1.2 Functional RG equations for Yang-Mills

Bootstrap: assume

$$Z_{ren}[J] \quad \text{is given}$$

$Z_{ren}[J]$ is the <u>renormalised</u>, <u>finite</u> generat. fct of <u>renormalised</u>, <u>finite</u> Green fcts.

$$\boxed{Z_k[J] := e^{\displaystyle -\underbrace{\tfrac{1}{2}\int d^dx \frac{\delta}{\delta J_i} R_{k\,ij} \frac{\delta}{\delta J_j}}_{\Delta S_k[\frac{\delta}{\delta J}]}} Z_{ren}[J]}$$

YM: $\frac{\delta}{\delta J_i} R_{k\,ij} \frac{\delta}{\delta J_j} e^{-\int J_i \phi_i} = \left(\frac{\delta}{\delta J^a_\mu} R^{ab}_{\mu\nu} \frac{\delta}{\delta J^c_\nu} + 2\frac{\delta}{\eta} R^{ab} \frac{\delta}{\delta \bar{\eta}}\right) e^{-\int J_i \phi_i} = A^a_\mu R^{ab}_{\mu\nu} A^b_\nu + 2\bar{c}^a R^{ab} c^c$

↑ p.5-18

- $Z_k[J]$ finite, as Z_{ren} admits by def. infinitely many derivatives; and $e^{-\Delta S_k[\frac{\delta}{\delta J}]} < 1$.
- no path integral is <u>needed</u>.

Flow equation for Z_k:

$$\partial_t Z_k[J] = -\left(\partial_t \Delta S_k\left[\frac{\delta}{\delta J}\right]\right) Z_k[J]$$

$$= -\frac{1}{2}\int d^dx \, \partial_t R_{k\,ij} \frac{\delta}{\delta J_j}\frac{\delta}{\delta J_i} Z_k[J]$$

Flow equation for W_k: $W_k = \ln Z_k$

$$\partial_t W_k[J] = -\frac{1}{2}\int d^dx \, \partial_t R_{k_{ij}}\left(W^{(2)}_{ji}[J] - W^{(1)}_i W^{(1)}_i \right)$$

with $\quad W^{(n)}_{i_1 \cdots i_n} = \frac{\delta}{\delta J_{i_1}} \cdots \frac{\delta}{\delta J_{i_n}} W$

Flow equation for $\Gamma_k = \sup_J \left\{ \int d^dx \, J\cdot\phi - W_k \right\} - \Delta S_k[\phi]$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left[\frac{1}{\Gamma^{(2)} + R_k}\right]_{ij} \partial_t R_{k\,ij}$$

Yang-Mills theories:

$\phi = (A, c, \bar{c})$

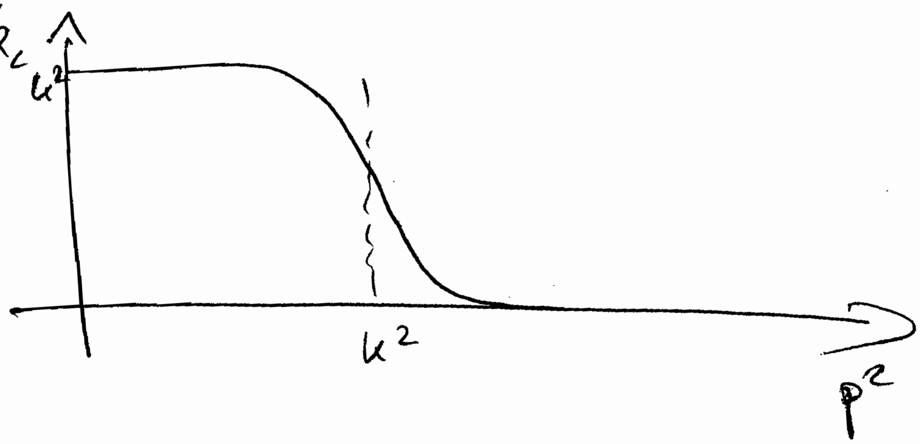
$$\mathcal{R}_k = \begin{pmatrix} R_{A\,\mu\nu}^{ab} & 0 & 0 \\ 0 & 0 & -R_c^{ab} \\ 0 & R_c^{ab} & 0 \end{pmatrix}$$

$$\Rightarrow \frac{1}{2}\int d^dx\, \phi_i\, \mathcal{R}_{k\,ij}\, \phi_j$$

$$= \frac{1}{2}\int d^dx\, A_\mu^a\, R_{A\,\mu\nu}^{ab}\, A_\nu^b$$

$$+ \int d^dx\, \bar{c}^a R_c^{ab}\, c$$

with

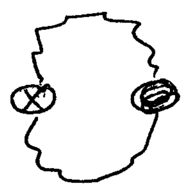


$$R_{A\,\mu\nu}^{ab} = \Gamma_{\mu\nu}^{ab}\, R_A$$

$$\mathcal{R}_c^{ab} = \Gamma_c^{ab}\, \mathcal{R}_c$$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\left[\frac{1}{\frac{\delta^2\Gamma_k}{\delta\phi^2}+R}\right]^{ab}_{AA\,\mu\nu}\partial_t R_{A\;\mu\nu}^{\;\;ab}$$

$$-\,\mathrm{Tr}\left[\frac{1}{\frac{\delta^2\Gamma_k}{\delta\phi^2}+R}\right]^{ab}_{c\bar{c}}\partial_t R_c^{\;ab}$$

