QCD

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Introduction

- The Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_f (i \not{D} - m_f) \psi_f$$

Yang-Mills matter

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Yang-Mills

matter

generators: 1,...,N_c^2-1

spinor: 1,...,4

flavour: 1,...,N_f

Lorentz: 1,...,4

colour: 1,...,N_c

In the SM: gauge group $SU(N_c = 3)$ with $N_f = 6$

Introduction

- quark masses (approximately)

u 3 MeV	c 1 GeV	t 170 GeV
d 3 MeV	s 100 MeV	b 4 GeV

so assume: $m_u \approx m_d \lesssim m_s \ll m_c, m_t, m_b$

i.e. $N_f = 3$ or $N_f = 2 + 1$

QCD vs. QED

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_f (i \not{D} - m_f) \psi_f$$

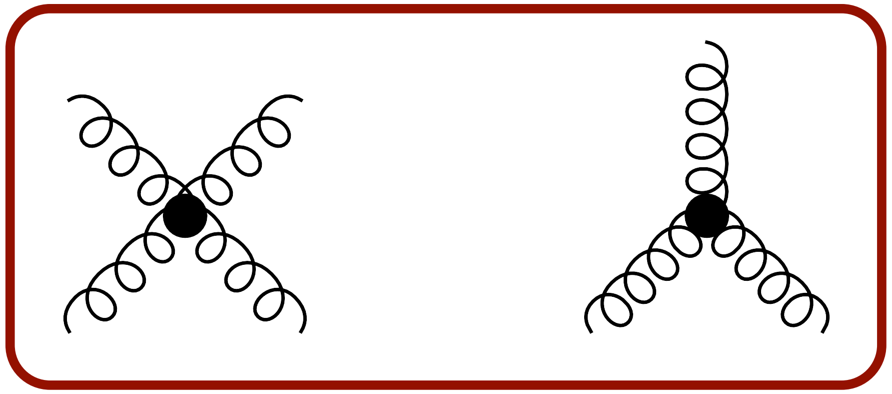
- the field strength tensor

QED: $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

QCD: $$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

coupling const (g) — structure const (f^{abc})

self-interactions:



QCD vs. QED

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_f (i \not{D} - m_f) \psi_f$$

- the Dirac operator

Gell-Mann matrices ($\lambda^a/2$ generators)

