Non-perturbative aspects of gauge theories
Exercise sheet 7 – Yang-Mills propagators I

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Exercise 13: Derivation of the flow equations for ghost and gluon

In this exercise you are going to derive explicit expressions for the flow equations of the propagators in Yang-Mills theory.

The generic outline for calculations like this is usually as follows:

- Write down an ansatz for the effective action, which contains the classical one
- Derive the dispersions and introduce suitable regulators
- Derive the expressions/Feynman rules for all n-point functions
- Derive/Chose projectotion operators for all quantities of interest
- Take appropriate function derivatives of the master equation to obtain equations for all objects of interest
- Insert the Feynman rules into the equations
- Project the equations onto the objects of interest with the derived projection operators
- If possible: Carry out all traces over internal groups
- If possible: Carry out all momentum integrals

The order is a guideline, but the list includes all necessary steps that have to be taken.

Turning to Yang-Mills theory, our ansatz for the effective action is the classical action (see 5.33 in the lecture notes) in Landau gauge together with full two-point functions. Start by introducing an appropriate parametrisation for both the gluon and the ghost as

$$\Gamma^{\varphi\varphi}(p) = Z_{\varphi\varphi}(p)(p^2 + m_{\varphi\varphi}^2) \tag{1}$$

which is compatible with their canonical dispersion relations, but pay attention to potential differences due to the non-trivial tensor structure in the gluon propagator. Choose a specific $m^2_{\bar{c}c}$, while we leave m^2_{AA} unspecified for now. Introduce appropriately dressed regulators as $R_\varphi(p) \sim Z_{\phi\phi}(p)p^2 r(x)$, with $x = p^2/k^2$, again paying attention to the tensor structures at hand. You can find more details on this in Section 5.4 in the lecture notes.

Proceed by finding suitable projection operators for $\left[\Gamma^{AA}(p)\right]^{ab}_{\mu\nu}$ and $\left[\Gamma^{\bar{c}c}(p)\right]^{ab}$ and also invert the two-point functions in order to obtain the propagators. Introduce the anomalous dimension $\eta_{\varphi\varphi}(p)$.

Derive Feynman rules from the ansatz for the effective action, up to potential sign you can find the result in Appendix F.1.

Take functional derivatives of the master flow equation, 5.45 (neglecting quarks) in the lecture notes. You should find the diagrams shown in Figure 5.4 (a) of the lecture notes.

Insert the Feynman rules, multiply the equations with their projection operators and carry out the colour and Lorentz trace. Introduce spherical coordinates for the integration and simplify the equations as much as possible, i.e. make sure to cancel all factors of $Z_{\varphi\varphi}(p)$.

Think about suitable choices for the gluon mass parameter in (1).

The solution of the equations will be the topic of a forthcoming exercise sheet.