$= \frac{1}{2}$ 

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 : $$\frac{1}{\frac{\delta^2\Gamma_k}{\delta c\,\delta\bar{c}}+R_c}$$

 : $$\frac{1}{\frac{\delta^2\Gamma_k}{\delta A^2}+R_A}$$

1.3 Gauge invariance

reminder II-40 *:

$$\text{STI:}\quad \Big(\underbrace{J^i \mathcal{G}^i[\tfrac{\delta}{\delta J}] + \overset{\overset{0}{\shortparallel}}{\frac{\delta \mathcal{G}^i}{\delta \phi^i}} - \frac{\delta S}{\delta \phi^i}\mathcal{G}^i}_{W_{\mathcal{G}}[J,\frac{\delta}{\delta J}]}\Big)_{\phi=\frac{\delta}{\delta J}} Z[J]=0$$

with $\mathcal{G} = (\mathcal{D}\,, g\, c\,, g\,\bar{c})$

$$\Rightarrow \quad W_{\mathcal{G}}[J,\tfrac{\delta}{\delta J}]\; Z[J] = W_{\mathcal{G}}[J,\tfrac{\delta}{\delta J}]\, e^{\Delta S_k[\frac{\delta}{\delta J}]}\, \underbrace{e^{-\Delta S_k[\frac{\delta}{\delta J}]}}_{Z[J]}$$

$$=0$$

$$= e^{\Delta S_k[\frac{\delta}{\delta J}]}\Big(W_{\mathcal{G}}[J,\tfrac{\delta}{\delta J}] - (\Delta S_k[\tfrac{\delta}{\delta J}]J^i)\,\mathcal{G}^i[\tfrac{\delta}{\delta J}]\Big)$$

$$\cdot\, Z_k[J]=0$$

$$\Rightarrow \boxed{\Big(\underbrace{W_{\mathcal{G}}[J,\tfrac{\delta}{\delta J}] - R^{ji}\frac{\delta}{\delta J^j}\,\mathcal{G}^i[\tfrac{\delta}{\delta J}]}_{W_{\mathcal{G}_k}[J,\frac{\delta}{\delta J}]}\Big) Z_k[J]=0}$$

mSTI , mWI

$$- R^{ji} \frac{\delta}{\delta J_j}\, \mathcal{G}^i\left[\frac{\delta}{\delta J}\right] Z_k[J]$$

$$= -\langle D^{ac}_{\mu} R_{A_k\,\mu\nu}{}^{cb} A^b_\nu \rangle - g\Big(f^{abc}\langle \bar{c}^d R_c^{db} c^c\rangle$$

$$\underbrace{+ f^{abc}\langle \bar{c}^c R_c^{bd} c^d\rangle}_{g\langle \bar{c}^b [t^a, R_c]^{bc} c^c\rangle}\Big)$$

$$= -\langle \delta^a \, \Delta S_k \rangle$$

For effective action:

$$\boxed{\delta^a \Gamma[\phi] = \langle \delta^a (S_{gf} + S_{gh})\rangle + \underbrace{\langle \delta^a \Delta S_k\rangle - \delta^a \Delta S_k}_{\text{loop-terms}}}$$

1 & 2 loop terms in full propagators

BRST-identity : $s(A_\nu, c, \bar{c}) = (D_\nu c, \frac{1}{2} f^{abc} c^b c^c, \frac{1}{\xi} \partial a$

without Reg.: $\langle s \, J \cdot \phi \rangle = 0 \quad \Leftarrow s \, S[\phi] = 0$

$$\boxed{\int \underbrace{\langle s \, \phi \rangle}_{} \frac{\delta \Gamma[\phi]}{\delta \phi} = 0}$$

↳ contains two-point functions

⇒ Sources for BRST-variations

$$Z[J] \to Z[J, Q] = \int d\phi \, e^{-S[\phi] + \int d^d x \; Q_\phi \cdot s\phi}$$

with,

$$\int d^d x \; Q_\phi \cdot s\phi = \int d^d x \left\{ K \cdot sA + L s C + \bar{L} \, s \bar{C} \right\}$$