QCD: $$i \not{D} = \gamma^\mu (i\partial_\mu + g A^a_\mu \frac{\lambda^a}{2})$$

coupling const

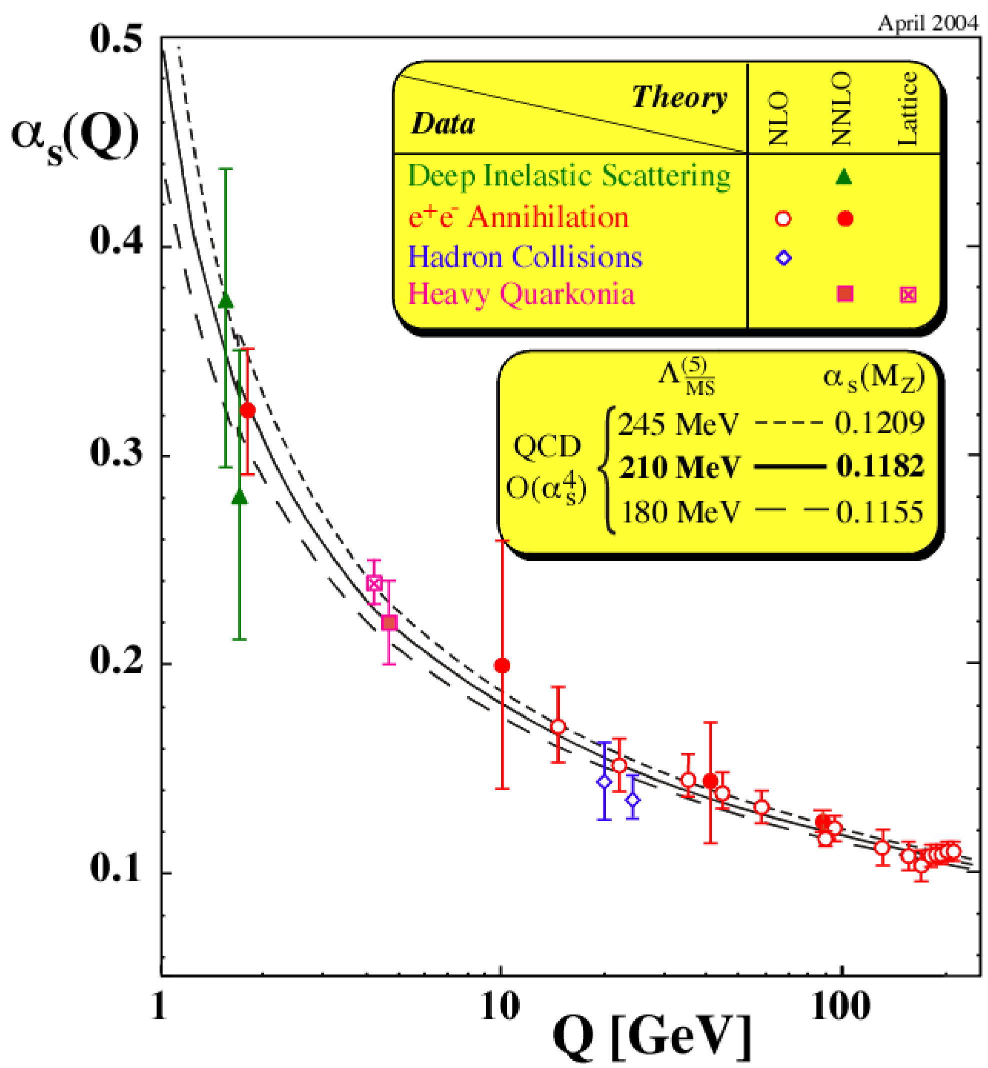
gauge potential

QED: $$i \not{D} = \gamma^\mu (i\partial_\mu - e A_\mu)$$

QCD: additional colour structure

Running coupling

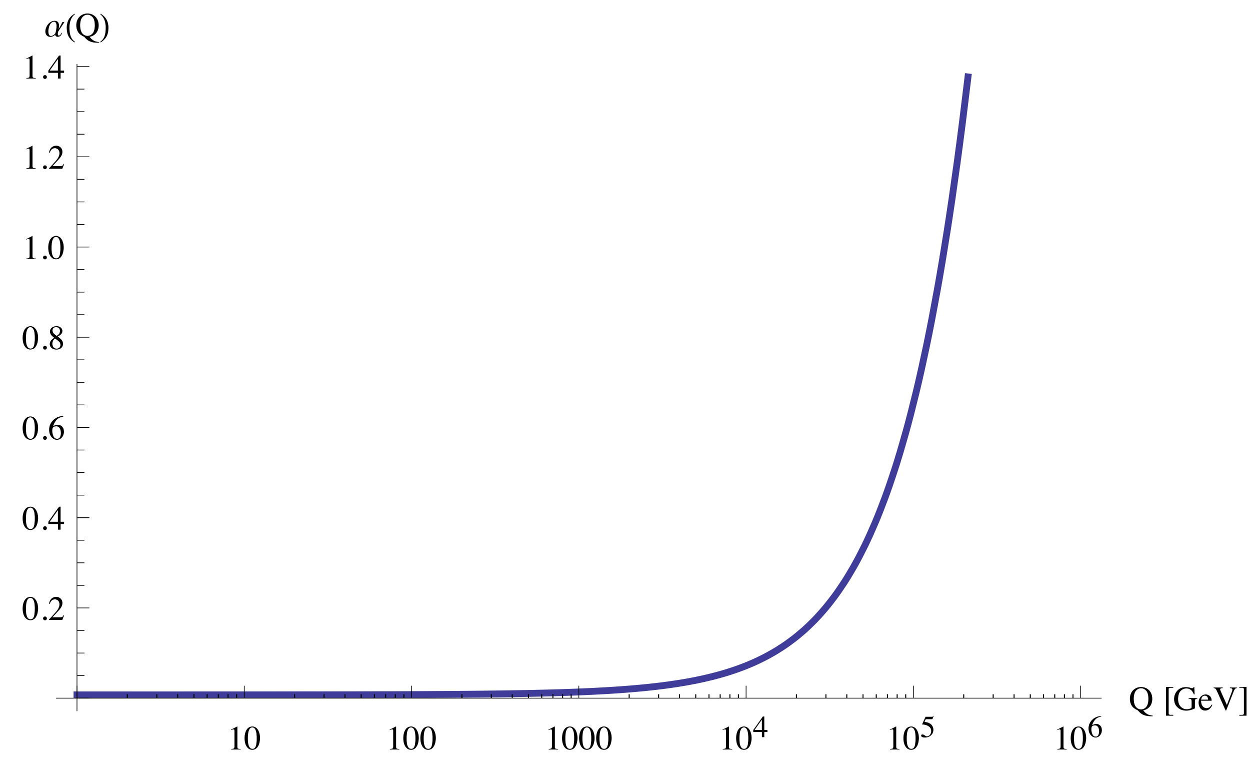
QCD



S. Bethke, hep-ex/0407021

$$\alpha_s(q) = \frac{g(q)^2}{4\pi}$$

QED



$$\alpha(q) = \frac{e(q)^2}{4\pi}$$

Running coupling

- coupling(s) not constant but modified by quantum fluctuations
- QCD and QED couplings have very different behavior
- QCD: coupling becomes large at low q $\rightarrow$ perturbation theory applicable?

The β-function

$$\beta(g) = \mu \frac{\partial g}{\partial \mu}$$

describes the change of the coupling strength with respect to the scale μ

QED:

$$\beta(e) = \frac{e^3}{12\pi^2}$$

QCD:

$$\beta(g) = -\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^3}{16\pi^2}$$

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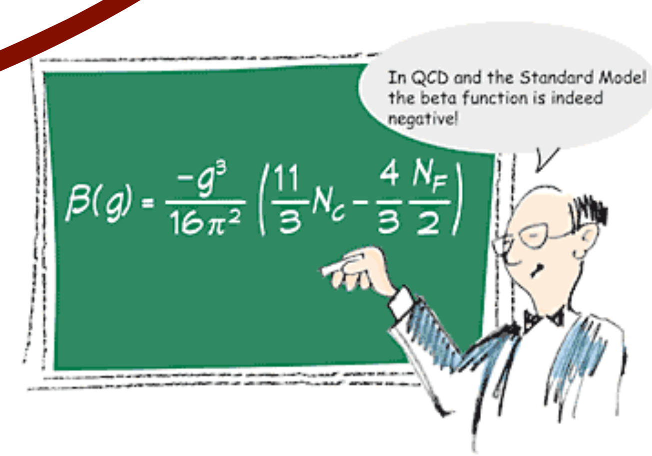
Nobel prize 2004:
Gross, Politzer, Wilczek



Running coupling α_s

$$\alpha_s(p^2) = \frac{g(p^2)^2}{4\pi}$$

expand β-function in powers of α_s:

$$\beta(\alpha_s) = \alpha_s\left(\beta_0\alpha_s + \beta_1\alpha_s^2 + \ldots\right)$$

solve differential equation:

$$\mu\frac{\partial}{\partial\mu}\alpha_s(\mu) = \beta(\alpha_s)$$

solution:

$$\alpha_s(p^2) = \frac{\alpha_{s_0}}{1 + \alpha_{s_0}\beta_0\log\left(\frac{p^2}{M^2}\right)}$$

