2.2 Phase transitions

We start with the effective action Γ, eq. (2.22) for a uniform field $M(\hat{x}) = M$. We loose the kinetic term and get from eq. (2.31);

$$\Gamma(M) = V\left(\frac{1}{2} r_0(T) M^2 + \frac{1}{4!} u_0 M^4\right) + \Delta\Gamma[M] \qquad (2.32)$$

with magnetic field, $\hat{M} = V \cdot M$ & $\Delta\Gamma = 0$ at lowest order

$$B = \frac{\partial \Gamma}{\partial \hat{M}} = \frac{1}{V} \frac{\partial \Gamma}{\partial M} = r_0(T) M + \frac{1}{6} u_0 M^3 \qquad (2.33)$$

Again, this gives us the mean-field results.

With $r_0 = \bar{r}_0 (T - T_c)$ (2.34)

we get

$$\boxed{M_0 = \left[\frac{6 \bar{r}_0}{u_0} (T_c - T)\right]^{1/2}} \quad \text{for } T < T_c \qquad (2.35)$$

The specific heat reads, $T < T_c$.

$$\boxed{C = -\frac{1}{V} T^2 \frac{d^2 T \Gamma[M_0]}{d T^2} = \frac{3 \bar{r}_0^2}{u_0} T^2 \left(T - 2(T - T_c)\right)} \qquad (2.36)$$

with $C(T > T_c) = 0$, and $\Delta C = C(T \to T_{c-}) - C(T \to T_{c+}) = \frac{3 \bar{r}_0^2}{u_0} T_c^3$ (2.37)

Eq. (2.33) - (2.37) summarise our mean-field results, where we have already inserted the non-universal parameter T_c in the Gibbs free energy via $r_0(T) = \bar{r}_0\,(T - T_c)$, triggering the spont. symmetry breaking.

The critical exponent $\alpha = 0$, which, together with $C(T<T_c) \neq 0$ entails a discontinuity for the specific heat.

We shall see later, that this behaviour is remedied by going beyond the Landau approximation (beyond lowest order saddle point appr.).

So far we have only adjusted our parameters in order to reproduce the mean-field results for the Ising model.

We would also like to emphasise that the Z_2-symmetry of the Ising model only determines the physics of the phase transition, if we get a 2^{nd} order transition.

Consider e.g. a mean-field potential

$$\frac{1}{V}\Gamma(M) = \frac{1}{2} r_0(T)\, M^2 + \frac{1}{4!} u_0 M^4 + \frac{1}{6!} v_0 M^6 \quad (2.38)$$

with $u_0 \gtrless 0$ and $r_0(T) = \bar{r}_0 (T - T_0)$. * T_c for $u_0 < 0$! The highest coupling is necessarily positive, $v_0 > 0$!