BRST variation: $s \int d^d x \; Q_\phi \, s\phi = \int d^d x \quad s^2 \bar{C}$

$$\text{cov. gauge} \to = \int d^d x \; \bar{L} \, \partial_\nu \, s A_\nu$$

$$\text{with} \quad s \; K \cdot sA = - K \overbrace{(sD)}^{DC} C - K \overbrace{D s C}^{\frac{1}{2} D C^2} = 0 \quad s(Dc) = 0$$

$$s \, L \, s \, C = L \underbrace{s^2 C}_{c^3} = 0$$

$$\Rightarrow \langle s\phi\rangle = \frac{\delta W[J,Q]}{\delta Q} \qquad \text{with } W[J,Q] = \ln Z[J,Q]$$

$$= -\frac{\delta\Gamma[\phi,Q]}{\delta Q}$$

BRST-identity:

$$\left\langle \int s\, J\cdot\phi + \int s\, Q_\phi \cdot s\phi \right\rangle$$

$$= \int \langle s J\cdot\phi\rangle + \frac{1}{\xi}\int \bar{c}\, \partial_\nu \langle s A_\nu\rangle = 0$$

Effective action:

$$\boxed{\int \frac{\delta\Gamma}{\delta Q_\phi}\cdot\frac{\delta\Gamma}{\delta\phi} + \frac{1}{\xi}\int \bar{c}\, \partial_\nu \frac{\delta\Gamma}{\delta K_\nu} = 0}$$

$\bar{c}\to\bar{c}+f(x)$: $\boxed{\partial_\nu \frac{\delta\Gamma}{\delta K_\nu} = \frac{\delta\Gamma}{\delta\bar{c}}}$

BRST-identity in the presence of regulator

$$\boxed{\int \frac{\delta\Gamma_k}{\delta Q_\phi}\cdot\frac{\delta\Gamma_k}{\delta\phi} + \frac{1}{\xi}\int \bar{c}\,\partial_\mu\frac{\delta\Gamma_k}{\delta a_\mu} + \underbrace{\langle s\Delta S_k\rangle - s\Delta S_k}_{\text{Loop - terms}} = 0}$$

$$\frac{\delta(\Gamma_k+\Delta S_k)}{\delta Q_\phi}\,\frac{\delta(\Gamma_k+\Delta S_k)}{\delta\phi}$$

with $\frac{\delta\Delta S_k}{\delta Q_\phi} = 0$

1-loop with full props, if using BRST-source-terms

$$\boxed{\partial_\mu\frac{\delta\Gamma_k}{\delta a_\mu} = \frac{\delta\Gamma_k}{\delta\bar{c}}}$$

$\langle s\Delta S_k\rangle - s\Delta S_k$: Reduction to 1-loop: II-53a*

$$\langle s \tfrac{1}{2} A_\mu^a R_{\mu\nu}^{ab} A_\nu^b \rangle - s \tfrac{1}{2} A_\mu^a R_{\mu\nu}^{ab} A_\nu^b$$

$$= \langle A_\mu^a R_{\mu\nu}^{ab} s A_\nu^b \rangle - A_\mu^a R_{\mu\nu}^{ab} s A_\nu^b$$

$$= \frac{\delta}{\delta K_\nu^b} \langle A_\mu^a R_{\mu\nu}^{ab} \rangle \, . \, .$$

$$= R_{\mu\nu}^{ab} \frac{\delta}{\delta J_\mu^a} \frac{\delta}{\delta K_\nu^b} W_k [J, Q]$$

$$= - R_{\mu\nu}^{ab} \frac{\delta}{\delta J_\mu^a} \frac{\delta}{\delta K_\nu^b} \Gamma_k [\Phi, Q]$$

$$= - R_{\mu\nu}^{ab} \frac{\delta \Phi_i}{\delta J_\mu^a} \frac{\delta}{\delta \Phi_i} \frac{\delta \Gamma_k [\Phi, Q]}{\delta K_\nu^a}$$

$$= - R_{\mu\nu}^{ab} \frac{\delta^2 W_k}{\delta J_\mu^a \delta J_i} \frac{\delta^2 \Gamma_k}{\delta \Phi_i \delta K_\nu^a}$$

$$= - R_{\mu\nu}^{ab} \left(\frac{1}{\Gamma^{(2)} + R} \right)_\mu^{a\ i} \frac{\delta^2 \Gamma_k}{\delta \Phi_i \delta K_\nu^a}$$