Running coupling α_s

$$\alpha_s(p^2) = \frac{\alpha_{s_0}}{1 + \alpha_{s_0}\beta_0 \log\left(\frac{p^2}{M^2}\right)}$$

M is momentum scale (UV) where α_{s_0} is defined (measured)

for $\quad p^2 = M^2 : \quad \alpha_s(p^2) = \alpha_{s_0}$

to remove arbitrary scale M, define mass scale which satisfies

$$1 = \alpha_{s_0}\beta_0 \log\left(\frac{M^2}{\Lambda^2_{QCD}}\right)$$

$$\Leftrightarrow \quad \Lambda^2_{QCD} = M^2 e^{-\frac{1}{\alpha_{s_0}\beta_0}}$$

Running coupling α_s

$$\Rightarrow \alpha_s(p^2) = \frac{\alpha_{s_0}}{1 + \alpha_{s_0}\beta_0 \log\left(\frac{p^2}{\Lambda_{QCD}^2} e^{-\frac{1}{\alpha_{s_0}\beta_0}}\right)}$$

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$$\Rightarrow \alpha_s(p^2 = \Lambda^2_{QCD}) \to \infty$$

perturbation theory breaks down!

$$\Lambda_{QCD} \approx 200 \text{ MeV}$$

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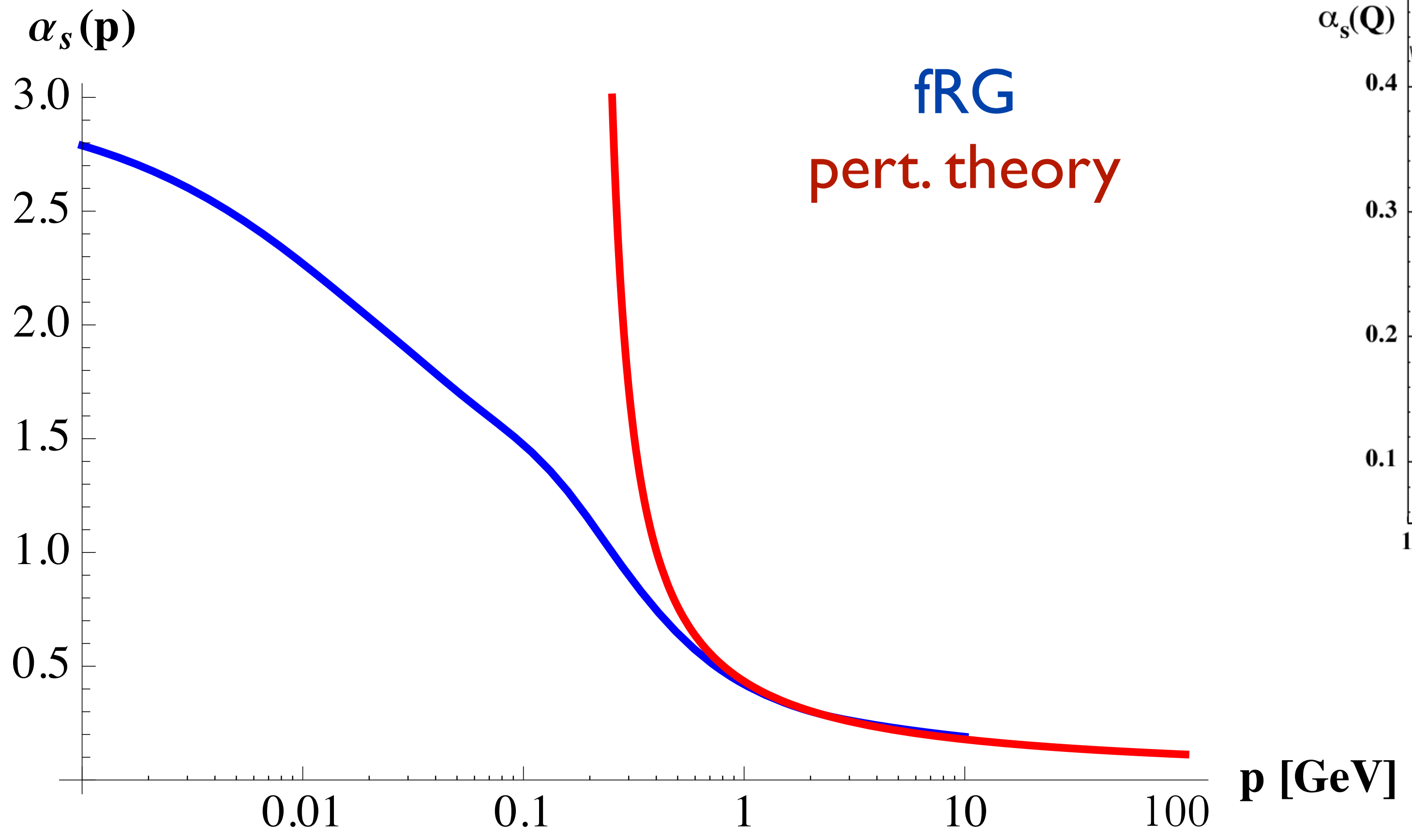
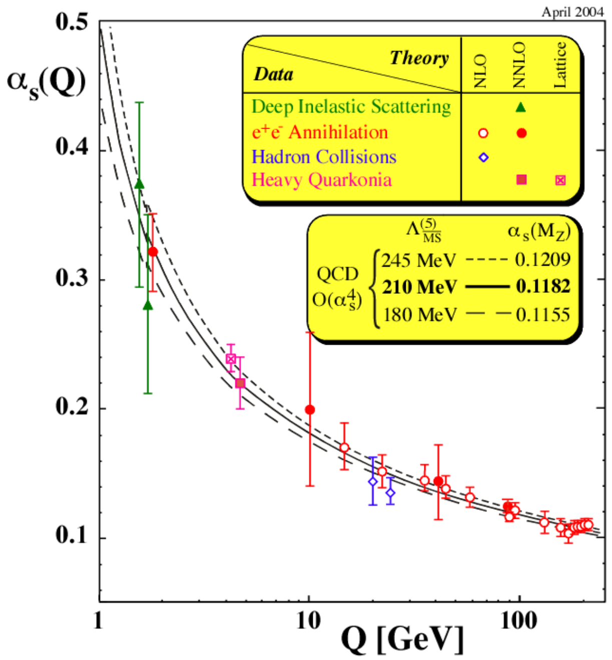
→ Need non-perturbative methods

