Transport coefficients from Kinetic Theory:

Bulk viscosity, Diffusion, Thermal conductivity

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Recap: Hydrodynamics of nearly perfect fluids

- Hydrodynamics: correlation functions at low energy and small momentum are governed by evolution of conserved charges
- Hydrodynamics can be derived as an expansion of derivatives of the fluid velocity and thermodynamic variables
- The leading order theory "ideal hydrodynamics" only depends on EoS and is exactly time reversible
- The next order theory "viscous hydrodynamics" involves transport coefficients, and describes dissipative, time irreversible phenomena

Non-relativistic hydrodynamic equations

- **Ideal:**

$$\begin{aligned} j^{\epsilon} &= v(\epsilon + P) \\ g &= \rho v \\ \Pi_{ij} &= P\delta_{ij} + \rho v_i v_j \end{aligned}$$

- **dissipative:**

$$\begin{aligned} j_i^{\epsilon} &= v_i(\epsilon + P) + v_j \delta\Pi_{ij} + Q_i \\ g &= \rho v \\ \Pi_{ij} &= P\delta_{ij} + \rho v_i v_j + \delta\Pi_{ij} \end{aligned}$$

where

$$\begin{aligned} \delta\Pi_{ij} &= -\eta(\nabla_i v_j - \nabla_j v_i - \frac{2}{3}\delta_{ij}\nabla\cdot v) - \zeta\delta_{ij}(\nabla\cdot v) \\ Q &= -\kappa\nabla T \end{aligned}$$

Determination of Transport Coefficients

- The transport coefficients can be extracted from experiment, or estimated from an underlying field theory
- "Linear Response Theory" connects transport coefficients and correlation functions in a field theory => Kubo formulae
- If interaction not weak, calculations based on Kubo formulae difficult => "Kinetic Theory" used to relate microscopic quasiparticles to hydrodynamics
- If interaction between quasiparticles is strong, Kinetic theory breaks down => "Holographic Method" used to extract transport properties from strongly coupled field theory

Representative Fluids

- Superfluid Helium : strongly coupled Bose fluid
- atomic Fermi gas near Feshbach resonance : strongly coupled Fermi liquid
- QGP : strongly coupled plasma
- cold relativistic quark matter at very high baryonic densities in the CFL Phase
- The phases of matter are quite different, but approximately scale invariant and their properties do not depend on detailed form the of the interaction

Transport Coefficients

- [x] shear viscosity η
- [ ] bulk viscosity ζ
- [ ] thermal conductivity κ
- [ ] diffusion D

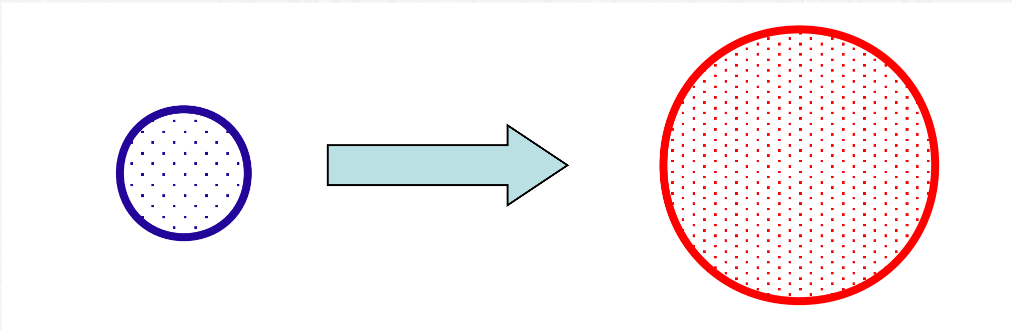
Bulk Viscosity ζ

- Bulk viscosity measures the energy dissipated as a fluid undergoes expansion or compression

- When a fluid is uniformly compressed, it is perturbed from equilibrium. The energy density rises, but pressure temporarily rises by more than what is predicted by EoS. Under uniform rarefaction, pressure temporarily falls further than is predicted by EoS. Bulk viscosity quantifies this extra shift in the pressure.