Applications:
gluonic mass term: $R_k=0$: STI: $p_\mu \Gamma^{(2)}_{\mu\nu} = \frac{1}{\xi} p^2 p_\nu$

parametr.: $\Gamma^{(2)ab}_{\mu\nu}(p^2) = \delta^{ab} p^2 \left[Z_{A_\perp}(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \left(Z_{A_L}(p^2) + \frac{1}{\xi}\right) \frac{p_\mu p_\nu}{p^2} \right]$

$$-i \frac{\delta}{\delta A^a_\mu(p)} \frac{\delta}{\delta C^b(q)} S\Gamma_k \Big|_{\phi=0} (p) = p_\nu \Gamma^{(2)}_{A;\nu\mu}(p,q)$$

$$= p_\mu p^2 \delta^{ab} Z_{A_L}(p^2) \delta(p-q) + \frac{1}{\xi} p^2 p_\nu$$

$$\Rightarrow \quad -i \frac{\partial}{\partial p_\nu} \frac{\delta}{\delta A^a_\nu(p)} \frac{\delta}{\delta C^b_q} S\Gamma_k \Big|_{\phi=0} (p=0) = 4 \; p^2 Z_{A_L}(p^2) \Big|_{p^2=0}$$

$$=: m^2_{k_L}$$

e.g. one loop: $[r_c = r_A]$ $\quad R_\phi = p^2 \cdot r_\phi(p^2/k^2)$ $\quad$ Landau

$$m^2_{k_L} = g^2 \frac{N}{16\pi^2} \int dy \, y^{d/2-2} \frac{r(y)}{(1+r(y))^2} \left(\frac{11}{2} - d - 5/d + \xi(1 - 1/d) + \left(7/2 - 6/d\right) \frac{y \partial_y r}{1+r(y)} \right)$$

$$\gtrless 0$$

Comparison with $\int m_k$ from flow

gives consistency check!?

Beware: $\boxed{m^2_{k_L} \boxed{\neq} p^2 Z_{A_\perp}(p^2)\big|_{p^2=0}}$

$\Gamma^{(2)}_{\mu\nu}(p^2)$
$= p^2 \{ Z_{A_\perp}(p^2)(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$
$+ (Z_{A_L}(p^2) + 1) \frac{p_\mu p_\nu}{p^2}$

Remarks on (m)STI's:

see also Fischer, Maas, JMP '08

Ward ID's / STIs' connect a longitudinal projection of a n-point fct $\Gamma^{(n)}_{\mu_1\cdots\mu_n}$ with $\Pi_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$

$$\Gamma^{(n)}_{L\,\mu_1\cdots\mu_n} = \Pi_{\mu_1\nu}(p_1)\Gamma^{(n)}_{\nu\,\mu_2\cdots\mu_n}(p_1,\ldots,p_n)$$

with other m-point fcts via a functional relation,

$$\boxed{\Gamma^{(n)}_{L\,\mu_1\cdots\mu_n} = F^{(n)}_{STI}\left[\{\Gamma^{(m)}_L\}, \{\Gamma^{(m)}_\perp\}\right]}$$

where $\Gamma^{(m)}_\perp$ are the fully transversal correlat. fcts.,

$$\boxed{\Gamma^{(n)}_{\perp\,\mu_1\cdots\mu_n} = \prod_{i=1}^{n} \Pi_{\mu_i\nu_i}(p_i)\,\Gamma^{(n)}_{\nu_1\cdots\nu_n}(p_1,\ldots,p_n)}$$

FRGs and DSEs in the Landau gauge connect $\Gamma^{(n)}_\perp$ with $\{\Gamma^{(m)}_\perp\}$ only:

$$\boxed{\Gamma^{(n)}_\perp = F^{(n)}_{FRG/DSE_\perp}\left[\{\Gamma^{(m)}_\perp\}\right]}$$

Proof: $F^{(n)}_{FRG/DSE\perp}$ are a sum of diagrams, constructed from the propagators

$$G_{\phi_1\phi_2} = \left[\frac{1}{\Gamma^{(2)}(+R_k)}\right]_{\phi_1\phi_2}$$

and vertices

$$\Gamma^{(n)}, \quad S^{(n)}$$

Due to the projections all external legs of $\Gamma^{(n_{ext})}$ in the diagrams are transversal. The gauge field propagator is transversal, hence all internal legs of $\Gamma^{(n)}$'s are transversal: all $\Gamma^{(m)}$ in the diagrams are fully transversal, the $\Gamma^{(2)}$ in G also is due to the Landau gauge. $\boxed{}$

Any solution of $F_{FRG/DSE\perp}$ can be put into F_{STI} which can be solved for $\Gamma_L^{(n)}$ in the approximation at hand.

This structure entails, that the STI's do not directly fix any $\Gamma^{(n)}_{\perp}$.

