Non-perturbative aspects of gauge theories
Exercise sheet 2 – FRG equations

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This exercise sheet will be rather similar to the first one, but we are considering a different method , i.e. the Functional Renormalisation Group, and a slightly different theory. The (Euclidean) Lagrangian for this theory is given by

$$\mathcal{L}[\phi] = \phi_a \frac{1}{2}(p^2 + m^2)\phi^a + \frac{\lambda}{4!}(\phi_a\phi^a)^2 \,, \tag{1}$$

which is $O(N)$-symmetric, i.e. $a \in (1, \dots N)$. Due to spontaneous symmetry breaking the invariant $\rho = \frac{1}{2}\phi_a\phi^a$ can assume a finite expectation value, which we assume to be space-time independent throughout this exercise sheet.

Exercise 4: Effective potential from the FRG

In the following we are going to calculate a differential equation for the effective potential in d space-time dimensions for a scalar theory. In order to truncate the infinite tower of equations we work in a derivative expansion, i.e. full correlation functions at vanishing momentum are given derivatives of the effective potential. This time we have to take the O(N) symmetry into account, therefore we introduce the invariant $\rho = \frac{1}{2}\phi_a\phi^a$. The propagator is again given by

$$\Gamma^{(2)}_{\phi_i\phi_j}(p) = p^2 + U^{(2)}(\rho) \,, \tag{2}$$

taking the substructure into account we arrive at

$$\frac{\delta}{\delta\phi_a}\frac{\delta}{\delta\phi_b}U(\rho) = \delta_{ab}U^{(1)}(\rho) + \delta_{1a}\delta_{1b}2\rho U^{(2)}(\rho) \,. \tag{3}$$

Therefore, you can explicitly work with a single component field σ and a (N-1) component field π. As a regulator we are using the Litim regulator

$$R_k(p) = (k^2 - p^2)\Theta\left(k^2 - p^2\right) \,. \tag{4}$$

Derive the partial differential equation for the effective potential. As an intermediate step you should find

$$\partial_t U(\rho)_k = \frac{1}{2}\int_q \left(\frac{N-1}{k^2 + U_k^{(1)}(\rho)} + \frac{1}{k^2 + U_k^{(1)}(\rho) + 2\rho U_k^{(2)}(\rho)} \right) \left(\partial_t R_k(q)\right) \,. \tag{5}$$

As a last step, solve the integral on the right-hand side of (5).

Exercise 5: FRG equation for the two-point function

Solving Exercise 6 renders this exercise trivial.
Derive the FRG equation for the two-point function for the same theory as in Exercise 4.

Exercise 6*: FRG equations for general theories

Derive the FRG equation for the two-point function in the superfield formalism. In order to do this, recall the definitions from *Exercise 3*. The master equation reads

$$\dot{\Gamma}_k[\Phi] = \frac{1}{2}\,\mathrm{Tr}\left\{G_{ab}\dot{R}^{ab}\right\}\,, \tag{6}$$

where the dot denote the derivative with respect to RG-time $t = \ln k/\Lambda$.