- Bulk viscosity will be non-zero whenever the trace of the stress energy tensor can differ from the equilibrium pressure. The inability to maintain equilibrium is assumed to derive from rapidly changing densities i.e. $\nabla . v \neq 0$

- $(P - P') = - \zeta (\nabla . v)$



Bulk Viscosity and Conformal Invariance

- In a conformal theory, even if the fluid is perturbed from equilibrium, the pressure still does not deviate from the value given by the EoS ($P = E/3$). This follows from the tracelessness of the stress-energy tensor in a conformal theory
- Uniform compression or rarefaction is the same as a dilatation transformation.
- In a conformal theory, a dilatation transformation is a symmetry, so the fluid will not leave equilibrium. Thus, in a non-interacting conformally invariant system in the normal phase, bulk viscosity vanishes
- When interaction is turned on, conformal symmetry could be broken to give finite bulk viscosity
- Bulk viscosity is proportional to the relaxation time, and to the deviations from breaking of conformal invariance in the EoS.

Relativistic hydrodynamics

- **Ideal:**

$$\partial_\mu T^{\mu\nu} = 0$$
$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu}$$

$$\eta^{\mu\nu} = diag(-1,1,1,1)$$

$$P = P(\epsilon)$$

- **dissipative:**

In the local rest frame: $T^{00} = \epsilon$, $T^{0i} = 0$.

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta^{(1)}T^{\mu\nu} + \delta^{(2)}T^{\mu\nu}..$$

$$\delta^{(1)}T^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta\delta^{\mu\nu}\partial\cdot u$$

where $\sigma^{\mu\nu} = \Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}\eta_{\alpha\beta}\partial\cdot u)$

$$j_\mu = nu_\mu + \delta j_\mu$$

with

$$\delta^{(1)}j_\mu = -\kappa\left(\frac{nT}{\epsilon+P}\right)^2\Delta_\mu\left(\frac{\mu}{T}\right)$$

Kinetic Theory: Boltzmann transport equation

- Viscous hydrodynamics based on assumption of short mfp: invalid at low densities, when the particle can traverse a significant fraction of reaction volume

- At low densities, the system can be treated as a hadronic gas undergoing binary collisions and Boltzmann treatments are justified

- For a Boltzmann description, one needs a phase space density f(p,x), which can be expanded about a local equilibrium as $$f_p(x) = f_p^0(x) + \delta f_p(x)$$

  where the Bose distribution is $$f_p^0(x) = \frac{1}{(e^{\beta(x) v_\mu(x) p^\mu} - 1)}$$

- The departure from equilibrium is determined by the Boltzmann equation $$\frac{\partial f}{\partial t} + v_p \cdot \nabla_x f = -C[f]$$

- For bulk viscosity, the departure from equilibrium is because of isotropic compression or rarefaction $\nabla \cdot v_p \equiv X_p$

- At linearized order, the departure from equilibrium $$\delta f_p(x) = -f_p^0(x)[1 + f_p^0(x)] X_p(x)$$

- In kinetic theory, the energy momentum tensor in a weakly interacting system is

$$T_{\mu\nu} = g_\pi \int \frac{d^3p}{(2\pi)^3} \frac{f_p(x)}{E_p} p_\mu p_\nu$$

- Deviation from thermal equilibrium

$$T_{\mu\nu} = T^0_{\mu\nu} + \delta T_{\mu\nu}, T^0_{\mu\nu} = (\epsilon + P) v_\mu v_\nu - P g_{\mu\nu}$$

- The shear and bulk viscosities are defined by small deviation away from equilibrium:

$$\delta T_{ij} = -\eta(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij} \nabla \cdot v) - \zeta \delta_{ij} \nabla \cdot v$$