Assuming uniformity, that is

$$\Gamma^{(n)}_{t\,\mu_1\cdots\mu_n} \simeq \Gamma^{(n)}_{L\,\mu_1\cdots\mu_n}$$

gives a relation. In a weak form this means

$$\left| \partial_{p_1} \Gamma^{(n)}(p_1,\dots,p_n)\Big|_{p_1=0} \right| < \infty$$

Note in this context that

$$\partial_{p_\rho} \Pi_{\mu\nu}(p) = -\left(\delta_{\rho\mu}\frac{p_\nu}{p^2} + \delta_{\rho\nu}\frac{p_\mu}{p^2}\right)$$

and $\left|\partial_{p_\rho}\Pi_{\mu\nu}(p)\right|_{p=0}$ is ill-defined!

In perturbation theory uniformity can be assumed, non-perturbatively it is at stake.

For more details see Fischer, Maas, JMP '08

Gribov-problem revisited

(1) $k \to \infty$: perturbation theory near the global minimum

$$\Gamma_k[\phi] \simeq S[\phi] + \text{rel. corrections}$$
$$+ \ 1/k\text{-corrections}$$

rel. correction: e.g. mass-term for A_μ

(2) full Flow preserves convexity

Litim, Pawlowski, Vergara '06

i.e. $\boxed{\frac{\delta^2}{\delta c \, \delta \bar{c}} \Gamma_k + R_{k,c} \geqslant 0}$

$\Leftrightarrow \boxed{\langle -\partial_\mu D_\mu \rangle \geq 0}$

- $\boxed{\Gamma_k \text{ 'stays' in first Gribov region}}$

- $\boxed{\text{no preference for specific copy}}$

1.4 Optimisation

$$\Gamma_k[\Phi] = \frac{1}{2}\int \frac{d^d p}{(2\pi)^d} A_\mu^a(p)\,\Gamma_A^{(2)\,ab}{}_{\mu\nu}(p)\,A_\nu^b(-p)$$

$$+ \int \frac{d^d p}{(2\pi)^d}\,\bar{C}^a(p)\,\Gamma_c^{(2)\,ab}(p)\,C(-p)$$

$$+ \sum_{\vec{i}} c_{\vec{i}} \prod_{j=1}^{3} \int \frac{d^d p_j}{(2\pi)^d}\,\phi_{i_j}(p_j)\,\Gamma^{(3)}_{i_1 i_2 i_3}(p_1,\dots,p_3)$$

$$+ \sum_{\vec{i}} c_i \prod_{j=1}^{4} \int \frac{d^d p_j}{(2\pi)^d}\,\phi_{i_j}(p_j)\,\Gamma^{(4)}_{i_1\dots i_4}(p_1,\dots,p_4)$$

$$+ \ \circ \ \circ \ \circ$$

Truncation: (i) $\Gamma^{(2)}(p) \Leftarrow$ full moment. dep. of propagators

(ii) approx. of $\Gamma^{(3)}$ and $\Gamma^{(4)}$

momentum dep. deduced from $\Gamma^{(2)}$; $\boxed{\Gamma^{(n>4)} \hat{=} 0}$

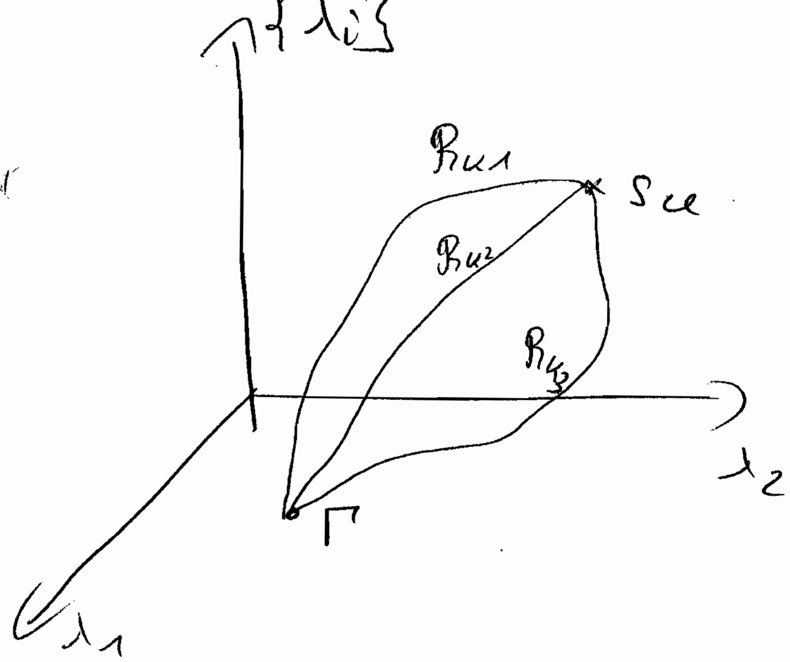
(iii) quarks / hadrons

effective pot.

Question: Can we accomodate for the left-out terms?

Are there regulators that optimise (maximise) the physical content of a given truncation?

