Critical Phenomena

Jan Martin Pawlowski, winter term 2010/2011

Tuesday & Friday, 11:15-13:00, kHS Pw 12 [LSF]

- Content
- Literature
- Exercises
- Script

Prerequisites: quantum mechanics, classical field theory, statistics, basic knowledge of QFT useful

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Content of lecture series

In the lecture course an introduction to the theory of critical phenomena is given. Critical phenomena are relevant for physics at all scales, and have been studied with a variety of approaches. The lecture course aims at giving both, an overview of the methods applied to the physics of critical phenomena, as well as a survey of interesting applications.

Outline in key words

- **Introduction:**
  Phase transitions, correlation functions, critical exponents, universality

- **Landau theory:**
  Gaussian model, Ginzburg-Landau hamiltonian, Ginzburg criterion

- **Renormalisation group:**
  block spins, fixed points, critical surface, beta-functions, non-universal aspects

- **Methods:**
  Transfer matrix, epsilon-expansion, 1/N expansion, integrability,

LINKS

Institute for Theoretical Physics

ExtreMe Matter Institute EMMI

DFG research group FOR 723

Research Training Group Simulational methods in Physics

Department of Physics and Astronomy

Graduate School of Fundamental Physics

Graduate Academy

functional RG equations

- **Applications:**
  Ising model, Heisenberg model, O(N)-models, non-linear sigma models, phase structure of selected physics phenomena.

Literature

Amit, Martin-Mayor	Field Theory, the Renormalization Group, and Critical Phenomena	World Scientific
Binney, Dowrick, Fisher, Newman	The Theory of Critical Phenomena, an Introduction to the Renormalization Group	Clarendon Press, Oxford
Cardy	Scaling and Renormalization in Statistical Physics	Cambridge UP
Le Bellac	Quantum and Statistical Field Theory	Cambridge UP
Mussardo	Statistical Field Theory	Oxford UP
Parisi	Statistical Field Theory	Addison-Wesley
Yeomans	Statistical Mechanics of Phase Transitions	Oxford UP
Zinn-Justin	Quantum Field Theory and Critical Phenomena	Oxford UP
Zinn-Justin	Phase transitions and and Renormalisation Group	Oxford UP

1 Introduction

This chapter serves as an introduction of the general concepts and ideas at the example of the ferromagnetic transition, also introducing the working horse of Statistical Field Theory / Critical Phenomena, the Ising model (> 13000 articles

1.1 The ferromagnetic transition

Certain materials can be magnetised at room temp. (iron, nickel, cobalt, ...). Microscopically, this is achieved by aligning the spins of (incomplete) shell electrons in one direction.

Consequently also their magnetic moments add-up, and lead, potentially, to a macroscopic magnetic moment.

$$\uparrow \; \uparrow \; \uparrow \; \uparrow \; \uparrow \; \ldots \qquad T < T_c$$

$$\vec{\mu}_S = -g_S \, \mu_B \cdot \vec{S}/(\hbar) \leftarrow \boxed{\hbar = 1} \tag{1.1}$$

gyromagnetic factor: $g_S = 2.0023\ldots$

and $\mu_B = e(\hbar)/2m_e$

If heated above a certain temperature T_c, (Curie temp.), the magnetisation disappears. Above this temperature (depends on the material), thermal ~~effects dominate over~~ the aligning interaction effects, the spins are disordered

$$\nwarrow \; \swarrow \; \uparrow \; \searrow \; \rightarrow \; \nearrow \; \ldots \qquad T > T_c$$

At T_c, we have indeed long range fluctuations.

Remark: Even though it is clear, that at sufficiently high temperature the spins are <u>not</u> aligned; it is a priori <u>not</u> clear, whether or not the spins are aligned below T_c.
They could be either

(a) aligned in small areas, that are disordered

(b) $T_c = 0$.