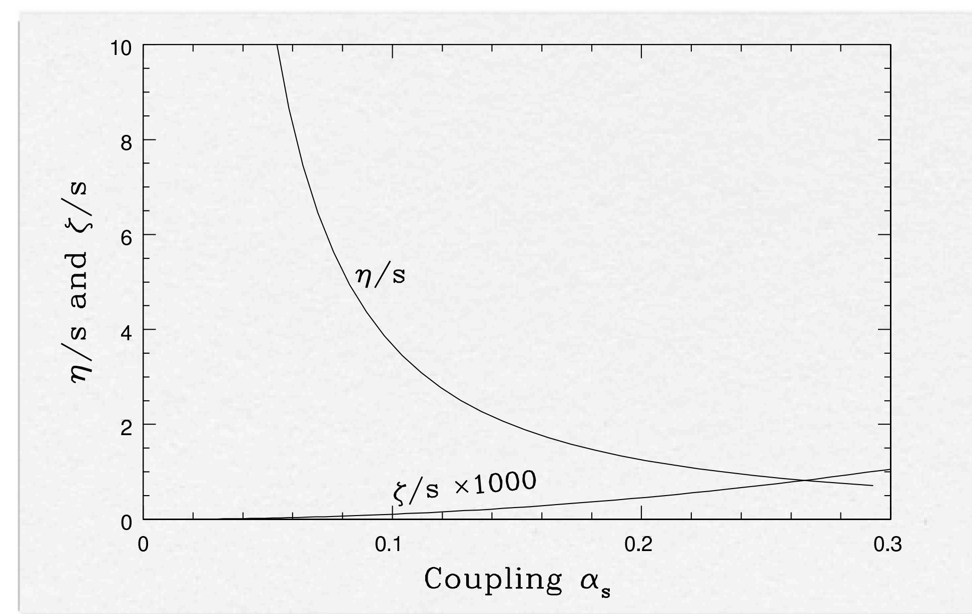
- For non-interacting systems, conformal symmetry requires $\delta T_{00} = 0 \Rightarrow \zeta = 0$.

- When interactions are turned on and conformal symmetry is broken, $T_{\mu\nu}$ equation has to be modified to include effect of the interaction to give non-vanishing ζ result

Bulk viscosity of high temperature QGP

- In a relativistic system, using dimensional arguments, both shear and bulk viscosity must scale as $\eta, \zeta \propto T^3$
- the parametric behaviour of shear viscosity in QCD is $\eta \sim \frac{T^3}{\alpha_s^2 log[1/\alpha_s]}$
- In a nearly conformal theory, ζ vanishes as 2nd power of breaking of conformal invariance: one power because departure from equilibrium is small, another because any departure from equilibrium has a small impact on pressure
- For massless QCD, conformal symmetry is broken by $\beta(\alpha_s) \sim \alpha_s{}^2$
- Bulk viscosity

$$\zeta \sim \frac{T^3}{\alpha_s^2 log[1/\alpha_s]} \times (\alpha_s^2)^2 \sim \frac{\alpha_s^2 T^3}{log[1/\alpha_s]}$$



Bulk viscosity of Pion Gas at low T

- In hadronic phase, dominant configuration of QCD with 2 flavors of massless quarks is a gas of massless pions

- In QCD with heavy quarks integrated out and light quark masses set to zero, conformal symmetry is broken in the quantum level. In the perturbative region of QCD, up to log corrections,

$$\zeta/s \propto \alpha_s^{-2}\left(\frac{1}{3}-v_s^2\right) \propto \alpha_s^2$$

while $\eta/s \propto \alpha_s^{-2}$

- ζ is smaller than η in the perturbative regime

- When temperature is reduced, η/s reaches minimum near T_c, while ζ/s rises sharply near T_c.