Theory space (space of couplings)



we shall see: optimal flow $\hat{=}$ shortest flow

Example: $O(N)$-Theory, fixed point properties

g. critical exponents in derivative expansion

$$\nu = \underset{\uparrow}{\nu_0} + \underset{\nearrow}{\nu_1} + \cdots$$

order derivative expansion

with $\nu_i = \nu_i(R_k)$

: $\boxed{\text{Correlat. length } \xi \sim (T-T_c)^{-\nu}}$

$O(N)$:

II-56b

$$\Gamma_k[\phi] = \int d^d x \left\{ \tfrac{1}{2} \vec{\phi} \, \overset{(-\partial^2)}{\overset{\shortparallel}{p^2}} \vec{\phi} + U_k[\underbrace{\tfrac{1}{2}\vec{\phi}\vec{\phi}}_{\rho}] \right\}$$

Flow: $\partial_t U_k[\rho] = 2^{d+1} \pi^{d/2} \Gamma[d/2] \left((N-1) \, l_0^d \left(\frac{U_k'(\rho)}{k^2} \right) + l_0^d \left(\frac{U_k'(\rho) + 2\rho U_k''(\rho)}{k^2} \right) \right)$

with

$$l_n^d(\omega) = (\delta_{n,0} + n) \int_0^\infty dy \, y^{d/2-1} \frac{-y^2 r'(y)}{(y(1+r) + \omega)^{n+1}}$$

Difference of orders, eg. $V_i - V_{i+1}$ ~ differences in flow:

$$\sim \frac{1}{\Gamma^{(2)}_{ki} + R_k} - \frac{1}{\Gamma^{(2)}_{k\,i+1} + R_k}$$

$$= \frac{1}{\Gamma^{(2)}_{ki} + R_k} \left(\Gamma^{(2)}_{k\,i+1} - \Gamma^{(2)}_{ki} \right) \frac{1}{\Gamma^{(2)}_{k\,i+1} + R_k}$$

⇒ **Every** expansion is an expansion in orders of the propagator!

General series: $S = \sum a_n \varepsilon^n$

Quickest convergence $\hat{=}$ maximise convergence radius

For flow:

$R_{k\,opt} = R_k \in \{R_k\}_{compare}$ with $\Gamma^{(2)} + R_k$ maximal $\Rightarrow \frac{1}{\Gamma^{(2)} + R}$ minimal

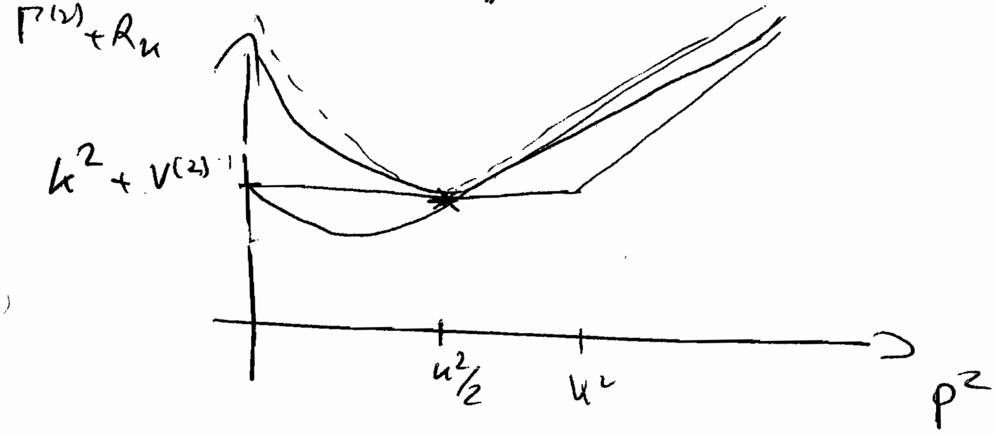
Litim '00

$\{R_k\}_{compare} = \{ R_k(k^2_{eff}) = c\, k^2_{eff} \}$ ($c = 1$ standard choice)

