# Non-perturbative aspects of gauge theories
# Exercise sheet 5 – Fermions

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### Exercise 10: Flow equations for a Yukawa theory

In this exercise we are deriving the flow equation for the effective potential in the LPA. The ansatz for the effective action is given by

$$\Gamma_k = \int_x \left\{ \bar\psi \left(\partial\!\!\!/ + m_\psi\right)\psi + \frac{h_\sigma}{\sqrt{2}}\bar\psi\left(\sigma + \mathrm{i}\,\gamma_5\pi\right)\psi_a + \left(\partial_\mu\phi\right)^2 + V(\phi)\right\} . \tag{1}$$

Start by obtaining the two-point functions in momentum space and proceed by inverting them in order to obtain the propagators. As regulators we use the usual Litim regulator

$$\begin{aligned} R_k^\phi(p) &= p^2 r_\phi(x) \equiv p^2\left(\frac{1}{x} - 1\right)\Theta\left(1-x\right) \\ R_k^\psi(p) &= \mathrm{i} p\!\!\!/\, r_\psi(x) \equiv \mathrm{i} p\!\!\!/ \left(\frac{1}{\sqrt{x}} - 1\right)\Theta\left(1-x\right) \ , \end{aligned} \tag{2}$$

where $x = p^2/k^2$. As regulated propagators you should find

$$\begin{aligned} G_{\bar\psi\psi}(p) &= \frac{-i\,\left(p\!\!\!/\left[1+r_\psi\right]\right) + M_\psi}{p^2\left[1+r_\psi\right]^2 + M_\psi^2} \\ G_{\pi\pi}(p) &= \frac{1}{p^2\left[1+r_\phi\right] + V'(\rho)} \\ G_{\sigma\sigma}(p) &= \frac{1}{p^2\left[1+r_\phi\right] + V'(\rho) + 2\rho V''(\rho)} \ , \end{aligned} \tag{3}$$

where $M_\psi = m_\psi + h_\sigma\rho$. Finally, the flow equation for the effective potential is given by

$$\partial_t V(\rho) = \frac{1}{2}\mathrm{Tr}\ G_{\pi\pi}\dot R_k^\phi + \frac{1}{2}\mathrm{Tr}\ G_{\sigma\sigma}\dot R_k^\phi - \mathrm{Tr}\ G_{\bar\psi\psi}\dot R_k^\psi \, . \tag{4}$$

Carry out the traces in (4)

### Exercise 11*: Beyond LPA

Consider the ansatz for the effective action in Exercise 10, i.e. (1), and introduce the wave function renormalisations $Z_\phi$ and $Z_\psi$ in an appropriate manner. Redefine the mass terms in a way that all propagators become inverse proportional to their wave-functions, i.e. $G_{\varphi\varphi} \sim Z_\varphi^{-1}$ with $\varphi = \phi, \psi, \dots$. As regulators we employ the once from (2), but dressed with their appropriate wave function renormalisations, i.e. $R_k^\varphi \to Z_\varphi R_k^\varphi$. In order to cancel the wave function renormalisations out of all equations introduce the anomalous dimension

$$\eta_\varphi = -\frac{\partial_t Z_\varphi}{Z_\varphi} \,. \tag{5}$$

Derive the flow equation for the effective potential.

*Hint: Take a detailed look at the regulator derivative.*

The equations for the anomalous dimensions can be obtained from the flow equations of the two-point functions by appropriate projections. Derive the flow equations the two-point functions $\Gamma^{(2)}_{\pi\pi}$ and $\Gamma^{(2)}_{\bar\psi\psi}$.

*Hint: The only difference to previous exercises is the signs from fermions.*

Find a suitable procedure, i.e. projection, in order to obtain algebraic equations for the anomalous dimensions.