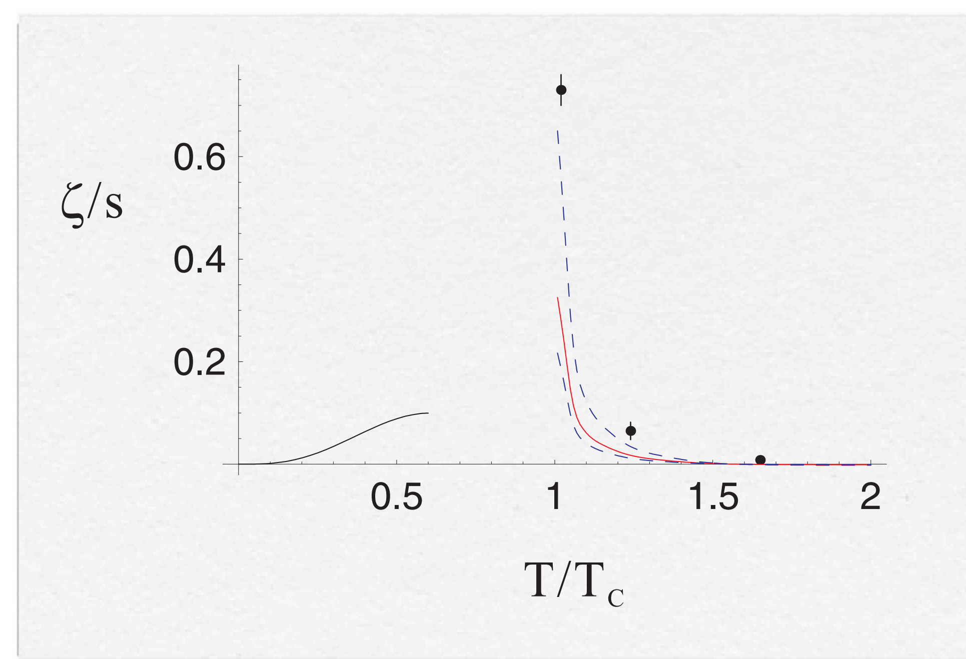
Bulk viscosity of Pion Gas at low T

- Bulk viscosity of massless pions can be obtained by solving the linearized Boltzmann equation. The Boltzmann equation describes the evolution of isospin averaged pion distribution function $f(x,p,t) = f_p(x)$:

$$\frac{p^\mu}{E_p}\partial_\mu f_p(x) =$$
$$\frac{g_\pi}{2}\int d\Gamma\, f_1 f_2 (1+f_3)(1+f_p) - (1+f_1)(1+f_2) f_3 f_p$$
$$\text{where } E_p = \sqrt{p^2+m_\pi^2},\; g_\pi = 3,$$
$$d\Gamma = \frac{|\tau|^2}{2E_p}\Pi\frac{d^3\vec{k}_i}{(2\pi^3)(2E_i)}(2\pi)^4\delta^4(k_1+k_2-k_3-p)$$

- $\Rightarrow \zeta \propto T^7/f_\pi^4$ [Chen and Wang 2007]

- For free pion gas, $s = 2\pi^2 g_\pi T^3/45$



Superfluidity

- Superfluidity is a property of quantum fluids related with existence of low energy excitations that satisfies the Landau criterion for superfluidity : $Min\ \epsilon(p)/p \neq 0$

- Superfluidity is due to appearance of a quantum condensate which spontaneously breaks a global symmetry of the system associated with conserved particle number $\Rightarrow$ phonon

- superfluidity discovered in ^4He below 2.17K is due to the B-E condensation of the bosonic atoms in the lowest quantum state

- Cooper Theorem ⇒ Fermionic superfluidity in quantum degenerate systems at low temperature when interaction between neutral fermions is attractive

Superfluid hydrodynamics

- Ideal:

$$
\begin{aligned}
j^\epsilon &= \rho s T v_n + (\mu + \frac{1}{2}v_s^2)(\rho_n v_n + \rho_s v_s) \\
&+ \rho_n v_n (v_n - v_s) \\
g &= \rho_n v_n + \rho_s v_s \\
\Pi_{ij} &= P\delta_{ij} + \rho_n v_{n,i} v_{n,j} + \rho_s v_{s,i} v_{s,j}
\end{aligned}
$$

- dissipative:

$$
\begin{aligned}
\delta\Pi_{ij} &= -\eta(\nabla_i v_{n,j} + \nabla_j v_{n,i} - \frac{2}{3}\delta_{ij}\nabla\cdot v_n) \\
&- \delta_{ij}(\zeta_1 \nabla\cdot(\rho_s(v_s - v_n)) + \zeta_2(\nabla\cdot v_n)) \\
\delta j_i^\epsilon &= v_{n,j}\delta\Pi_{ij} + \rho_s(v_{s,i} - v_{n,i})H + Q_i
\end{aligned}
$$

where

$$
\begin{aligned}
H &= -\zeta_3 \nabla\cdot(\rho_s(v_s - v_n)) - \zeta_4 \nabla\cdot v_n \\
Q_i &= -\kappa \nabla_i T
\end{aligned}
$$