Running coupling α_s

non-pertrubative method: the Renormalisation Group





S. Bethke, hep-ex/0407021

J. Braun, L. M. Haas, J. M. Pawlowski; work in progress

no Landau pole, at very low momenta α_s is finite

Thermodynamics

partition function $$Z(\beta) = \mathrm{Tr}\, e^{-\beta H}$$

free energy $$\Omega(\beta) = -T \ln Z(\beta)$$

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in QFT $$Z \sim \int e^{iS}$$

information of initial state is lost due to thermal fluctuations of heat bath

time scale: $$\Delta t \sim \frac{1}{T} = \beta$$

Finite temperature

$$T = 0 \qquad T \neq 0$$

$$\mathbb{R}^4 \rightarrow S \times \mathbb{R}^3$$

→ breaks O(4) to O(3), p^0 is discretised to Matsubara frequencies

$$\int \frac{d^4 p}{(2\pi)^4} \rightarrow T \sum_n \int \frac{d^3 k}{(2\pi)^3}$$

where
$$\omega_n = \begin{cases} (2n+1)\pi T & \text{fermions} \\ 2n\pi T & \text{bosons} \end{cases}$$

this means that A^0 is distinguished from spacial components

Thermodynamics

partition function $$Z(\beta) = \mathrm{Tr}\, e^{-\beta H}$$

free energy $$\Omega(\beta) = -T \ln Z(\beta)$$

propagator

where $$\ln Z(\beta) = \frac{1}{2} \mathrm{Tr}\, \ln \Delta_F$$

pressure $$P = -\frac{\partial \Omega}{\partial V} = \frac{\pi^2}{90} T^4$$

ultrarelativistic ideal gas of spinless particles

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Stefan-Boltzmann law

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weak coupling expansion of equation of state:

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gluon dofs
(dim of adjoint repr.)

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$$c_0 = \frac{\pi^2}{90} \left(2 \left(N_c^2 - 1 \right) + 4 N_c N_f \frac{7}{8} \right) \qquad c_2 = -\frac{N_c^2 - 1}{144} \left(N_c + \frac{5}{4} N_f \right)$$

polarisations

quark dofs

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Stefan-Boltzmann law

Debye mass

in weak coupling limit (g<1, high temperatures) have following contributions to gluon propagator:

$$\Pi^{ab\,\mathrm{L}}(q) = \frac{\delta^{ab}}{\vec{q}^2 + m_D^2}$$

electric

$$\Pi^{ab\,\mathrm{T}}(q) = \frac{\delta^{ab}}{\vec{q}^2 - i\frac{\pi}{4} m_D^2 \frac{\omega}{|\vec{q}|}}$$

magnetic

where

$$m_D^2 = g^2 T^2 \left(1 + \frac{N_f}{6}\right)$$

Debye mass

screening of static electric interaction at distances $r \sim m_D^{-1} \sim \frac{1}{gT}$

Landau damping

screening of dynamical magnetic interactions: Landau damping due to strong interactions of gluons and particles in plasma

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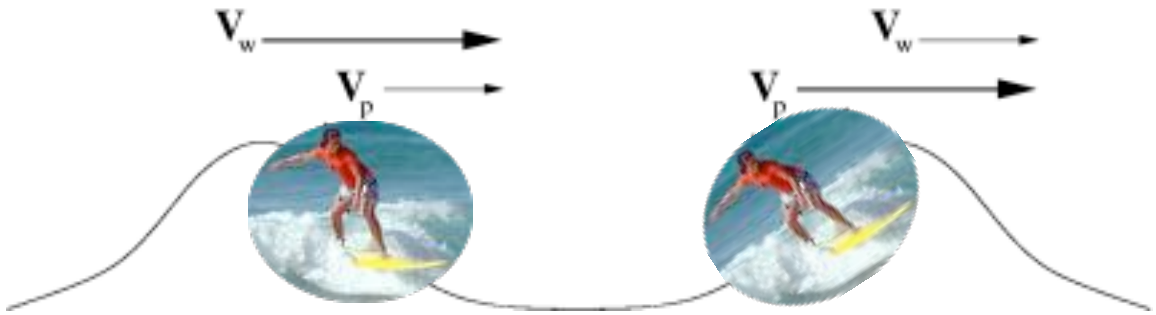
physical interpretation from Wikipedia:



particles = surfers

Langmuir waves = waves in sea

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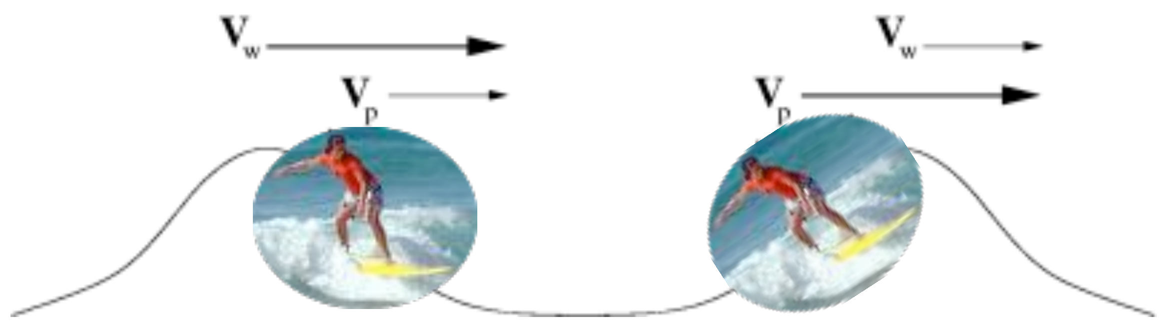

particles = surfers

Langmuir waves = waves in sea

$v_{\mathrm{surfer}} < v_{\mathrm{wave}}$: surfer will be caught & pushed along (gaining energy)

$v_{\mathrm{surfer}} > v_{\mathrm{wave}}$: surfer pushes on wave (losing energy)

Plasma oscillations

Langmuir waves ≡ rapid oscillations of e^- (quark) density in plasma

quantise oscillations: obtain quasiparticle/ collective excitations

$q \gg gT :$ 2 transv. modes, $\omega \simeq q$

$q < gT :$ 2 transv., 1 longit. mode (plasmon)

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$q \gg gT:$ 2 transv. modes, $\omega \simeq q$

$q < gT:$ 2 transv., 1 longit. mode (plasmon)

for $q \to 0:$

plasma frequency $\omega = \omega_p = \dfrac{m_D}{\sqrt{3}}$

damping constant $\gamma = 6.64 \dfrac{g^2 N_c T}{24\pi}$

Limits

$E \to \infty$: perturbation theory ($\mathbb{R}^4$)

$T \to \infty$: $S \times \mathbb{R}^3 \to \mathbb{R}^3$ as $\beta = \frac{1}{T} = 0$

$\to$ spatial $\mathbb{R}^3$ theory (not perturbation theory)

Limits

$E \to \infty$: perturbation theory ($\mathbb{R}^4$)

$T \to \infty$: $S \times \mathbb{R}^3 \rightarrow \mathbb{R}^3$ as $\beta = \frac{1}{T} = 0$

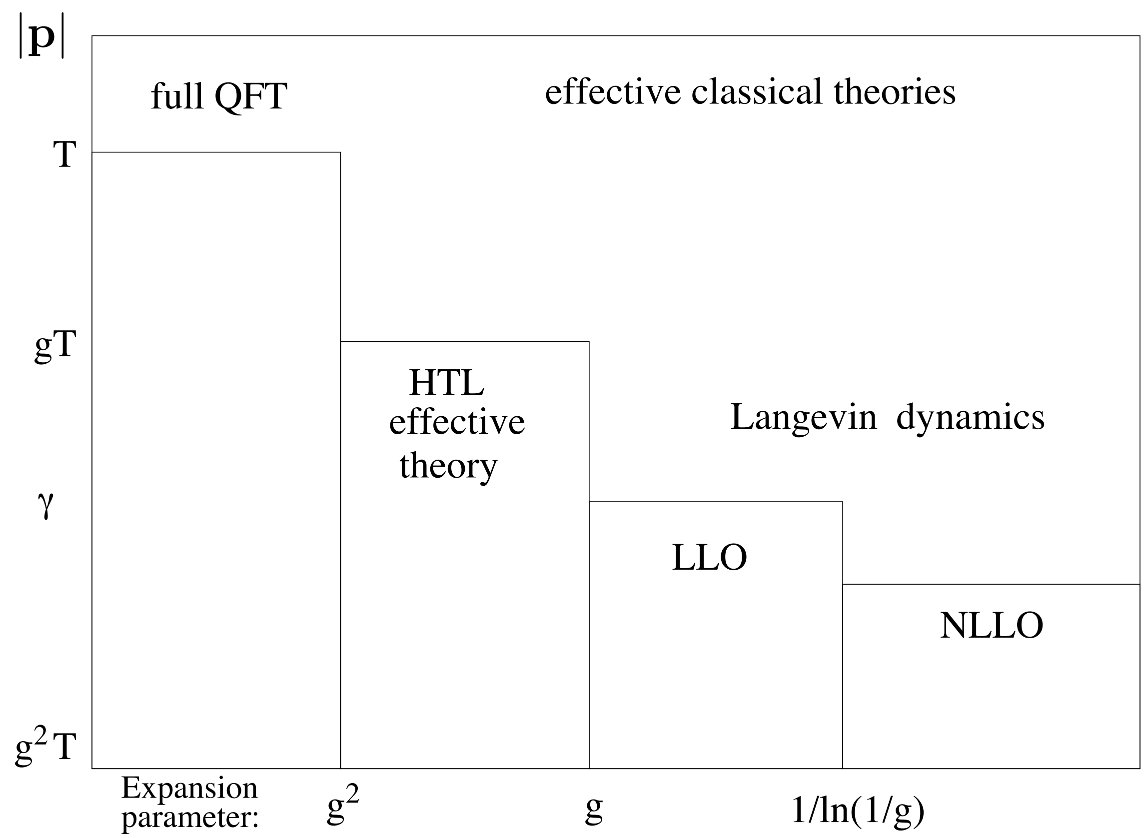
→ spatial $\mathbb{R}^3$ theory (not perturbation theory)

finite T: screening of static electric interactions,
no screening of static magnetic interactions

