Non-perturbative aspects of gauge theories
Exercise sheet 10 – Chiral symmetry breaking

Lectures: Jan Pawlowski j.pawlowski@thphys.uni-heidelberg.de
Tutorials: Nicolas Wink n.wink@thphys.uni-heidelberg.de
Institut für Theoretische Physik, Uni Heidelberg due date: 14 January 2019

Exercise 18: Chiral symmetry breaking

In this exercise we consider the mechanism behind chiral symmetry breaking. For this we use the ansatz

$$S_{\mathrm{M}} = \int_x \left\{ \bar{\psi} \left(\mathrm{i}\partial\!\!\!/ + \bar{g} A \right) \psi + \bar{\lambda}_{\alpha\beta\gamma\delta} \bar{\psi}_\alpha \bar{\psi}_\beta \psi_\gamma \psi_\delta \right\} . \tag{1}$$

Proceed by determining the dimensions of the couplings $\bar{\lambda}$ and $\bar{g}$ and introduce suitable dimensionless couplings λ and g. Determine the relevant diagrams contributing at one-loop order to λ and write down schematically the β_λ-function, you should find

$$\beta_\lambda = \partial_t \lambda = (d-2)\lambda - a\,\lambda^2 - b\,\lambda g^2 - c\,g^4 . \tag{2}$$

In the following you can assume $a, b, c > 0$, as well as $\partial_t g = 0$. Examine the fixed points of (2) and find the critical value of the gauge coupling g^2_{cr} for which there is only a single fixed point. Plot the β_λ-function for a gauge coupling below, above and at the critical value and discuss the relation to chiral symmetry breaking in QCD.