Non-perturbative aspects of gauge theories
Exercise sheet 13 – Scalars in Quantum Gravity

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Exercise 21: Scalars in Quantum Gravity

In this exercise we consider N_s scalar fields minimally coupled to quantum gravity. The ansatz for the effective action is given by

$$\Gamma = \Gamma_{\text{gravity}} + \sum_{i=1}^{N_\mathrm{s}} \frac{Z_{\varphi^i}}{2} \int \mathrm{d}^4x \sqrt{g}\, g^{\mu\nu} \partial_\mu \varphi^i \partial_\nu \varphi^i \,. \tag{1}$$

We want to estimate qualitatively the influence of the scalar fields on the fixed-point value of the Newton's coupling and of the cosmological constant. In a fluctuation computation we would obtain this from the scalar contribution to graviton n-point functions. Draw all diagrams that contribute to the graviton two- and three-point function. You should find that all diagrams contain a closed scalar loop and hence all contributions are proportional to N_s.

For simplicity we compute the scalar contributions to the beta functions from the background field approximation. Compute with heat-kernel methods the trace of right-hand side of the Wetterich equation

$$\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k}\right]_{\varphi\varphi} \partial_t R_k \,. \tag{2}$$

In straight analogy to the exercise from last week, choose the regulator function as

$$R_k = \Gamma^{(2)}_{\varphi\varphi} \cdot r_k\left(\frac{\bar\Delta}{k^2}\right) \qquad \text{with} \qquad r_k(x) = \left(\frac{1}{x} - 1\right)\Theta(1-x)\,. \tag{3}$$

The flow involves contributions from the scalar anomalous dimension $\eta_\varphi = -\partial_t \ln Z_\varphi$. We could determine those from the flow of scalar two-point function $\partial_t \Gamma^{(2)}_{\varphi\varphi}$, but here we set them simply to zero, $\eta_\varphi = 0$, which is indeed a good approximation is most cases.

In straight analogy to last week, compare the $\sqrt{g}$ and the $\sqrt{g}R$ terms in order to obtain the contributions to the beta function. You should find

$$
\begin{aligned}
\partial_t g_k &= \beta_{g,\mathrm{gravity}} + g_k^2 \frac{N_\mathrm{s}}{6\pi} \left(1 - \frac{\eta_\varphi}{4}\right) \,, \\
\partial_t \lambda_k &= \beta_{\lambda,\mathrm{gravity}} + g_k \frac{N_\mathrm{s}}{4\pi} \left(1 - \frac{\eta_\varphi}{6}\right) \,.
\end{aligned}
\tag{4}
$$

In which direction are the fixed point values g_k^* and λ_k^* pushed? How do these changes influence the pure gravity contributions $\beta_{g,\mathrm{gravity}}$ and $\beta_{\lambda,\mathrm{gravity}}$, and how does this back-react on the fixed-point values? (See exercise sheet 12 for the precise form of these contributions.) Make a guess: which effect wins in the end?