Non-perturbative aspects of gauge theories
Exercise sheet 3 – FRG equations II

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Exercise 7: Perturbation Theory

Go through the section in the script that discusses perturbation theory as a truncation of FRG equations. In particular, recollect the derive eq. (2.72) (the one-loop correction to the the effective potential) and eq. (2.74) (the one-loop beta function of the quartic coupling λ).

Exercise 8: Vertex expansion

Consider a simple ϕ^4 theory, as in *Exercise 1*, but this time within a vertex expansion. The task to derive the flow equation for $\Gamma^{(3)}_{k,\phi\phi\phi}(p_1, p_2)$. In order to keep the equation traceable we consider $\Gamma^{(n\geq 5)}_{k,\phi^n}(p_1, \ldots, p_{n-1}) = 0$.

For this exercise it is beneficial to work in DeWitt notation. As an exercise keep track of the explicit momentum contractions, i.e. the index contractions within this notation. The key relation you need the reads

$$\frac{\delta}{\delta\phi_i} G_{jk} = -G_{ja}\Gamma_k^{aib} G_{bk} \, . \tag{1}$$

You can also (1) in a graphical manner, keeping track of the index contractions can simply be done by labelling the external propagators with indices. For this theory you can use the invariance of the n-point functions under the exchange of legs to reduce the number of diagrams/terms in the end.

As an additional bonus exercise (and demonstration of the power of the notation above), you can also derive the diagrams for the different three-point functions in the $O(N)$ theory of *Exercise 4* in the broken phase. Keep in mind that every vertex should contain a even number of legs of the Goldstone modes.