Transport Coefficients in Superfluids

- According to fluid mechanics, a normal gas (or Fermi gas above critical temperature) has 3 kinetic coefficients: shear viscosity, bulk viscosity and thermal conductivity
- Below critical temperature, Fermi gas is in the superfluid phase, and the number of kinetic coefficients is 5: shear viscosity, thermal conductivity, and 3 bulk viscosities

Bulk Viscosity in superfluids

- In a superfluid at non-zero temperature (using Two-fluid description of Landau), there are 2 independent motions, one normal and the other superfluid. The transport properties depend on shear viscosity, three independent bulk viscosities and thermal conductivity
- Dissipative processes lead to positive entropy production ⇒ $\kappa, \eta, \zeta_2, \zeta_3 > 0$ *and* $\zeta_1^2 \leq \zeta_2\zeta_3$
- ζ_2 plays the role of standard bulk viscosity coefficient, ζ_1 and ζ_3 provide a coupling between hydrodynamic equations of the 2 components
- In a conformally invariant system in the superfluid phase, it has been shown [Son 2007] that two of the three bulk viscosities vanish: $\zeta_1 = \zeta_2 = 0$.
- In low temperature regime $T << T_c$, transport properties of superfluids are determined by phonons: $\zeta_1^2 = \zeta_2\zeta_3$ (saturated) and there are only 2 independent bulk viscosities

Unitary Fermi Gases

- In experiments with trapped cold atomic gases, by varying magnetic-field controlled interaction, fermionic pairing is observed to undergo BEC to BCS crossover

- In weak coupling BCS region, the system is characterized by formation of Cooper pairs. In strong coupling limit, the system can be described by BEC dilute gas.

- The unitarity limit is reached when the magnetic field is tuned at the Feshbach resonance, where the two-body scattering length diverges. Far from unitarity, properties are well understood using Mean Field Theory, but not reliable close to unitarity (scattering length >> inter-particle distance), no small parameter in Lagrangian to expand in

- Close to unitary region, understanding of phases comes from MC simulations

- Different method: At sufficiently low temperature $T << T_C$, only active DOF are phonons, assuming contribution of other DOF are thermally suppressed

Kinetic Theory of superfluid phonons

- The dispersion law of phonons is given by $\epsilon_p = c_s p + B p^3 + \mathcal{O}(p^5)$
- Various thermodynamic quantities (entropy density, phonon number) can be computed starting from phonon distribution function n.
- Kinetic equation for evolution of out-of-equilibrium phonon distribution function (Boltzmann transport eqn):

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial r} \cdot \frac{\partial H}{\partial p} - \frac{\partial n}{\partial p} \cdot \frac{\partial H}{\partial r} = C[n] \text{ where } H = \epsilon_p + \vec{p} \cdot \vec{v}_s$$

- At equilibrium phonons obey Bose-Einstein distribution : $n_{eq}(\epsilon_p) = \frac{1}{e^{\epsilon_p/T} - 1}$

  and the collision term vanishes.

- For small departures from equilibrium, the collision term can be linearized on the deviation $\delta n = n - n_{eq}$. The transport coefficients can be obtained by solving the kinetic equation
- Various numerical approaches (variational methods, orthonormal polynomials etc). Approx expression using RTA (relaxation time approximation): $\delta C = - \delta n / \tau_{rel}$
- RTA -> correct parametric dependence, but inaccurate numerical factors

Effective Field Theory

- The properties of the phonons can be extracted from the Effective Field Theory
- The Effective phonon Lagrangian L_{eff} can be determined from the pressure using EoS, by demanding general coordinate invariance and conformal invariance
- From L_{eff} the phonon dispersion law is obtained, and the self couplings (c_s , B) depend on some universal and dimensionless constants
- Transport coefficients can be determined from L_{eff} considering appropriate phonon scattering process
- It was shown that Fermi gas at unitarity is exactly conformal, and bulk viscosity vanishes in normal phase: $\zeta_1=\zeta_2=0$, while $\zeta_3 \neq 0$ [Escobedo, Mannarelli, Manuel 2009]
- Close to unitarity, scale invariance is broken $\Rightarrow$ additional terms in $L_{eff} \Rightarrow$ the phonon dispersion law and self-couplings are modified $\Rightarrow$ first non-vanishing corrections to bulk viscosity
- Comparison with Bose superfluids showed that T dependence for bulk viscosity in these two superfluids is the same