Hence the question is raised, how the above situation is described. Indeed the microscopic physics does not single out a specific direction in space, it is rotational invariant (i.e. the hamiltonian)

$\Rightarrow$ $\boxed{\text{spontaneous symmetry breaking}}$

In the present ferromagnetic example we classify the different <u>phases</u>
$T > T_c$, $T = T_c$, $T < T_c$ with an appropriate observable, the spontaneous magnetisation $\mathcal{M}$.

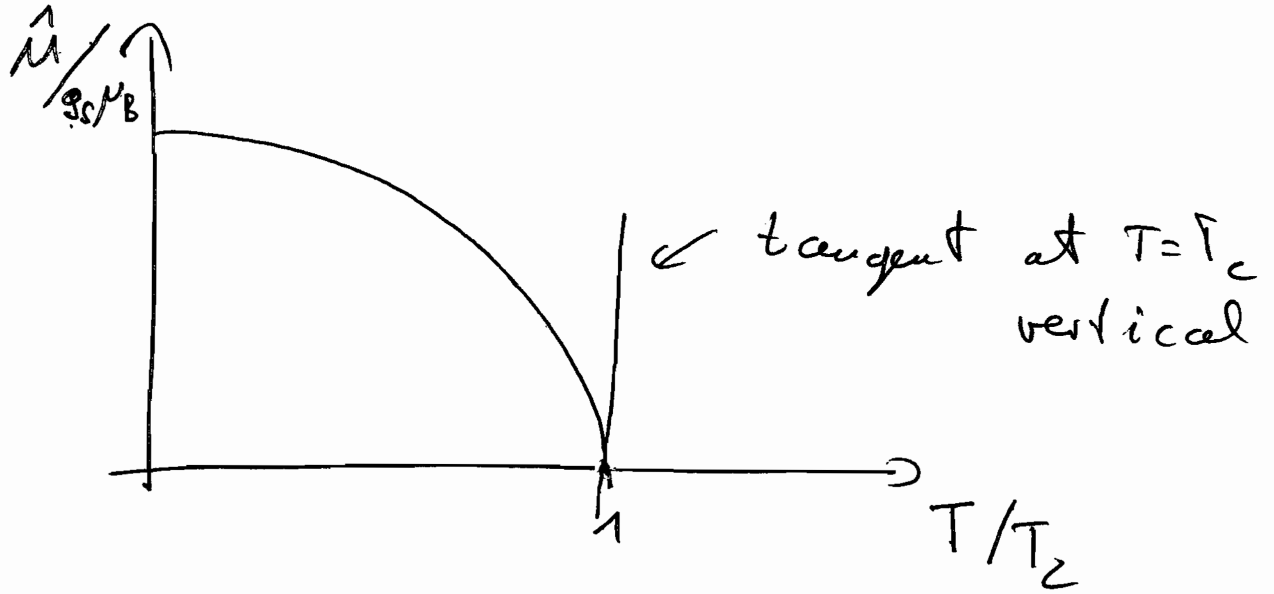
$$\hat{\mathcal{M}} = \left|\sum_i \langle \vec{\mu}_i \rangle\right|, \quad \vec{m} = n\langle \vec{\mu} \rangle \quad (1.2)$$

where $n = N/Vol$ is the density of magnetic moments/spins per volume (or area/line).
$\vec{\mathcal{M}}$ is the <u>order parameter</u> of the ferromagnetic phase transition.

Phases:

$$T > T_c \qquad \hat{M} = 0$$
$$T = T_c \qquad \hat{M} = 0 \qquad (1.3)$$
$$T < T_c \qquad \hat{M} \neq 0$$

Temperatur dependence of spont. magnet. $\vec{M}$.



Remarks:

(i) The vertical tangent indicates that

$$\hat{M} \sim |1 - T/T_c|^{\beta}$$
$$\Rightarrow T\frac{\partial \hat{M}}{\partial T} \sim \beta \underbrace{|1 - T/T_c|}_{\text{reduced temp.}}{}^{\beta - 1} \quad \text{with } \beta < 1 \qquad (1.4)$$

(ii) correlation length ξ diverges (2nd order phase transition)