Example: 0th order derivative expansion

$$\Gamma^{(2)}(p) + R_k(p^2) = \boxed{p^2 + R_k(p^2)} + V^{(2)}$$

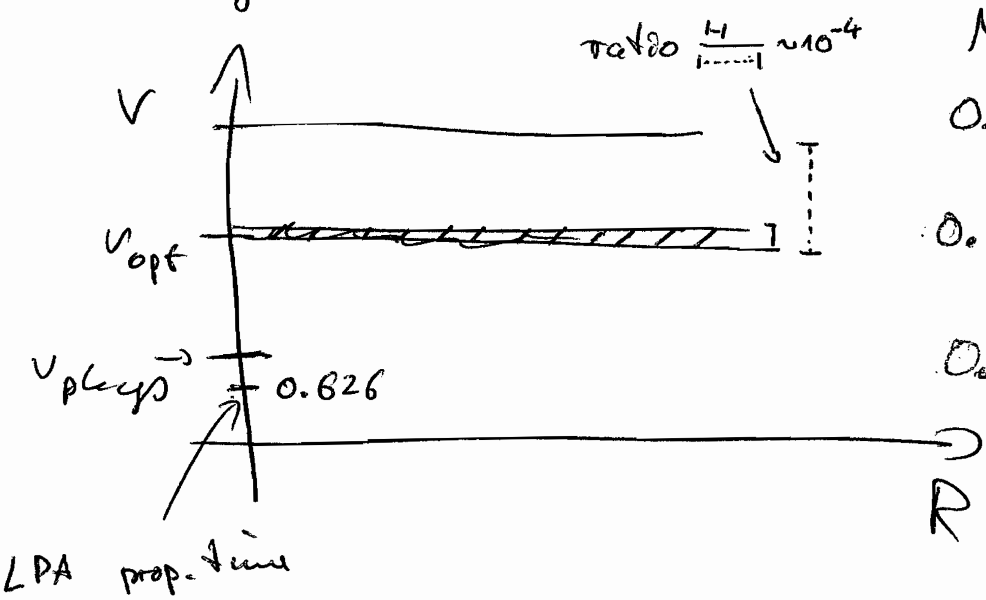
$$k_{eff}^2 = \frac{1}{2} k^2$$



$\Rightarrow$ $\boxed{\text{optimal regulators: } p^2 + R_k(p^2) > k^2}$

+ flatness: $R_{flat}(p^2) = (k^2-p^2)\,\Theta(k^2-p^2)$ Litim '00

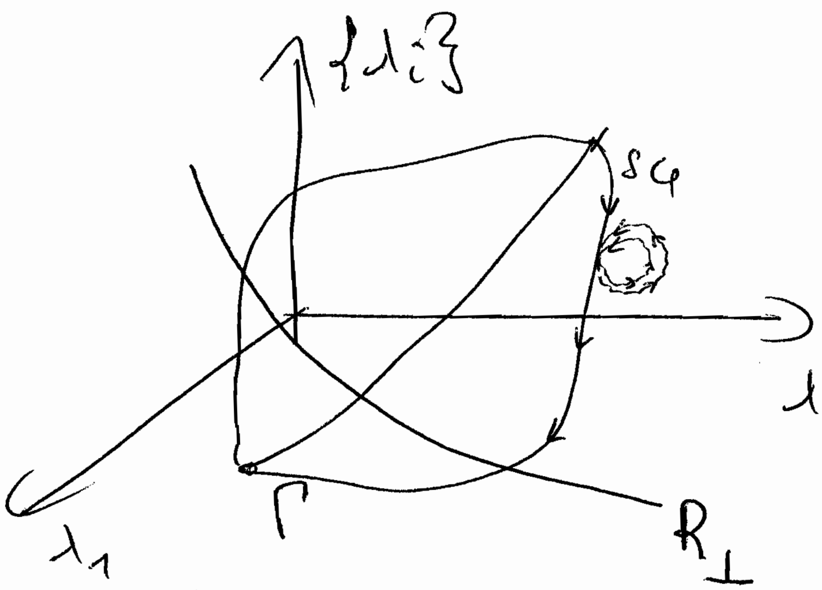
(schematically



N = 1, Ising	
0.69	0.6895
0.65	0.6496
0.63	0.6301
	↑ LPA PTRG $m = 10$

why is R_{flat} optimal?

Reformulate convergence/stability idea:



- minimise the flow:

$$\frac{\delta \dot{\Gamma}}{\delta \phi} = -\frac{1}{2}\,\underbrace{\mathrm{Tr}\left[\frac{1}{\Gamma^{(2)}+R}\,\dot{R}\,\frac{1}{\Gamma^{(2)}+R}\,\frac{\delta}{\delta\phi}\frac{\delta}{\delta\phi}\right]}_{\text{flow operator}}\frac{\delta\Gamma}{\delta\phi}$$

$\Rightarrow$ minimise the flow operator.

$$\boxed{\left.\frac{\delta}{\delta R_\perp}\right|_{R_{opt}} \frac{1}{\Gamma^{(2)}+R}\,\dot{R}\,\frac{1}{\Gamma^{(2)}+R} = 0}$$

$\{R_\perp\}$: set of [illegible]