screening of dynamical magnetic interactions:
non-perturbative effect

Perturbative methods at high T

D. Litim, C. Manuel, hep-ph/0110104



$p \sim T$: hard modes need full thermal QCD

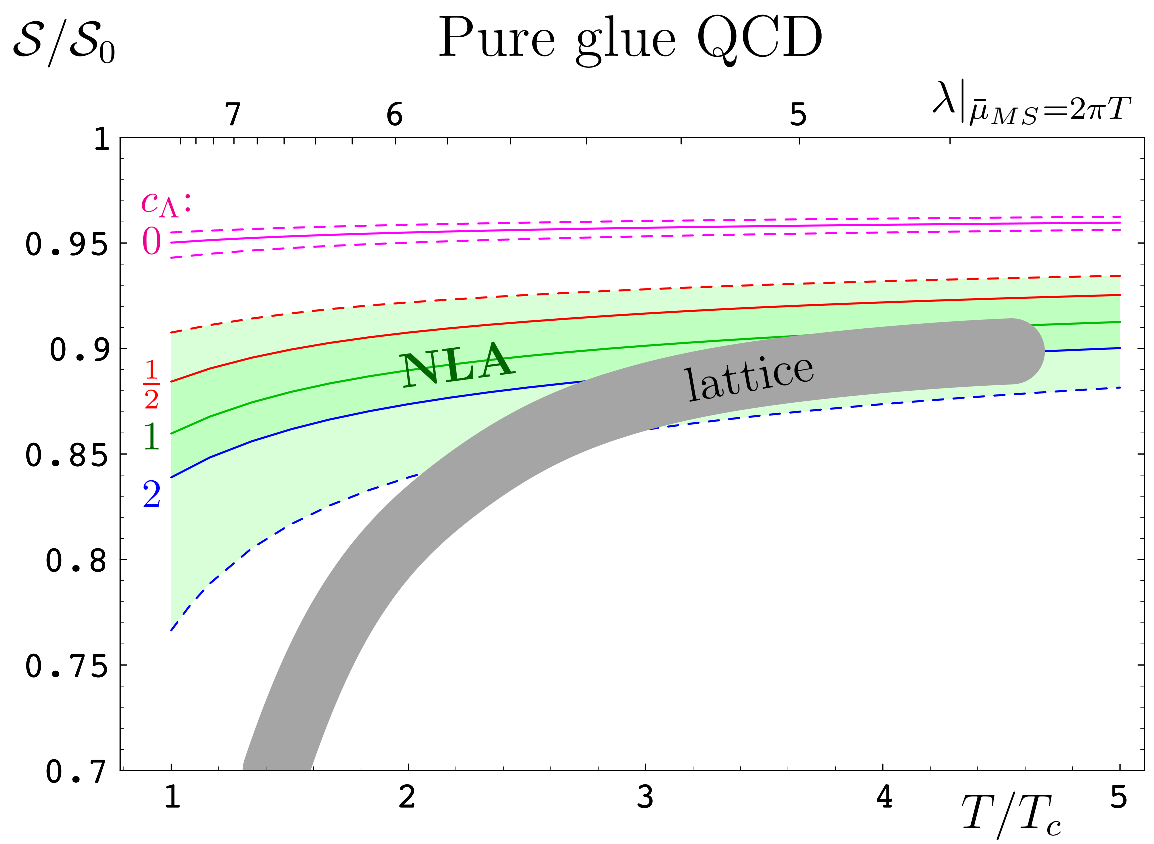
$p \ll T$: effective classical theories, integrate out $p \sim gT$

$p \ll gT$: ultra-soft modes, integrate out $p \sim g^2 T \log(1/g)$

for nice review see: J.-P. Blaizot, E. Iancu, A. Rebhan, hep-ph/0303185

Perturbative methods at high T

J.-P. Blaizot, E. Iancu, A. Rebhan;
G. Boyed et al.



- solid lines: perturb. methods, c_Λ non-pert. parameter
- dashed lines: error band (vary QCD renormalisation scale)

agree well above $T/T_c \approx 2$

The QCD phase diagram

introduce quark chemical potential μ:

$$\mathcal{L}_{matter} = \bar{\psi}(i \not{D} + \mu\gamma_0 - m)\psi$$

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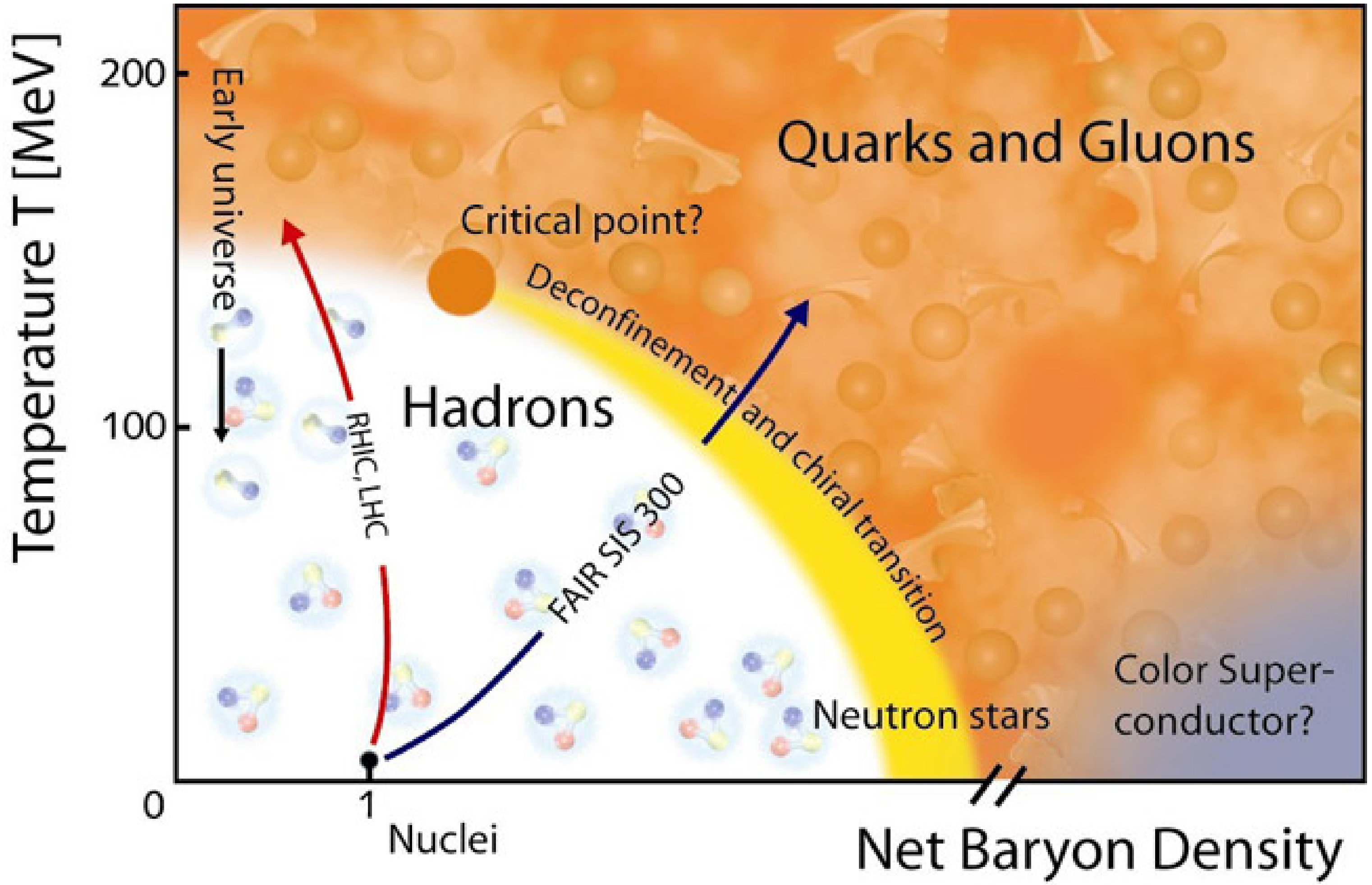
obtain phase diagram



credits: GSI Darmstadt

Chiral symmetry

consider matter sector of QCD: $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

$$\mathcal{L}_{matter} = \bar{\psi}^f_L i \not{D} \psi^f_L + \bar{\psi}^f_R i \not{D} \psi^f_R - m_f \bar{\psi}^f_L \psi^f_R - m_f \bar{\psi}^f_R \psi^f_L$$

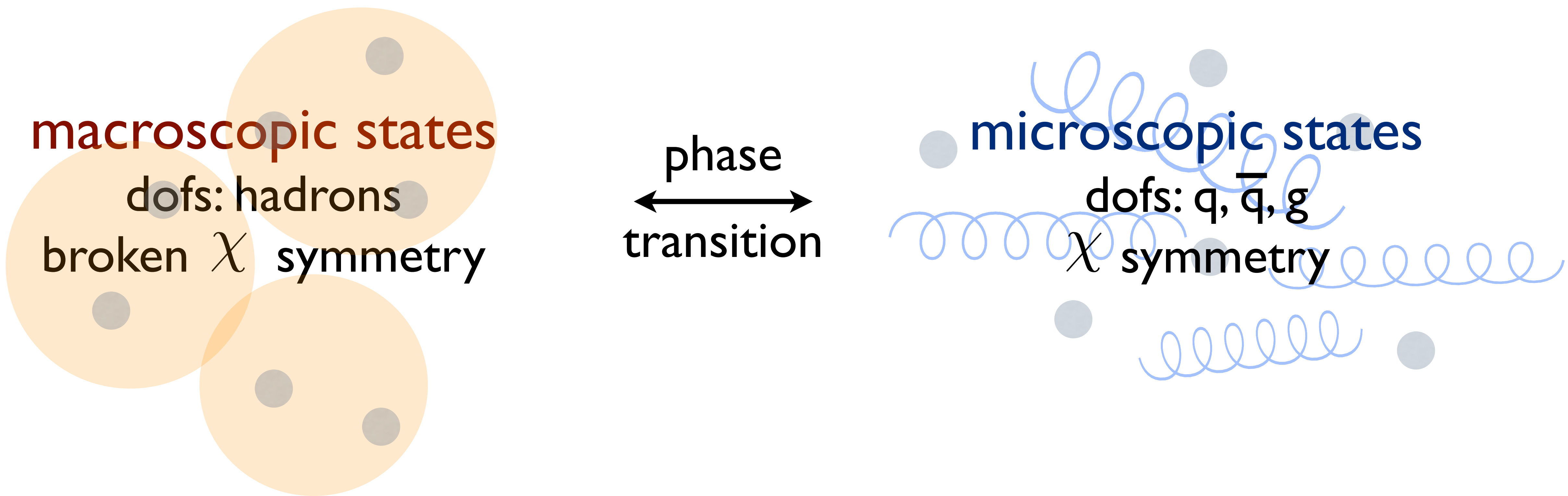
if $m = 0$:

left and right handed parts of Dirac spinor transform independently under

$$\begin{aligned} \psi^f_L &\to g^{ff'} \psi^{f'}_L \\ \psi^f_R &\to g^{ff'} \psi^{f'}_R \end{aligned} \quad \text{where} \quad \begin{aligned} g^{ff'} &\in U(N_f)_L \\ g^{ff'} &\in U(N_f)_R \end{aligned}$$

$\Rightarrow$ Lagrangian is symmetric under such trafos (chiral symmetry)