In exact conformal limit:

$$P = c_0 m^4 \mu_0^{5/2}$$

where $c_0 = \frac{2^{5/2}}{15\pi^2 \xi^{3/2}}$, $\mu_0 = \xi \frac{E_F}{m}$

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{LO} + \mathcal{L}_{NLO} \\ &= c_0 m^{3/2} X^{5/2} + c_1 m^{1/2} \frac{(\nabla X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} (\nabla^2 \phi)^2 \sqrt{X} \end{aligned}$$

where $X = m\mu_0 - \partial_0 \phi - \frac{(\nabla \phi)^2}{2m}$

$$\epsilon_p = c_s \left(p - \pi^2 \sqrt{2\xi} (c_1 + \frac{3}{2} c_2) \frac{p^3}{k_F^2} \right)$$

$$\Rightarrow \quad B = -\pi^2 c_s \sqrt{2\xi} (c_1 + \frac{3}{2} c_2) \frac{1}{k_F^2}$$

$$\zeta_1 = \zeta_2 = 0$$

$$\zeta_3 \simeq 3695.4 \left(\frac{\xi}{\mu_0} \right)^{9/4} \frac{(c_1 + \frac{3}{2} c_2)^2}{m^8} T^3$$

Close to conformal limit:

$$\begin{aligned} P &= P_0 + P_{CB} \\ &= c_0 m^4 \mu_0^{5/2} + \frac{d_0 m^3 \mu_0^2}{a} \end{aligned}$$

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{LO} + \mathcal{L}_{NLO} + \mathcal{L}_{CB} \\ &= c_0 m^{3/2} X^{5/2} + c_1 m^{1/2} \frac{(\nabla X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} (\nabla^2 \phi)^2 \sqrt{X} \\ &+ d_0 \frac{m X^2}{a} + .. \end{aligned}$$

Defining $y = \frac{d_0 \pi^2 \xi^{3/2}}{am\sqrt{2\mu_0}}$

$$\Rightarrow \quad c_{s,B} = c_s (1 + \frac{y}{2})$$

$$B_{CB} = -\frac{\pi^2 \xi^{3/2}}{\sqrt{3\mu_0} m^2} \left(c_1 (1 - \frac{3y}{2}) + \frac{3}{2} c_2 (1 - \frac{5y}{2}) \right)$$

$$\zeta_1 \simeq -264.7 c_2 (c_1 + \tfrac{3}{2} c_2) \xi^3 \frac{T^3}{m^4 \mu_0^3} y$$

$$\zeta_2 \simeq 19 c_2^2 \frac{\xi^{3/2} T^3}{\mu_0^{3/2}} y^2$$

$$\zeta_3 \simeq 3695.4 \left(\frac{\xi}{\mu_0} \right)^{9/4} \frac{(c_1 + \frac{3}{2} c_2)^2}{m^8} T^3 (1 - \frac{66 c_1 + 135 c_2}{8 c_1 + 12 c_2} y)$$

Experimental detection of transport coefficients: Unitary Fermi gas

1. Damping of radial breathing modes depends on bulk and shear viscosity. Since ζ_2 vanishes, bulk viscosity enters only in the presence of a difference in velocity between the normal and superfluid components, and is in general negligible. To determine ζ_3, one should produce oscillations where the normal and superfluid component oscillate out of phase

2. Transport coefficients also enter into the damping rate for propagation of first and second sound in a superfluid. The damping of first sound, α_1, depends on shear viscosity and on ζ_2, while the damping of second sound, α_2, depends on all dissipative coefficients