R_{opt} minim. flow $\longrightarrow \left.\frac{\delta \Gamma^{(2)}}{\delta R_\perp}\right|_{R_{opt}} = 0$

$$= \left.\frac{\delta}{\delta R_\perp}\right|_{R_{opt}} \left.\frac{\partial}{\partial t}\right|_{\Gamma^{(2)}} \frac{1}{\Gamma^{(2)}+R} = \left.\frac{\partial}{\partial t}\right|_{\Gamma^{(2)}} \left.\frac{\delta}{\delta R_\perp}\right|_{R_{opt}} \frac{1}{\Gamma^{(2)}+R} = 0$$

$$\Rightarrow \boxed{\left.\frac{\delta}{\delta R_\perp}\right|_{R_{opt}} \frac{1}{\Gamma^{(2)}+R} = 0}$$

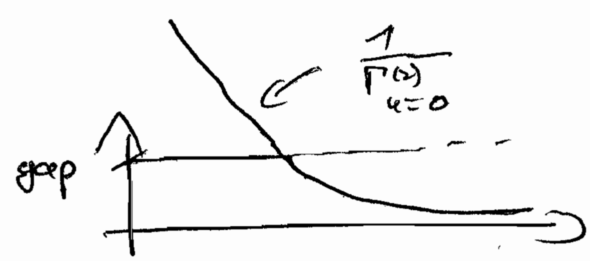
choice of $\{R_\perp\}$: (i) $R(k_{eff}^2) = c\, k_{eff}^2$

(ii) $\max \frac{Z}{\Gamma^{(2)}_\phi + R_\perp} = \max \frac{Z'}{\Gamma^{(2)}_\phi + R'_\perp} = \text{gap}$

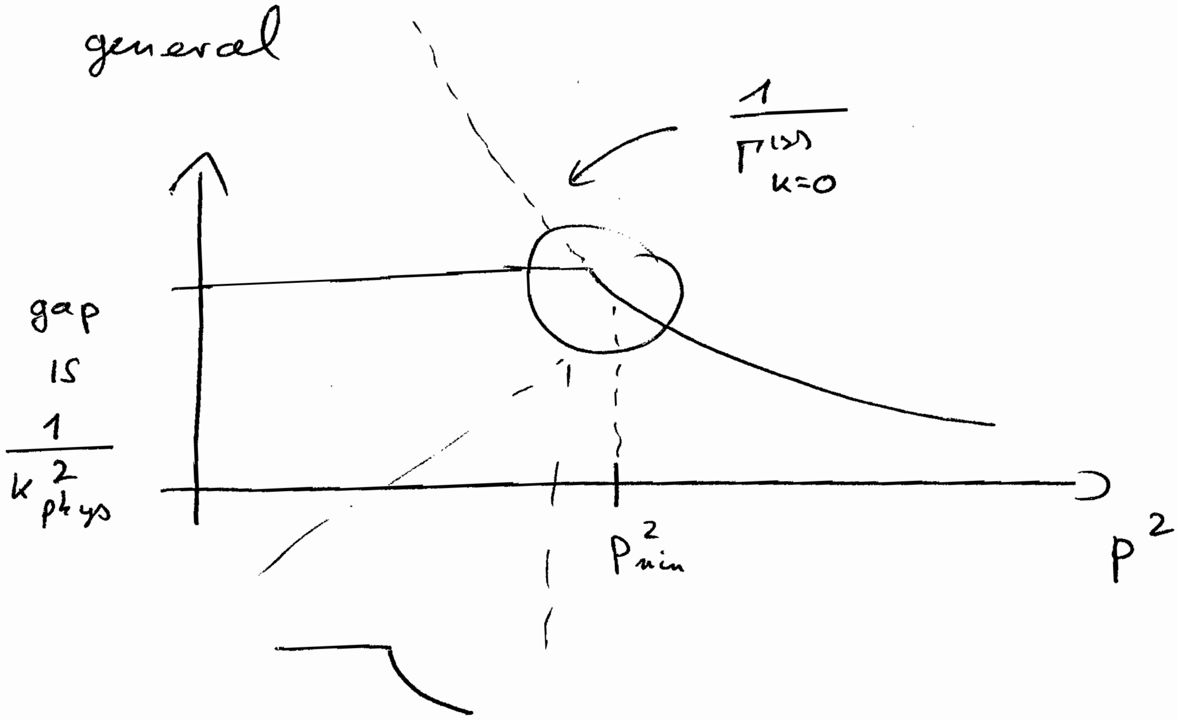
with (ii) it follows for 0th order derivative expansion:

$$\left.\frac{1}{p^2 + R_\perp(p^2)}\right|_{R_{opt}} \quad \text{maximal}$$

$$\Rightarrow R_{opt} = (k^2 - p^2)\,\Theta(k^2 - p^2)$$



In general



$$\left\| \frac{\delta}{\delta R_\perp} \frac{1}{\Gamma^{(2)}+R} \right\| = 0$$

$\Rightarrow$ derivative exp. : $\|\cdot\|$ sensitive to derivat.

Flow acts on $\mathcal{O}$ with Taylorexp.