Chiral phase transition



order parameter: chiral condensate $\langle\bar{\psi}\psi\rangle$

$$\langle\bar{\psi}\psi\rangle = \begin{cases} 0 & T > T_{c,\chi} \\ > 0 & T < T_{c,\chi}. \end{cases}$$

The sigma model

- want to model phenomenologically chiral symmetric field theory of strong interactions

- scalar fields: elementary fields, their interactions arranged to produce spontaneous breakdown of chiral symmetry

- elementary fields: nucleons, pions, sigma (meson)

- symmetric under rotations

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QCD		sigma model
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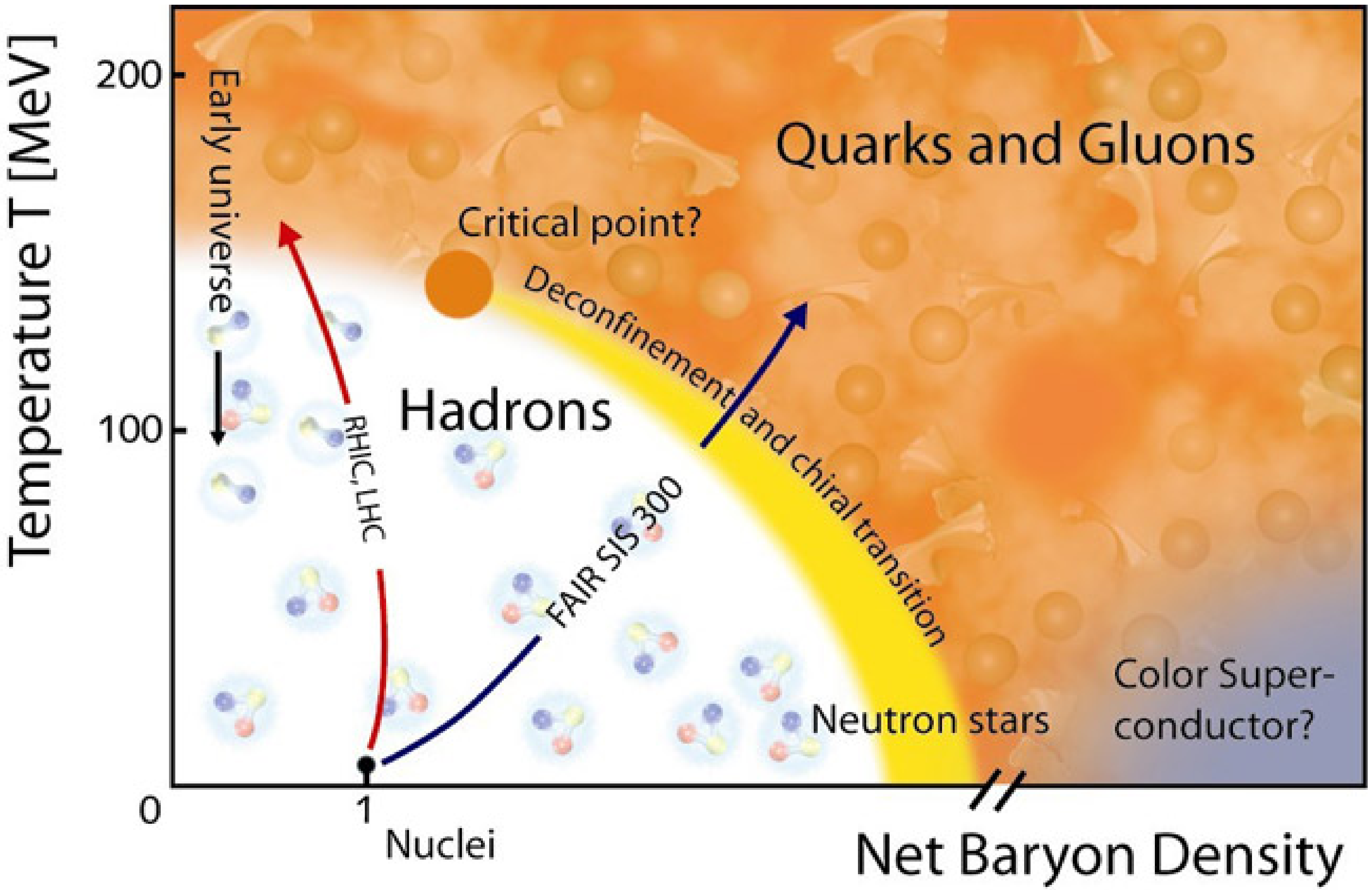
QCD $\qquad\qquad$ sigma model

$$SU(2) \times SU(2) \;\xleftrightarrow{\text{isomorphic}}\; SO(4)$$

QCD and sigma model have same symmetry group

Conclusions



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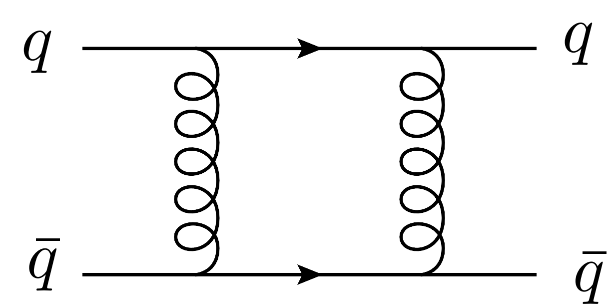
From QGP to Cold Atoms

$\bar{\psi}\,\not{D}\psi \longrightarrow$ 

$\longrightarrow (\bar{\psi}\psi)^2$

quark & anti-quark $\bar{\psi}\psi \longrightarrow \phi$ scalar

interpretation: condensate $\bar{\psi}\psi \quad \mathrm{vs.} \quad \psi\psi$

$\bar{\psi}\psi\phi + \ldots$	$\psi\psi\phi + \ldots$
QCD	Cold Atoms