Relativistic superfluid hydrodynamics

- Ideal:

$$\begin{aligned}\partial_\mu T^{\mu\nu} &= 0\\ T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu}\end{aligned}$$

$$\eta^{\mu\nu} = diag(-1, 1, 1, 1)$$

$$P = P(\epsilon)$$

$$\partial_\mu n^\mu = 0$$

$$\partial_\mu s^\mu = 0$$

$$\Psi_\mu = \partial_\mu \phi$$

- dissipative: $T_d^{ij} = -\zeta\delta^{ij}\nabla\cdot v$

Bulk viscosity of cold relativistic quark matter

- Relativistic superfluid matter in Neutron stars: high baryonic density and low temperature
- Deconfined quark matter: preferred phase in presence of three light quark flavors is CFL
- In CFL, baryon symmetry is spontaneously broken, and CFL quark matter becomes a superfluid as in B-E condensates
- In the regime where T is smaller than all energy gaps of quasi-particles (mesons, quarks, gluons), transport coefficients in CFL are dominated by collisions of superfluid phonons $\Rightarrow$ EFT for superfluid phonons can be applied.
- Besides phonons, kaons may contribute to the transport coefficients

Bulk viscosity of cold relativistic quark matter

- Using EFT, dispersion law of phonons can be obtained:

$$c_s = \sqrt{\frac{1}{3}} \text{ and } B = -\frac{11 c_s}{540\Delta^2}$$

- At v high chemical potentials, $m_q << \mu$: CFL is approx scale invariant, leads to vanishing bulk viscosity:

$$\zeta_1 = \zeta_2 = 0$$

$$\zeta_3 \sim \frac{1}{T}\frac{\mu^6}{\Delta^8}$$

- Scale breaking effects due to a non vanishing value of strange quark mass included in L_{eff}. After identifying the leading collisional process relevant for bulk viscosity, Boltzmann equation is written down for the phonon, and linearized in small deviations around equilibrium. The collision term is explicitly written down and bulk viscosity coefficients are numerically computed.

Astrophysical detection of transport coefficients: Compact stars

Relativistic superfluid phases: If superfluidity occurs in the interior of compact stars, there should be possible signatures:

- Glitches: sudden spin-up of pulsars, relies on existence of superfluid component in the interior of the neutron star, rotating faster than the solid crust
- Evolution of r-mode oscillations in compact stars: non-radial oscillations of the star with Coriolis force as the restoring force. When dissipative phenomena damp these r-modes, the star can rotate without losing angular momentum to gravitational waves. It is necessary to consider in detail all the dissipative processes and to compute corresponding transport coefficients

Thermal Conductivity κ with Kinetic Theory

- The form of dissipative terms in relativistic hydrodynamics depend on the definition of rest frame of the fluid. In Landau frame (energy three-flux vanishes), heat conduction does not enter as energy flux T^{0i}, but in the rest frame of the fluid as baryon current

$$\delta^{(1)}j_\mu = -\kappa\left(\frac{nT}{\epsilon+P}\right)^2 \Delta_\mu\left(\frac{\mu}{T}\right)$$

- Eckart frame: baryon three-current vanishes. Boost velocity $v_E = v/n$ to go from Landau frame.

- In Eckart frame: $T_E^{0i} = -(\epsilon+P)v_E^i = -\kappa\left(\frac{nT^2}{\varepsilon+P}\right)\partial_i\left(\frac{\mu}{T}\right)$

- Boltzmann equation: $p^\mu \partial_\mu f_a = C_a(f)$ describes evolution of phase space densities $f_a(x,p)$.
  $C_a(f) = 0$ for equilibrium distributions f_0 of B-E or F-D forms. Close to equilibrium $f_H = f_0 + \delta f$.

- In terms of δf, dissipative corrections are

$$\delta T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{p_0}\delta f$$

$$\delta j^\mu = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p_0}\delta f$$

- For heat conduction, we are interested in δT^{0i} and δj^i: $\quad T_E^{0i} = -\frac{\epsilon+P}{n}\delta j^i + \delta T^{0i}$

Thermal conductivity in superfluids

- Distribution function of elementary excitations satisfies the kinetic equation :

$$\frac{\partial n}{\partial t}+\frac{\partial n}{\partial r}\cdot\frac{\partial H}{\partial p}-\frac{\partial n}{\partial p}\cdot\frac{\partial H}{\partial r} = C[n] \text{ where } H = \epsilon_p+\vec{p}\cdot\vec{v}_s$$

- At equilibrium: $n_{eq}(\epsilon_p) = \frac{1}{e^{\epsilon_p/T}-1}$

- For small departures from equilibrium, distribution function : $n = n_0 + n_1$ where $n_0 << n_1$. The problem is "linearized" ⇒ The transport coefficients can be obtained by solving the kinetic equation

