# Non-perturbative aspects of gauge theories
# Exercise sheet 11 – Gauge fixing in Quantum Gravity


Lectures: Jan Pawlowski — j.pawlowski@thphys.uni-heidelberg.de
Tutorials: Nicolas Wink — n.wink@thphys.uni-heidelberg.de
Manuel Reichert — reichert@cp3.sdu.dk
Institut für Theoretische Physik, Uni Heidelberg


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### Exercise 19: Gauge fixing in Quantum Gravity

In this exercise we consider the gauge fixing and the ghost-graviton interactions in quantum gravity. We start from the gauge-fixing condition

$$F_\mu[\bar{g}, h] = \bar{\nabla}^\alpha h_{\alpha\mu} - \frac{1+\beta}{4} \bar{\nabla}_\mu \bar{g}^{\alpha\beta} h_{\alpha\beta} \,. \tag{1}$$

Compute the Faddeev-Popov operator

$$\begin{aligned} M_{\mu\nu}[\bar{g}, h] &= \frac{\partial F_\mu}{\partial h_{\alpha\beta}} \frac{\partial}{\partial \omega^\nu} \mathcal{L}_\omega(g_{\alpha\beta}) \\ &= \bar{\nabla}^\rho \left( g_{\mu\nu} \nabla_\rho + g_{\rho\nu} \nabla_\mu \right) - \frac{1+\beta}{2} \bar{\nabla}_\mu \bar{g}^{\gamma\rho} g_{\nu\gamma} \nabla_\rho \,. \end{aligned} \tag{2}$$

The Lie-derivative acting on a rank two tensor is given by $\mathcal{L}_\omega T_{\alpha\beta} = \omega^\mu \nabla_\mu T_{\alpha\beta} + (\nabla_\alpha \omega^\mu) T_{\mu\beta} + (\nabla_\beta \omega^\mu) T_{\alpha\mu}$. Throughout $\nabla_\mu$ is constructed with respect to the full metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$, while $\bar{\nabla}_\mu$ is constructed with respect to the background metric $\bar{g}_{\mu\nu}$.

This results in the ghost-graviton interactions that are described by the action

$$S_{\text{gh}}\left[\bar{g}, h, c, \bar{c}\right] = \int \mathrm{d}^4 x \sqrt{\bar{g}}\, \bar{c}^\mu M_{\mu\nu} c^\nu \,. \tag{3}$$

The interaction between gravitons and ghost is linear in the graviton, i.e., $c\bar{c}h$. Find a good argument or proof by computing the functional derivatives of (3).