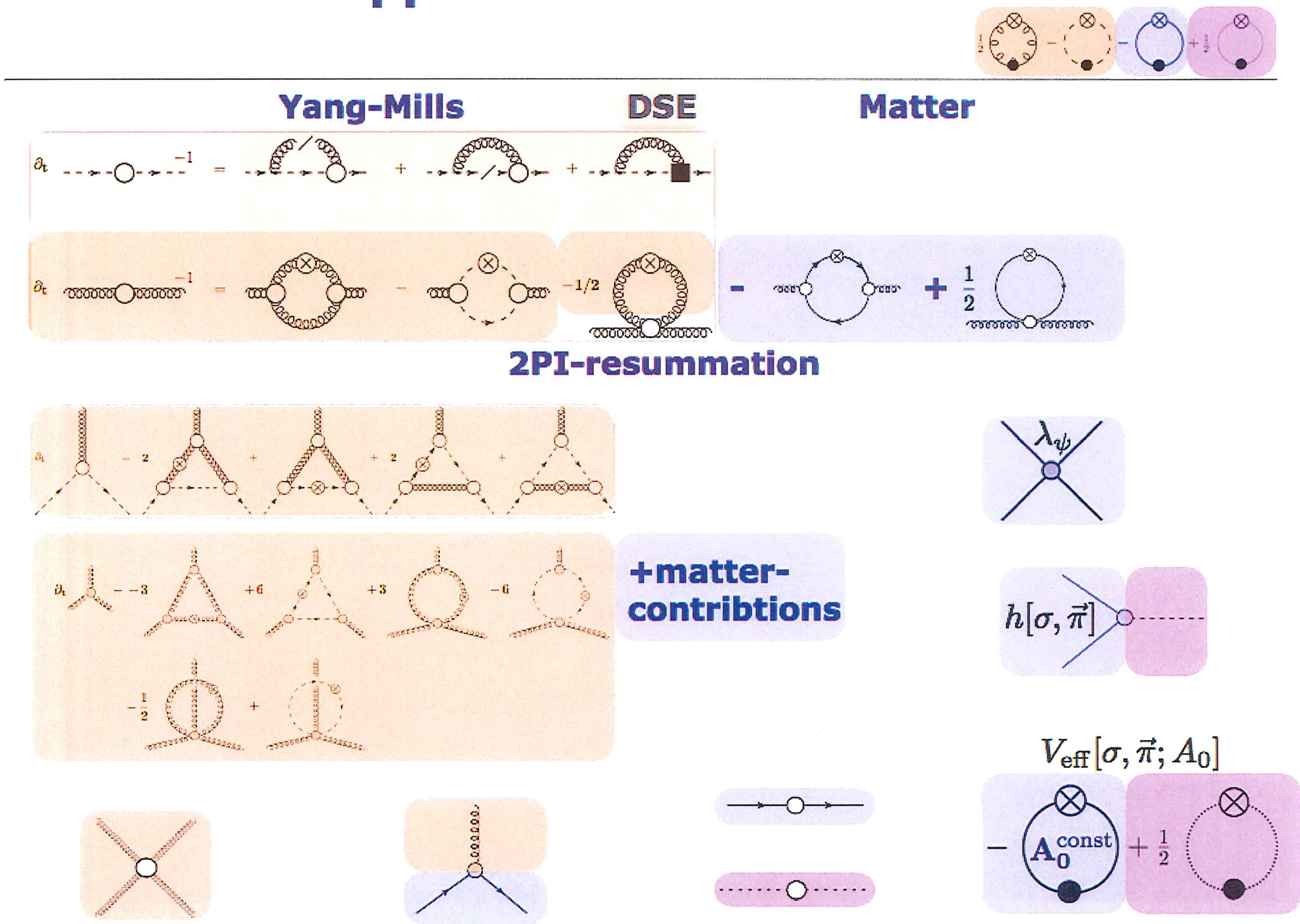


Dynamical hadronisation

Full bosonisation $\hat{\lambda}_\psi = 0$



Approximation scheme



Bonus: dynamical creation of Hadrons

Challenges:

(1) QCD flow at all scales

(2) finite μ

$\Rightarrow$ bound state spectrum of QCD

(3) Non-equilibrium