- Inserting n into LHS of kinetic equation: terms containing second viscosity in the superfluid, term with temperature gradient ∇T associated with heat conduction and a term related to first viscosity

Thermal Conductivity : Results

- In superfluids: thermal transport takes place along the normal component by convection, where the superfluid moves relative to the normal fluid. The entropy and heat are carried by the normal component. The convective distribution is controlled by shear viscosity of the normal fluid. Thermal conductivity of the normal fluid is dominated by roton and phonon scattering

- in normal fluids: At high temperatures, diffusive process, determined by scattering of atoms. At large T (similar to shear viscosity): $\kappa \sim T^s$ with $s = \frac{1}{2} + \frac{2}{\nu - 1}$

- in unitary Fermi gas: Thermal conductivity from phonons of unitary gas → not yet determined

- In QGP: In the limit μ>>T: $\kappa \sim \dfrac{\mu^2}{\alpha_s^2}$

- In the limit T>> μ : $\kappa \sim \dfrac{T^4}{\alpha_s^2 \mu^2}$

Diffusion D

- Due to multiparticle interactions, particles do not move along streamlines but instead exhibit fluctuating motions as they tumble around each other, leading to a net migration of particles down gradients in particle concentration

- If fluid is composed of quasi-particles, then diffusion of impurities and shear viscosity are closely linked, as both are related to momentum diffusion. They have same dependence on coupling constants, and similar temperature dependence up to kinematic factors

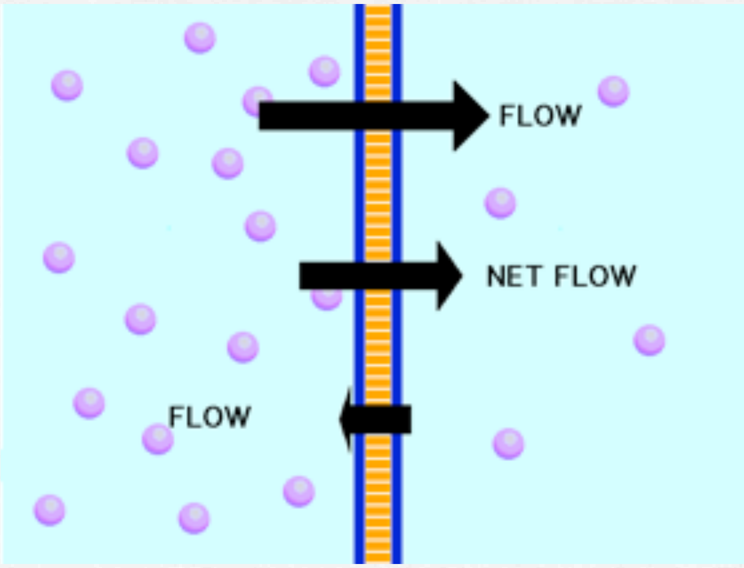
- If the number density of impurity particles is conserved, it satisfies the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot j = 0$$

- For smoothly varying number density,

$$j = -D\nabla n$$

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$



Diffusion : Results

- In Superfluids: phonon scattering off ^3He is the dominant mechanism. The mobility μ of ^3He quasi-particles in superfluid ^4He can be estimated from the theory of elastic scattering: $\mu \propto T^8$

  Einstein relation $D = \mu k_B T \Rightarrow D \propto T^{-7}$

- in normal fluids: scattering between atoms: $D \propto T^{(1+s)}$ with $s = \frac{1}{2} + \frac{2}{\nu - 1}$

  The T dependence is identical to that of shear viscosity

- in unitary Fermi gas: calculation involves quasi-particle scattering amplitudes. The system is considered as ideal gas of majority (up) atoms mixed with minority (down) atoms, whose elementary excitations are quasi-particles with effective mass $m^*_\downarrow$. The momentum relaxation time can be estimated from thermodynamic arguments. Not related to viscosity.

- in QGP: dominated by heavy quark scattering on light quarks and gluons $qQ \to qQ$ and $gQ \to gQ$. As in case of shear viscosity, most important Feynman diagrams involve t-channel gluon exchanges. D has same parametric dependence on the coupling as shear viscosity

Thank you!